Accelerating black holes in higher-derivative theories of gravity

Marina David

Based on work in progress with
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Opening new horizons

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  - Originally found in 1917 [e.g. Weyl, Levi-Civita, Ehlers and Kundt 1962, …]
  - Describes accelerating black hole(s) [Kinnersley and Walker 1970]
  - Conical deficit represents a cosmic string, driving the acceleration [Gregory, Hindmarsh 9506054]

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  - Accelerating black hole solutions outside of/beyond General Relativity [Meng, Zhao 1601.07634, Lim 1604.07485 Zhang, Mann, 1908.05118, Cisterna, Neira-Gallegos, Oliva, Rebolledo-Caceres 2101.03628]

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Part 1

But first, a quick review

Schwarzschild Black Holes

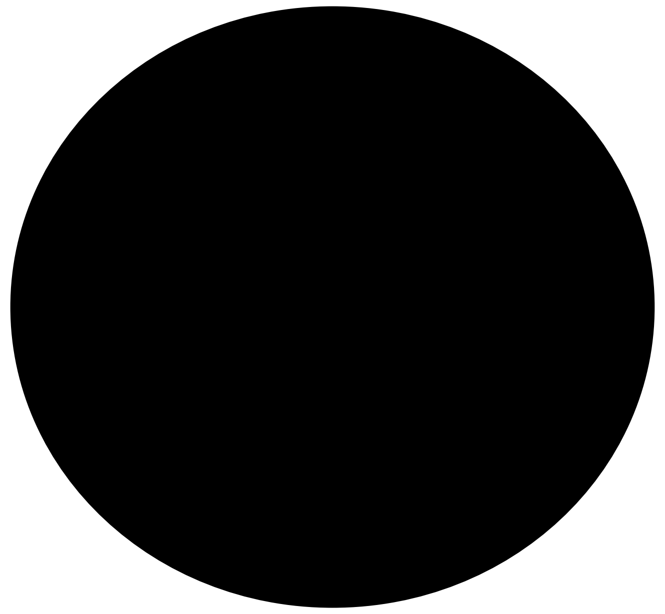
m : mass parameter

- Metric

$$ds^2 = -\mathcal{F}(r)dt^2 + \frac{dr^2}{\mathcal{F}(r)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

- Described by

$$\mathcal{F}(r) = 1 - \frac{2m}{r}$$



m

Schwarzschild-AdS Black Holes

m : mass parameter
L : AdS length scale

- Metric

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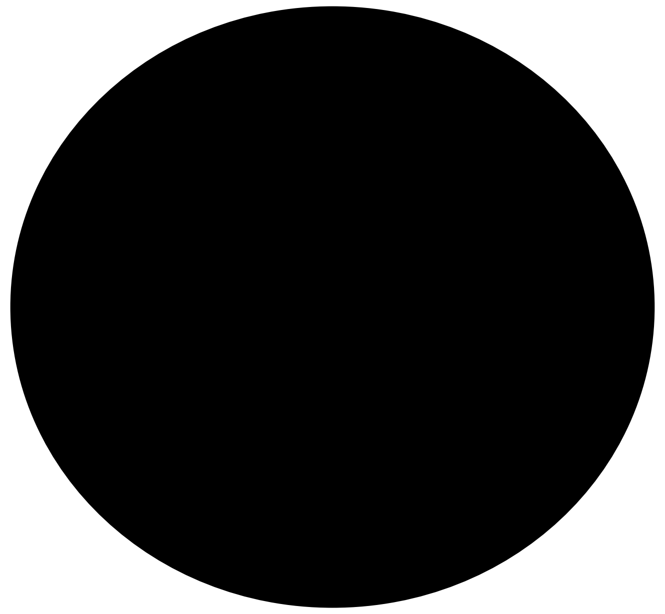


m

Schwarzschild-AdS Black Holes with a deficit

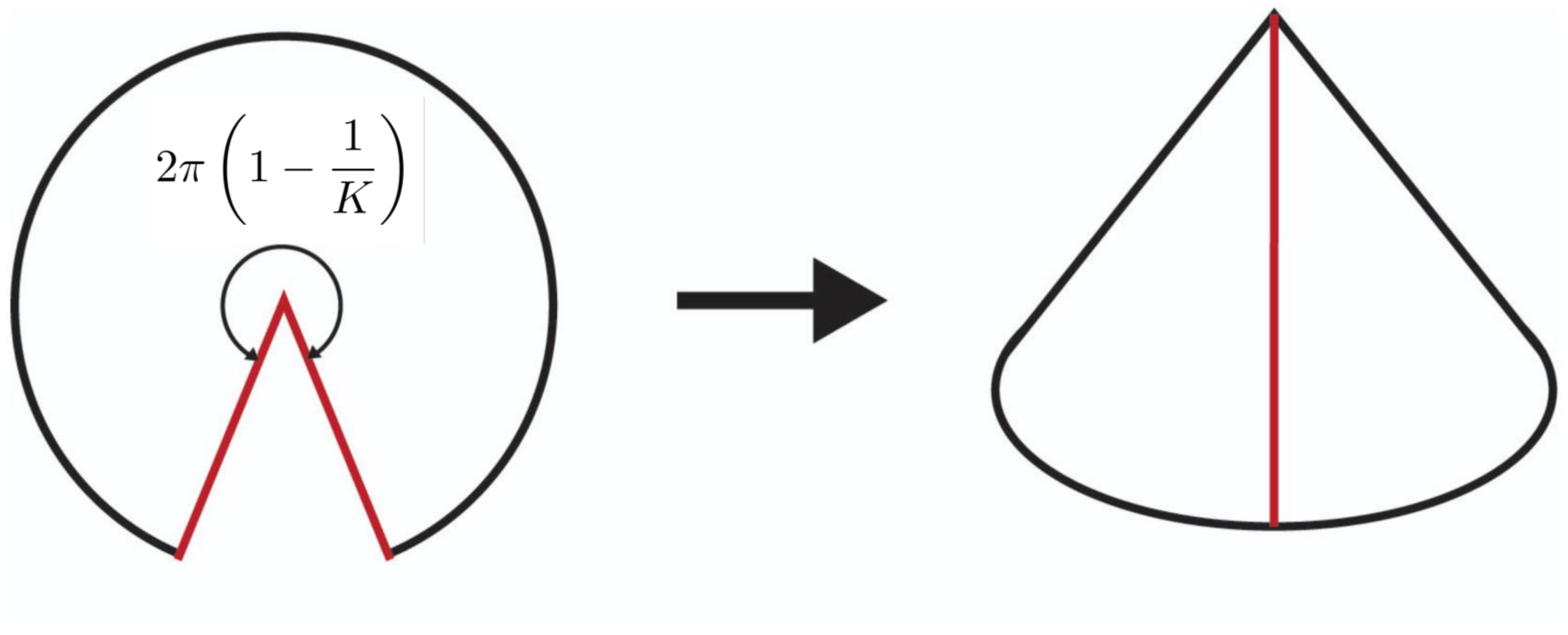
m : mass parameter
L : AdS length scale
K : conical deficit parameter

- Schwarzschild black hole with a **conical deficit**

$$ds^2 = -\mathcal{F}(r)dt^2 + \frac{dr^2}{\mathcal{F}(r)} + r^2 d\theta^2 + r^2 \sin^2\theta \frac{d\phi^2}{K^2}$$

- Conical deficit of

$$\delta = 2\pi\left(1 - \frac{1}{K}\right)$$



$m \;\; K$

Accelerating Black Holes in AdS

m : mass parameter
L : AdS length scale
K : conical deficit parameter
A : acceleration parameter

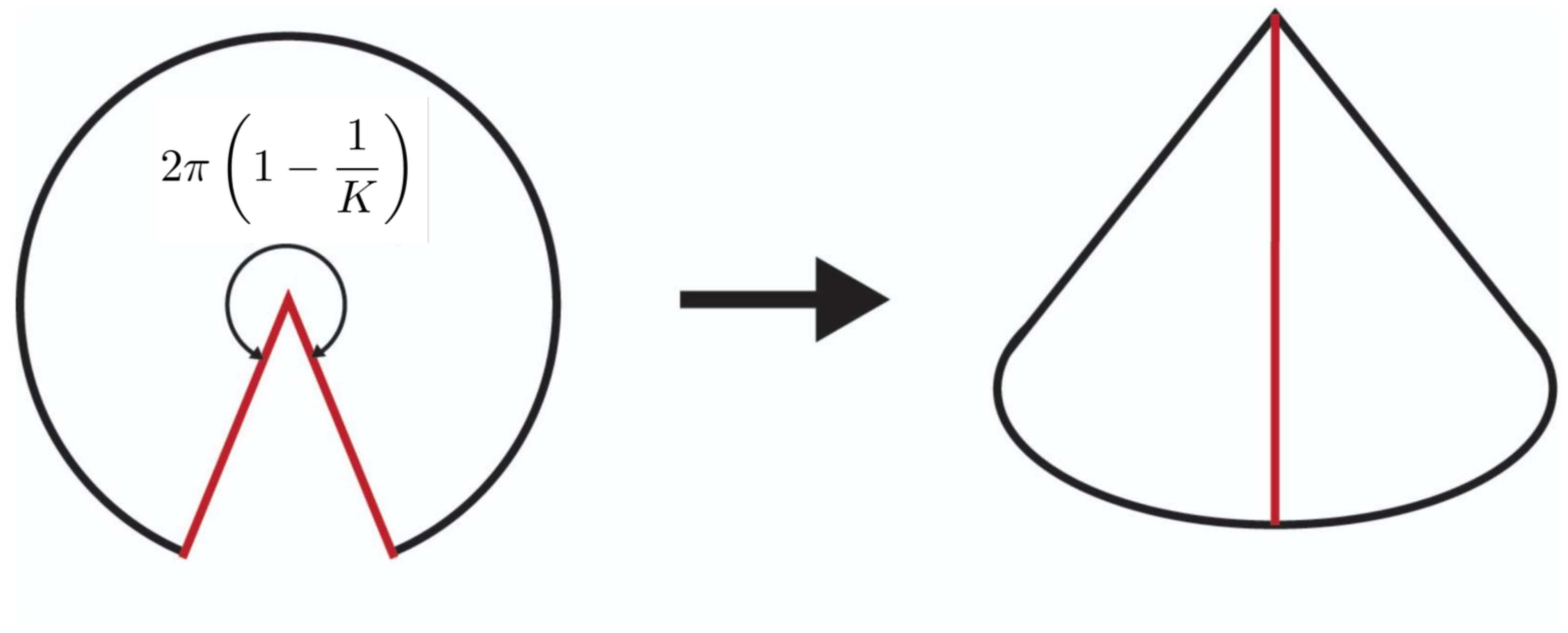
- Metric

$$ds^2 = \underbrace{\frac{1}{\omega(r,x)^2}}_{\text{conformal factor}}\left(-\mathcal{F}(r)dt^2 + \frac{dr^2}{\mathcal{F}(r)} + \frac{r^2 d\theta^2}{\mathcal{G}(\theta)} + r^2\mathcal{G}(\theta)\sin^2\theta\frac{d\phi^2}{K^2}\right)$$

- Described by

$$\mathcal{F}(r) = (1 - \underline{A^2r^2})\left(1 - \frac{2m}{r}\right) + \frac{r^2}{L^2}$$

$$\mathcal{G}(\theta) = 1 + \underline{2Am\cos\theta} \qquad \omega(r,\theta) = 1 + \underline{Ar\cos\theta}$$



$m\ K\ A$

Part 2

Interpreting the solution

New coordinate system

- m : mass parameter
- L : AdS length scale
- K : conical deficit parameter
- A : acceleration parameter

- Coordinate change

$$t \to At, \quad y \to \frac{1}{Ar}, \quad x \to \cos\theta, \quad z \to \frac{\phi}{K}$$

- Metric in **"polynomial form"**

$$ds^2 = \underbrace{\frac{1}{\Omega(y,x)^2}}_{\text{conformal factor}} \left(-F(y)dt^2 + \frac{dy^2}{F(y)} + \frac{dx^2}{G(x)} + G(x)dz^2 \right)$$

- Described by

$$F(y) = -\left(1 - y^2\right)\left(1 - 2Amy\right) + \frac{1}{A^2L^2}$$

$$G(x) = \left(1 - x^2\right)\left(1 + 2Amx\right)$$

$$\Omega(y,x) = A(x+y)$$

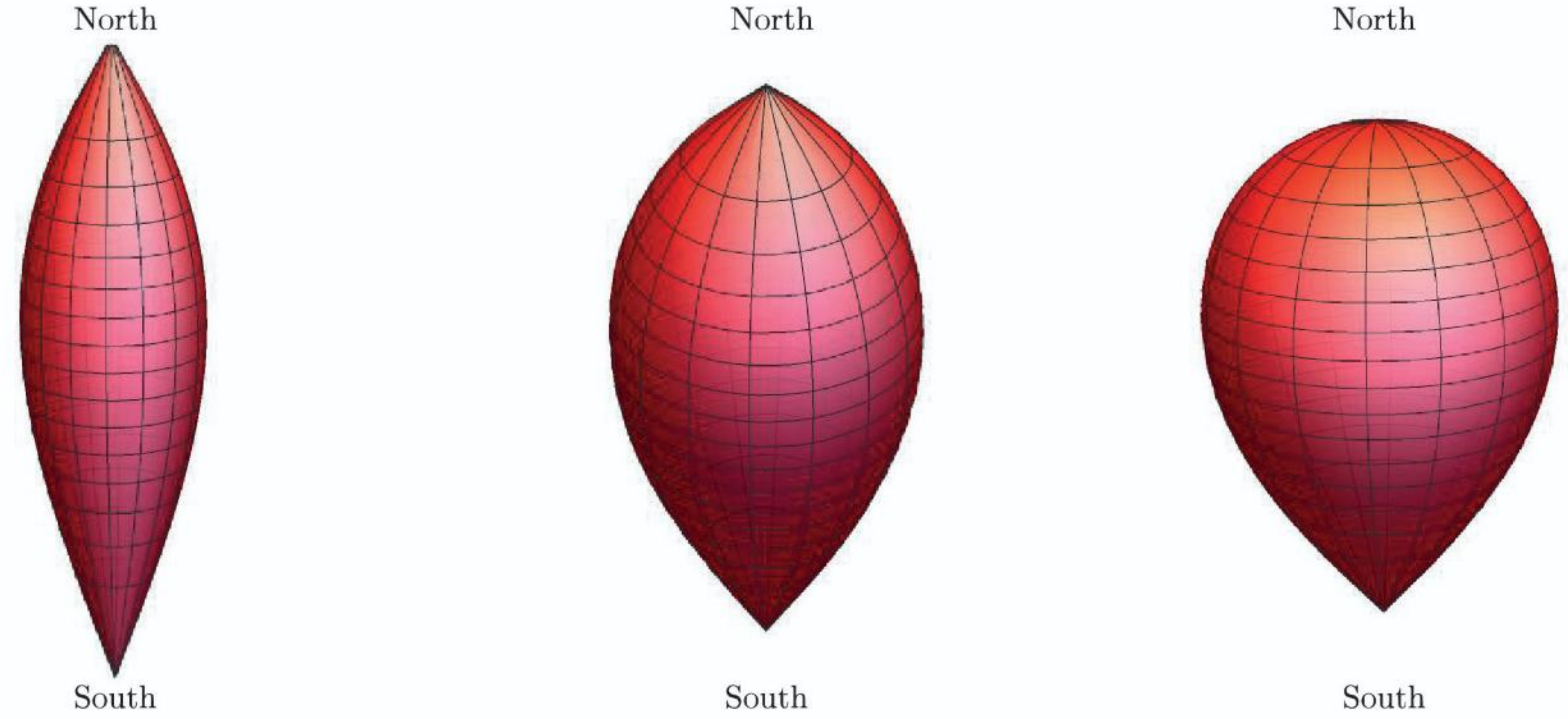
Feature 1: the conical deficit

Consider a circle around the pole which has the ratio: $\dfrac{\text{Circumference}}{\text{radius}} = \dfrac{2\pi \left.\frac{d\sqrt{G}}{dx}\right|_{x=\pm 1}}{K \left.\frac{1}{\sqrt{G}}\right|_{x=\pm 1}} = \dfrac{\pi}{K}\left|\dfrac{dG}{dx}\right|_{x=\pm 1}$

Conical deficit for each pole is given by $\delta_{\pm} = 2\pi\left(1 - \dfrac{2}{K}\left|\dfrac{dG}{dx}\right|_{x=\pm 1}\right) = 2\pi\left(1 - \dfrac{1 \pm 2Am}{K}\right)$

String tension

$$\mu_{\pm} = \frac{\delta_{\pm}}{8\pi}$$



$K = 2(1 - 2Am)$ $K = 1.2(1 - 2Am)$ $K = (1 - 2Am)$

Feature 2: conformal boundary

$$ds^2 = \underbrace{\frac{1}{\Omega(y,x)^2}}_{\text{conformal factor}} \left(-F(y)dt^2 + \frac{dy^2}{F(y)} + \frac{dx^2}{G(x)} + G(x)dz^2 \right)$$

m : mass parameter
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$$\Omega(y,x) = A(x+y)$$

- **Conformal infinity:** $x + y = 0$

- **Restrict ourselves to the physically most important case:**

$$x \in (-1, 1) \qquad x + y > 0$$

Feature 3: acceleration horizon(s)

Roots of $F(y)$ determine the horizons

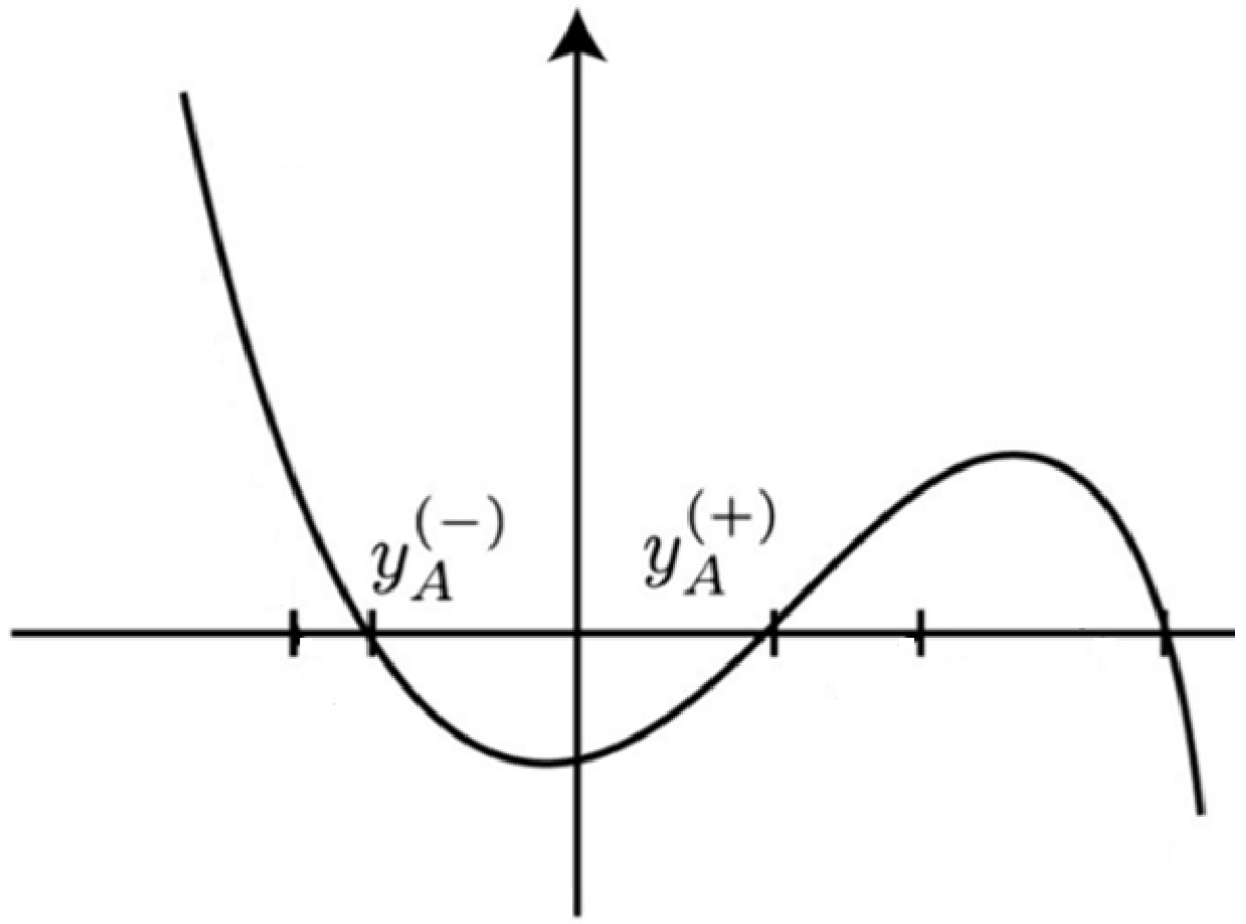
$$F(y) = -\left(1 - y^2\right)(1 - 2Amy) + \frac{1}{A^2 L^2}$$

Feature 3: acceleration horizon(s)

y_A : acceleration horizon

Roots of $F(y)$ determine the horizons

$AL > 1$



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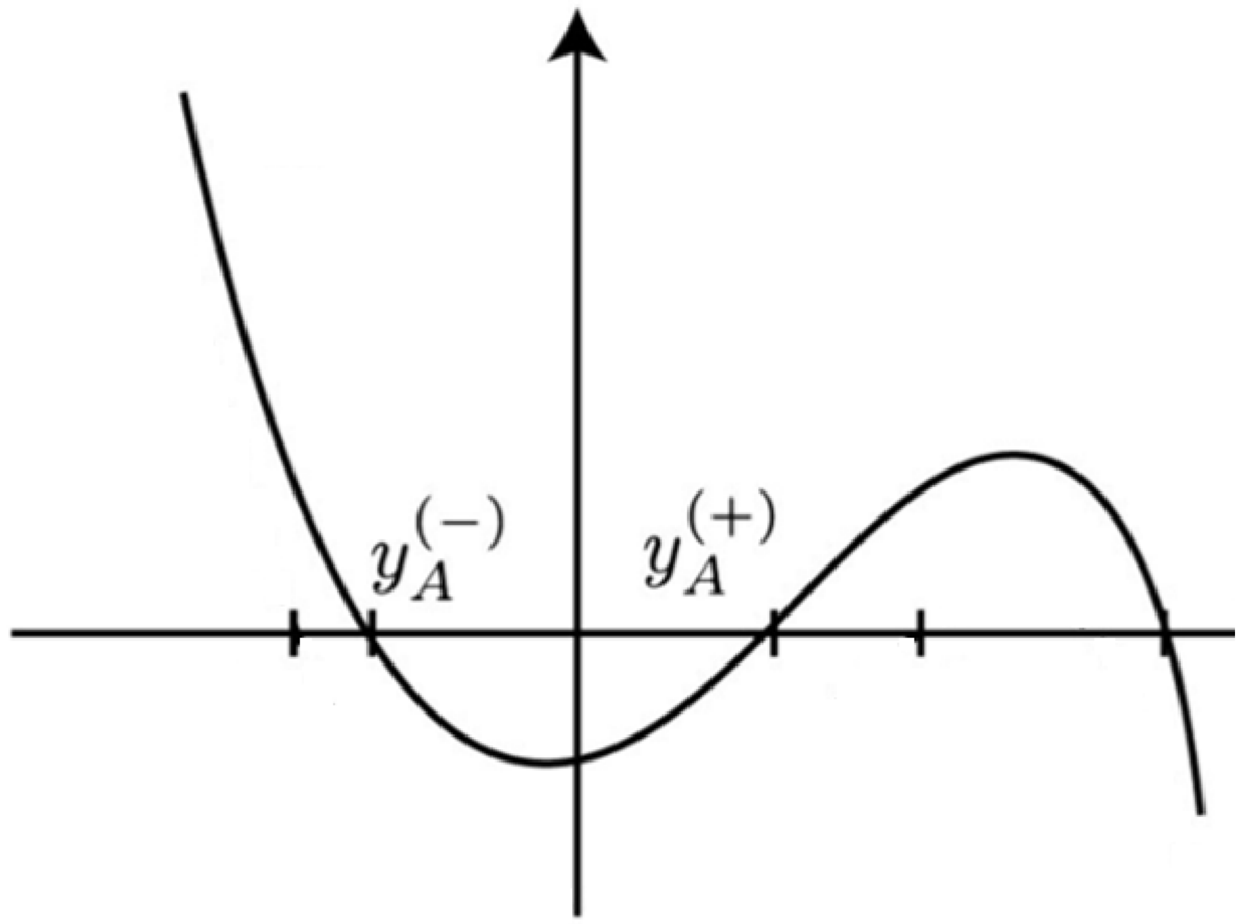

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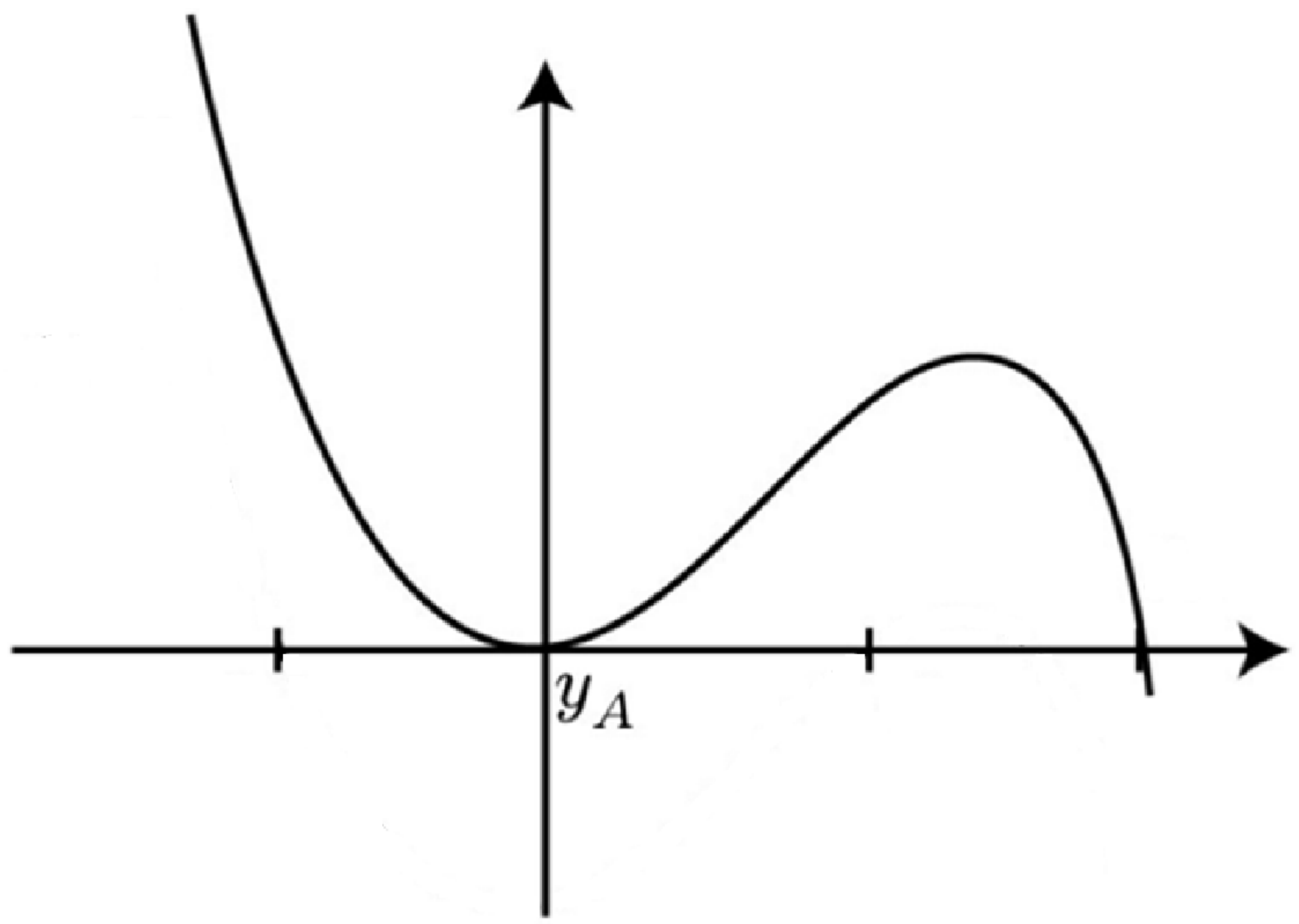

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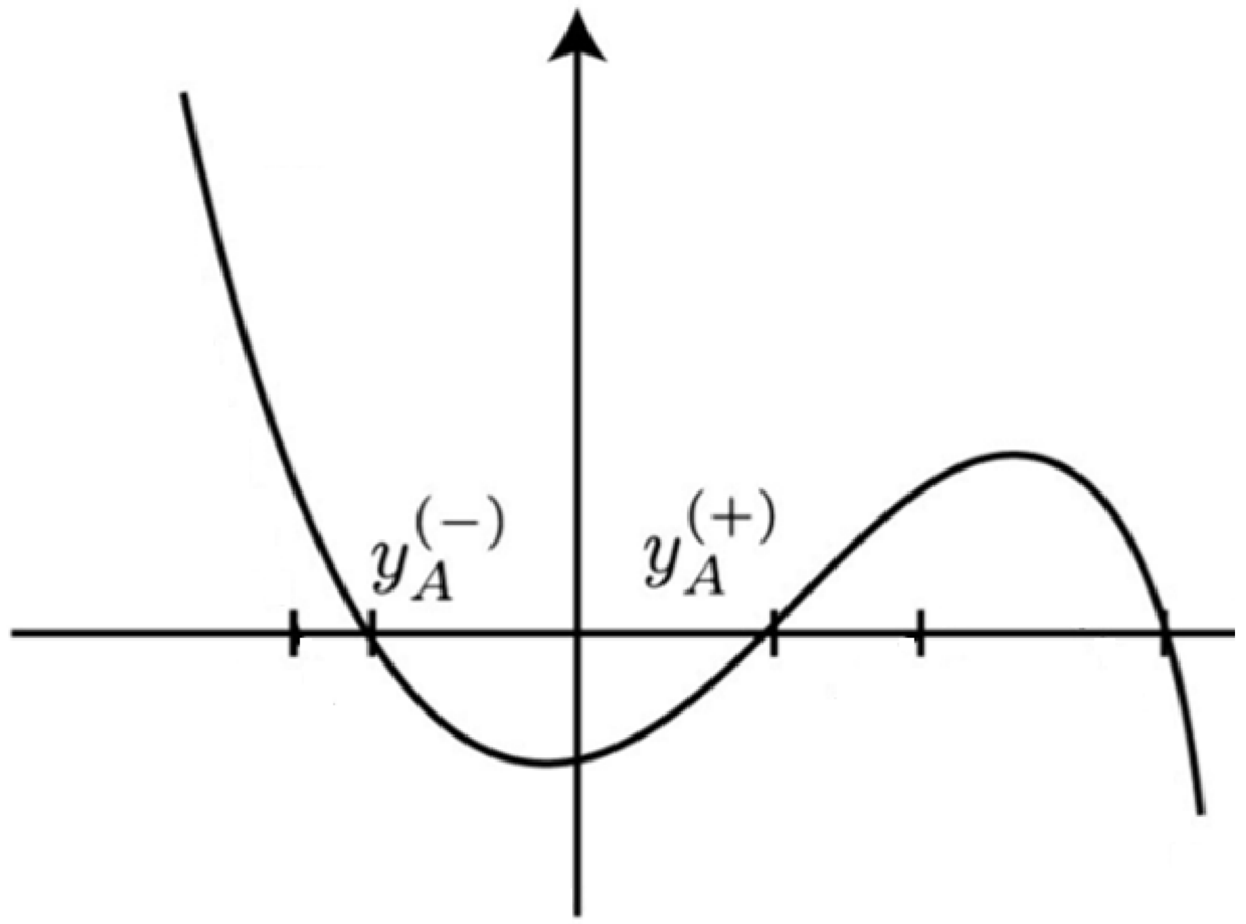

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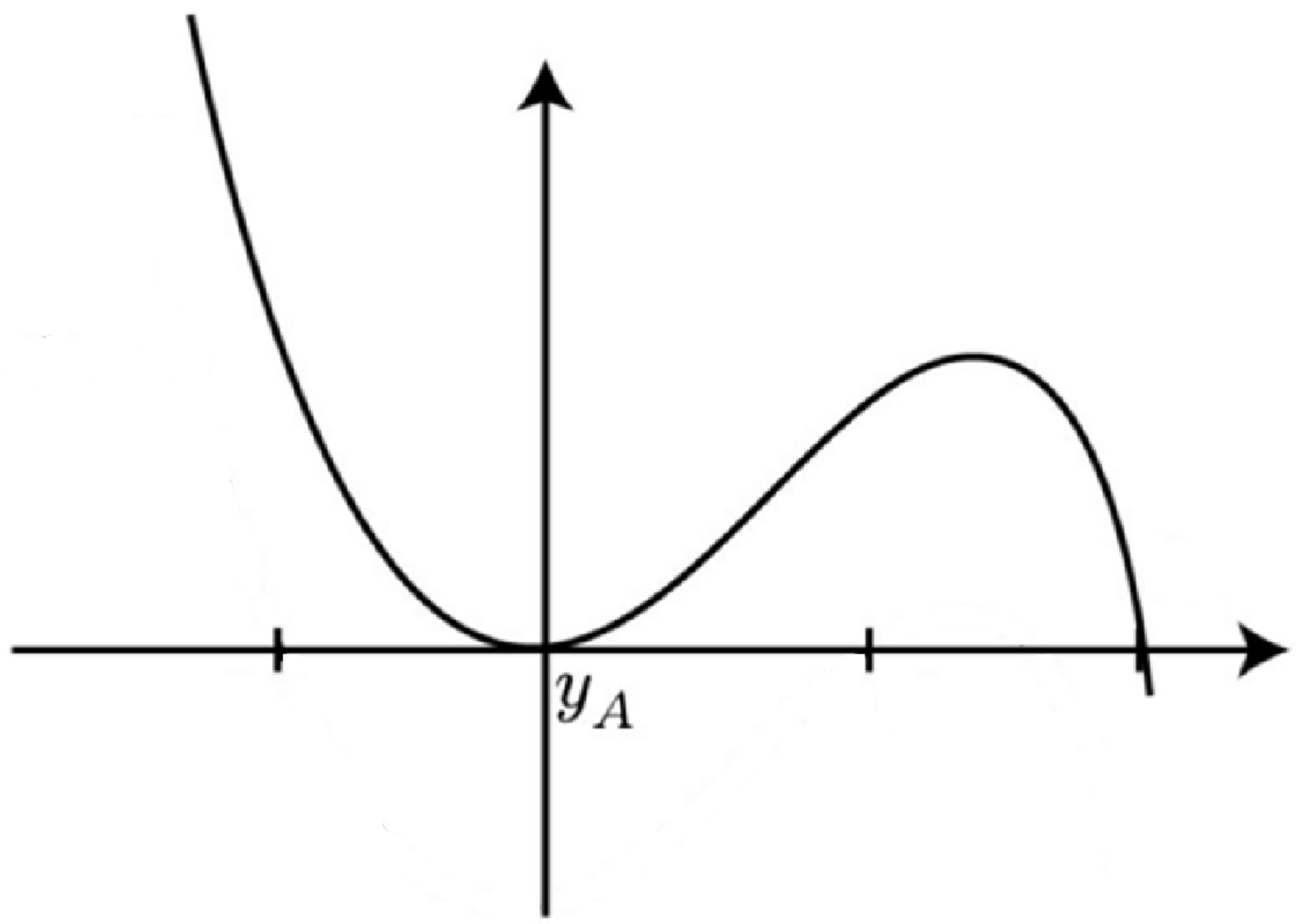
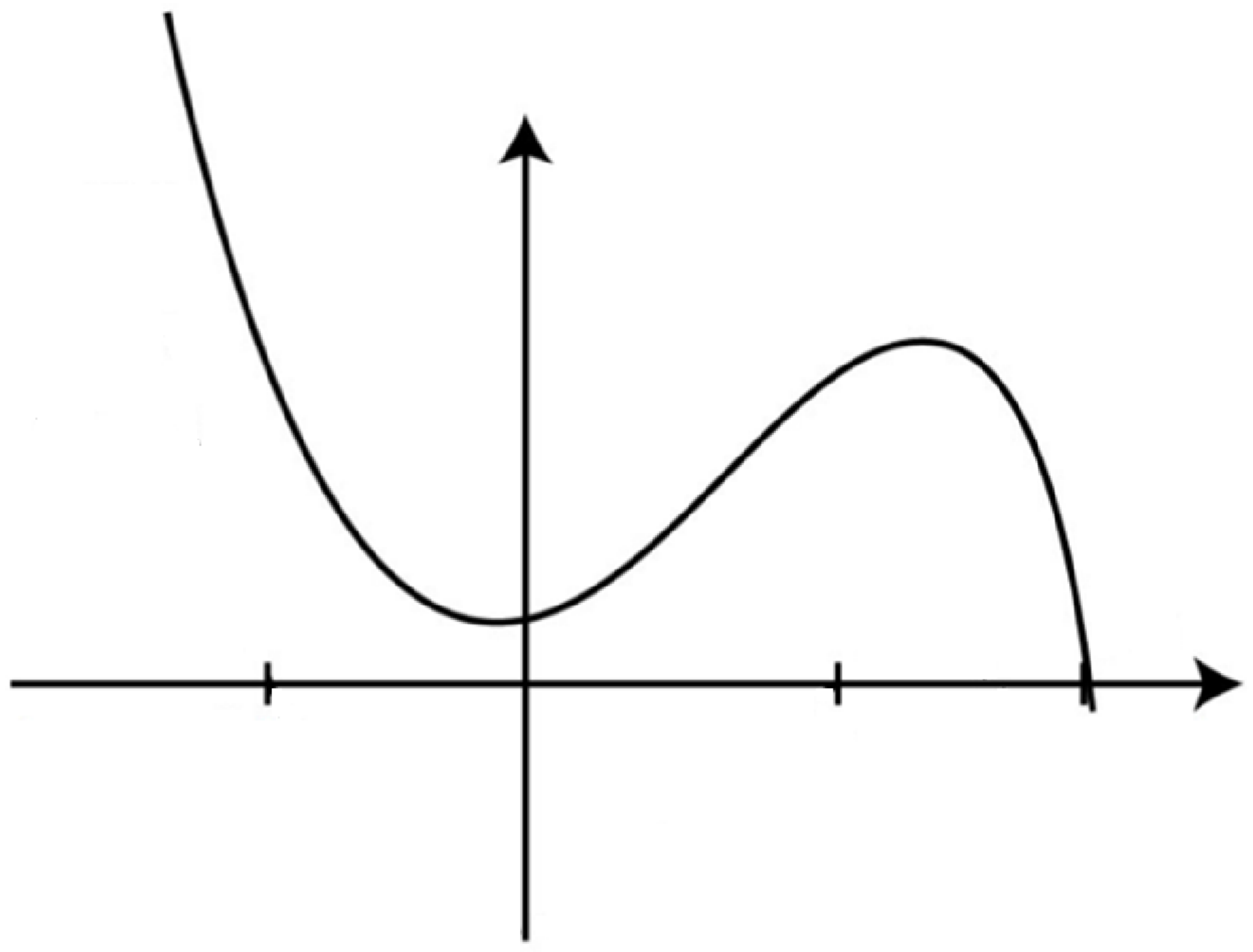


$A < 1/L$



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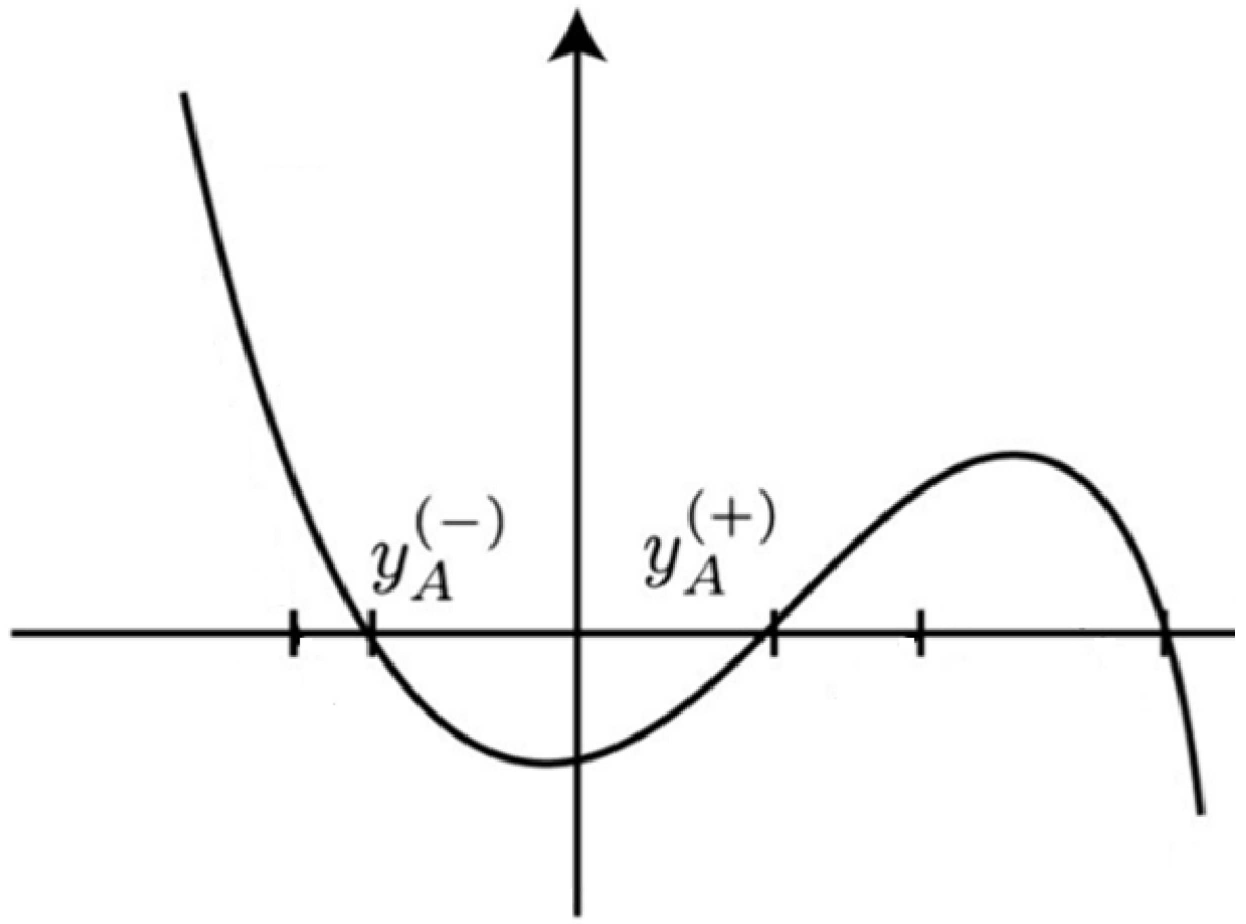

✗

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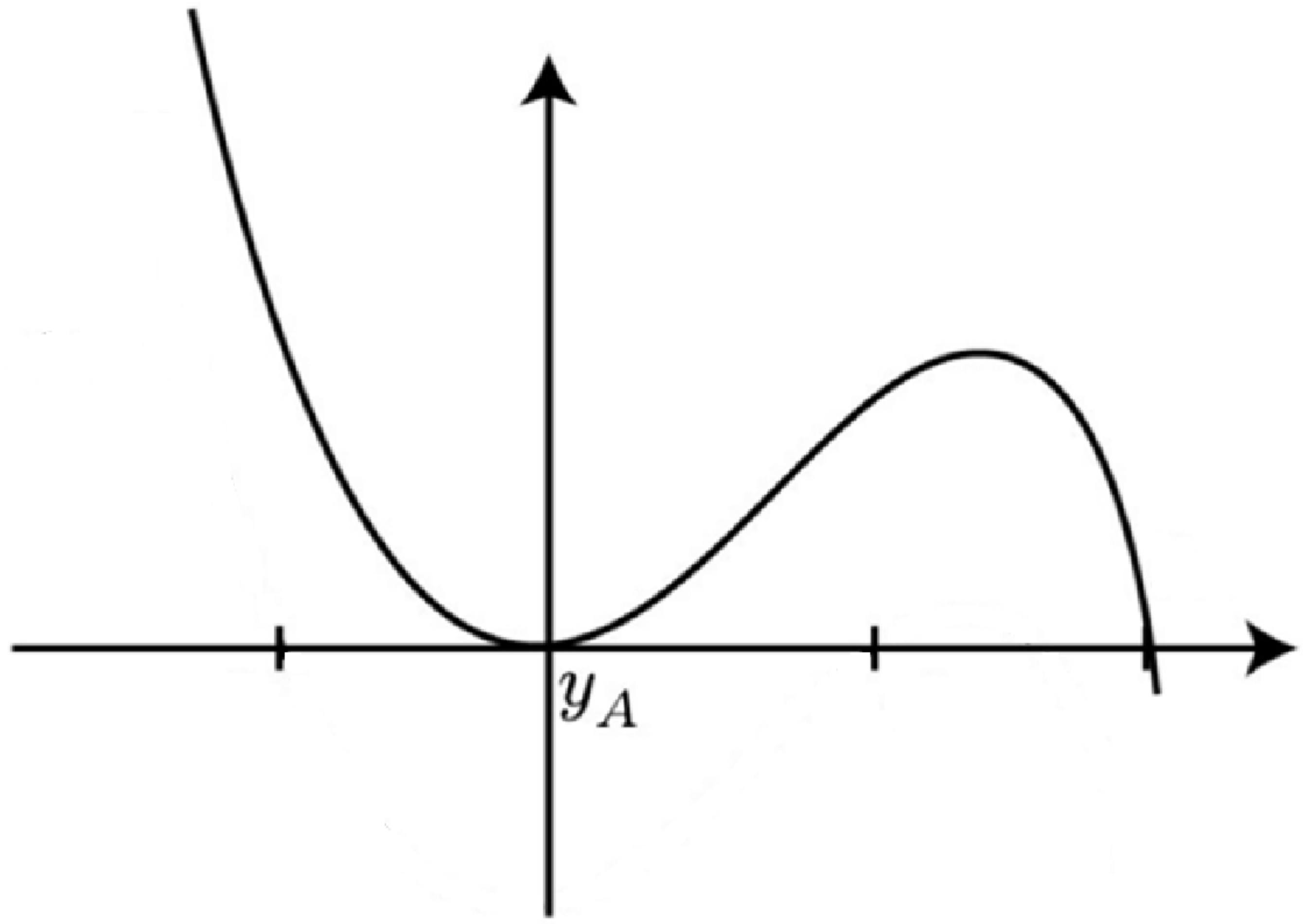

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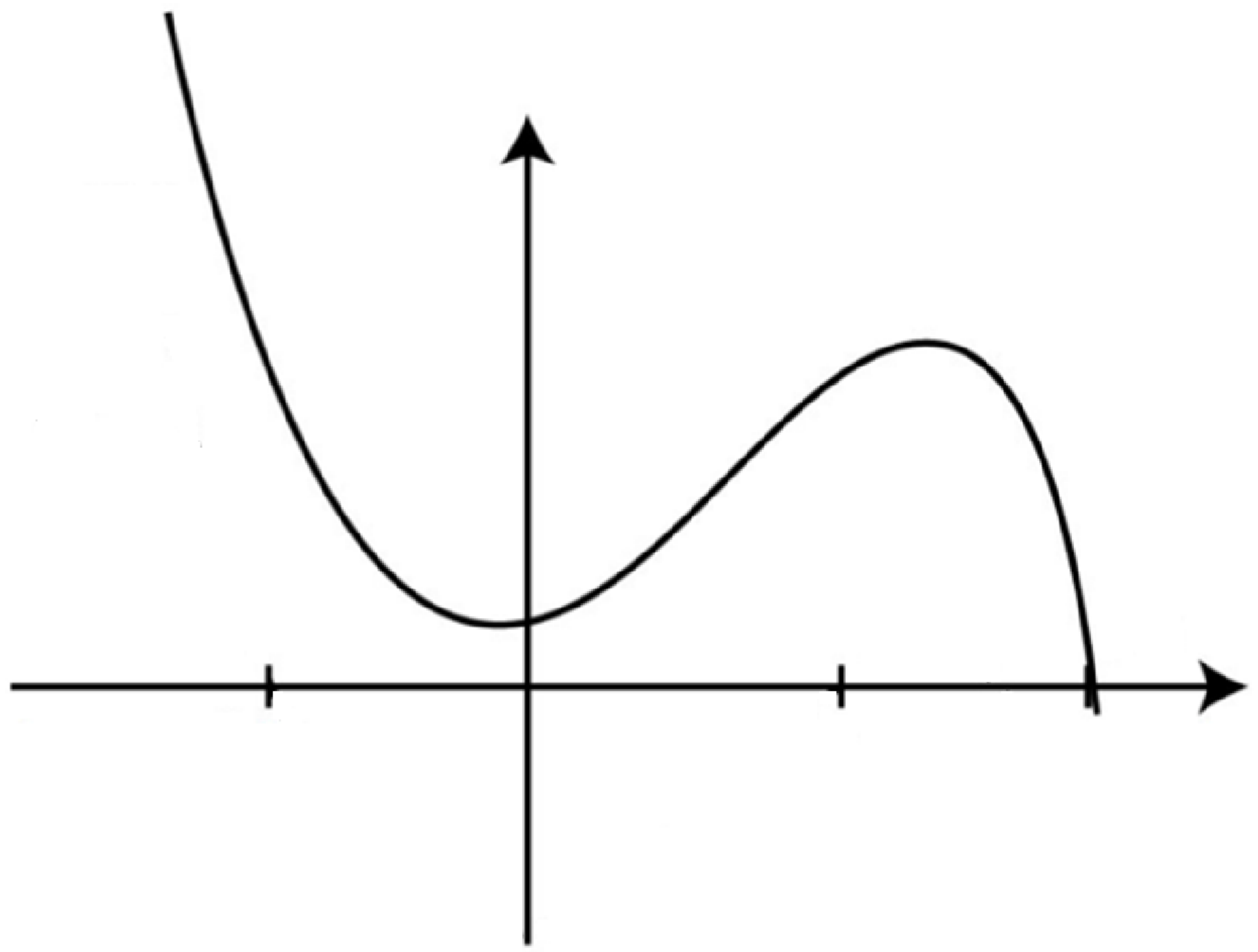

✓

slowly accelerating black hole

$$F(y) = -\left(1 - y^2\right)(1 - 2Amy) + \frac{1}{A^2L^2}$$

Part 3

Going Beyond General Relativity

Going Beyond General Relativity

- General Relativity is an **effective theory**…

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left(R - 2\Lambda + \lambda_{\text{ev}} R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} R_{\delta\gamma}{}^{\mu\nu} + \lambda_{\text{odd}} R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} \tilde{R}_{\delta\gamma}{}^{\mu\nu}\right)$$

- First non-trivial terms come at six derivatives

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- First non-trivial terms come at six derivatives
- Goals of this talk:
  - Are there accelerating black hole solution in this theory?
  - How do we deal with the boundary conditions?

Part 3

Finding the Solution

The Even-Parity Ansatz

- Correction to **diagonal** terms in the metric

$$ds^2 = \frac{1}{\Omega(y,x)^2}\Big\{-\left(1+\underline{\lambda_{\rm ev} f(y,x)}\right)F(y)dt^2 + \left(1+\underline{\lambda_{\rm ev} h(y,x)}\right)\frac{dy^2}{F(y)} + \left(1+\underline{\lambda_{\rm ev} j(y,x)}\right)\frac{dx^2}{G(x)} + \left(1+\underline{\lambda_{\rm ev} k(y,x)}\right)G(x)\frac{d\phi^2}{K^2}\Big\}$$

The Even-Parity Ansatz

**coefficients depend on parameters of the solution

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- **Trick**: guess a polynomial

$$f(y,x) = \sum_{n=0}^{n_{\rm max}}\sum_{m=0}^{m_{\rm max}} C^{(f)}_{nm} x^n y^m, \qquad h(y,x) = \sum_{n=0}^{n_{\rm max}}\sum_{m=0}^{m_{\rm max}} C^{(h)}_{nm} x^n y^m,$$

$$j(y,x) = \sum_{n=0}^{n_{\rm max}}\sum_{m=0}^{m_{\rm max}} C^{(j)}_{nm} x^n y^m, \qquad k(y,x) = \sum_{n=0}^{n_{\rm max}}\sum_{m=0}^{m_{\rm max}} C^{(k)}_{nm} x^n y^m.$$

The Odd-Parity Ansatz

- Correction to **off-diagonal** terms in the metric

$$ds^2 = \frac{1}{\Omega(y,x)^2}\left\{F(y)dt^2 + \frac{dy^2}{F(y)} + \frac{dx^2}{G(x)} + G(x)\frac{d\phi^2}{K^2} + \underline{\lambda_{\mathrm{odd}}p(y,x)}dtd\phi\right\}$$

The Odd-Parity Ansatz

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- Correction to **off-diagonal** terms in the metric

$$ds^2 = \frac{1}{\Omega(y,x)^2}\left\{F(y)dt^2 + \frac{dy^2}{F(y)} + \frac{dx^2}{G(x)} + G(x)\frac{d\phi^2}{K^2} + \underline{\lambda_{\text{odd}}p(y,x)}dtd\phi\right\}$$

- **Trick**: guess a polynomial

$$p(y,x) = \sum_{n=0}^{n_{\max}}\sum_{m=0}^{m_{\max}} C^{(p)}_{nm}x^n y^m$$

Features of the Solution

- All corrections **truncate** (unlike Kerr) and are functions of both coordinates

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- All corrections **truncate** (unlike Kerr) and are functions of both coordinates
- Corrections are polynomial & up to order 6 in **both x and y**
- There are additional gauge choices, but they don’t change the overall form of the solution
- Odd-parity term induces rotation

(A Part) of the Solution

- For example:

$$
\begin{aligned}
f(y,x) = &-\frac{4}{L^4} + \frac{8}{3}A^5 m x \left(15 - \frac{74}{A^2L^2} + \frac{24}{A^4L^4}\right) - \frac{8A^4m^2}{3L^2}x^2\left(7A^2L^2+96\right) + 88A^5mx^3 \\
&+ \frac{16A^4m^2}{L^2}x^4\left(A^2L^2+12\right) - 40A^6m^2x^6 + y\left[-\frac{32A^3m}{L^2}\left(A^2L^2+1\right) + \frac{32A^4m^2}{15L^2}x^3\left(30-151A^2L^2\right)\right. \\
&\left.-272A^6m^2x^5\right] + y^2\left[40A^6m^2 + \frac{A^3m}{L^2}x\left(16 - \frac{376A^2L^2}{5}\right) + \frac{64A^4m^2x^2\left(8A^2L^2+15\right)}{5l^2} - \frac{968}{5}A^5mx^3\right. \\
&\left.-\frac{1}{5}3368A^6m^2x^4\right] + y^3\left[48A^5m + \frac{64A^4m^2}{3L^2}x\left(11A^2L^2+3\right) - 144A^5mx^2 - \frac{1}{5}1568A^6m^2x^3\right] \\
&+ y^4\left(\frac{304}{5}A^6m^2x^2\right) + y^5\left(16A^6m^2x\right),
\end{aligned}
$$

Part 5

Black Hole Thermodynamics

The First Law

- First law now depends on the tension and length of the string [Appels, Gregory, Kubizňák, 1702.00490]

$$dM = TdS - \underline{\lambda_+ d\mu_+} - \underline{\lambda_- d\mu_-}$$

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- **Goal:** compute the thermodynamic quantities and verify the first law

** odd parity: **no corrections** to the thermodynamics at first order

** even parity: **non-trivial corrections** to the thermodynamics at first order

The Entropy and String Tension

$$dM = TdS - \lambda_+ d\mu_+ - \lambda_- d\mu_-$$

r_+ : black hole horizon

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- **Entropy:** Iyer-Wald Formula

$$S = -2\pi \int_{\mathcal{H}} d^2x \sqrt{\gamma} P^{\mu\nu\alpha\beta} \epsilon_{\mu\nu} \epsilon_{\alpha\beta}$$

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- **Entropy:** Iyer-Wald Formula

$$S = -2\pi \int_{\mathcal{H}} d^2x \sqrt{\gamma} P^{\mu\nu\alpha\beta} \epsilon_{\mu\nu} \epsilon_{\alpha\beta}$$

- relative correction

$$\delta S = \frac{\pi r_+^2}{K(1 - A^2 r_+^2)} \lambda_{\text{ev}} \left(-\frac{46A^4}{3} + \frac{10A^2}{r_+^2} - \frac{6\left(A^4 r_+^4 - 7A^2 r_+^2 + 3\right)}{L^4 \left(A^2 r_+^2 - 1\right)^2} + \frac{4\left(22A^4 r_+^4 - 51A^2 r_+^2 + 18\right)}{3L^2 r_+^2 \left(A^2 r_+^2 - 1\right)} + \frac{2}{r_+^4} \right)$$

The Entropy and String Tension

r_+ : black hole horizon

$$dM = \underline{TdS} - \lambda_+ \underline{d\mu_+} - \lambda_- \underline{d\mu_-}$$

- **Entropy:** Iyer-Wald Formula

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- relative correction

$$\delta S = \frac{\pi r_+^2}{K(1 - A^2 r_+^2)} \lambda_{\text{ev}} \left(-\frac{46A^4}{3} + \frac{10A^2}{r_+^2} - \frac{6\left(A^4 r_+^4 - 7A^2 r_+^2 + 3\right)}{L^4 \left(A^2 r_+^2 - 1\right)^2} + \frac{4\left(22A^4 r_+^4 - 51A^2 r_+^2 + 18\right)}{3L^2 r_+^2 \left(A^2 r_+^2 - 1\right)} + \frac{2}{r_+^4} \right)$$

- **String tension:** Find the ratio between the circumference and radius

The Entropy and String Tension

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- **String tension:** Find the ratio between the circumference and radius

$$\delta_\pm = 2\pi \left[1 - \left(\frac{1 \pm 2MA}{K} \right) \left(1 + \frac{\lambda_{\text{ev}}}{2} \left(k\left(y, x_\pm\right) - j\left(y, x_\pm\right) \right) \right) \right] \longrightarrow \mu_\pm = \frac{\delta_\pm}{8\pi}$$

Subtleties...

- First law

$$dM = TdS - \lambda_+ d\mu_+ - \lambda_- d\mu_-$$

- #1: What is the appropriate time coordinate?
- #2: String lengths – first principles derivation is ***not known*** (even in GR)
- #3: Mass – correction to conformal mass formula is ***not known*** (but WIP)

Subtlety 1: The Time Normalization

- What is the appropriate time coordinate? [Anabalón, Gray, Gregory, Kubizňák, Mann, 1811.04936]

Subtlety 1: The Time Normalization

$$L_{\text{eff}} = L - \frac{2}{L^3}\lambda_{\text{ev}}$$

- What is the appropriate time coordinate? [Anabalón, Gray, Gregory, Kubizňák, Mann, 1811.04936]
- Take the **massless** limit: AdS with modified radius

$$ds^2 = \frac{1}{\Omega^2}\frac{L_{\text{eff}}^2}{L^2}\left[-\left(1+\frac{r^2}{L^2}\left(1-A^2L^2\right)\right)dt^2 + \frac{dr^2}{1+\frac{r^2}{L^2}\left(1-A^2L^2\right)} + r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)\right]$$

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- Perform an appropriate coordinate transformation

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- Therefore:

$$\tau = \alpha t \quad \longrightarrow \quad \alpha = \frac{L_{\text{eff}}}{L}\sqrt{1-A^2L^2}$$

The Temperature

r_+ : black hole horizon

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- **Temperature:** demanding the regularity of the Euclidean metric at the horizon

$$T = \frac{\mathcal{F}'(r_+)\left[1 + \frac{1}{2}\lambda_{\rm ev}(f(r_+,x) - h(r_+,x))\right]}{4\pi\alpha}$$

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Angular dependencies cancel exactly such that the temperature is **constant** over the horizon

$$\delta T = T_0 \lambda_{\text{ev}} \left(-\frac{46A^4}{3} + \frac{4A^2\left(4A^2r_+^2 - 15\right)}{3L^2\left(A^2r_+^2 - 1\right)} + \frac{10A^2}{r_+^2} + \frac{2\left(5A^4r_+^4 + 5A^2r_+^2 - 1\right)}{L^4\left(A^2r_+^2 - 1\right)^2} + \frac{2}{r_+^4} \right)$$

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- Compute the mass and string lengths via the first law
- Completely fixed once we demand

1. Mass is zero when $m = 0$
2. Demanding the charges are equal for $A \leftrightarrow -A$

Conclusion

Final Remarks

Future directions

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Thank you for your attention!