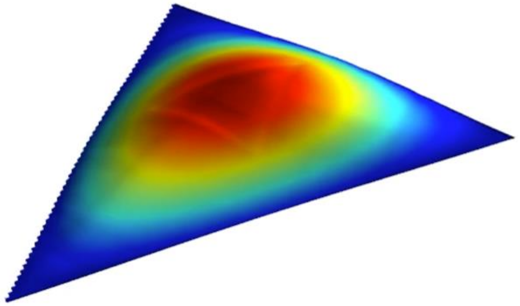
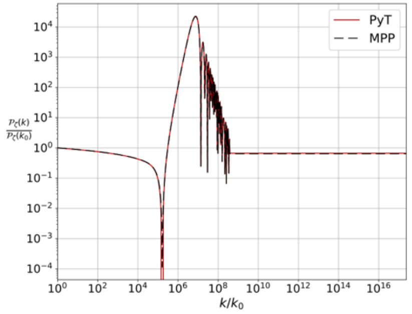
Taylor expansion methods for inflation, loops, and the decoupling of scales

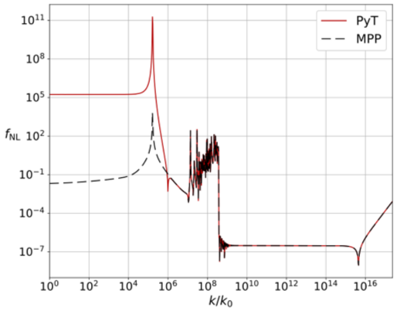
David Mulryne

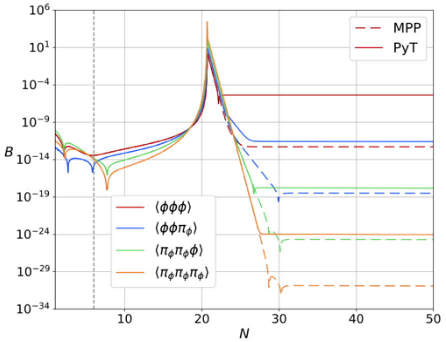
2601.14229 , with Laura Iacconi, David Seery; and 2503.15194 with Laura Iacconi, Andrea Constanti; ++

PACOS 2026, Sheffield









Plan

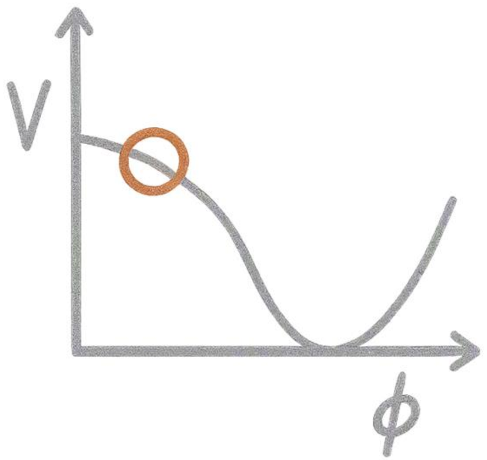
- Inflation, and the observational signatures of inflation
- Inflationary dynamics for short scale signatures (c.f. Oksana Iarygina talk)
- Why might loops be important and a **recent debate**: the coupling of large and short scales at loop level (c.f. quantum correction talk from Lucien Heurtier)
- A segway into "Taylor expansion" methods (and a numerical implementation)
- Addressing the debate and the decoupling of scales

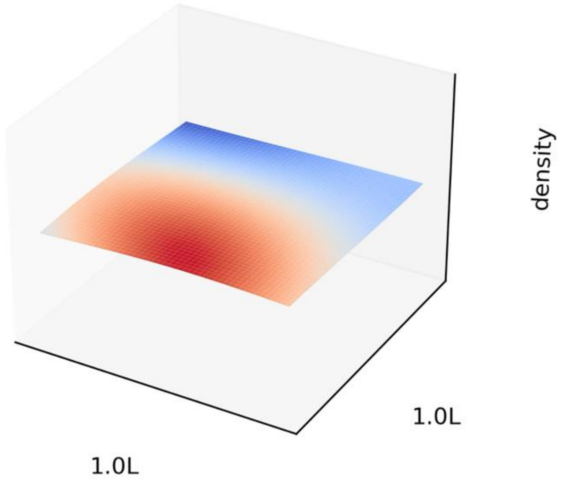
Context

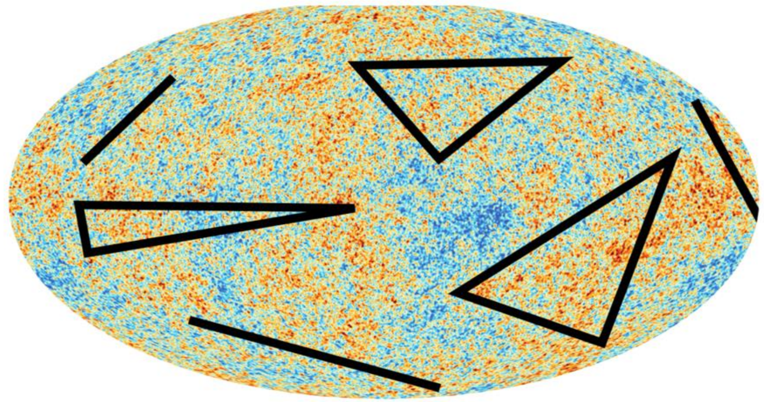
- To test models of inflation we want to calculate the statistics of the fluctuations they produce
- To do so reliably having different tools is useful, both analytic and numerical
- Numerical tools are needed for precision predictions, but can be challenging or slow for some models (c.f. Guillermo Fernando Quispe Peña talk)
- Sometimes different tools are very efficient for analytic calculations, or reveal different analytic structures (e.g. δN)
- Recently strong focus on very short scales, PBHs, and SIGWs

Inflation

- Postulate new field (fields), ϕ, field can mimic a cosmological constant
- Quantum mechanics implies $\phi(t) \rightarrow \phi(t) + \delta\phi(t, \mathbf{x})$
- Inflation makes $\delta\phi$ “long lived”, and retains information about fields present during inflation, a “cosmological collider”







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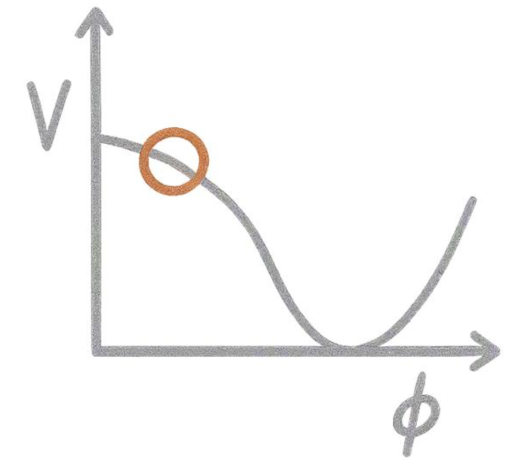

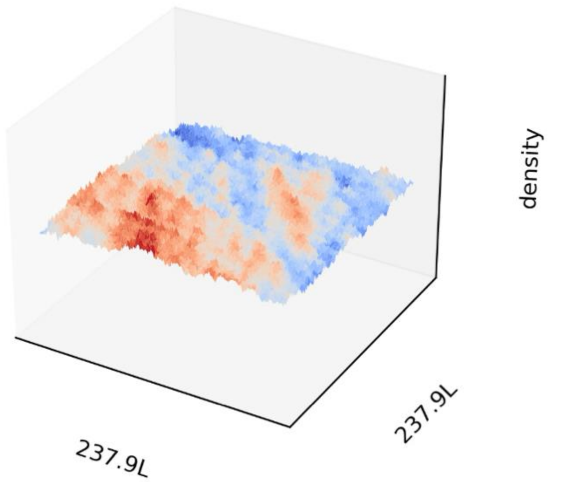

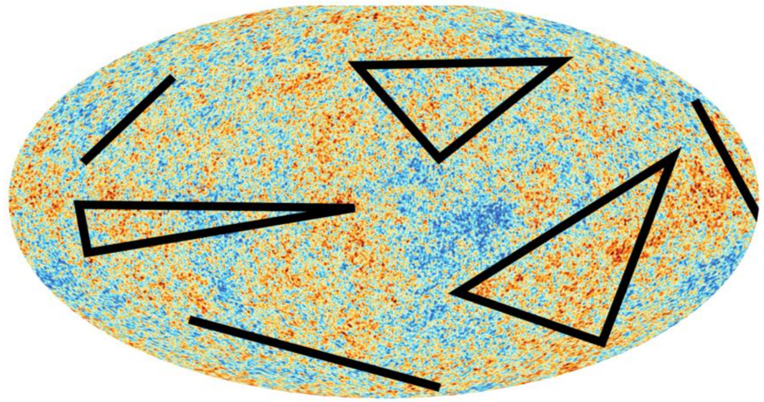

Calculating statistics

- We have a code for that! (PyTransport, 2016 onwards, DJM, Ronayne, Constantini, Iacconi, Seery; Dias, Frazer; also other code e.g. CppTransport, Seery 2016; CosmoFlow, Werth, Pinol, Renaux-Petel, 2024)
- $\langle \zeta(\mathbf{x_1})\zeta(\mathbf{x_2})\rangle \longrightarrow \langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\rangle = (2\pi)^3\delta(\mathbf{k_1}+\mathbf{k_2})\frac{2\pi^2}{k^3}\mathscr{P}_\zeta(k)$
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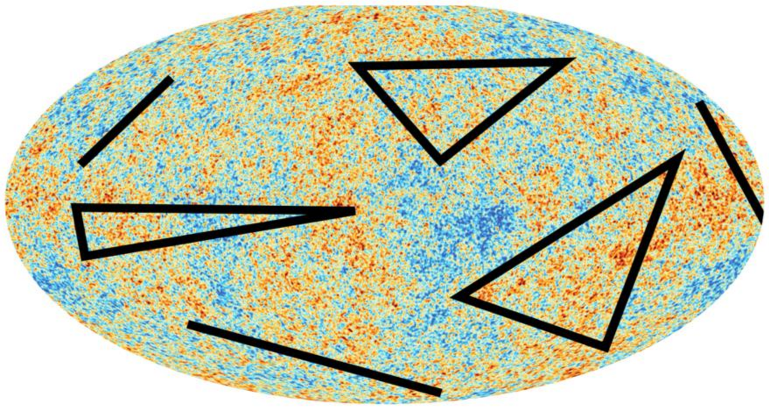
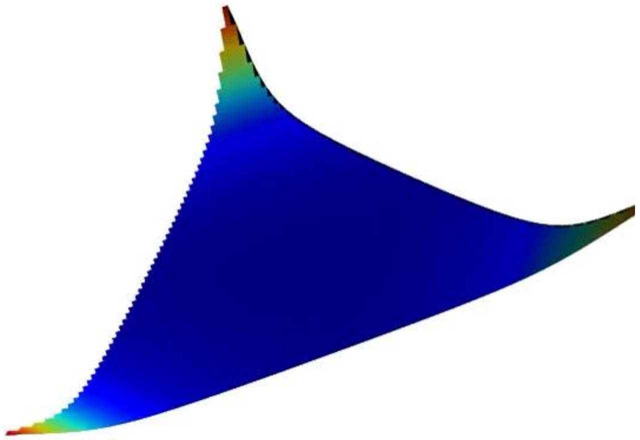
$$\mathcal{P} \propto k^{n_s-1}$$

$$f_{\rm nl} < \text{a few}$$

1807.6211
Planck collaboration





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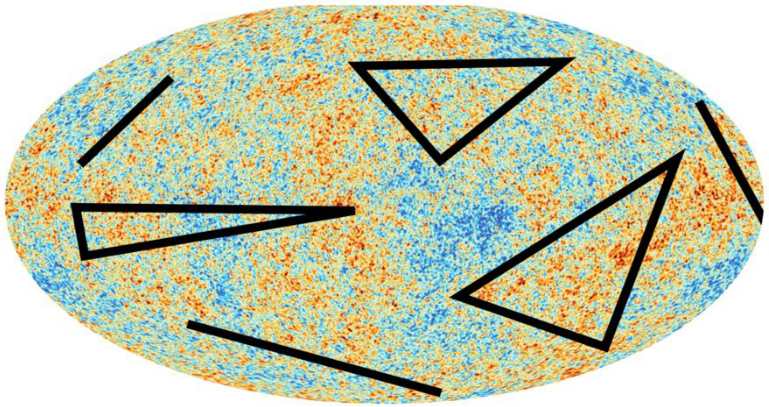
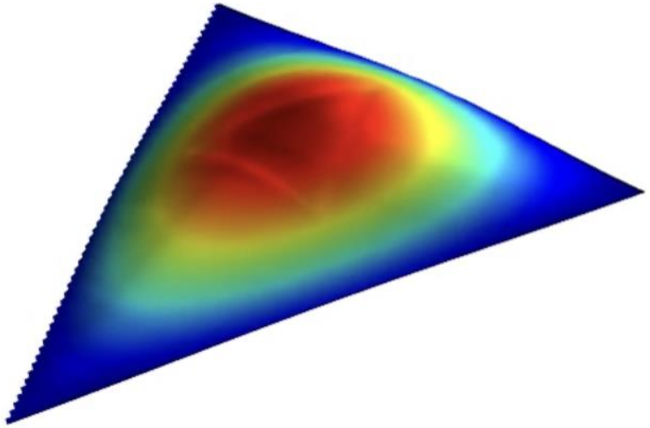
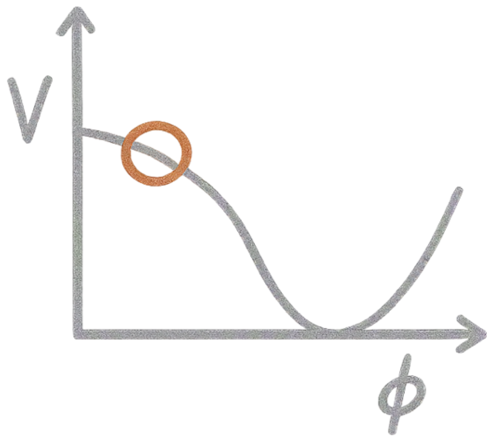




From large to short scales (PBHs, SIGWs)

- Large Scales



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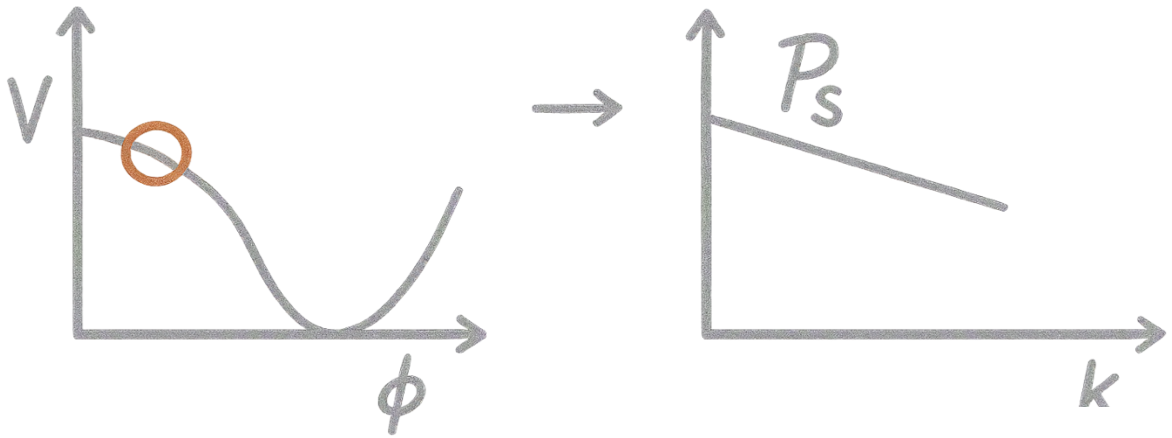

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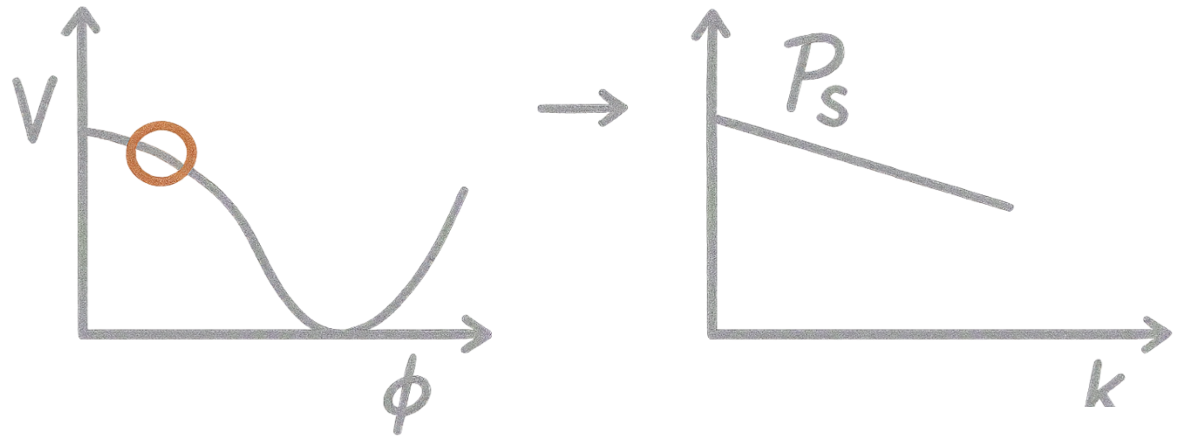
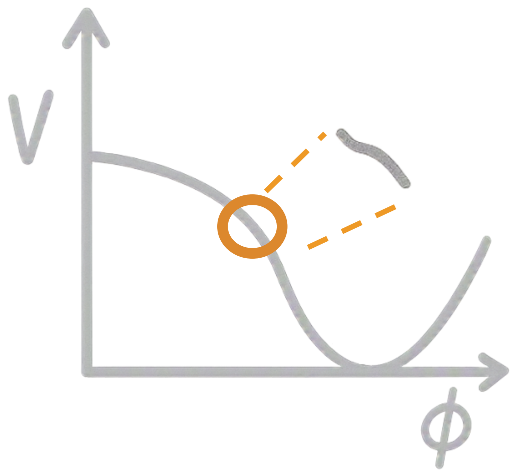


- Short Scales
  - Features, or multiple field dynamics?
  - ➢ PBHs
  - ➢ Induced Gravitational Waves



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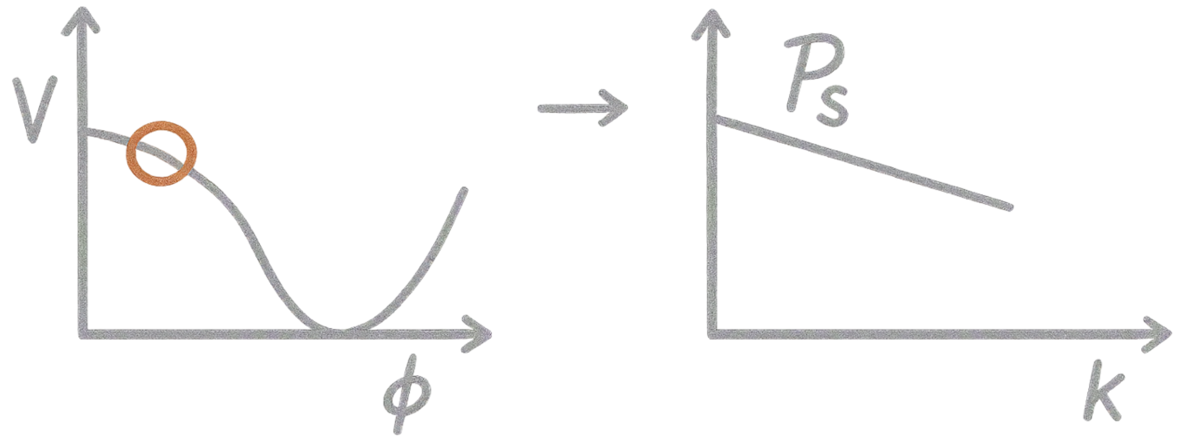

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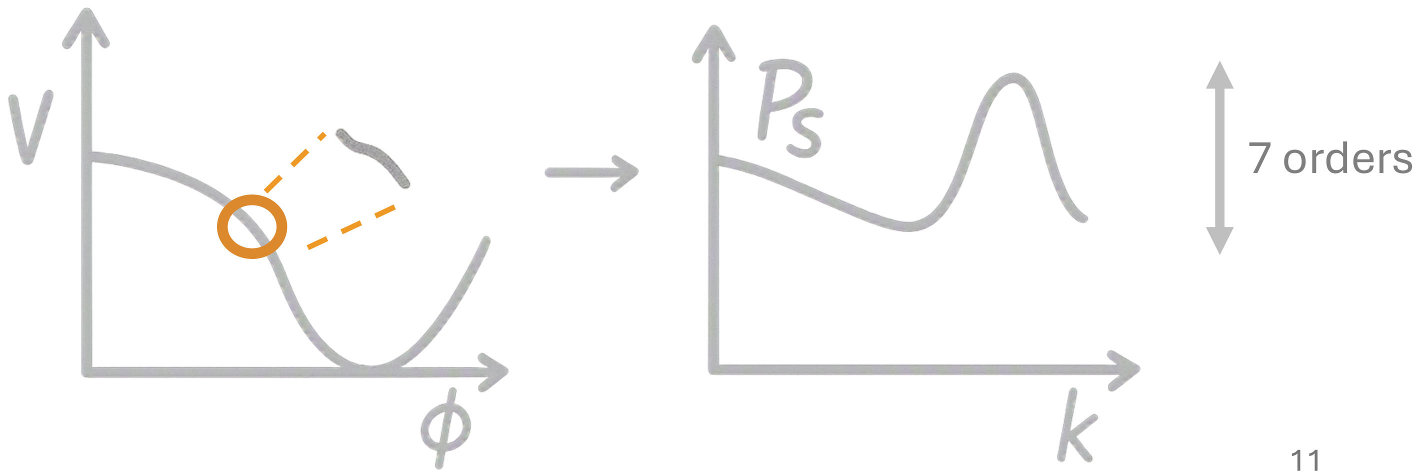

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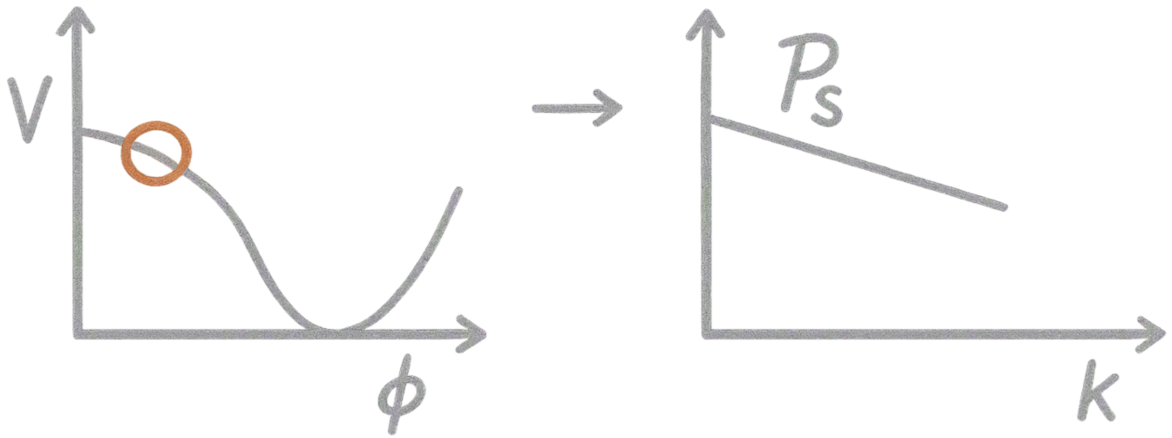

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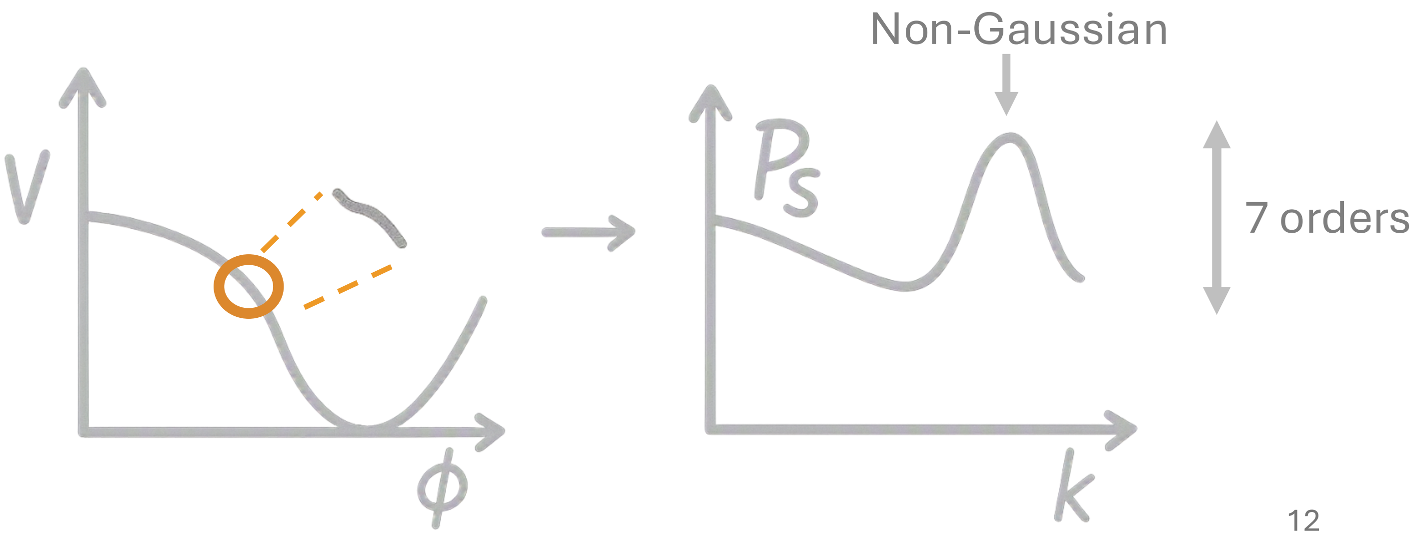

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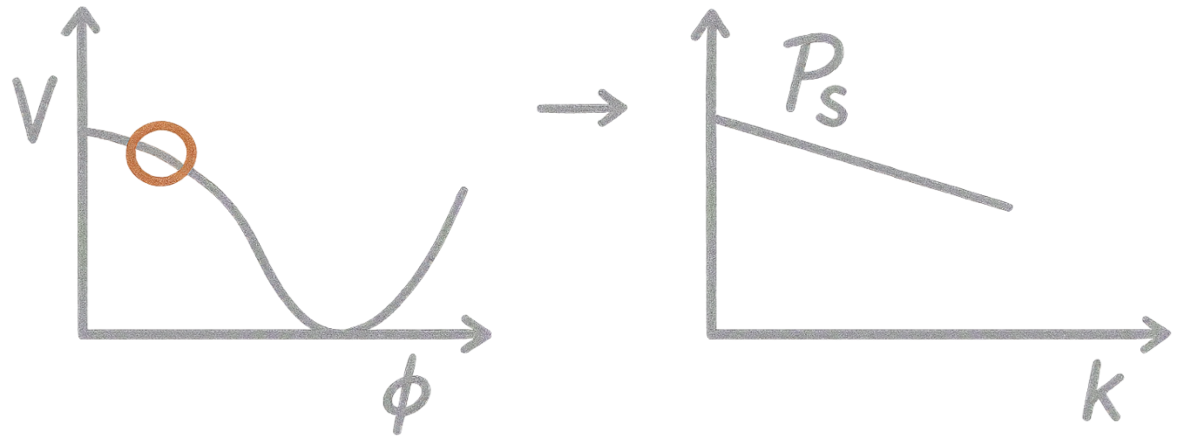

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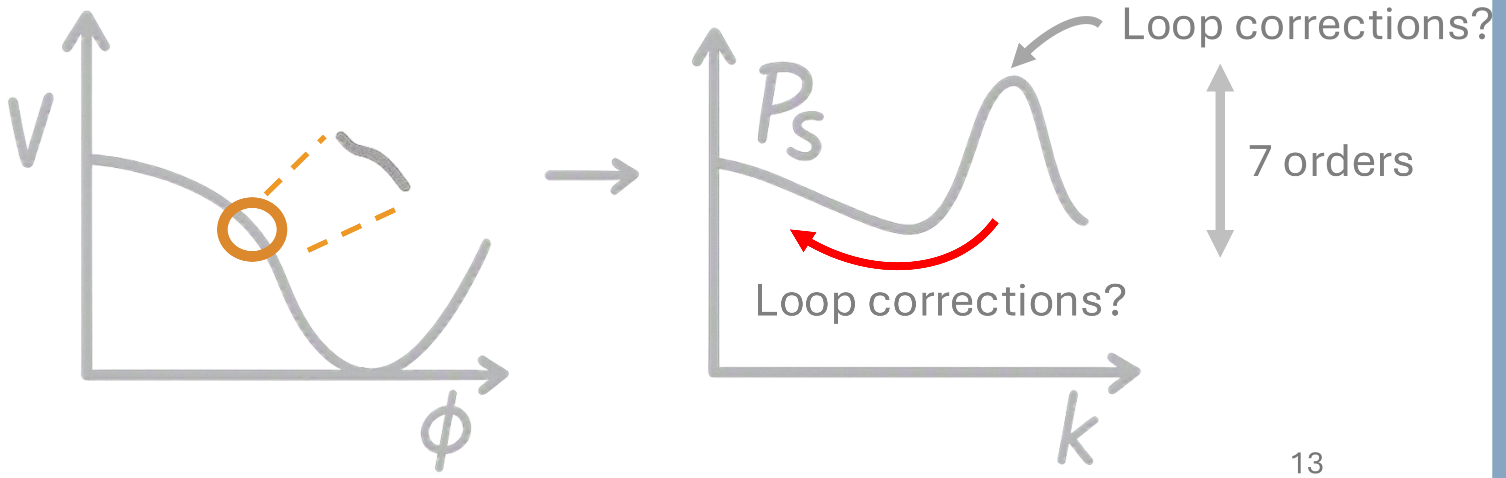
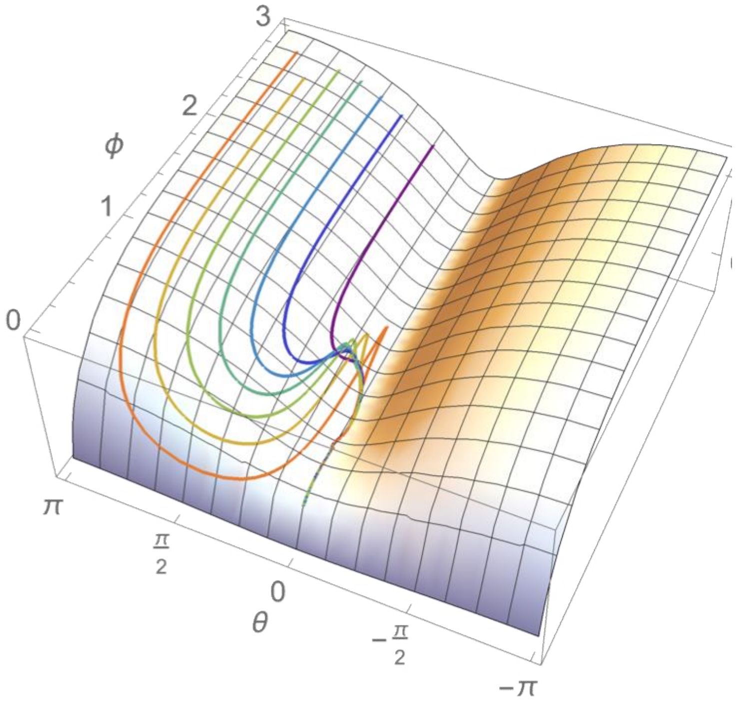
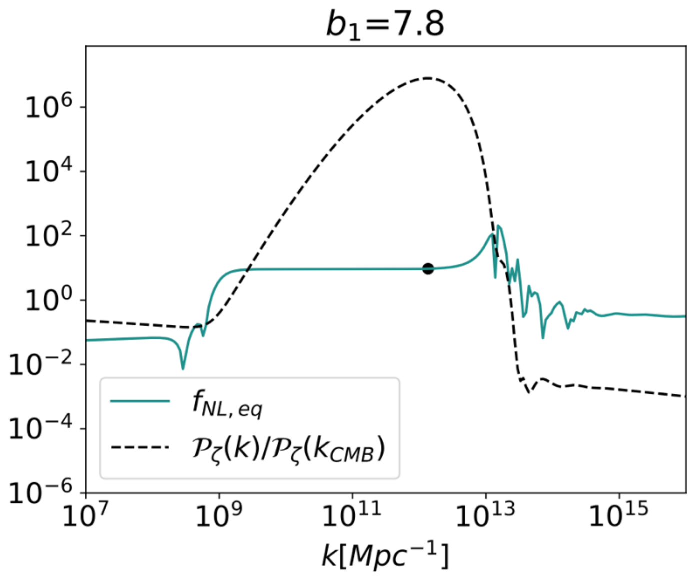
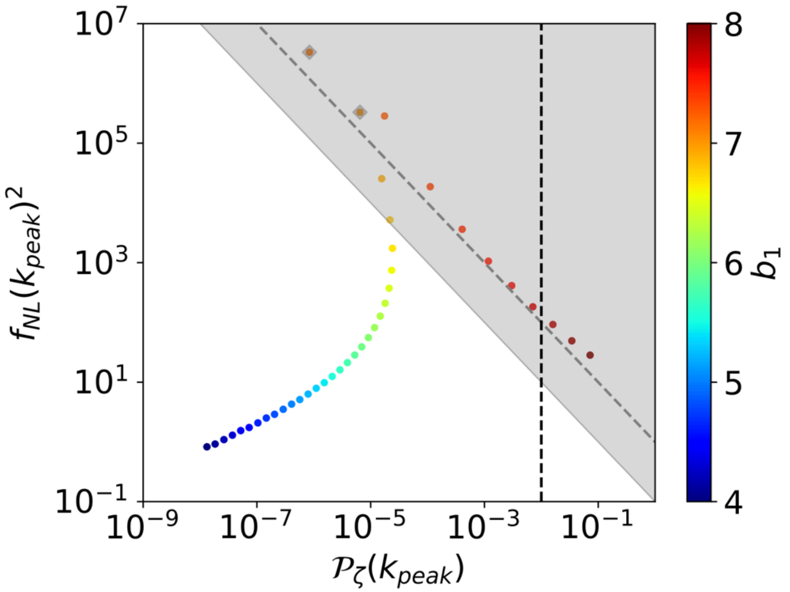


A concrete example



$$\mathcal{S} = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{{M_{Pl}}^2}{2}R - \frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{2}e^{2b_1\phi}\left(\partial\chi\right)^2 - V\left(\phi,\,\chi\right)\right]$$

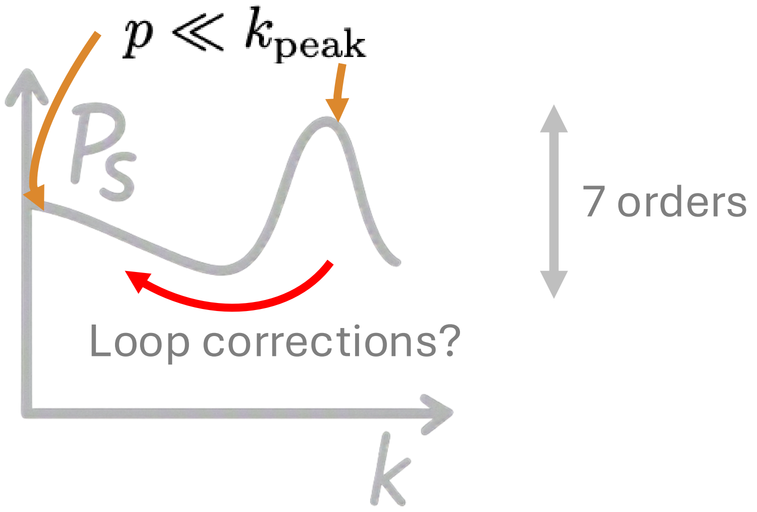




Palma+, Fumagalli+, Braglia+, Iacconi+, …+++

Iacconi, DJM, 2023, c.f. others e.g. Fumagalli et al 2023, Caravano et al. 2024 + , Garcia-Saenz et al. 2025, ++

Loops on separated scales (a recent debate)



- ***Original claim***: backreaction is so strong, that peak can never be high enough for PBHs
  Kristiano & Yokoyama, 2022, then many many others challenging or supporting, e.g. Riotto, Firouzjahi, Kawaguchi, Fumagalli, Tada et al.

$$
\begin{aligned}
P_\zeta(p;\,\tau)_{\text{1-loop}} = \frac{1}{4}\int_{-\infty}^{\tau} \mathrm{d}\tau_1\, \epsilon(\tau_1)\eta'(\tau_1)a^2(\tau_1) \int_{-\infty}^{\tau} \mathrm{d}\tau_2\, \epsilon(\tau_2)\eta'(\tau_2)a^2(\tau_2) \int \frac{\mathrm{d}^3k}{(2\pi)^3} \times \\
\zeta_p^*(\tau)\zeta_p(\tau)\Big[4\zeta_p'(\tau_1)\zeta_k(\tau_1)\zeta_q(\tau_1)\zeta_p'^*(\tau_2)\zeta_k^*(\tau_2)\zeta_q^*(\tau_2) + 8\zeta_p(\tau_1)\zeta_k'(\tau_1)\zeta_q(\tau_1)\zeta_p^*(\tau_2)\zeta_k^*(\tau_2)\zeta_q'^*(\tau_2) \\
+ 8\zeta_p(\tau_1)\zeta_k'(\tau_1)\zeta_q(\tau_1)\zeta_p^*(\tau_2)\zeta_k'^*(\tau_2)\zeta_q^*(\tau_2) + 8\zeta_p'(\tau_1)\zeta_k(\tau_1)\zeta_q(\tau_1)\zeta_p^*(\tau_2)\zeta_k^*(\tau_2)\zeta_q'^*(\tau_2) \\
+ 8\zeta_p(\tau_1)\zeta_k'(\tau_1)\zeta_q(\tau_1)\zeta_p'^*(\tau_2)\zeta_k^*(\tau_2)\zeta_q^*(\tau_2)\Big] \\
- \frac{1}{2}\int_{-\infty}^{\tau} \mathrm{d}\tau_1\, \epsilon(\tau_1)\eta'(\tau_1)a^2(\tau_1) \int_{-\infty}^{\tau_1} \mathrm{d}\tau_2\, \epsilon(\tau_2)\eta'(\tau_2)a^2(\tau_2) \int \frac{\mathrm{d}^3k}{(2\pi)^3} \times \\
\zeta_p(\tau)^2\, \Re\Big[4\zeta_p'^*(\tau_1)\zeta_k(\tau_1)\zeta_q(\tau_1)\zeta_p'^*(\tau_2)\zeta_k^*(\tau_2)\zeta_q^*(\tau_2) + 8\zeta_p^*(\tau_1)\zeta_k(\tau_1)\zeta_q'(\tau_1)\zeta_p^*(\tau_2)\zeta_k^*(\tau_2)\zeta_q'^*(\tau_2) \\
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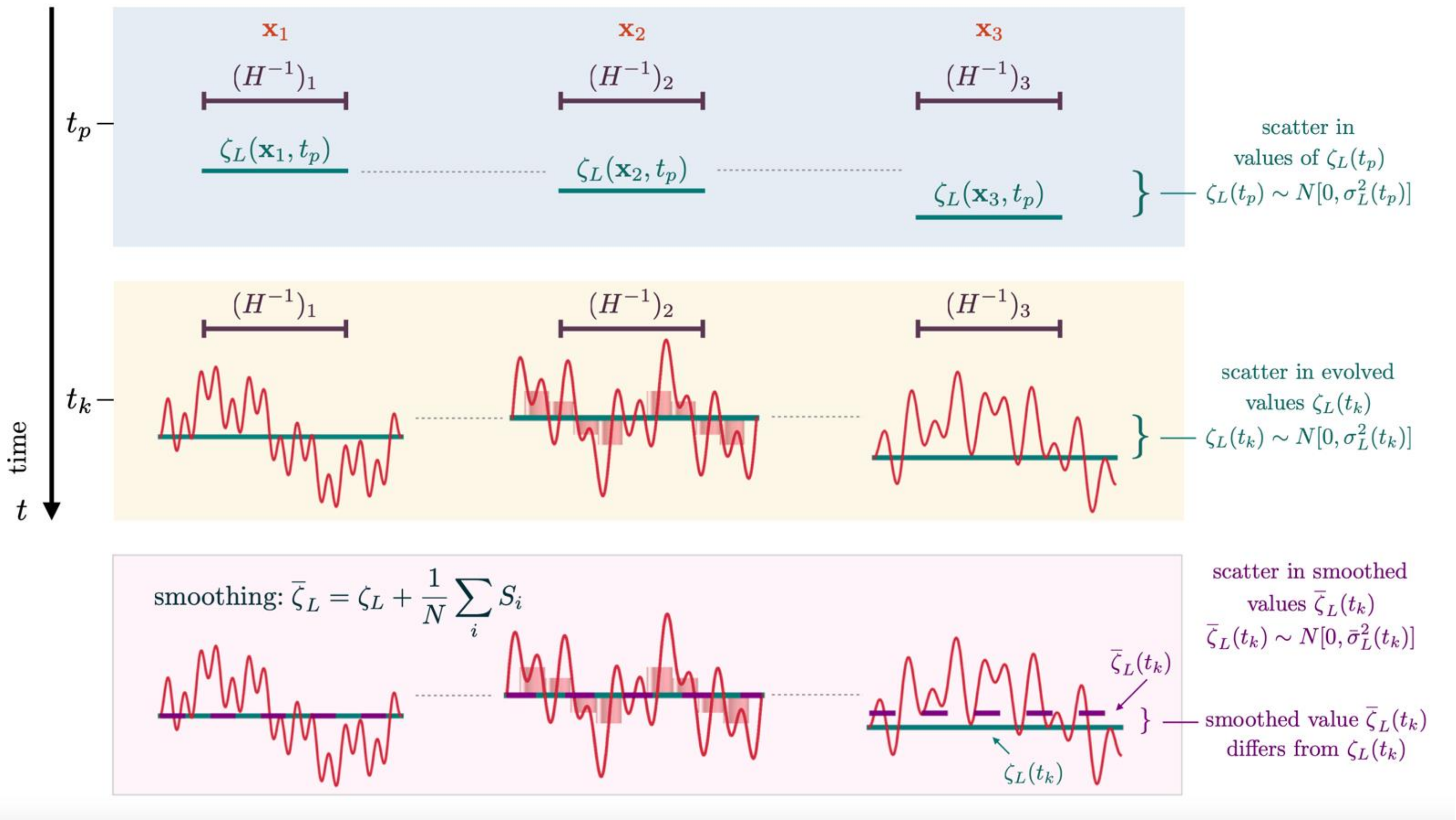
\end{aligned}
$$

Alternative approach

- Is there a more intuitive approach? (Iacconi, DJM, Seery, 2023)

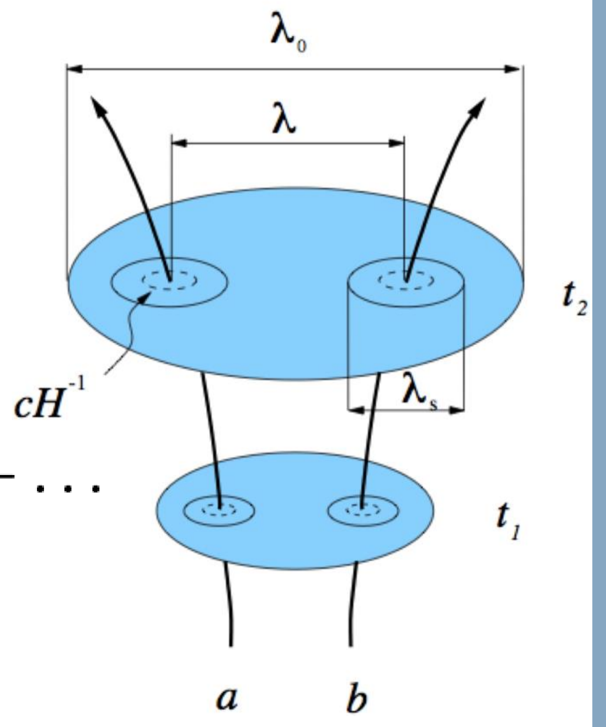


Taylor expansion methods: δN and MPPs (1)

- In δN cosmological perturbation theory (on super-horizon scales) becomes a Taylor expansion in the number of e-folds (equivalent to the curvature) in terms of field fluctuations, schematically:

$$\zeta(\mathbf{k}) = \delta N = N_I\,\delta\phi^I + \frac{1}{2}N_{IJ}\,\delta\phi^I \star \delta\phi^J + \frac{1}{6}N_{IJK}\,\delta\phi^I \star \delta\phi^J \star \delta\phi^k + \dots$$

$$N_I = \frac{\partial N}{\partial \phi_I}\,, \quad \text{etc}$$



Liddle, Lyth, Malik Wands, Sasaki, +++, Bynes and Wands, Seery and Lidsey

- Moreover, soft limits are a Taylor expansion, schematically:

$$\delta\phi(\text{short}) = \delta\phi(\text{short})_0 + \frac{\partial\delta\phi(\text{short})_0}{\partial\phi_{\text{back}}}\delta\phi(\text{long})$$

Taylor expansion methods: δN and MPPs (2)

- It is also possible to make a δN like Taylor expansion in field space, and even for it to be valid at the quantum (operator) level

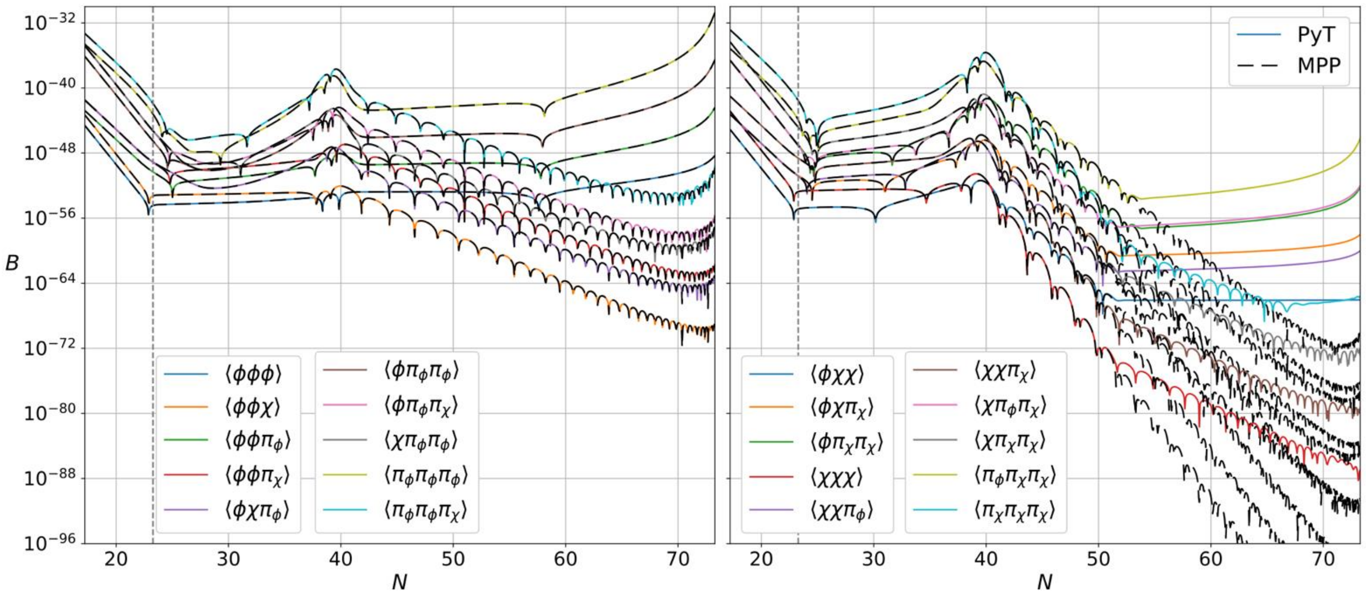
- This is known as a Multi-point propagator (MPP) schematically:
  (Constantini, Iacconi, DJM, 2025; c.f. Seery, DJM, Frazer Ribeiro, 2010; Bernardeau et al. 2008)

$$\delta\phi^I(t_2,\mathbf{k}) = \Gamma^I_J\delta\phi^I(t_1,\mathbf{k}) + \frac{1}{2}\Gamma^I_{JK}\delta\phi^J(t_1)\star\delta\phi^K(t_1) + \frac{1}{3!}\Gamma^I_{JKL}\delta\phi^J(t_1)\star\delta\phi^K(t_1)\star\delta\phi^L(t_1)$$

$$\frac{\partial\delta\phi^I(t_2,\mathbf{k})}{\partial\delta\phi^J(t_1,\mathbf{k}')} = \delta(\mathbf{k}-\mathbf{k}')\Gamma^I_J(k)\,, \quad \text{etc}$$

MPPs work: An MPP code

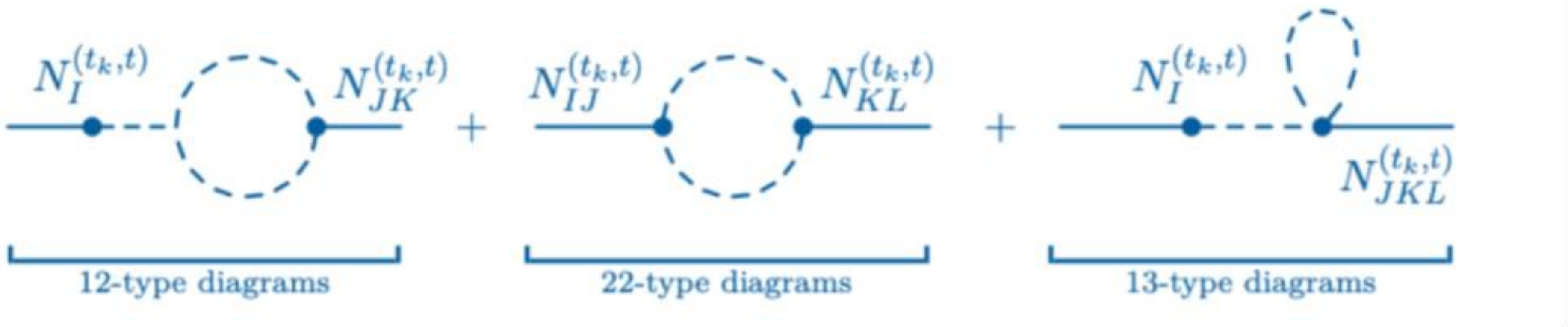
- The MPP coefficients satisfy ordinary differential equations
- We can evolve them numerically, and, use them to calculate the statistics of field perturbation (or of zeta) starting deep inside the horizon, with some advantages! (Constantini, Iaconni, DJM, 2025)



δN loops (1) Iacconi, DJM, Seery, 2023, 2026

- Mathematically we can study our intuitive picture using using δN or related methods by forming loops (for earlier loop work see e.g. Byrnes et al. 2007)

$$\langle\zeta_{\mathbf{p}}(t)\zeta_{-\mathbf{p}}(t)\rangle'_{\text{1-loop}} = \underbrace{\langle\zeta_{\mathbf{p}}\zeta_{-\mathbf{p}}\rangle'_{11}}_{\substack{\text{1-loop in}\\ \text{initial conditions}}} + \underbrace{\langle\zeta_{\mathbf{p}}\zeta_{-\mathbf{p}}\rangle'_{12} + \langle\zeta_{\mathbf{p}}\zeta_{-\mathbf{p}}\rangle'_{22} + \langle\zeta_{\mathbf{p}}\zeta_{-\mathbf{p}}\rangle'_{13}}_{\text{1-loop due to non-linearity of } \delta N}$$



$$P_\zeta(p;t)_{12} = N_I^{(t_i,t)} N_{JK}^{(t_i,t)} \int \mathrm{d}^3q\ \alpha^{IJK}(p,q,q;t_i)$$

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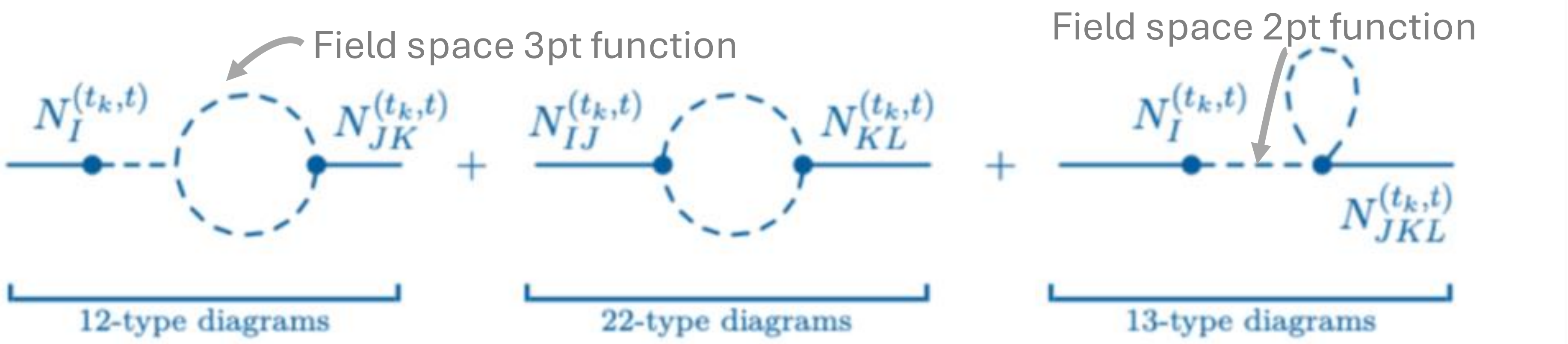

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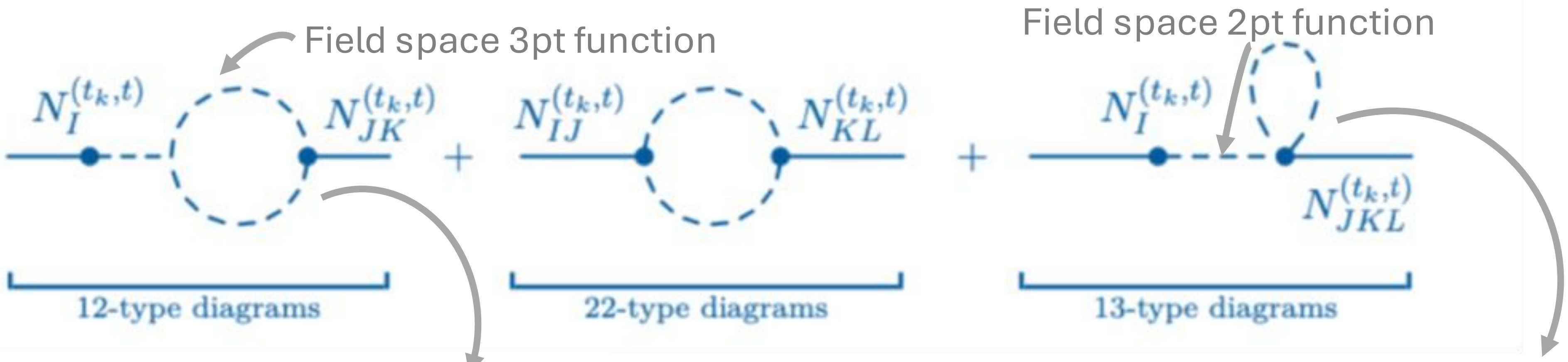

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δN loops (2)

- We can use soft limit expression: (c.f. single field consistency, Maldacena, 2002)

$$\alpha^{IJK}(p,q,q;t_i) \supseteq P^{IL}(p;t_i)\frac{\partial P^{JK}(q;t_i)}{\partial X^L(t_i)}$$

X represents field and field momentum

- Then $$P_\zeta(p;t)_{12+13} = N_I^{(t_i,t)}P^{IL}(p;t_i)\int \mathrm{d}^3q\left[N_{JK}^{(t_i,t)}\frac{\partial P^{JK}(q;t_i)}{\partial X^L(t_i)} + N_{LJK}^{(t_i,t)}P^{JK}(q;t_i)\right]$$

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δN loops (2)

- We can use soft limit expression: (c.f. single field consistency, Maldacena, 2002)

$$\alpha^{IJK}(p,q,q;t_i) \supseteq P^{IL}(p;t_i)\frac{\partial P^{JK}(q;t_i)}{\partial X^L(t_i)}$$

X represents field and field momentum

- Then $P_\zeta(p;t)_{12+13} = N_I^{(t_i,t)} P^{IL}(p;t_i) \int \mathrm{d}^3 q \left[N_{JK}^{(t_i,t)} \frac{\partial P^{JK}(q;t_i)}{\partial X^L(t_i)} + N_{LJK}^{(t_i,t)} P^{JK}(q;t_i) \right]$

$$P_\zeta(p;t)_{12+13} = N_I^{(t_i,t)} \int \mathrm{d}^3 q \, P^{IL}(p;t_i) \frac{\partial}{\partial X^L(t_i)} \left[N_{JK}^{(t_i,t)} P^{JK}(q;t_i) \right]$$

For single field, p mode adiabatic

$$\Delta \mathcal{P}_\zeta(p;t)_{12+13} = \int_{q_{\min}}^{q_{\max}} \mathrm{d}\ln q \, \frac{\mathrm{d}}{\mathrm{d}\ln q} \left[N_{JK}^{(t_i,t)} \mathcal{P}^{JK}(q;t_i)_{\text{tree}} \right]$$

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For single field, p mode adiabatic

$$\Delta\mathcal{P}_\zeta(p;t)_{12+13} = \int_{q_{\min}}^{q_{\max}} \mathrm{d}\ln q\,\frac{\mathrm{d}}{\mathrm{d}\ln q}\left[N_{JK}^{(t_i,t)}\mathcal{P}^{JK}(q;t_i)_{\mathrm{tree}}\right]$$

- Also MPP loop: $$\mathcal{P}_\zeta(p;t)_{11} \propto \int \mathrm{d}\ln q\,\frac{\mathrm{d}}{\mathrm{d}\ln q}\left[\Gamma^J{}_{NO}(p,q,q)^{(t^*,t_i)}\mathcal{P}^{NO}(q;t^*)\right] + (I\leftrightarrow J)$$

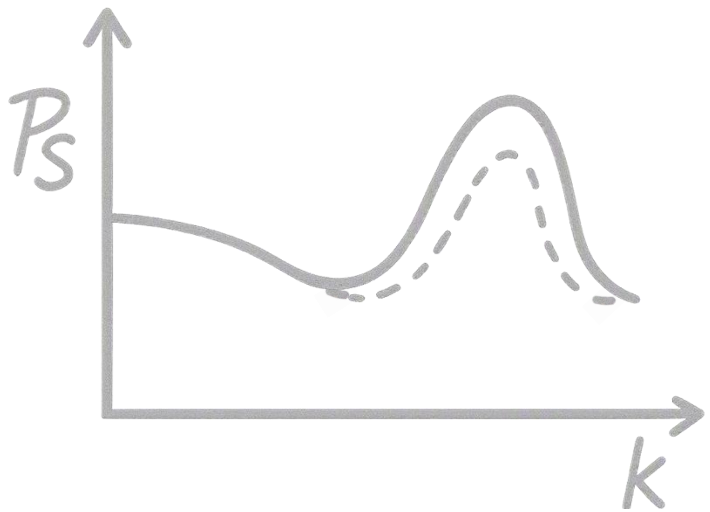
Decoupling of scales

- We have $$\Delta\mathcal{P}_\zeta(p;t)_{12+13} = \int_{q_{\min}}^{q_{\max}} \mathrm{d}\ln q \, \frac{\mathrm{d}}{\mathrm{d}\ln q}\left[N^{(t_i,t)}_{JK}\mathcal{P}^{JK}(q;t_i)_{\text{tree}}\right]$$

$$\Downarrow$$

$$\Delta\mathcal{P}_\zeta(p;t)_{12+13} = N^{(t_i,t)}_{JK}\mathcal{P}^{JK}(q;t_i)_{\text{tree}}\Big|_{q=q_{\max}} - N^{(t_i,t)}_{JK}\mathcal{P}^{JK}(q;t_i)_{\text{tree}}\Big|_{q=q_{\min}}$$

- Hence, only boundary terms, and not the peak structure, enters the answer (similar conclusions found by other authors, Tada et al 2024, Fumagalli 2025, Kawaguchi et al. 2024)



Conclusions

- Taylor expansions, such as MPP methods, can be numerically advantageous, as well analytically simpler for some problems or limits
- We have, for example, a new version of our numerical multi-field inflation code that uses them
- Loop order contributions to correlators can be built using δN or MPPs – equivalent to In-In loops
- This provides insight into the structure of loop terms in the limit of separated scales
- Could we also calculate loops using numerical MPP codes?