Rapid Turn Multifield Cosmology with Gauss-Bonnet Term

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work in progress

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Motivation

Multifield cosmological models:

(Multiple scalars minimally coupled to Einstein gravity)

- Motivated by string compactifications, ‘swampland’ conjectures
- Lead to novel effects compared to single field models, due to rapid turn regime

Higher derivative terms:

- Motivated by fundam. considerations: string compactifications, quantum corrections
- Promising in view of current observational puzzles...

Goal: Combine the two; start with GB term

Multifield Cosmological Models

(broadly motivated by quantum gravity)

Action:

$$S = \int d^4x \sqrt{-\det g}\left[\frac{R}{2} - \frac{1}{2} G_{IJ}(\phi)\, g^{\mu\nu}\, \partial_\mu \phi^I\, \partial_\nu \phi^J - V(\phi)\right] ,$$

$g_{\mu\nu}(x)$ - spacetime metric , $\mu, \nu = 0, ..., 3$

$G_{IJ}(\phi)$ - field space metric , $I, J = 1, 2, ..., n$

Standard background Ansatze:

$$ds_g^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad , \quad \phi^I = \phi_0^I(t) \quad ,$$

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} \quad - \quad \text{Hubble parameter}$$

Conceptual note:

In single-field models potential $V(\phi)$ plays key role:

Always: field redefinition $\rightarrow$ canonical kinetic term

(Can transfer complexity to the potential)

In multi-field models:

Cannot redefine away the curvature of G_{IJ} !

(I.e., kinetic term becomes important !)

$\Rightarrow$ Can have: - Genuine multi-field trajectories in field space even when $\partial_{\phi^I} V = 0$ for some I

- New phenomena due to non-geodesic background trajectories in field space

Characteristics of a background trajectory:

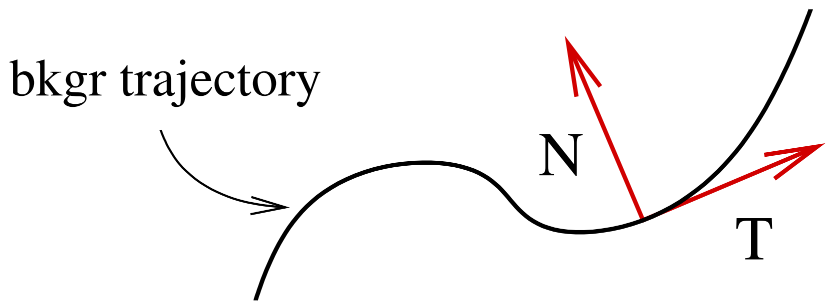
From now on consider 2 scalars, i.e. $I, J = 1, 2$

Background trajectory $(\phi_0^1(t), \phi_0^2(t))$ in field space:

– Tangent and normal vectors:

$$T^I = \frac{\dot{\phi}_0^I}{\dot{\sigma}} \quad , \quad N_I = (\det G)^{1/2} \epsilon_{IJ} T^J \quad , \quad \dot{\sigma}^2 = G_{IJ} \dot{\phi}_0^I \dot{\phi}_0^J$$

(Note: $N_I T^I = 0 \;\; , \;\; T_I T^I = 1 \;\; , \;\; N_I N^I = 1$)



Characteristics of a background trajectory:

– Turning rate of the trajectory:

$$\Omega = -N_I D_t T^I \quad , \quad D_t T^I \equiv \dot{\phi}_0^J \, \nabla_J T^I$$

(measure for deviation from geodesic)

– Slow-roll parameters: [Cespedes, Atal, Palma 2012]

$$\varepsilon = -\frac{\dot{H}}{H^2} \quad , \quad \eta^I = -\frac{1}{H\dot{\sigma}} \, D_t \dot{\phi}_0^I$$

Expand: $\eta^I = \eta_{||} T^I + \eta_{\perp} N^I \quad \rightarrow \quad \eta_{||} = -\frac{\ddot{H}}{2H\dot{H}} \;\; , \;\; \eta_{\perp} = \frac{\Omega}{H}$

Note: $\varepsilon \, , \eta_{||}$ - same as Hubble slow-roll parameters in single-field case

Rapid Turn Regime

(strongly non-geodesic field-space trajectories)

Pheno viability and perturbative stability:

In the past:

Slow roll & slow turn: $\varepsilon\,, |\eta_{||}| \ll 1 \quad \& \quad |\eta_\perp| \ll 1$

[Sasaki, Stewart 1996; Gordon, Wands, Bassett, Maartens 2001; Groot Nibbelink, van Tent 2002; D. Langlois and S. Renaux-Petel 2008; Peterson, Tegmark 2011;...]

Recently also:

Slow roll & rapid turn: $\varepsilon\,, |\eta_{||}| \ll 1 \quad \& \quad \eta_\perp^2 \gg 1$

[Cespedes, Atal, Palma 2012; Achucarro, Atal, Germani, Palma 2017; Garcia-Saenz, Renaux-Petel, Ronayne 2018; Bjorkmo, Ferreira, Marsh 2019;...]

Slow roll on steep potentials:

Slow roll $\&$ rapid turn: [Achucarro, Palma 2019]

$$\varepsilon \;=\; \varepsilon_V \left(1 + \frac{\eta_\perp^2}{9}\right)^{-1} \quad ,$$

$$\varepsilon_V = \tfrac{1}{2}\frac{G^{IJ}V_I V_J}{V^2} \quad , \quad V_I = \partial_{\phi_0^I} V$$

$\Rightarrow$ For large $\eta_\perp^2$ can have simult.: small ε and large ε_V

$\rightarrow$ Spacetime expansion in slow-roll regime: decoupled from flatness of scalar potential

– easy to evade swampland constraints

– useful for realizing inflation in fundam. set-ups

Primordial black holes (PBHs): natural candidate for DM

A generation mechanism:

Large enough fluctuations during inflation can seed PBHs

- Single-field models: PBH-formation is a challenge...

- Two-field models: (recall: $\eta_\perp = \Omega/H$)

Power spectrum of curvature perturbation ζ:

$$\mathcal{P}_\zeta \sim \mathcal{P}_0 \, e^{c\,|\eta_\perp|} \quad , \qquad c = const > 0$$

For PBH generation, need δt with: $\mathcal{P}_\zeta / \mathcal{P}_0 \sim 10^7$

$\rightarrow$ easy to achieve with a brief rapid turn

[Palma, Sypsas, Zenteno 2020; Anguelova 2021;
Fumagalli, Renaux-Petel, Ronayne, Witkowski 2023]

Multifield dark energy:

Key characteristics of dynamical dark energy:

- equation-of-state parameter w_{DE}
- dark energy perturbations' speed of sound c_s^{DE}

In single-scalar (quintessence) models: [speed of light: $c = 1$]

$c_s^{DE} \equiv 1 \quad \rightarrow \quad$ observ. distinctions from Cosm. Const. Λ only for w_{DE} significantly $\neq -1$

In multifield (rapid-turn) case: [Akrami, Sasaki, Solomon, Vardanyan 2021; Anguelova, Dumancic, Gass, Wijewardhana 2022, 2024]

Can have $c_s^{DE} < 1$ even for $w_{DE} \approx -1$

$\rightarrow$ dynamical DE: observ. distinguishable from Λ

Rapid Turn with Gauss-Bonnet Term

Action:

$$S = \int d^4x \sqrt{-\det g}\left[\frac{R}{2} - \frac{1}{2} G_{IJ}(\phi)\, g^{\mu\nu}\, \partial_\mu \phi^I\, \partial_\nu \phi^J - V(\phi) + \frac{h(\phi)}{8} R^2_{GB}\right],$$

$$R^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

R^2_{GB} : by itself - topological; coupled to scalar(s) - dynamical

Standard background Ansatze:

$$ds^2_g = -dt^2 + a(t)^2 d\vec{x}^2 \quad , \quad \phi^I = \phi^I_0(t) \quad , \quad H = \dot{a}/a$$

Consider field-space metric G_{IJ}:

$$ds^2_G = d\varphi^2 + f(\varphi) d\theta^2$$

Background equations of motion:

Equations for the scalar fields:

$$D_t\dot{\phi}_0^I + 3H\dot{\phi}_0^I + G^{IJ}V_J - 3G^{IJ}h_J H^2(H^2+\dot{H}) = 0 \quad ,$$

$$V_J \equiv \partial_{\phi_0^J} V \quad , \quad h_J \equiv \partial_{\phi_0^J} h \quad ,$$

$$D_t\dot{\phi}_0^I \equiv \dot{\phi}_0^J \nabla_J \dot{\phi}_0^I = \ddot{\phi}_0^I + \Gamma^I_{JK}\dot{\phi}_0^J\dot{\phi}_0^K$$

Turning rate:

Using $\Omega = -N_I D_t T^I$ and the EoMs:

$$\Omega = \frac{N_I V^I}{\dot{\sigma}} - \frac{3N_I h^I}{\dot{\sigma}} H^2(H^2+\dot{H})$$

$\Rightarrow$ Can have $\Omega \neq 0$ even for $V^I = 0$!

Background equations of motion:

From Einstein equations:

$$3H^2 = \frac{1}{2}G_{IJ}\dot{\phi}_0^I\dot{\phi}_0^J + V - 3h_I\dot{\phi}_0^I H^3$$

Slow roll:

$$\varepsilon = -\tfrac{\dot{H}}{H^2} \ll 1 \;,\; \delta = \dot{h}H \ll 1 \quad \rightarrow \quad 3H^2 \approx V$$

$$\Rightarrow \quad \Omega \approx \frac{N_I\left(V^I - \frac{1}{3}h^I V^2\right)}{\dot{\sigma}} \quad , \quad \dot{\sigma}^2 = G_{IJ}\dot{\phi}_0^I\dot{\phi}_0^J$$

Still have $\eta_\perp = \frac{\Omega}{H}$ due to kinematics $\Rightarrow$ Rapid turn regime:

$\eta_\perp^2 \gg 1$ (at least, $\eta_\perp^2 \gg \varepsilon, |\eta_{||}|, |\delta|$) - much easier to realize

$\rightarrow$ Embedding in string compactifications?...

Effective equation of state:

Can have $w_{eff}(\{\phi^I\}) < -1$ depending on $h(\{\phi^I\})$

$\rightarrow$ NEC effectively violated, but NO exotic matter

Although, generically, phantom fluids are unstable, with GB term can have transient phantom-like cosmic acceleration

Note: Causal structure - can be different due to GB term

(Could be useful for solving the information loss paradox [Izumi 2014]...)

Other higher curvature/derivative terms:

Same kind of features $\rightarrow$ Novel realizations of rapid turn regime with pheno-interesting equation-of-state parameter w_{eff} ...

Summary

Multifield cosmology with higher derivative terms:

– GB term $\rightarrow$ Decoupling of rapid turn regime from (very) steep potentials

- Novel realizations of rapid turn (maybe, easier to embed in string compactifications?...)

– Effective equation of state parameter w_{eff} : can vary below -1 depending on GB coupling function

Phenomenological implications of Rapid Turn with GB term?...

Thank you!