Coalescence-Inspired Sum Rules in a Hydrodynamic Framework

Priya, Tribhuban Parida, Sandeep Chatterjee

priya23@iiserbpr.ac.in, tribhubanp18@iiserbpr.ac.in, sandeep@iiserbpr.ac.in

Abstract

Quark coalescence models predict sum rules where a hadron's flow coefficient v_n is the sum of its constituent quarks' flows, leading to Number of Constituent Quark (NCQ) scaling. We investigate the emergence of analogous sum rules within a pure hydrodynamic framework, where hadronization is described by the Cooper-Frye freezeout. Despite the different mechanism, we find that v_2 sum rules hold, with coefficients determined by hadron masses rather than quark content. Our analytical derivation from hydrodynamics and numerical results show consistent agreement, particularly at high transverse momentum.

1. Coalescence Sum Rules

In the quark coalescence model, the flow harmonic v_n of a hadron is constructed from the flow of its valence quarks. For example, the v_2 of a $\pi^+(u\bar{d})$ is:

$$v_2^{\pi^+} = v_2^{\bar{d}} + v_2^{u}$$

NCQ Scaling: If constituent quarks share the same flow and transverse momentum ($p_T^q = p_T^H/n_q$), the Number-of-Constituent-Quark scaling emerges:

$$v_2^H(p_T^H) = n_q \, v_2^q(p_T^H/n_q)$$

We define specific hadronic combinations to test, assuming $v_2^u = v_2^d$ and $v_2^{\bar{u}} = v_2^{\bar{d}}$:

Set 1	$v_2^{\Lambda^0} = \frac{2}{3}v_2^p + \frac{1}{3}v_2^{\Omega^-}$
Set 2	$v_2^{\Lambda^0} = \frac{1}{2}v_2^p + \frac{1}{2}v_2^{\Xi^-}$
Set 3	$v_2^{\Lambda^0} = v_2^p + \frac{1}{3}v^{K^-}$

2. Hydrodynamic Sum Rules

We test if similar sum rules hold for hadrons produced via **Cooper-Frye freezeout** in hydrodynamic simulations, where hadronization is a thermodynamic process, not a coalescence one.

5. Conclusion

- Sum rules for elliptic flow v_2, inspired by QC, are also emerging through hydrodynamic Cooper-Frye freezeout.
- The weighting coefficients in the hydro sum rule are determined by the masses of the involved hadrons, not their quark content.
- While the coefficients differ from the naive coalescence expectation, the fundamental linear relationship holds.
- The agreement with numerical hydrodynamic simulations is excellent, validating our analytical derivation. The validity of these rules improves at high p_T.

3. Analytical Derivation from Hydrodynamics

An analytical expression for v_2 from Cooper-Frye freezeout is given by [1]:

$$v_2(p_T) = \frac{\alpha}{T}(p_T - u\, m_T) \qquad (1)$$

where $m_T = \sqrt{m^2 + p_T^2}$ is the transverse mass.

To relate v_2 of different species, we posit a linear sum rule for hadrons A, B, and C:

$$v_2^A(p_T) = \alpha_1 v_2^B(p_T) + \alpha_2 v_2^C(p_T) \qquad (2)$$

Substituting the hydrodynamic form (1) into (2) and matching coefficients of p_T and m_T leads to the system:

$$\alpha_1 + \alpha_2 = 1 \qquad (3a)$$
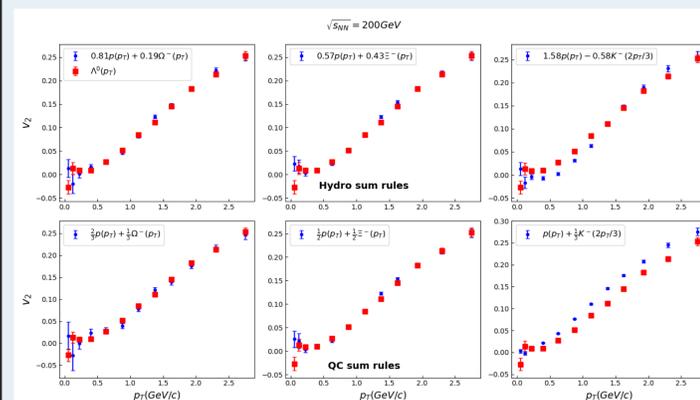$$\alpha_1 m_B^2 + \alpha_2 m_C^2 = m_A^2 \qquad (3b)$$

Solving this system yields the hydrodynamic coefficients α_1 and α_2 purely from the hadron masses.

$$\alpha_1 = \frac{m_A^2 - m_C^2}{m_B^2 - m_C^2}, \quad \alpha_2 = 1 - \alpha_1 \qquad (4)$$
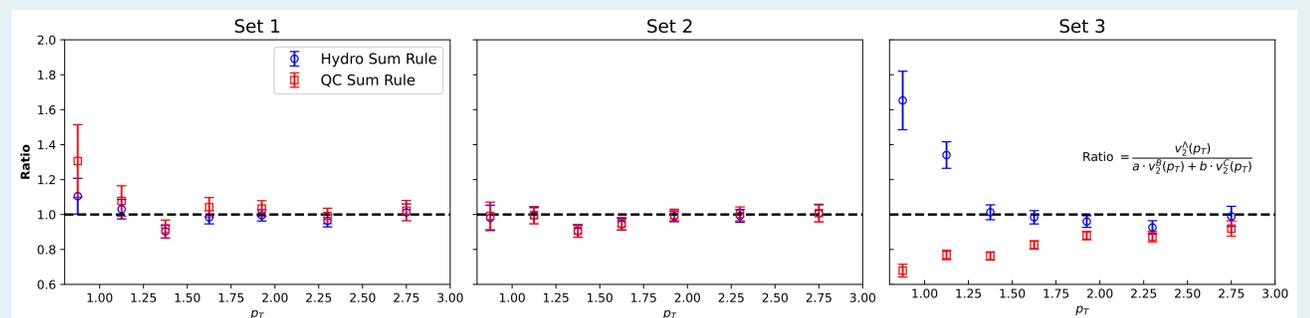
Coefficient Comparison

Set	Hydro (α_1, α_2)		Coalescence (α_1, α_2)	
Set 1	0.81	0.19	2/3	1/3
Set 2	0.57	0.43	1/2	1/2
Set 3	1.58	-0.58	1	1/3

4. Results and Discussion

(a) Validation of Sum Rules

- Hydrodynamics results confirm the validity of the analytically derived sum rules.
- The mass-derived coefficients from hydrodynamics differ from the quark-counting ones, yet the sum rule holds.
- **Set 3:** Shows significant deviation. The assumption of equal p_T scaling for strange quarks (in K^- and p) in the coalescence sum rule is not valid.

(b) Ratio of LHS to RHS for the sum rules. Agreement improves at high p_T, as per our assumption in deriving mass dependent coefficients.

6. References

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[3] Kishora Nayak, Shusu Shi, and Zi-Wei Lin. Coalescence sum rule and the electric charge- and strangeness-dependences of directed flow in heavy ion collisions. *Phys. Lett. B*, 849:138479, 2024.