A cosmological no-go theorem in bumblebee gravity

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Based on: C. van de Bruck, M.I. Gorji, NAN, M.C. Pookillath, M. Yamaguchi, 2509.11647, to appear in JCAP

Introduction

Bumblebee models = *"Vector-tensor theories with spontaneous spacetime-symmetry breaking"*

- Ubiquitous in the literature - 256 papers (as of yesterday) in the last 10 years
- Vast majority of papers related to black holes
- Kostelecky, Övgün, Casana, Li, Maluf, Ding, Xi, Shao, and many more

Cosmology with the bumblebee model:

- Capelo+Páramos PRD 2015 - Background analysis
- Reyes et al PRD 2024+JCAP 2025 - background analysis
- Xu et al `2504.10297` - CMB analysis with linear potential

Black hole solutions with $\Lambda \to$ dS/AdS $\sim$ cosmology.

Timeline

1988 Kostlecky+Samuel PRD & PRL: Spontaneous Lorentz violation in string theory

1989 Kostelecky+Samuel PRD: Introduction of the bumblebee model

2004 Kostelecky PRD: non-minimal coupling to gravity

2015 The black-hole boom starts

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Systematic construction of the bumblebee model

A commonly used action is

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \xi B_\mu B_\nu R^{\mu\nu} - V_B(B^2 \pm b^2)\right]$$

- Several terms need to be added

Operator	Mass dim. (d)	Derivative exp. (D)
B	1	0
∇B	2	1
R	2	2
$B^2\nabla B$	4	1
$(\nabla B)^2$, B^2R	4	2
$R\nabla B$	4	3
R^2	4	4

General action

The final, most general action, can be written as

$$S_B = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{\gamma}{4}B_{\mu\nu}B^{\mu\nu} - V(B^2) + \eta B^2\nabla_\mu B^\mu + \xi B^\mu B^\nu + \sigma B_\mu B^\mu + \varsigma(\nabla_\mu B^\mu)^2 + \upsilon\nabla_\mu B^\mu R\right],$$

- Several terms propagate ghosts, as is **well known** from generalised Proca theory [Heisenberg 1402.7026] $\longrightarrow$ ξ and σ **not independent**

Cosmological perturbations of bumblebee gravity

- We work on a flat FLRW background and take $\bar{B}_\mu \to \{\bar{B}_0, \vec{0}\}$ and the spatially flat gauge

$$ds^2 = -\left(1 + 2\,\alpha\right) dt^2 + 2a\left(\partial_i\,\beta + \beta_i\right) dt dx^i + a^2\left(\delta_{ij} + h_{ij}\right) dx^i dx^j$$

$$B_\mu = (\bar{B}_0 + \delta B_0\,,\ \partial_i\,\delta B_s + \delta\,\delta B_i^{\perp})$$

- Scalar dofs: $\alpha,\ \beta,\ \delta B_0,\ \delta B_s = 4$
- Vector dofs: $\beta_i,\ \delta B_i^{\perp} = 4$
- Tensor dofs: $h_{ij} = 2$

Tensor modes

Due to the non-minimal coupling, tensor modes are modified. Take the quadratic action

$$S^{(2)}_{B,T} = \frac{M_{\rm Pl}^2}{8}\int d^3x\, dt\, a^3 \mathcal{K}_T \left(\dot{h}_{ij}\dot{h}^{ij} - \frac{c_T^2}{a^2}\partial_i h_{jk}\partial^i h^{jk}\right)$$

$$c_T^2 \approx 1 + 2\xi \tilde{B}_0^2\,; \qquad \xi, \sigma, \upsilon \ll 1$$

- Speed of tensor modes at late times strongly constrained to $|c_T - 1| \sim \mathcal{O}(10^{-15})$
- For a healthy tensor sector, we must also demand that

$$\mathcal{K}_T > 0, \qquad c_T^2 > 0$$

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2 tensor dofs remain (10 dofs left)

Vector modes

Only $\delta B_s^{\perp}$ are dynamical. The quadratic action reads

$$S^{(2)}_{B,V} = \frac{\gamma}{2}\int d^3x\,dt\,a\left(\delta\dot{B}^{\perp}_i\,\delta\dot{B}^{\perp i} - \frac{c_V^2}{a^2}\partial_i\delta B^{\perp}_j\partial^i\delta B^{\perp j}\right)$$

$$c_V^2 \approx 1 + \frac{2\xi^2\tilde{B}_0^2}{\gamma\mathcal{K}_T}\,; \qquad k \gg aH$$

- For stability, we must impose

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4-2=2 vector dofs remain (8 dofs left)

Scalar modes

1. Integrate out β straight away $\rightarrow \ddot{\alpha}$ and $\delta\ddot{B}_0$ appear

---

υ term: $\upsilon R \nabla_\mu B^\mu$.

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2. Perform a variable redefinition $\tilde{\alpha} \equiv \delta B_0/\bar{B}_0$

$$S^{(2)}_{B,S} = \int \frac{d^3k}{(2\pi)^3} dt \left(\mathcal{L}^{(2)}_2 + \mathcal{L}^{(2)}_\upsilon\right)$$

$$\mathcal{L}^{(2)}_2 = \mathcal{L}^{(2)}_2\left(\tilde{\alpha}, \dot{\tilde{\alpha}}, \delta B_0, \delta\dot{B}_0, \delta B_s, \delta\dot{B}_s\right); \quad \mathcal{L}^{(2)}_\upsilon = \mathcal{L}^{(2)}_\upsilon\left(\tilde{\alpha}, \dot{\tilde{\alpha}}, \ddot{\tilde{\alpha}}, \delta B_0, \delta\dot{B}_0, \delta B_s, \delta\dot{B}_s\right)$$

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$$S_{B,S}^{(2)} = \int \frac{d^3k}{(2\pi)^3} dt \left(\mathcal{L}_2^{(2)} + \mathcal{L}_\upsilon^{(2)}\right)$$

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3. Introduce a new variable $Q \equiv \dot{\alpha}$, $\quad \mathcal{L}_{\rm tot}^{(2)} = \mathcal{L}_2^{(2)} + \mathcal{L}_\upsilon^{(2)} + \lambda Q + \dot{\lambda}\tilde{\alpha} + \text{T.D.}$

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4. Integrate out α and find

$$\mathcal{L}_{\rm tot}^{(2)} = \mathcal{L}_{\rm tot}^{(2)}\big(\dot{\lambda}, \lambda, \dot{Q}, Q, \delta B_0, \delta\dot{B}_0, \delta B_s, \delta\dot{B}_s\big)$$

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Scalar modes

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2. Perform a variable redefinition $\tilde{\alpha} \equiv \delta B_0/\bar{B}_0$

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Quadratic action now in "normal" form, $\{\ddot{\tilde{\alpha}}, \dot{\tilde{\alpha}}, \tilde{\alpha}\}$, $\to \{\dot{\lambda}, \lambda, \dot{Q}, Q\}$, standard analysis possible.

υ term: $\upsilon R \nabla_\mu B^\mu$.

Scalar modes II

$$\mathcal{L}_{\rm tot}^{(2)} = \frac{1}{2} a^3 \left(\dot{\mathcal{V}}_4^\dagger \, \mathbf{K}_4 \, \dot{\mathcal{V}}_4 + \dot{\mathcal{V}}_4^\dagger \, \mathbf{N}_4 \, \mathcal{V}_4 - \mathcal{V}_4^\dagger \, \mathbf{X}_4 \, \mathcal{V}_4 \right), \quad \mathcal{V}_4^\dagger \equiv (\lambda, Q, \delta B_0, \delta B_s)$$

- Study the rank of the kinetic matrix $\mathbf{K}_4$

$$\max[\mathrm{rank}(\mathbf{K}_4)|_{\upsilon=0}] = 2\,, \quad \max[\mathrm{rank}(\mathbf{K}_4)|_{\eta=\xi=\sigma=\varsigma=0}] = 3 \implies \upsilon = 0$$

ς term: $\varsigma\,(\nabla_\mu B^\mu)^2$

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- Now $\mathbf{K}_4 \to \mathbf{K}_3 = \begin{pmatrix} K\bar{B}_0^2 & -K\bar{B}_0 & 0 \\ -K\bar{B}_0 & K & 0 \\ 0 & 0 & \gamma\frac{k^2}{a^2} \end{pmatrix}, \quad K \equiv -\frac{2}{\varsigma}(\xi + 2\sigma)(\xi + 2\sigma + 2\varsigma)$

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4-2=2 scalar dofs remain (6 dofs left)

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4-2=2 scalar dofs remain (6 dofs left)

$\xi + 2\sigma = 0$ and $\varsigma = 0$

$\Rightarrow$ **2-1=1 scalar dofs remain (5 dofs left)**

ς term: $\varsigma\,(\nabla_\mu B^\mu)^2$

Final action

$$S_B = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - V(B^2) + \eta B^2\nabla_\mu B^\mu + \xi B^\mu B^\nu G_{\mu\nu}\right]$$

Operator	Der. order (D)	d.o.f.	Correct d.o.f.
$\eta B^2\nabla_\mu B^\mu$	1	1S+2V+2T	$\boxed{\text{YES}}$
$\varsigma(\nabla_\mu B^\mu)^2$	2	2S+2V+2T	NO
$\xi B^\mu B^\nu R_{\mu\nu}$	2	2S+2V+2T	NO
$\sigma B^2 R$	2	2S+2V+2T	NO
$\upsilon\nabla_\mu B^\mu R$	3	3S+2V+2T	NO
$\# B^\mu B^\nu G_{\mu\nu}$	2	1S+2V+2T	$\boxed{\text{YES}}$

This has been largely overlooked in the bumblebee literature

See for example Higgs inflation 1003.2635, vector DM 2310.03862, and generalised Proca theory 1402.7026 for similar features. The model has very interesting features at the background level.

Cosmological perturbations of the final action

Repeat the same exercise as before and find

$$S^{(2)}_{B,S} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\, dt\, a^3 \left(\mathcal{K}\delta\dot{B}_s^2 - \mathcal{G}\frac{k^2}{a^2}\delta B_s^2\right)$$

$$\mathcal{K} \equiv 2M_{\rm pl}^2 H^2 \mathcal{K}_T \frac{3\eta\bar{B}_0 H\left(M_{\rm pl}^2 + 3\xi\bar{B}_0^2\right) + 3\eta^2\bar{B}_0^4 + 2\bar{B}_0^2\left(6\xi^2 H^2 - \left(M_{\rm pl}^2 - \xi\bar{B}_0^2\right)V''\right)}{\left[\eta\bar{B}_0^3 - H\left(M_{\rm pl}^2 - 3\xi\bar{B}_0^2\right)\right]^2},\; k \ll aH$$

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Impose the background equations

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Impose the background equations

$$\boxed{\mathcal{K} = \mathbf{0}}$$

$\mathcal{K}$ may appear at cubic order. The model cannot be studied perturbatively

The no-go theorem

(i) The most general marginal action → Eq. (2.9) – ✓

(ii) Isotropic and homogeneous background → $B_\mu = B_0(t)\delta^0_\mu$ → Eq. (3.3) – ✓

(iii) No extra *propagating* d.o.f. on FLRW (1S+2V+2T) → Eqs. (4.23) & (4.35) (and Table 3) – ✓

(iv) Healthy cosmological perturbations (no ghost or gradient instabilities + no strong coupling) → Eq. (4.38) → $\mathcal{K} = 0$ – FAILS,

} No-go result

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Match to generalised Proca theory

- The bumblebee model with degeneracy condition is a subset of generalised Proca with

$$\boxed{G_2 = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - V_B\left(-2X\right), \quad G_3 = -2\eta X, \quad G_4 = \frac{M_{\rm Pl}^2}{2} + \xi X\,; \quad X \equiv -\frac{1}{2}B_\mu B^\mu}$$

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All stability conditions can be rewritten for arbitrary G_i's as

$$\mathcal{K} = \mathcal{Q}_T\frac{H^2}{X}\cdot\frac{\mathcal{Q}_T\left[3\bar{B}_0 HXG_{3,X} - 6H^2\big(G_4 + 2XG_{4,X}\big) + 2X^2G_{2,XX}\right] + 3\big(2HG_4 - \bar{B}_0XG_{3,X}\big)^2}{\left[\bar{B}_0XG_{3,X} + 2H\big(G_4 - 4XG_{4,X}\big)\right]^2}$$

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Once we impose the background equations (for general G_i's, **not** for our model), we find

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Take-home: "Strong coupling of scalar perturations on dynamical cosmological background is a generic feature for this branch of generalised Proca".

Summary

1. We systematically constructed the most general bumblebee theory for the first time
2. The action propagates scalar ghosts unless a set of degeneracy conditions $\upsilon = \varsigma = 0, \sigma = -\xi/2$ are imposed $\rightarrow$ becomes subset of generalised Proca theory
3. The final action is strongly coupled and linear perturbations are pathological
4. The same result applies to full generalised Proca theory

Results to be taken in a special, inertial observer frame

Thank you for your attention

Thank you for your attention

⟶

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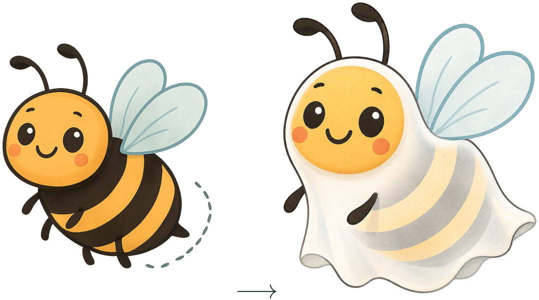

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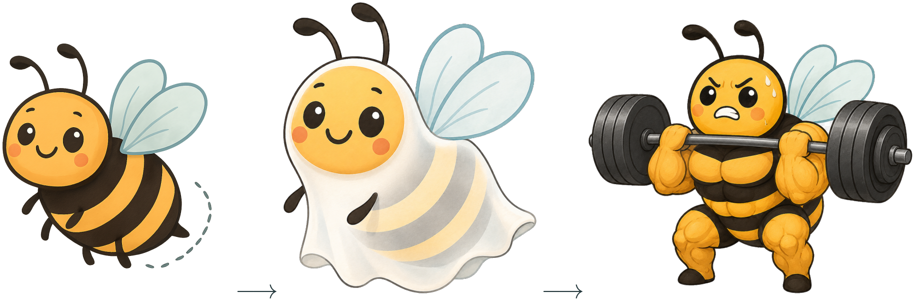

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