Natural Higgs Mass from Power-Law Running

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based on 2603.15081, 2506.18667

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The Hierarchy Problem

- Higgs mass measured at LHC (ATLAS, CMS 2012):

$$m_h \approx 125 \text{ GeV}$$

- Standard Model expected to be completed at $M_{\text{GUT}} \sim 10^{16}$ GeV

$$\frac{m_h^2}{M_{\text{GUT}}^2} \sim 10^{-28}$$

- Every loop correction from a field of mass M_{GUT}:

$$\delta m_h^2 \sim M_{\text{GUT}}^2$$

- This overwhelms the observed mass and requires a cancellation to 1 part in 10^{28}
  [Susskind 79] [Veltman 81]
- **Why is the Higgs so light compared to the GUT scale?**

't Hooft's Symmetry Argument

- 't Hooft (1980): a parameter is *natural* if setting it to zero increases the symmetry
- **Fermion:** $m_f \to 0$ restores chiral symmetry $\Rightarrow$ protected

$$\delta m_f \propto m_f \log M^2$$

- **Scalar:** $m_h \to 0$ restores no symmetry $\Rightarrow$ unprotected

$$\delta m_h^2 \propto M^2 (?)$$

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- Traditional conclusion: a light Higgs requires a new symmetry to stabilize it
  - Formally the same as Λ^2, and the mass is super-renormalizable.
  - This is naive dimensional analysis. **Precise calculation required.**

New: Power-law running

- The renormalized self-energy yields the same scaling:

$$\delta m_f \propto m_f \log q^2 \qquad \text{fermion}$$
$$\delta m_h^2 \propto q^2 \log q^2 \qquad \text{scalar}$$

- This work: the equations hold **at every energy scale,** not at a UV scale [KSC 2604.04569] [KSC 2506.18667]
- q^2 is the physical probe scale as in the running strong coupling $\alpha_s(q^2)$.

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- q^2 is the physical probe scale as in the running strong coupling $\alpha_s(q^2)$.
- The running Higgs mass $m^2(q^2)$ as a function of external momentum q^2 is

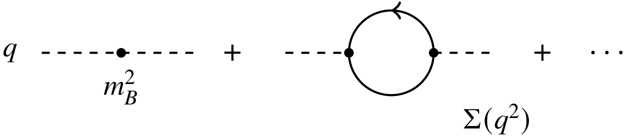
$$m^2(m_{EW}^2) = \mathcal{O}(m_{EW}^2) \quad \text{and} \quad m^2(M_{\text{GUT}}^2) = \mathcal{O}(M_{\text{GUT}}^2).$$

- **The absence of symmetry protection produces the power-law running.**
- **No new physics:** physical (on-shell) renormalization.

Higgs Self-Energy

- The mass is defined as the parameter m_B^2 in the EFT Lagrangian, but not observable
- The **observable is the combination** of the bare mass plus the self-energy: [Fleischer, Jegerlehner 81]

$$m^2(q^2) = m_B^2 + \Sigma(q^2)$$



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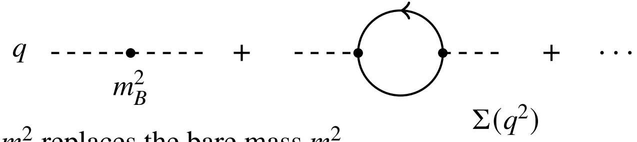


- The pole mass m_h^2 replaces the bare mass m_B^2

$$m_h^2 = m_B^2 + \Sigma(m_h^2) \quad \Longrightarrow \quad m_B^2 = m_h^2 - \Sigma(m_h^2)$$

- Equivalent description in a new basis

$$m^2(q^2) = m_h^2 + \Sigma(q^2) - \Sigma(m_h^2)$$

- We need further field strength renormalization.

The running mass $m^2(q^2)$

- On-shell renormalization defines a mass at every external momentum q: [KSC 2604.04569],[Coleman 18] [Cheng, Li 84]

$$
\begin{aligned}
m^2(q^2) &= m_h^2 + \Sigma(q^2) - \Sigma(m_h^2) - (q^2 - m_h^2)\frac{d\Sigma}{dq^2}(m_h^2) \\
&\equiv m_h^2 + \Sigma_{\text{ren}}(q^2)
\end{aligned}
$$

- The resulting propagator is the unique function with gauge-invariance unit residue and canonical kinetic term

$$
\frac{i}{p^2 - m^2(q^2)}
$$

- The *function* $m^2(q^2)$ runs
  - Analogous to the running coupling $\alpha_s(q^2)$, physically measurable at every scale
  - Natural interpretation: $m^2(q^2)$ is the RG running of the mass from the pole m_h^2 up to the probe scale q^2.

Finite and regularization Independent

- The mass function has a schematical form, in any regularization,

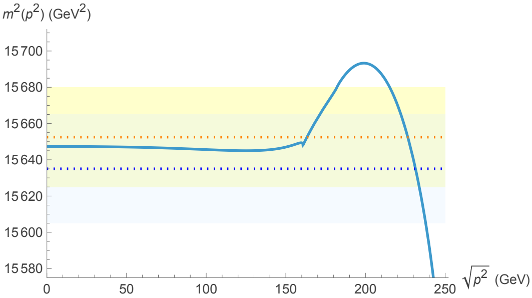
$$m^2(q^2) = \int_0^1 dx \left[A + \Delta \log \frac{B}{\Delta}\right], \quad \Delta = (M^2 - x(1-x)p^2).$$

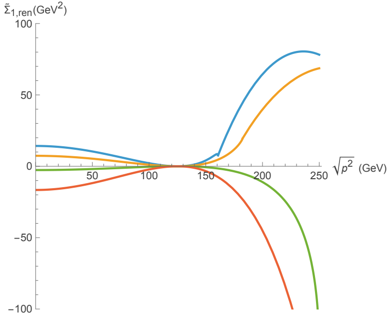
  where A, B may carry divergent parts
  - Cutoff: $A = \Lambda^2$, $\log B \sim \log \Lambda^2$; dimreg: $A = 0$, $\log B \sim 2/\epsilon$

- The **divergent** parts of Σ cancel in the difference of the same quantity. [Bogoliubov, Parasiuk 57] [Hepp 66] [Zimmermann 69]
- The mass function is finite, $m^2(q^2) = m_h^2 + \Sigma(q^2) - \Sigma(m_h^2) - (q^2 - m_h^2)\frac{d\Sigma}{dq^2}(m_h^2)$:
  - $-\Sigma(m_h^2)$ removes A.
  - $-(q^2 - m_h^2)\frac{d\Sigma}{dq^2}(m_h^2)$ removes $\log B$.
- Finite up to $\mathcal{O}\left(\frac{(q^2-m_h^2)^2}{\Lambda^2}\right)$, insensitive to $\hbar/\Lambda \sim a$ the "lattice size".

One-loop correction within the Standard Model

- Off-shell Higgs $m^2(q^2)$ away from the pole [KSC 2506.18667] [Kauer, Passarino 12].
- Dominant contributions $W \approx Z > t > h$.





Decoupling

We may worry about a possible large loop correction by a UV field with mass M (in)directly coupling to the Higgs. e.g. $\mathcal{L} \supset -\frac{1}{2}M^2X^2 - \frac{1}{4}\kappa h^2X^2$.

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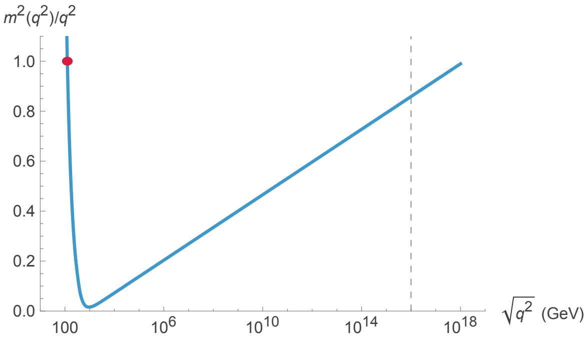
A careful calculation of $m^2(q^2)$, with $M^2 \gg q^2$

$$\Sigma^{\text{heavy}}_{\text{ren}}(q^2) = \mathcal{O}\left(\frac{(q^2 - m_h^2)^2}{M^2}\right)$$

cf. $m^2(q^2) = m_h^2 + \Sigma(q^2) - \Sigma(m_h^2) - (q^2 - m_h^2)\frac{d\Sigma}{dq^2}(m_h^2)$

- *Not* $\mathcal{O}(M^2)$; extends the Appelquist–Carazzone theorem to super-renormalizable masses [Appelquist, Carazzone 75] [KSC 2408.06406]
- q^2 is the scale at which the mass is probed, a physical external momentum
- At $q^2 \sim m_h^2$, **we cannot see UV physics.**

The Running Ratio $r = m^2(q^2)/q^2$



Red dot: measured pole mass m_h = 125 GeV

Dashed line: $M_{\text{GUT}} \sim 10^{16}$ GeV

For large q^2,

$$\frac{\Sigma^{\text{SM}}_{\text{ren}}(q^2)}{q^2} \approx \sum_{a=t,W,Z,h} \frac{c_0^a}{16\pi^2} \log \frac{q^2}{m_a^2} - const.$$

c_0 the avg. anomalous dimension

- **Quasi-conformal** $m^2(q^2)/q^2$ is $\mathcal{O}(1)$ at every scale
- **Prediction:** $r \to 1$ near 10^{18} GeV: upper bound on the SM's perturbative domain

$\mathcal{O}(1)$ at the GUT Scale

- Complete one-loop SM result at $\sqrt{q^2} = M_{\mathrm{GUT}} = 10^{16}$ GeV: [Georgi, Quinn, Weinberg 74]

$$\frac{m^2(M_{\mathrm{GUT}}^2)}{M_{\mathrm{GUT}}^2} = \underbrace{+1.135}_{\mathrm{top}} \underbrace{-0.169}_{W} \underbrace{-0.109}_{Z} \underbrace{+0.002}_{h} = +0.858$$

- Order one, no fine-tuning
  - Dominant contribution from the top quark Yukawa coupling $y_t \approx 1$
  - W and Z partially cancel the top (spin-statistics: fermion loops carry (-1)) [Pauli 40]
  - This partial cancellation is structural, not accidental; it persists at two loops

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- Net slope of $\Sigma^{\rm SM}_{\rm ren}/q^2$:

$$\frac{\Sigma^{\rm SM}_{\rm ren}}{q^2} \approx \frac{c_0}{16\pi^2}\log\frac{q^2}{m_h^2} - 0.060, \qquad c_0 \approx 2.24$$

Explaining the Electroweak Scale

- GUT–SM matching at M_{GUT} defines the boundary condition:

$$r \equiv \frac{m^2(M_{\text{GUT}}^2)}{M_{\text{GUT}}^2} = \mathcal{O}(1), \qquad 0 < r \leq 1$$

- Inverting gives the Higgs pole mass as a prediction:

$$m_h^2 = r\, M_{\text{GUT}}^2 - \Sigma_{\text{ren}}^{\text{SM}}(M_{\text{GUT}}^2)$$

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- **No fine tuning!**
- For $m_h^2 \ll M_{\rm GUT}^2$, with $\frac{\Sigma_{\rm ren}^{\rm SM}}{q^2} \approx \frac{c_0}{16\pi^2} \log \frac{q^2}{m_h^2} - 0.060$:

$$0 = r - \frac{c_0}{16\pi^2} \log \frac{M_{\rm GUT}^2}{\color{red}{m_h^2}} + 0.060$$

Exponential Suppression

- For $m_h^2 \ll M_{\mathrm{GUT}}^2$, the boundary relation reads

$$r = \frac{c_0}{16\pi^2} \log \frac{M_{\mathrm{GUT}}^2}{m_h^2} - 0.060$$

- Solving for the Higgs mass gives the closed form:

$$m_h^2 \approx e^{-16\pi^2 (r+0.06)/c_0} M_{\mathrm{GUT}}^2, \qquad c_0 \approx 2.24$$

- An $\mathcal{O}(1)$ boundary value r at M_{GUT} is mapped exponentially down to m_h
  - Dimensional transmutation, with the slow logarithm playing the role of the running coupling [Bardeen 95]

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- The 28 orders of magnitude measure the **smallness of the anomalous dimension** $c_0/16\pi^2 \approx 0.014$, not a tuning

Tuning

Higgs mass

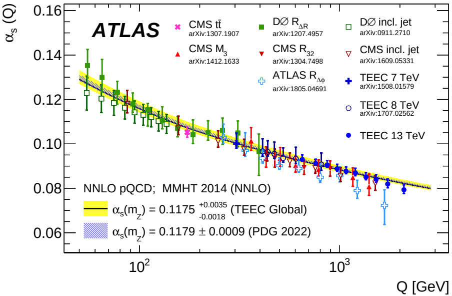
$$m_h^2 \approx M_{\mathrm{GUT}}^2 \, e^{-16\pi^2 (r+0.06)/c_0}$$

- $r = 0.858, \quad \text{net } c_0 \approx 2.24$
- $d\log(m_h^2/M_{\mathrm{GUT}}^2)/dr = -16\pi^2/c_0 \approx -70$

QCD scale [Gross, Wilczek 73] [Politzer 73]

$$\Lambda_{\mathrm{QCD}} \sim m_Z \, e^{-8\pi^2/(b_0 g_s^2)}$$

- $\alpha_s(m_Z) = 0.1180$
- $\Delta\alpha_s = 0.001 \implies \Delta\Lambda/\Lambda \sim 10\%$



Both are $\mathcal{O}(1)$ couplings whose specific values determine exponential hierarchies.

Why we could not have seen the power running?

- Recall the dimreg: [Bardeen, Buras, Duke, Muta 78]

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - \Delta(q^2)} = \frac{i}{16\pi^2} \left[-\frac{2}{\epsilon} + \gamma_E - \log 4\pi + \log \frac{\Delta(q^2)}{\mu^2} \right] \Delta(q^2), \quad \Delta(q^2) = M^2 - x(1-x)q^2$$

- **No quadratic divergence from ϵ. No quadratic dependence in μ.**
- In $\overline{\text{MS}}$, we cannot see the quadratic dependence.
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- **No quadratic divergence from** ϵ. **No quadratic dependence in** μ.
- In $\overline{\text{MS}}$, we cannot see the quadratic dependence.
- The quadratic growth survives in q, through $\Delta = M^2 - x(1-x)q^2$.
- In a mass-independent scheme the two must be joined by a **threshold correction** at $\mu \sim M$, **indistinguishable from the** M^2 **quadratic sensitivity**
  - $q^2 \ll M^2$: $\Delta \approx M^2$, contribution $\propto M^2$ (would-be quadratic sensitivity)
  - $q^2 \gg M^2$: $\Delta \approx -x(1-x)q^2$, contribution $\propto q^2$ (power-law running in q)
- DimReg is okay to keep the $\Delta \log \Delta$.

Summary

1. The renormalized Higgs mass function $m^2(q^2)$ is finite, physical and runs as q^2
2. Heavy fields decouple properly: $\Sigma^{\text{heavy}}_{\text{ren}} = \mathcal{O}((q^2 - m_h^2)^2/M^2)$, not M^2
3. $m^2(q^2)/q^2 = \mathcal{O}(1)$ at every scale from m_{EW} to M_{GUT}
4. The 28 orders of magnitude are generated by the SM loop factor:

$$m_h^2 \approx e^{-16\pi^2(r+0.06)/c_0}\, M_{\text{GUT}}^2, \qquad c_0 \approx 2.24$$

5. An order-one GUT boundary condition suffices to produce the electroweak scale; no new symmetry is needed

The absence of symmetry protection is not the disease.
It provides an explanation of the hierarchy.

Backup

From the GUT Scale Down to the Standard Model

- **Boundary at** $M_{\text{GUT}} = 10^{16}$ GeV (complete one-loop SM):

$$r \equiv \frac{m^2(M_{\text{GUT}}^2)}{M_{\text{GUT}}^2} = \underbrace{+1.135}_{\text{top}} \underbrace{-0.169}_{W} \underbrace{-0.109}_{Z} \underbrace{+0.002}_{h} = +0.858 = \mathcal{O}(1)$$

- **Running down** the desert with the net slope:

$$\frac{\Sigma_{\text{ren}}^{\text{SM}}(q^2)}{q^2} \approx \frac{c_0}{16\pi^2} \log \frac{q^2}{m_h^2} - 0.060, \qquad c_0 \approx 2.24$$

- **Pole-mass matching** $m^2(q^2) = m_h^2 + \Sigma_{\text{ren}}^{\text{SM}}(q^2)$ at $q^2 = M_{\text{GUT}}^2$:

$$r = \frac{m_h^2}{M_{\text{GUT}}^2} + \frac{c_0}{16\pi^2} \log \frac{M_{\text{GUT}}^2}{m_h^2} - 0.060 \xrightarrow{m_h^2 \ll M_{\text{GUT}}^2} r \approx \frac{c_0}{16\pi^2} \log \frac{M_{\text{GUT}}^2}{m_h^2} - 0.060$$

- **Closed form**:

$$m_h^2 \approx e^{-16\pi^2 (r+0.06)/c_0} M_{\text{GUT}}^2$$

- An $\mathcal{O}(1)$ boundary at M_{GUT} flows to m_{EW}, the 28 orders set by the small loop slope $c_0/16\pi^2 \approx 0.014$, not by tuning

Formulation

- One-loop self-energy integrals diverge $\delta m_h^2 \sim \int^\Lambda d^4k \frac{1}{k^2}$:

$$\begin{aligned} \delta m_f &\propto m_f \log \Lambda^2 && \text{fermion} \\ \delta m_h^2 &\propto \Lambda^2 && \text{scalar} \end{aligned}$$

- Λ is not related to interaction; setting the limitation of the description.
- Renormalization absorbs Λ-dependent terms; all observables are Λ-independent
- Old "hierarchy problem" appears as an artifact of this identification

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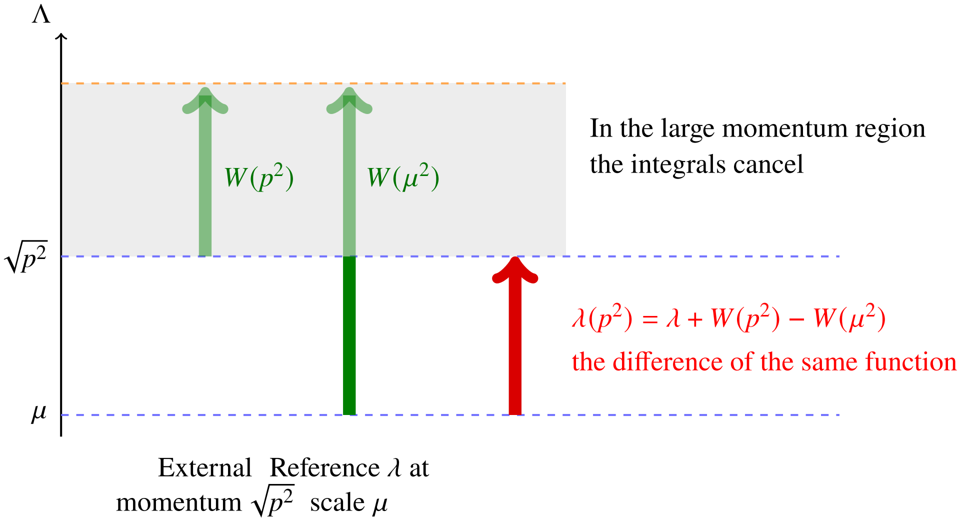
- Old "hierarchy problem" appears as an artifact of this identification
- Modern formulation: a new physics
- e.g. a UV field with mass M coupling to the Higgs mass.
- $\frac{1}{2}M^2X^2 + \frac{1}{4}\kappa h^2X^2$.

Finite renormalization, relative coupling



cf. Wilsonian renormalization [KSC] 2310.10004

Regularization Independence

- The one-loop fermion self-energy, after Feynman parameterization:

$$\int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - \Delta(q^2)}$$

with $\Delta(q^2) \equiv M_f^2 - x(1-x)q^2$.

- **Cutoff regularization** $(k_E^2 < \Lambda^2)$:

$$\frac{i}{16\pi^2} \int_0^1 dx \left[\Lambda^2 - \Delta(q^2) \log \frac{\Lambda^2}{\Delta(q^2)} \right] + \mathcal{O}\left(\frac{(p^2 - m_h^2)^2}{\Lambda^2} \right)$$

- **Dimensional regularization** $(d = 4 - \epsilon)$:

$$\frac{i}{16\pi^2} \int_0^1 dx \left[-\frac{2}{\epsilon} + \gamma_E - \log 4\pi + \log \frac{\Delta(q^2)}{\mu^2} + \mathcal{O}(\epsilon) \right] \Delta(q^2)$$

- The divergent pieces $(\Lambda^2, 1/\epsilon)$ differ between schemes
- **Statement:** After applying the on-shell subtraction, the renormalized mass $m^2(q^2)$ is finite [BPHZ].