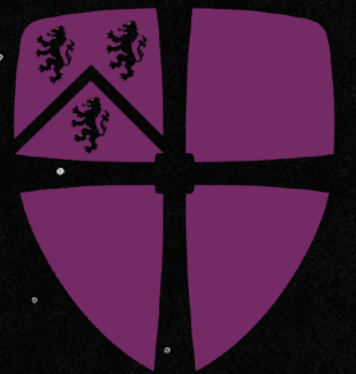


Probing small-scale cosmology with primordial relics

Djuna Lize Croon (IPPP Durham)

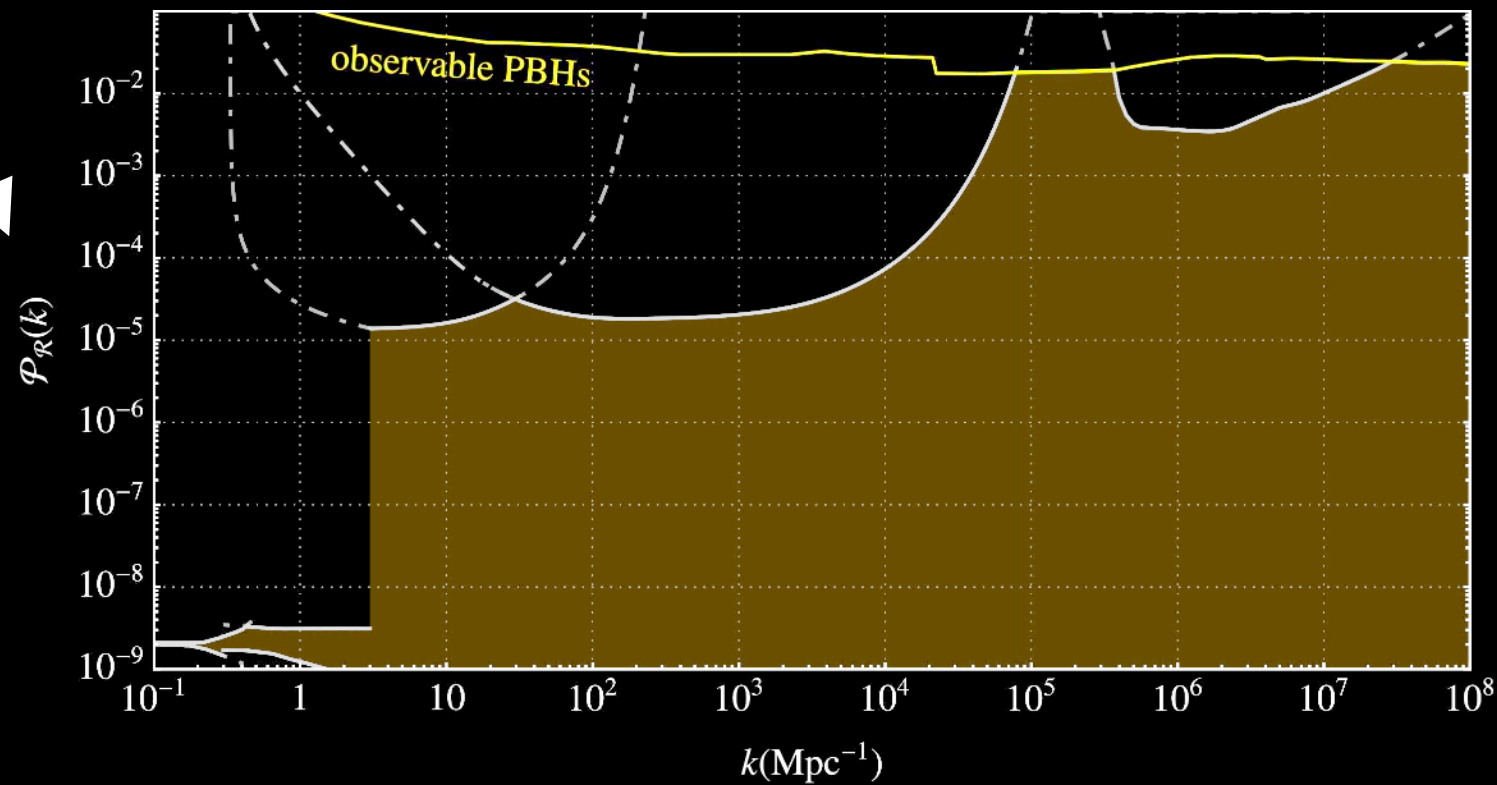
PASCOS, June 2026

djuna.l.croon@durham.ac.uk | djunacroon.com

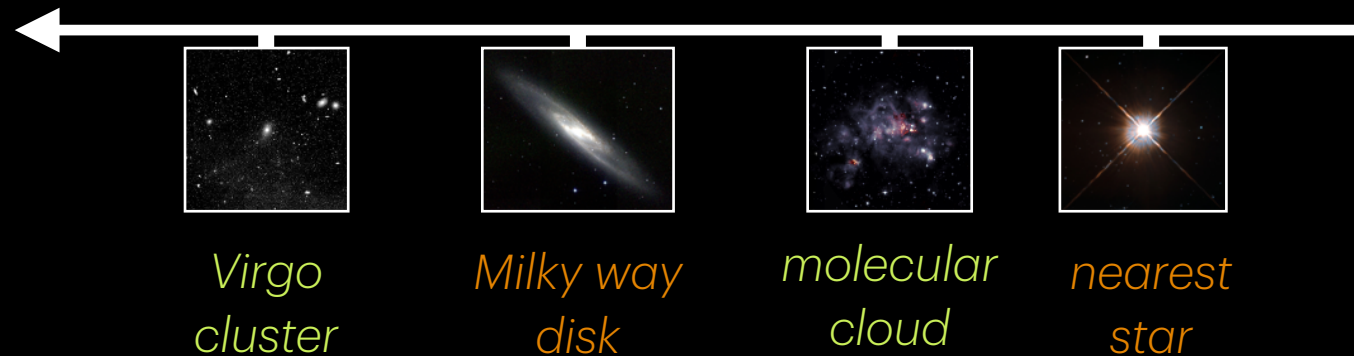


Cosmology's small scale frontier

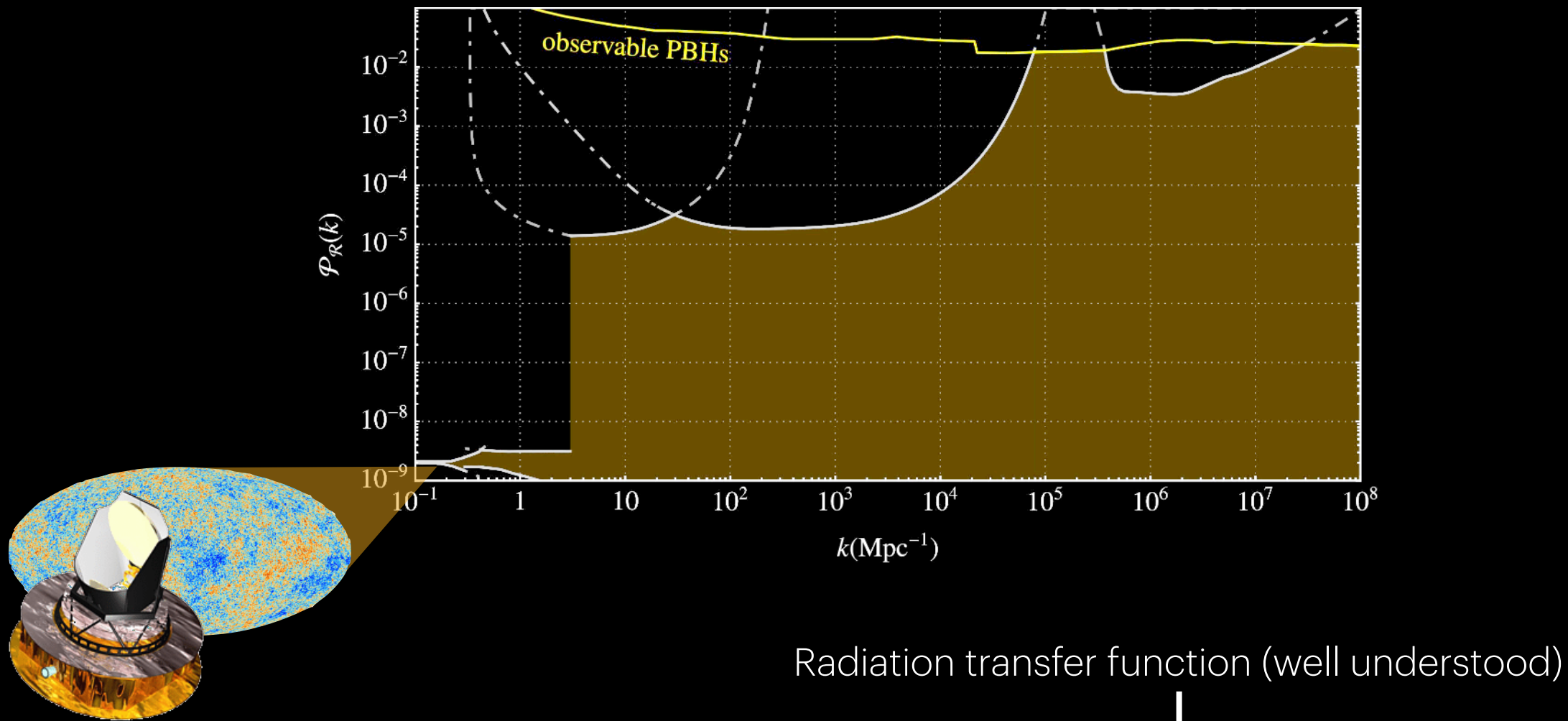
Dimensionless primordial power spectrum of curvature perturbations



larger scales



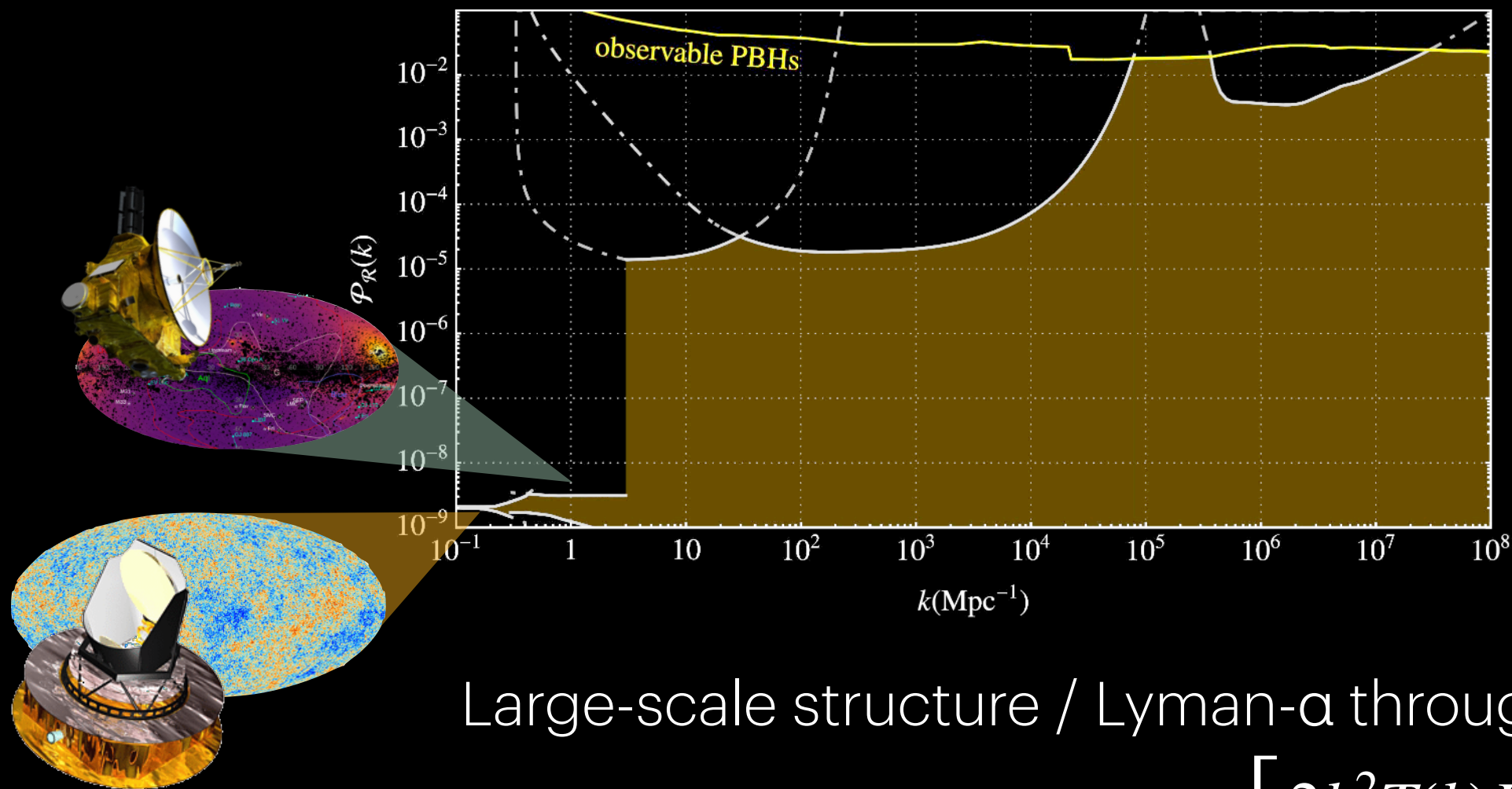
Cosmology's small scale frontier



CMB anisotropies: $C_{\ell}^{TT} = 4\pi \int d \ln k \mathcal{P}_{\mathcal{R}}(k) \Delta_{\ell}^T(k)^2$

Silk (diffusion) damping suppresses anisotropies for $k \gtrsim 0.2 \text{ Mpc}^{-1}$

Cosmology's small scale frontier



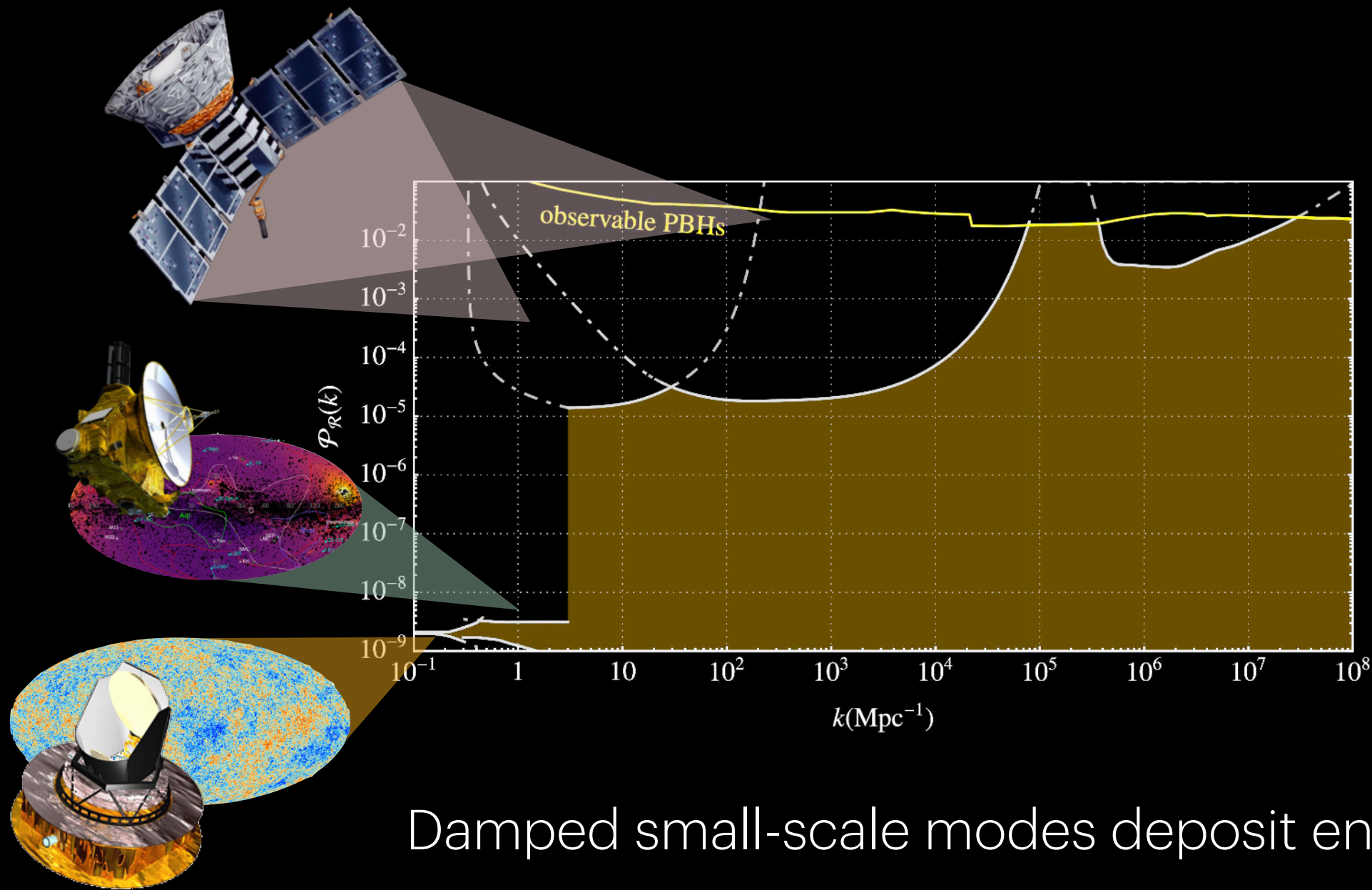
Large-scale structure / Lyman- α through the **matter**

power spectrum $\mathcal{P}_m(k, z) = \left[\frac{2k^2 T(k) D(z)}{5\Omega_m H_0^2} \right]^2 \mathcal{P}_R(k),$

(astrophysical) modelling systematics for $k \gtrsim 3 \text{ Mpc}^{-1}$

see also Olga Garcia Gallego's talk earlier today

Cosmology's small scale frontier



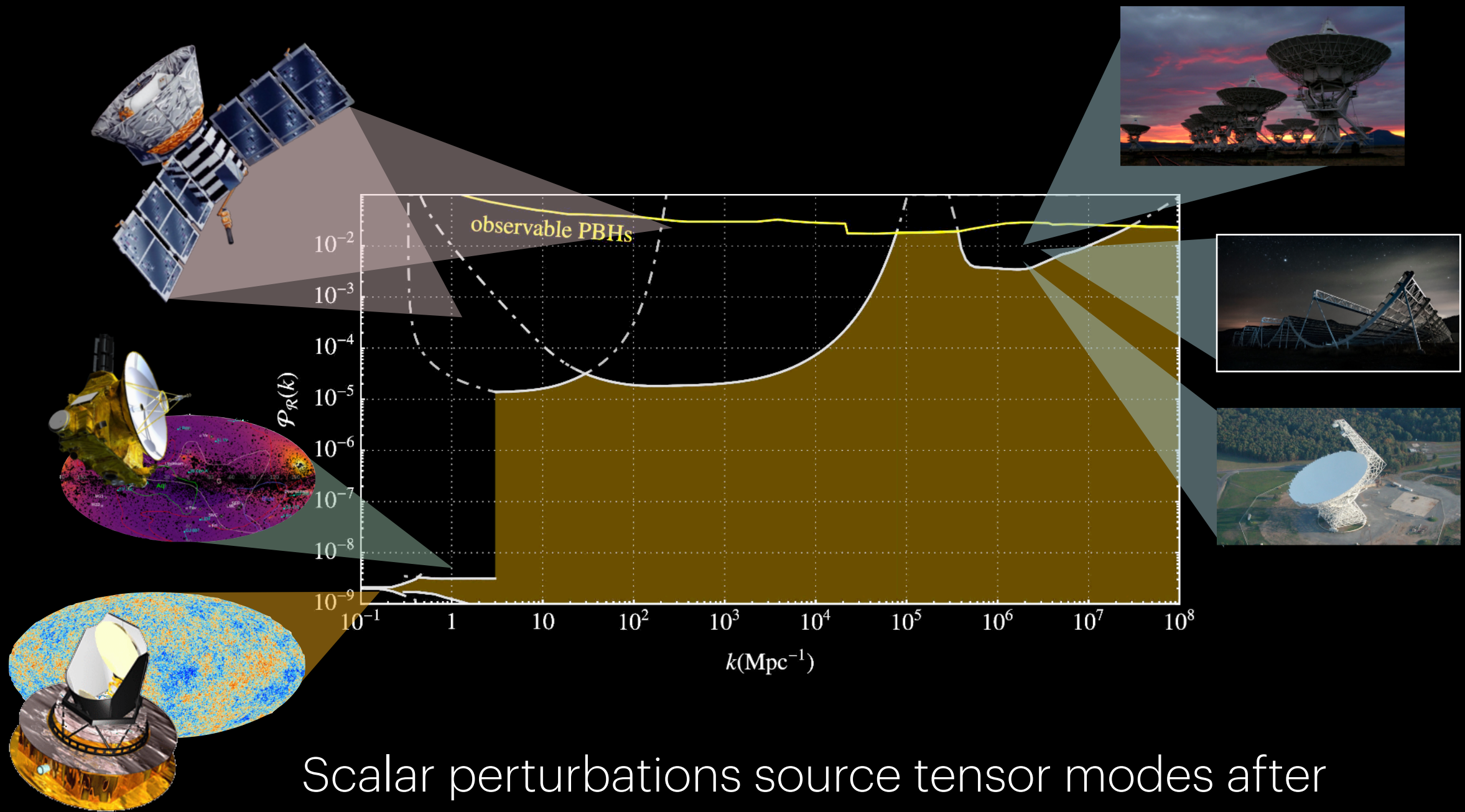
Damped small-scale modes deposit energy

→ spectral distortions to the CMB blackbody spectrum

$$\text{e.g. } \mu \simeq \int d \ln k \, W_{\mu}(k) \mathcal{P}_{\mathcal{R}}(k)$$

↑
Window function

Cosmology's small scale frontier

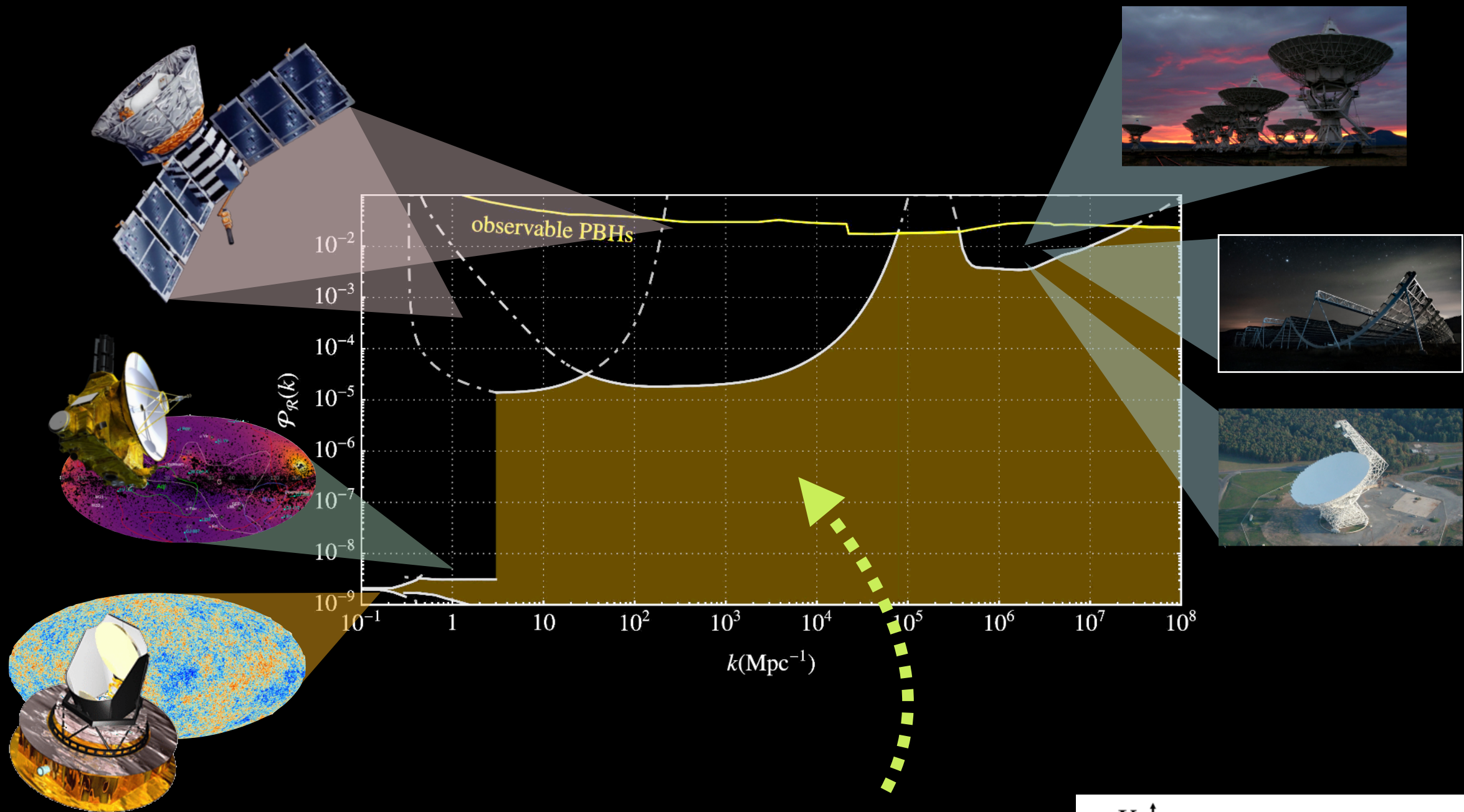


Scalar perturbations source tensor modes after horizon re-entry (**scalar induced gravitational waves**)

$$\Omega_{\text{GW}}(f) \sim \int d \ln k_1 d \ln k_2 \underset{\uparrow}{W}(f; k_1, k_2) \mathcal{P}_{\mathcal{R}}(k_1) \mathcal{P}_{\mathcal{R}}(k_2) .$$

Kernel containing mode coupling, transfer functions, redshift, time integral...

Cosmology's small scale frontier



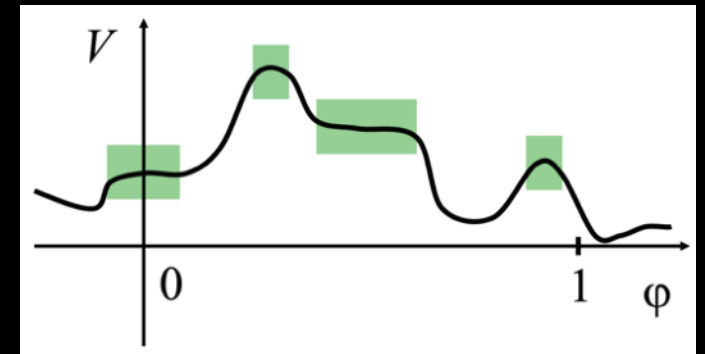
Small-scale structure?

Inflationary running/ultra-slow-roll

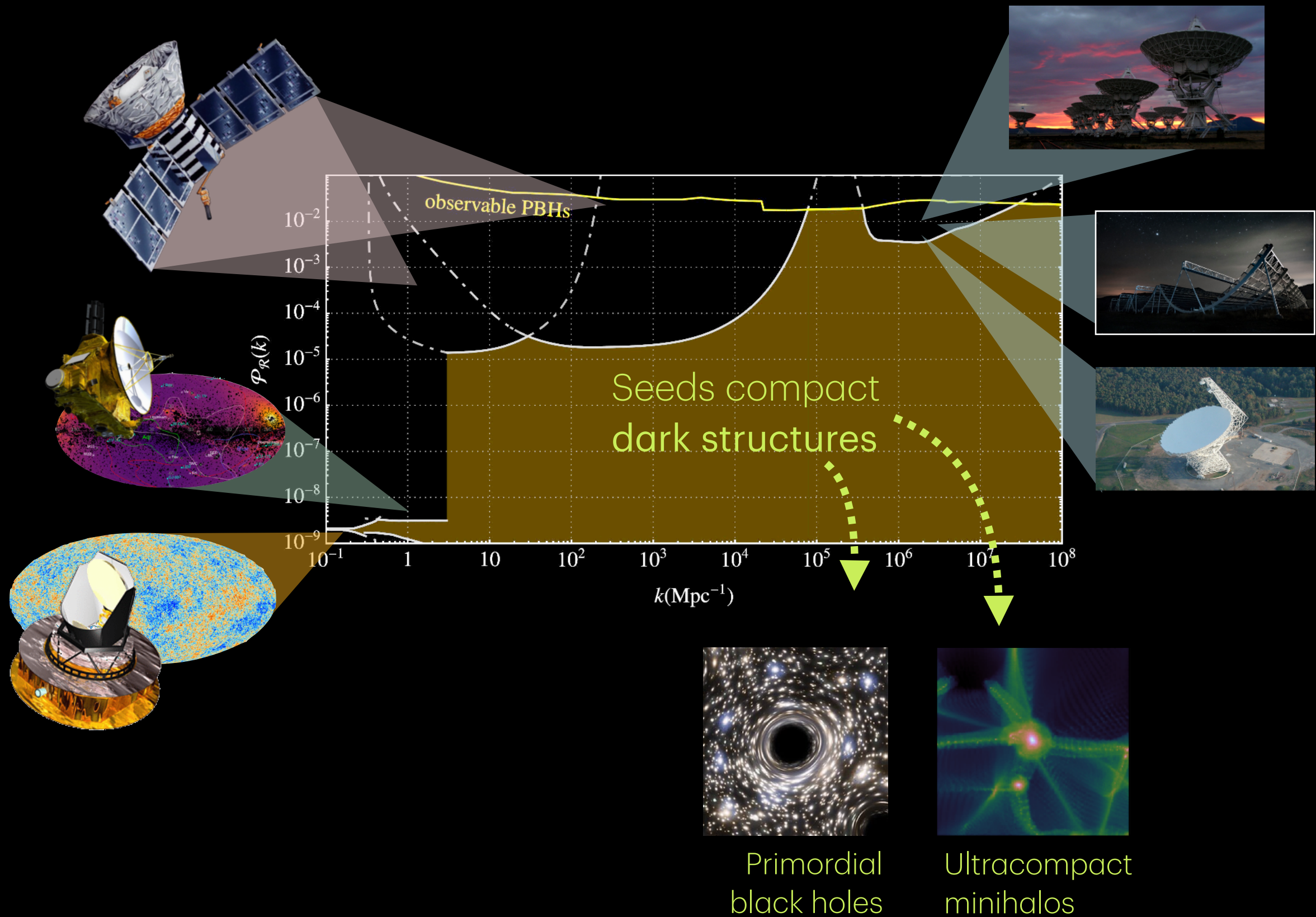
Curvaton/spectator-field models

First order phase transitions

...



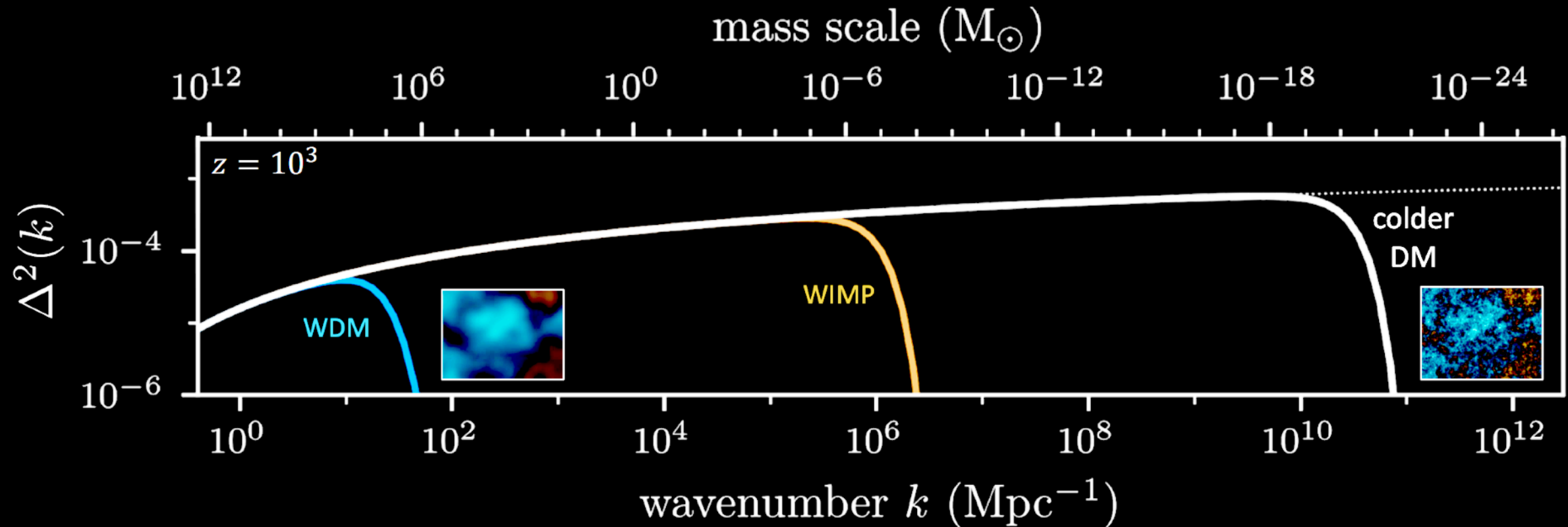
Cosmology's small scale frontier



Dark matter substructure

Small scales are a discriminator of DM physics

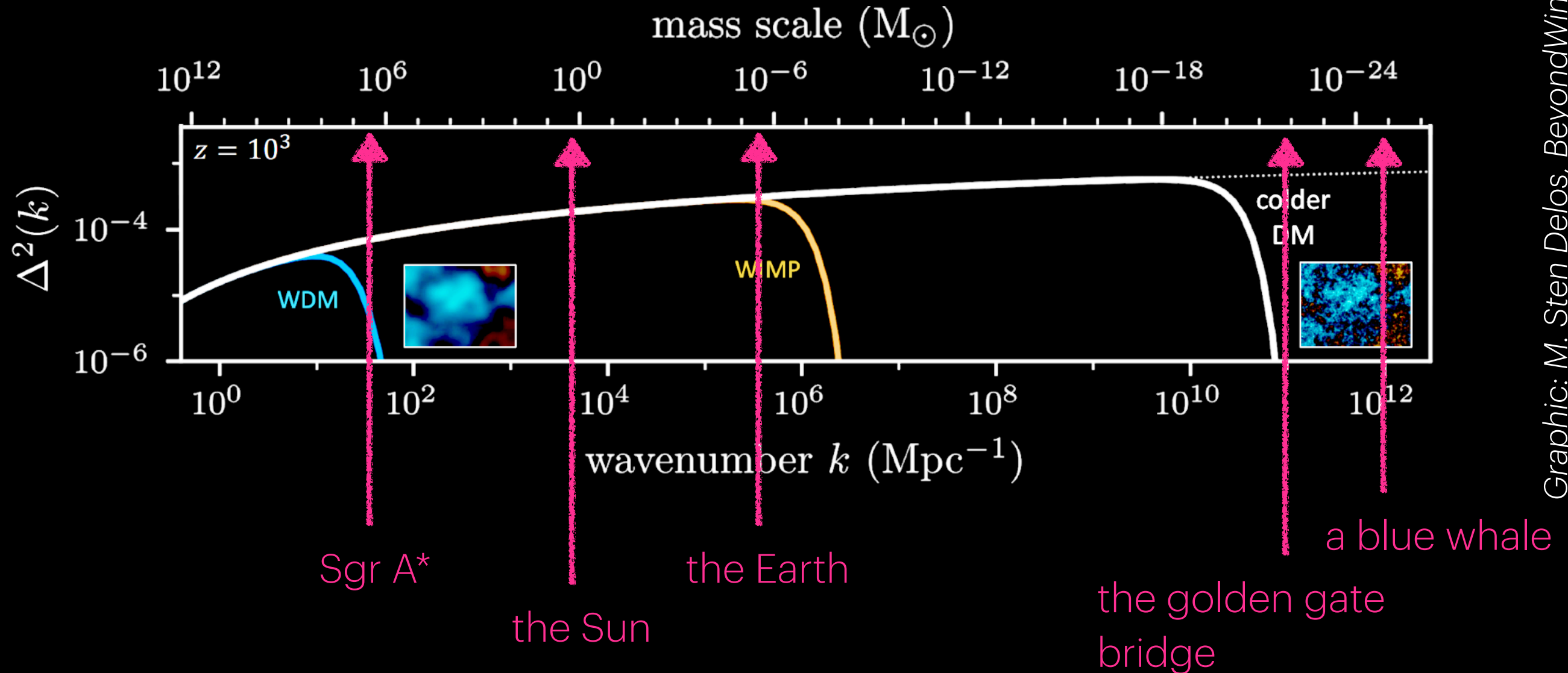
- Λ CDM: hierarchical structure $\rightarrow$ many small subhalos



Dark matter substructure

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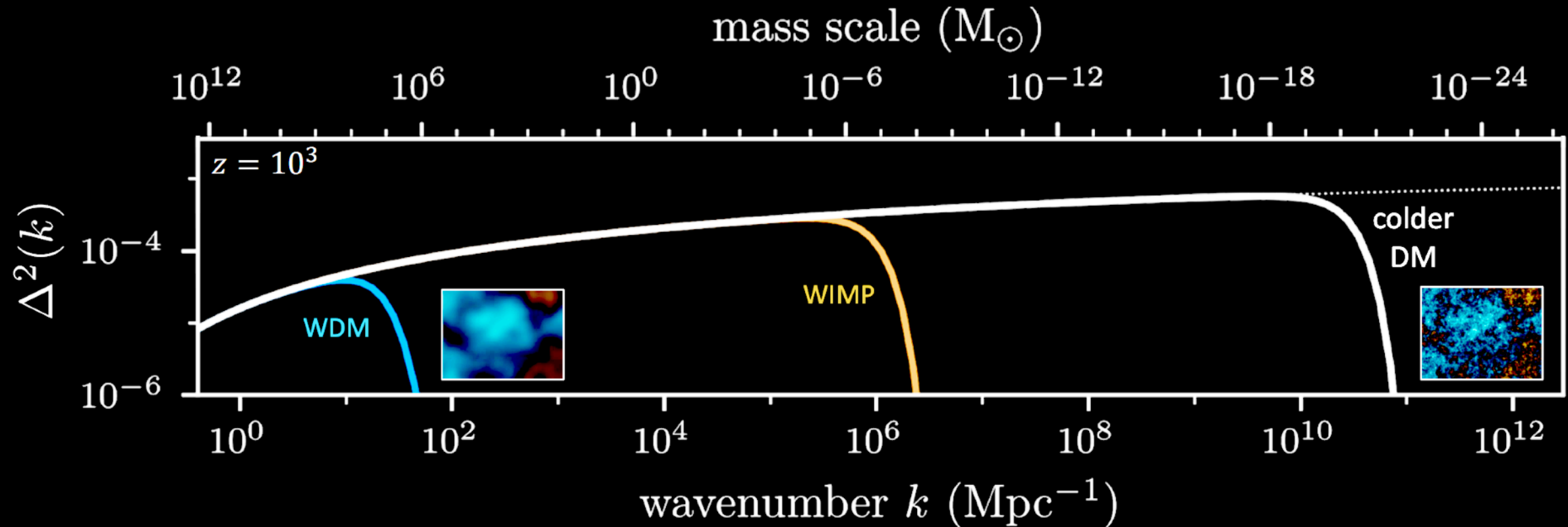
- Λ CDM: hierarchical structure $\rightarrow$ many small subhalos



Dark matter substructure

Small scales are a discriminator of DM physics

- Λ CDM: hierarchical structure $\rightarrow$ many small subhalos



- Small-scale enhancement of the primordial power spectrum: high z formation of ultracompact minihalos

Compact object formation



Collapse of overdensities

- Primordial small-scale overdensities collapse:

Large overdensity ($\delta = (\rho - \bar{\rho})/\bar{\rho} \gtrsim 0.5$):

1. *Enters* the horizon during radiation domination
2. *Collapses* to a *PBH* soon after

Collapse of overdensities

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Moderate overdensity ($\delta \gtrsim 10^{-3}$):

3. *Enters* the horizon during radiation domination
4. *Grows weakly* until matter-radiation equality
5. *Grows faster* after equality
6. *Collapses* shortly after equality to form a *UCMH*

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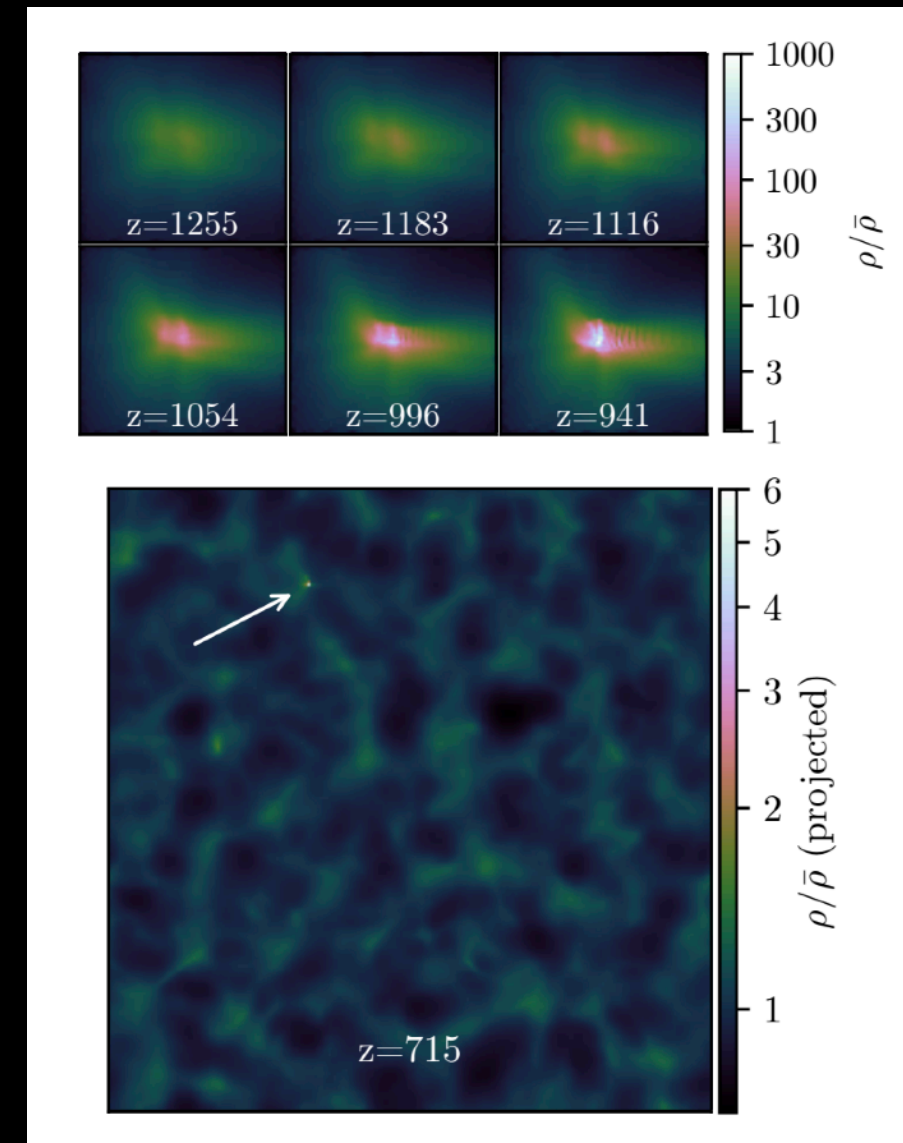
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- Simulations: for individual collapse and internal structure, resolution limited for abundances



Collapse of overdensities: UCMH

- How many overdensities collapse of each mass?
 - Assume a Gaussian spectrum
 - Find perturbations with $\delta_c \sim 1.686$ by z_c



Linear extrapolation of
density contrast for
spherical collapse

Collapse of overdensities: UCMH

- How many overdensities collapse of each mass?
 - Assume a Gaussian spectrum
 - Find perturbations with $\delta_c \sim 1.686$ by z_c

- $\mathcal{P}_{\mathcal{R}}(k) \longrightarrow \sigma^2(R, z)$



$$\sigma_0^2(R, z) = \frac{16}{81} \int d \ln k (kR)^4 W^2(kR) T^2(k, z) \mathcal{P}_{\mathcal{R}}(k)$$

Variance, with window function W and transfer function T describing sub-horizon evolution

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$$\bullet \mathcal{P}_{\mathcal{R}}(k) \longrightarrow \sigma^2(R, z) \longrightarrow \beta(R)$$

Collapse probability at
smoothing scale R

$$\sigma_0^2(R, z) = \frac{16}{81} \int d \ln k (kR)^4 W^2(kR) T^2(k, z) \mathcal{P}_{\mathcal{R}}(k)$$

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DM subfraction

$$\sigma_0^2(R, z) = \frac{16}{81} \int d \ln k (kR)^4 W^2(kR) T^2(k, z) \mathcal{P}_{\mathcal{R}}(k)$$

Variance, with window function W and transfer
function T describing sub-horizon evolution

Collapse of overdensities: abundance

Press-Schechter

$$\beta(R) = \int_{\delta_{\min}}^{\delta_{\text{PBH}}} d\delta \frac{1}{\sqrt{2\pi}\sigma_0(R)} \exp \left[-\frac{\delta^2}{2\sigma_0^2(R)} \right]$$

$\delta_{\min}(R, z_c)$ to reach $\delta_c \sim 1.686$ by z_c

Has a cloud-in-cloud problem ($\rightarrow$ post-hoc factor of 2), see Kushawa & Suyama 2509.25871

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Single scale estimate

Localised feature in the powerspectrum

$$f_{\text{UCMH}}^{\text{ss}} \simeq \beta_{\text{UCMH}}(R_{\text{ss}}), \text{ with } R_{\text{ss}} = \arg \max_R \beta_{\text{UCMH}}(R)$$

Monochromatic UCMH population

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Peaks theory (BBKS)

Uses higher moments σ_1, σ_2 :
Collapsed objects are *local maxima* of the smoothed field above threshold

Counts $n_{\text{pk}}(\delta_{\min}, \sigma_0, \sigma_1, \sigma_2)$

Still has a peak-in-peak problem

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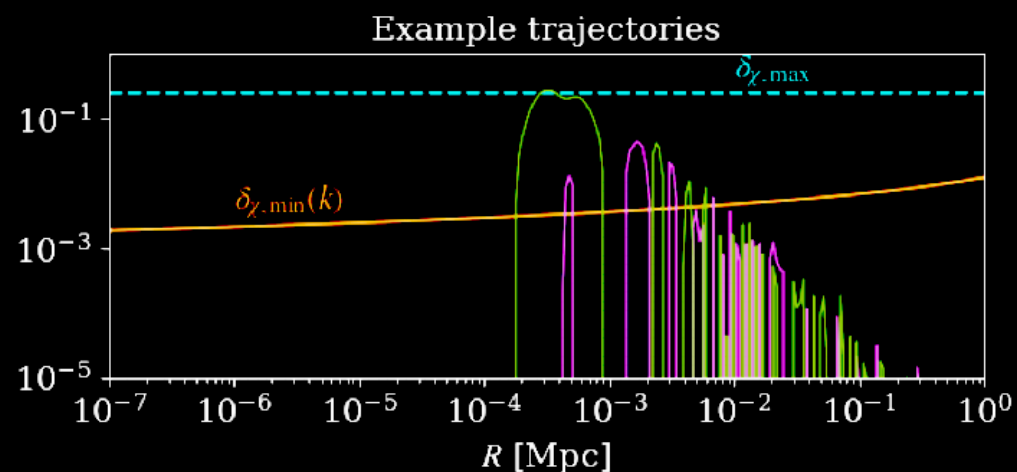
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Excursion set theory

Generate N trajectories, associate UCMH with first crossing of $\delta_{\min}(R, z_c)$



Single scale estimate

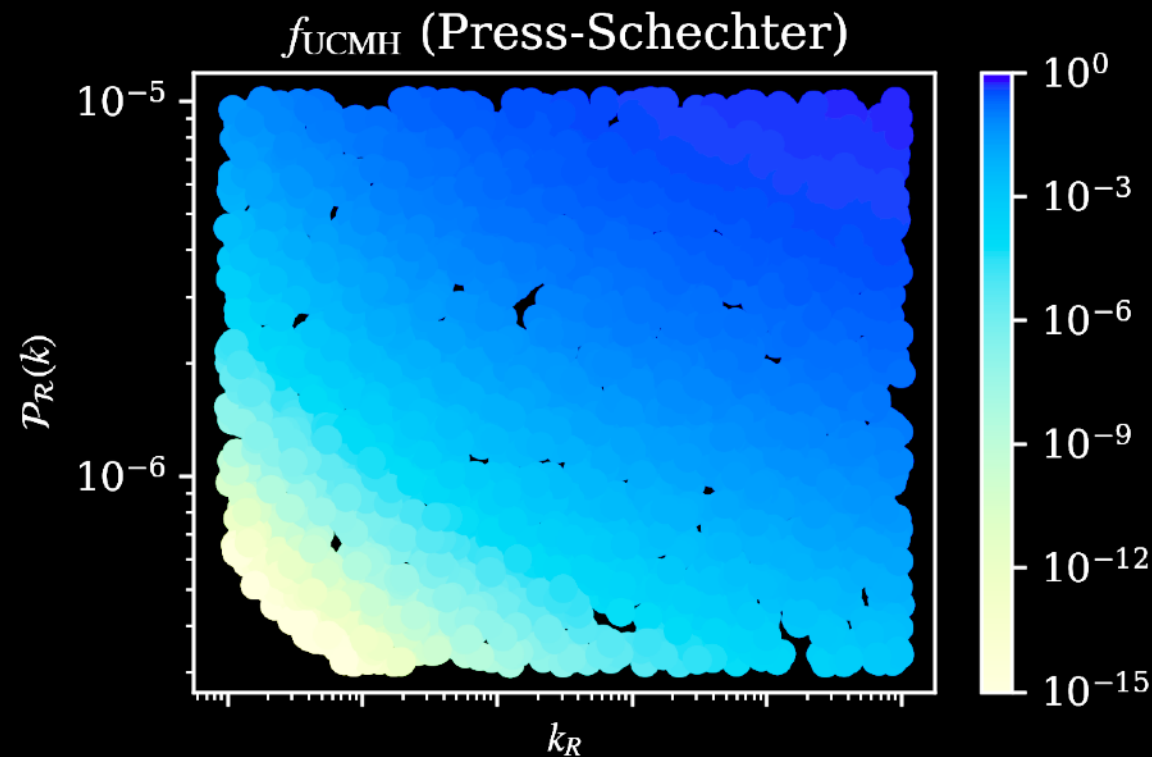
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Monochromatic UCMH population

Collapse of overdensities

$z_c \geq 1000$, $\mathcal{P}_{\mathcal{R}}(k)$ flat
between $k_R/3 < k < 3k_R$



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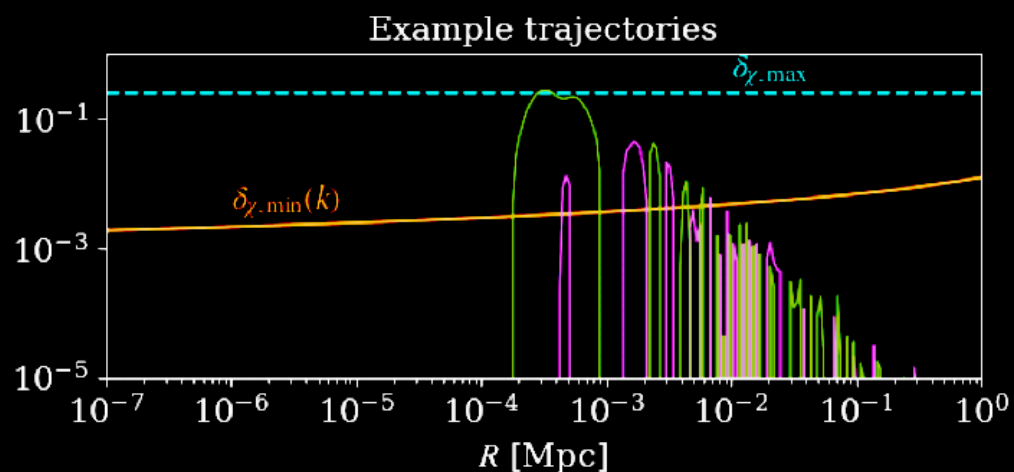
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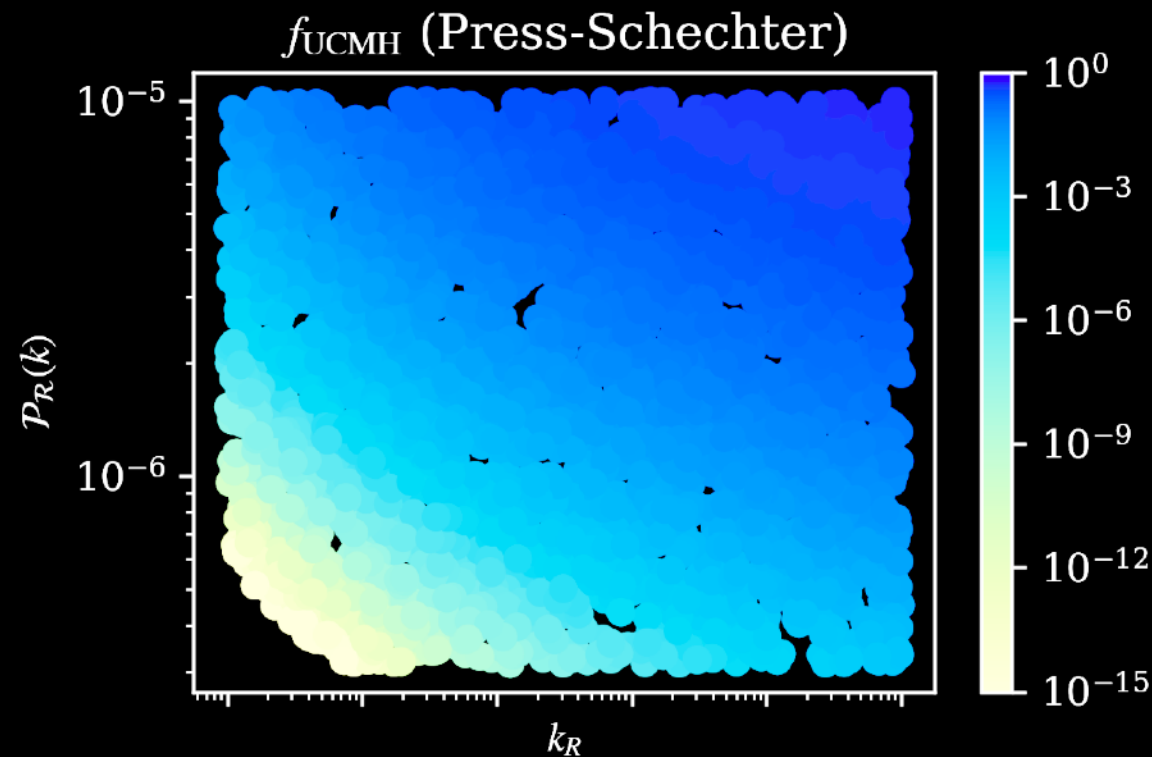
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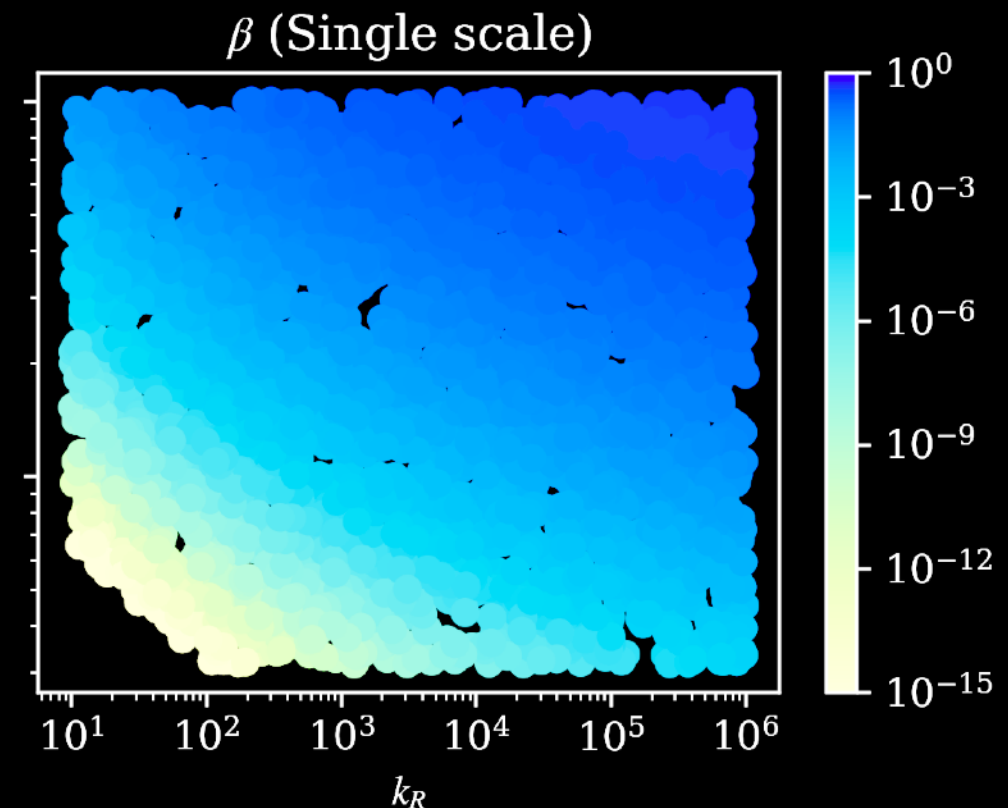
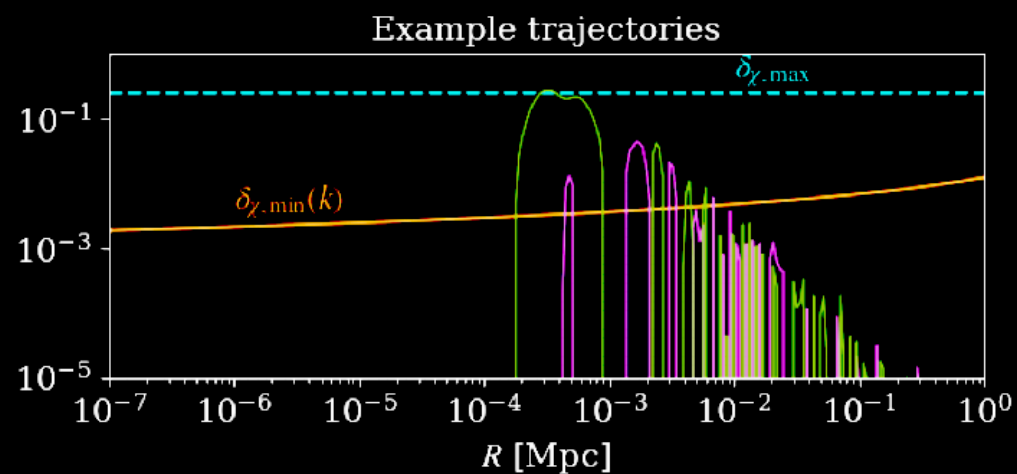
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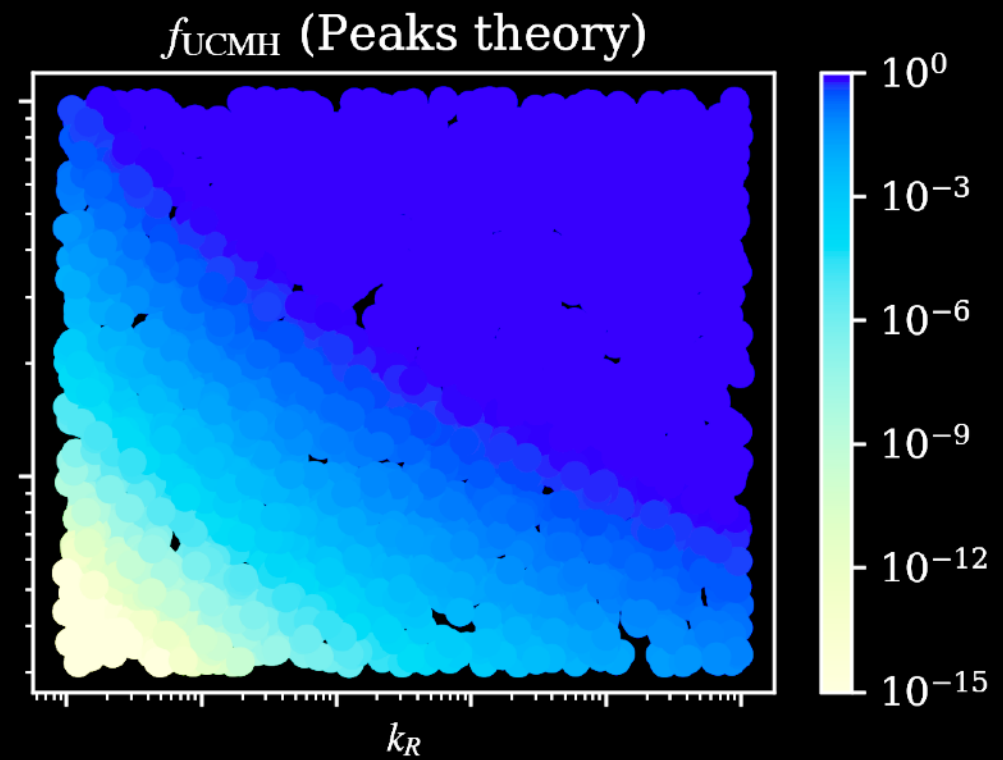
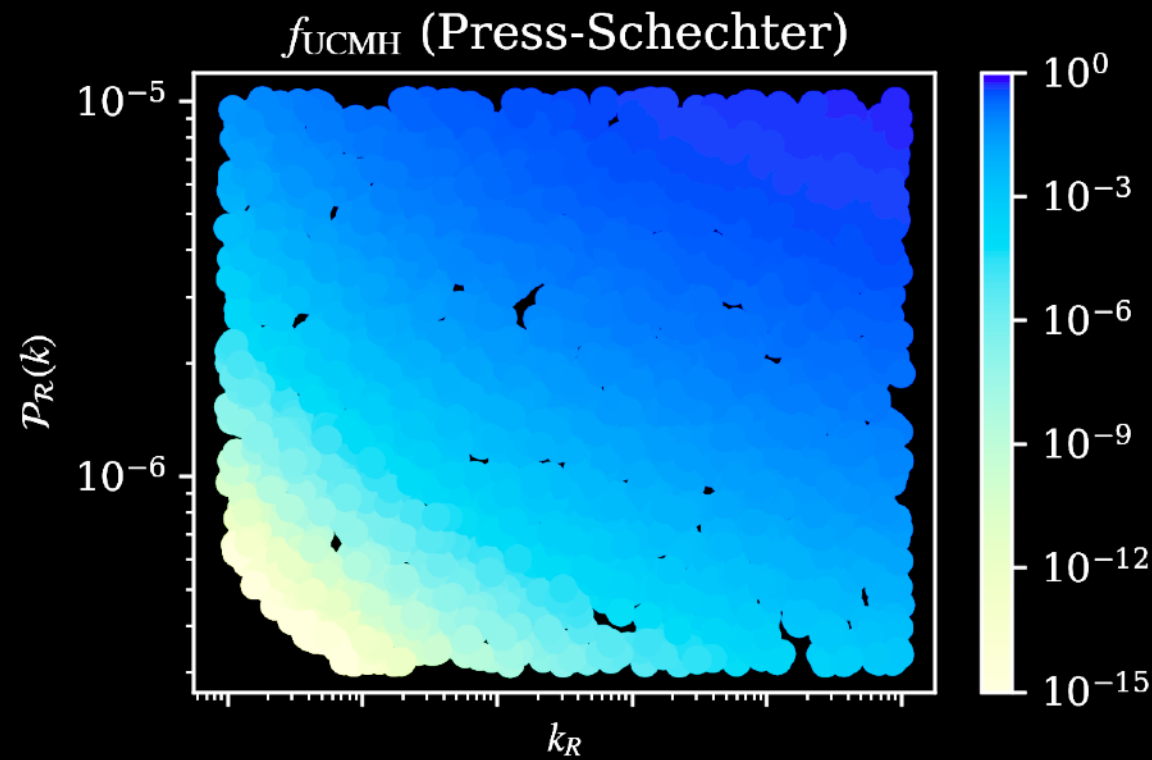
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Generate N trajectories, associate UCMH with first crossing of $\delta_{\text{min}}(R, z_c)$



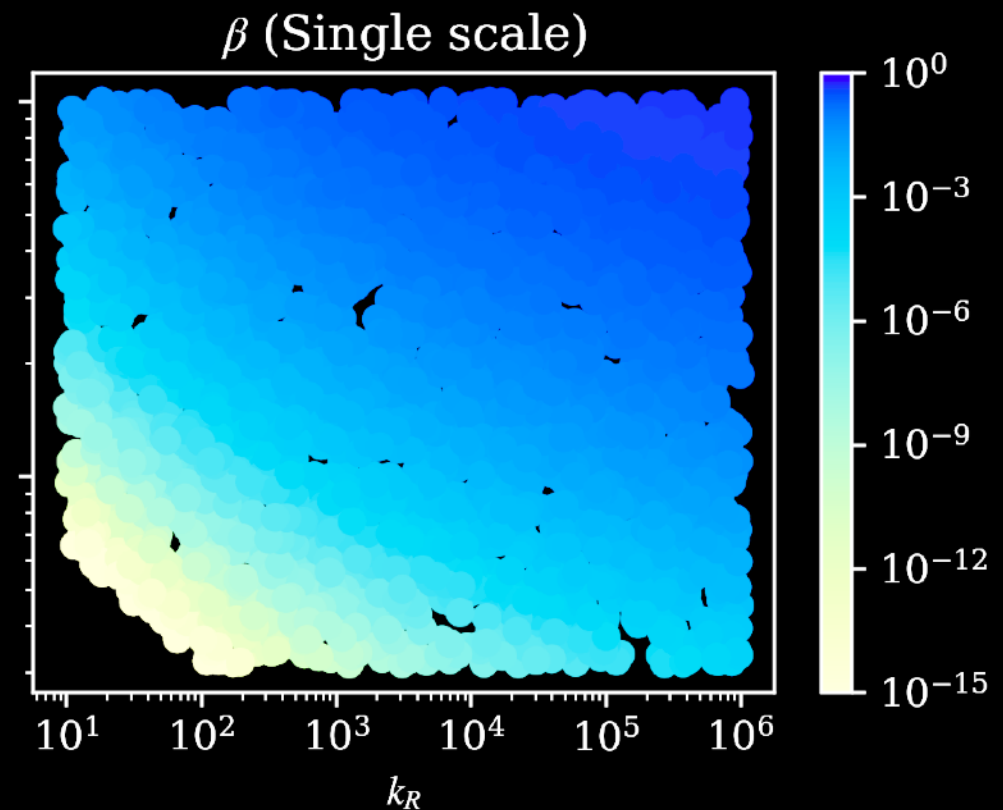
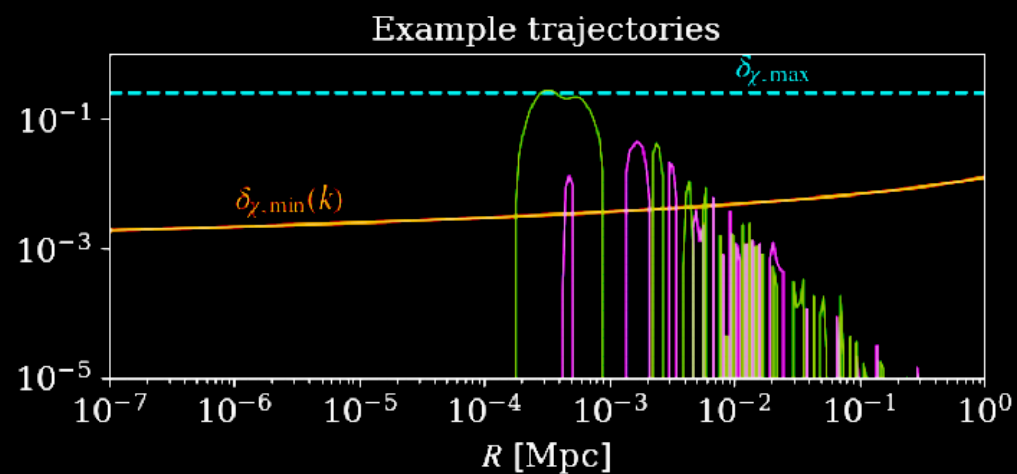
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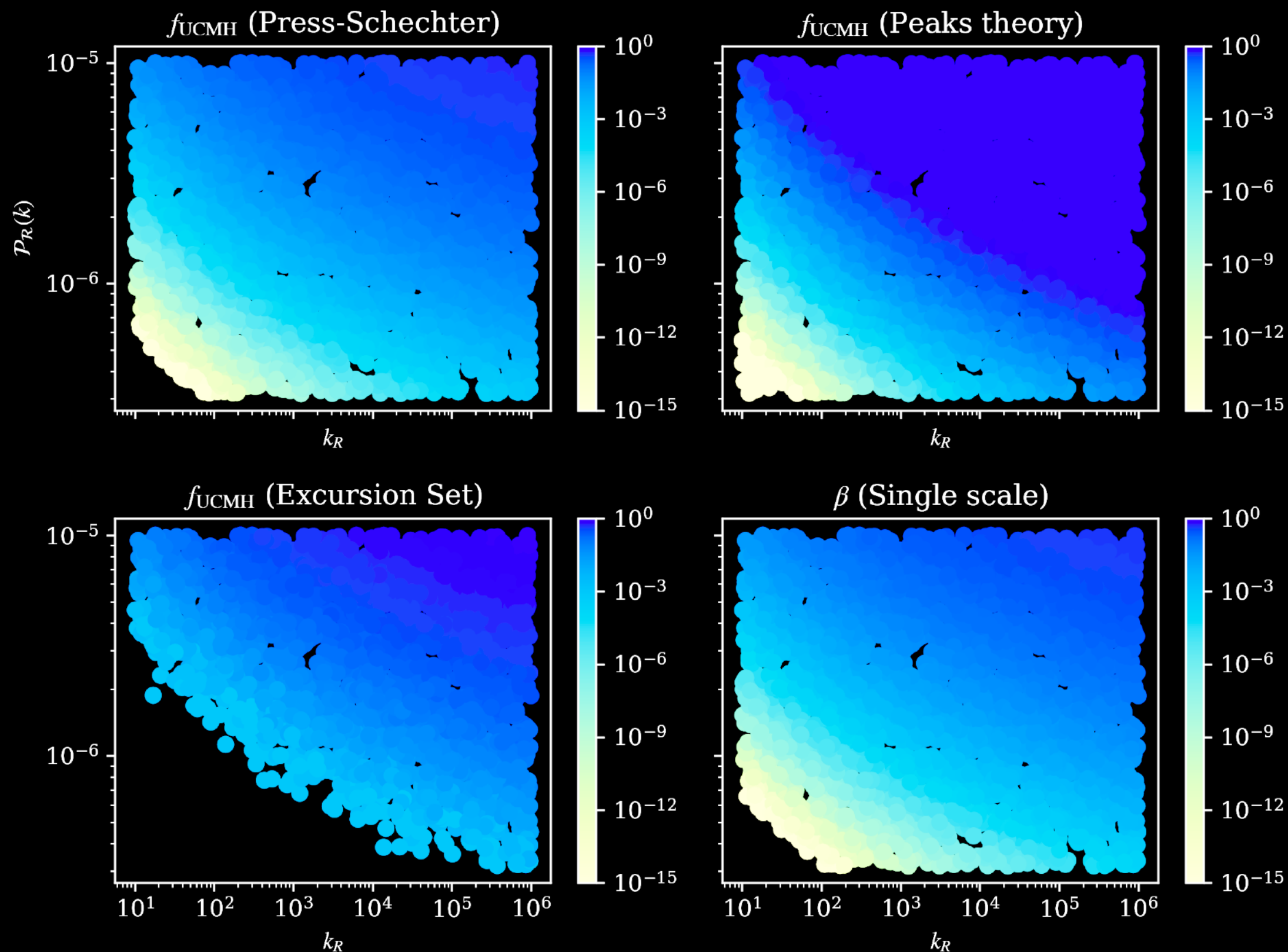
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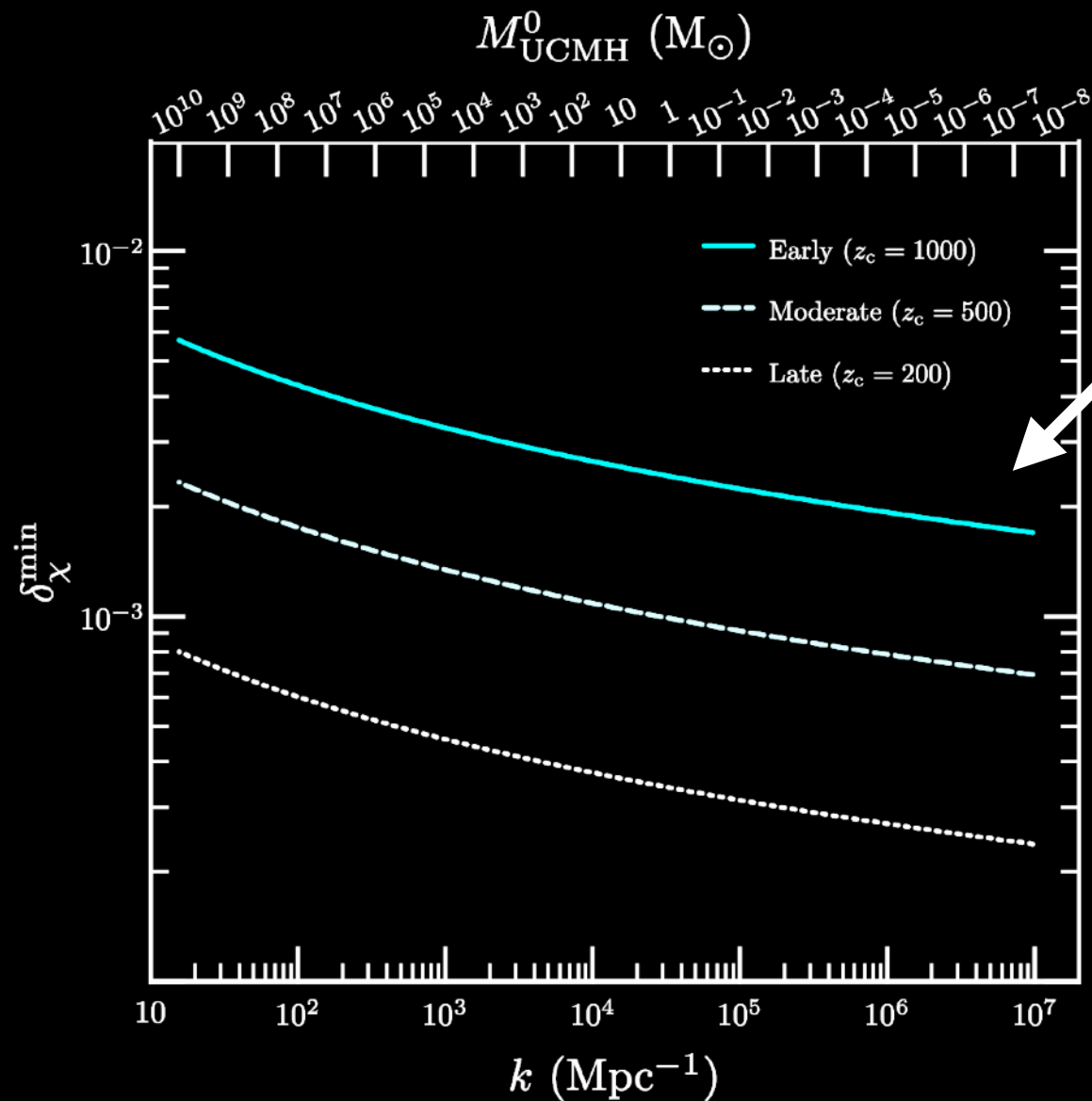
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Collapse of overdensities: abundance

Collapse probability is exponentially sensitive to $\delta_{\min}(R, z_c)$

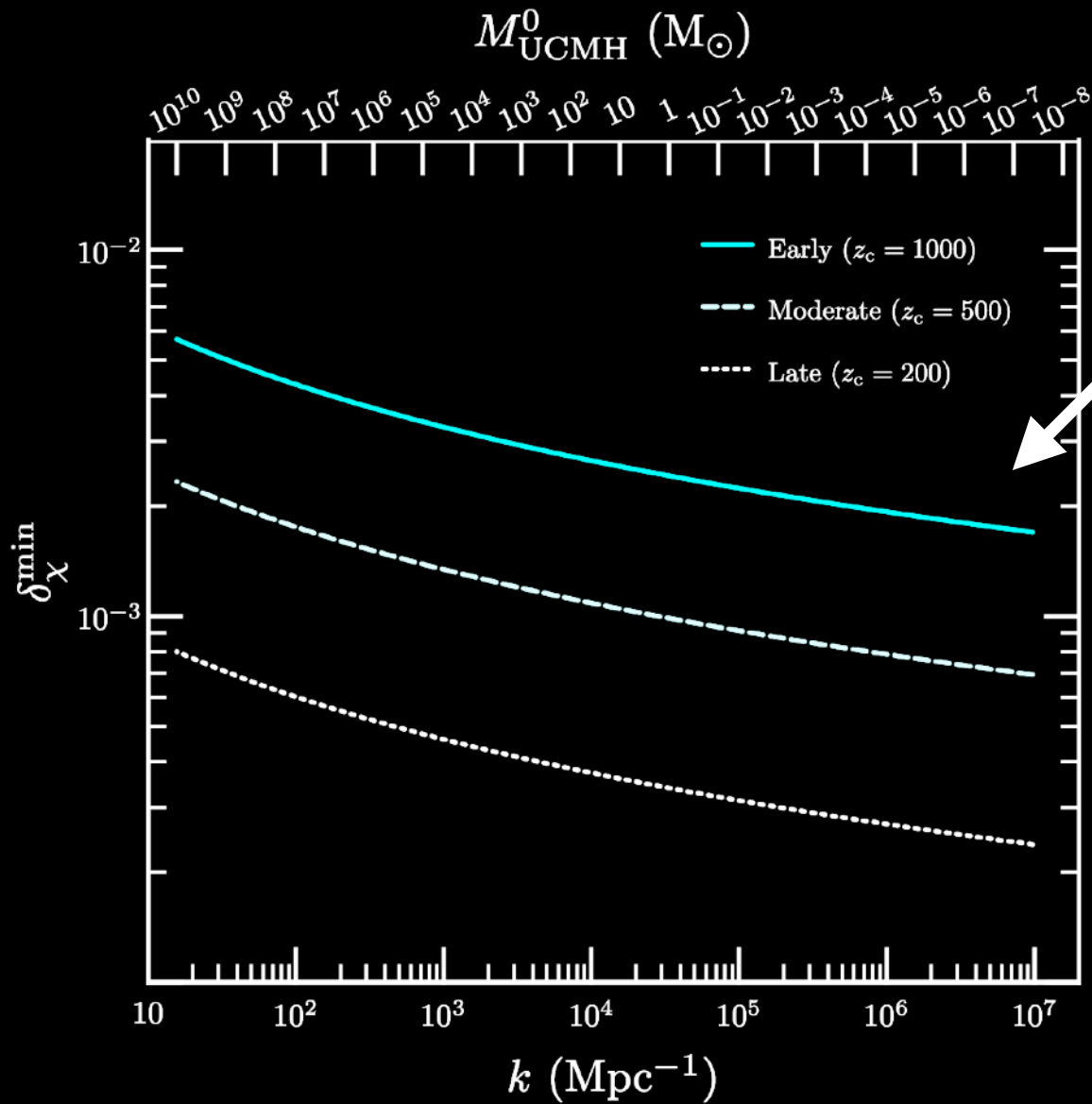


$\delta \propto \alpha \propto (1+z)^{-1}$ during matter domination

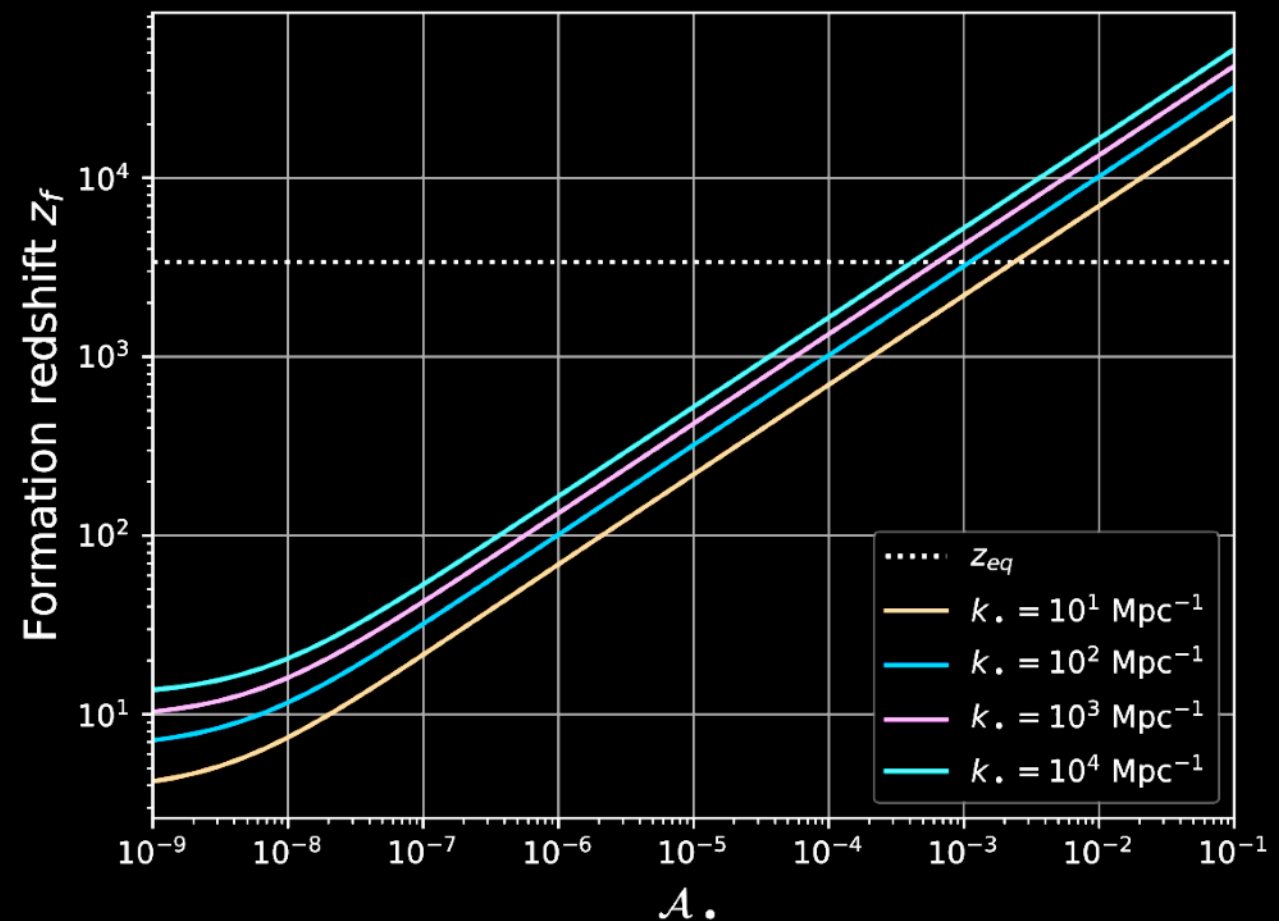
Smaller $z_c \rightarrow$ much larger abundance

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Smaller $z_c \rightarrow$ much larger abundance



Bringmann, Scott, Akrami, PRD, arXiv:1110.2484

Note that this assumption changes derived constraints significantly across the literature!

Pare & Lavalle, arXiv:2606.14827

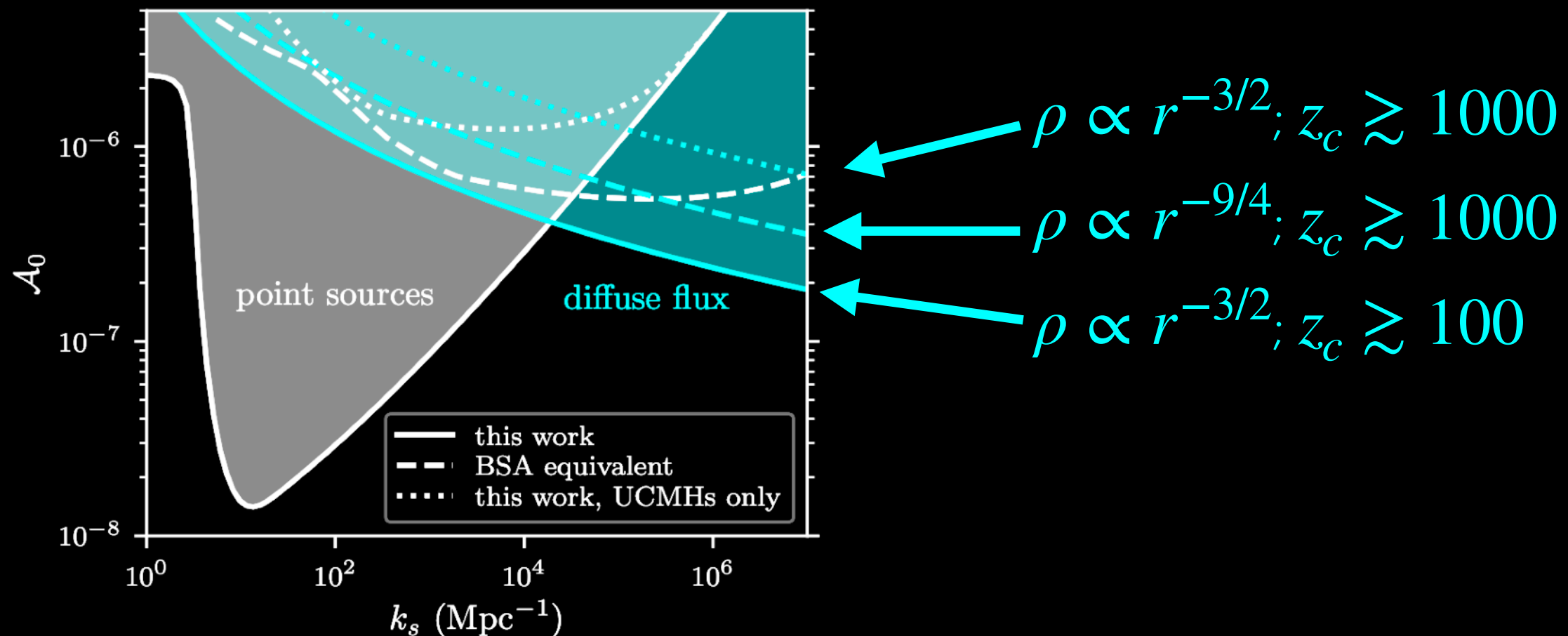
Probes of UCMH



Dark matter annihilation inside UCMH

Strong constraints for WIMP-like DM ($\chi\bar{\chi} \rightarrow b\bar{b}$, $m_\chi = \text{TeV}$)

- Gamma-ray searches using Fermi-LAT
- Depends on UCMH profile ($\Gamma \propto \rho_\chi^2$)



Delos, Erickcek, Bailey, Alvarez, PRD, arXiv:1806.07389

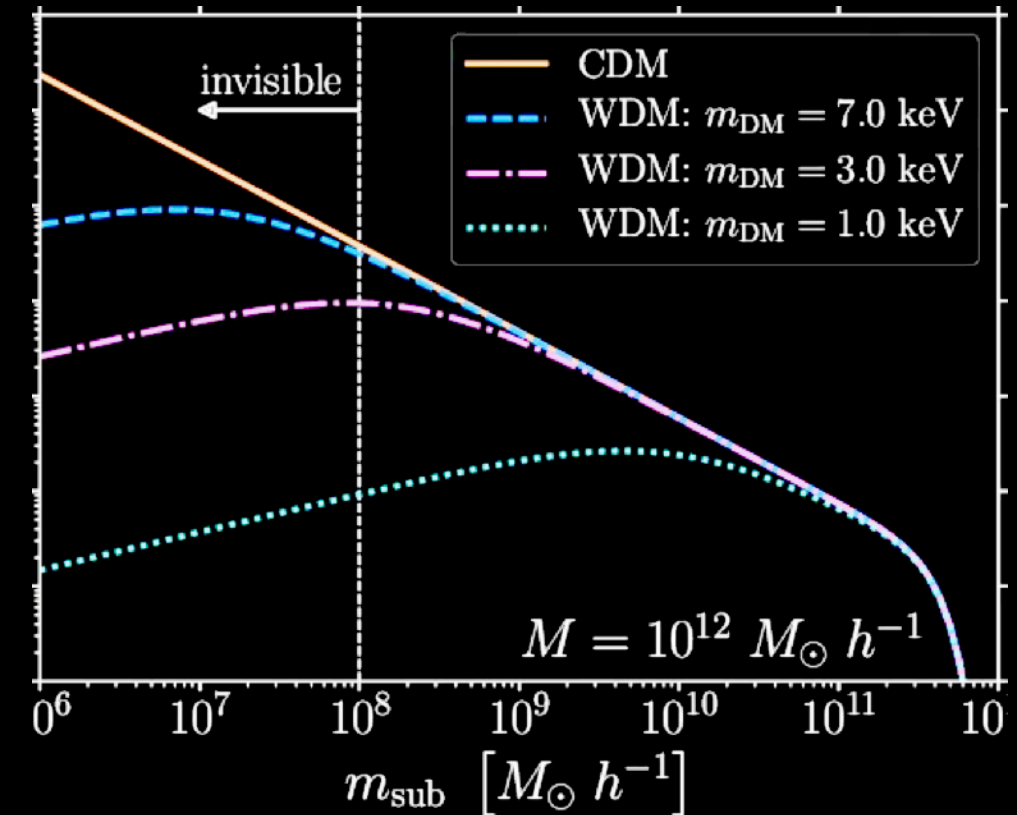
BSA = Bringmann, Scott, Akrami, PRD, arXiv:1110.2484

see also Josan & Green, PRD, arXiv:1006.4970

Gravitational probes

- Typically have resolution limits:
 - Strong lensing: $\gtrsim 10^8 M_\odot$
 - Stellar streams: $\gtrsim 10^6 M_\odot$

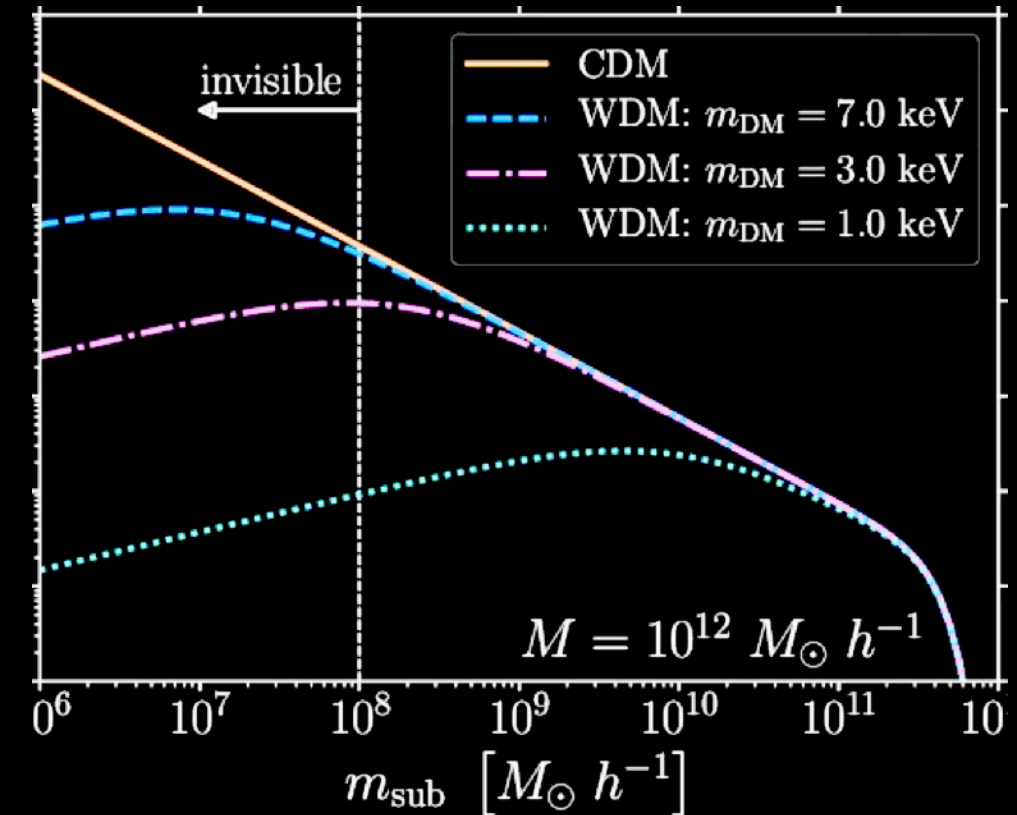
Chianese, arXiv: 1910.06617



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- UCMH have a characteristic mass-radius relationship

$$\frac{R(z_c)}{\text{pc}} = 0.02 \left(\frac{1000}{z_c + 1} \right) \left(\frac{M(z_c)}{M_\odot} \right)^{1/3} \quad \text{with} \quad M(z_c) = \frac{z_{\text{eq}} + 1}{z_c + 1} M_i$$

Effective radius where UCMH density has dropped to $3 \times$ the background density

- Many PBH probes don't apply (galactic microlensing, gravitational waves from binary mergers)

Gravitational probes

What remains?

- Wide binary **heating** (Gaia)

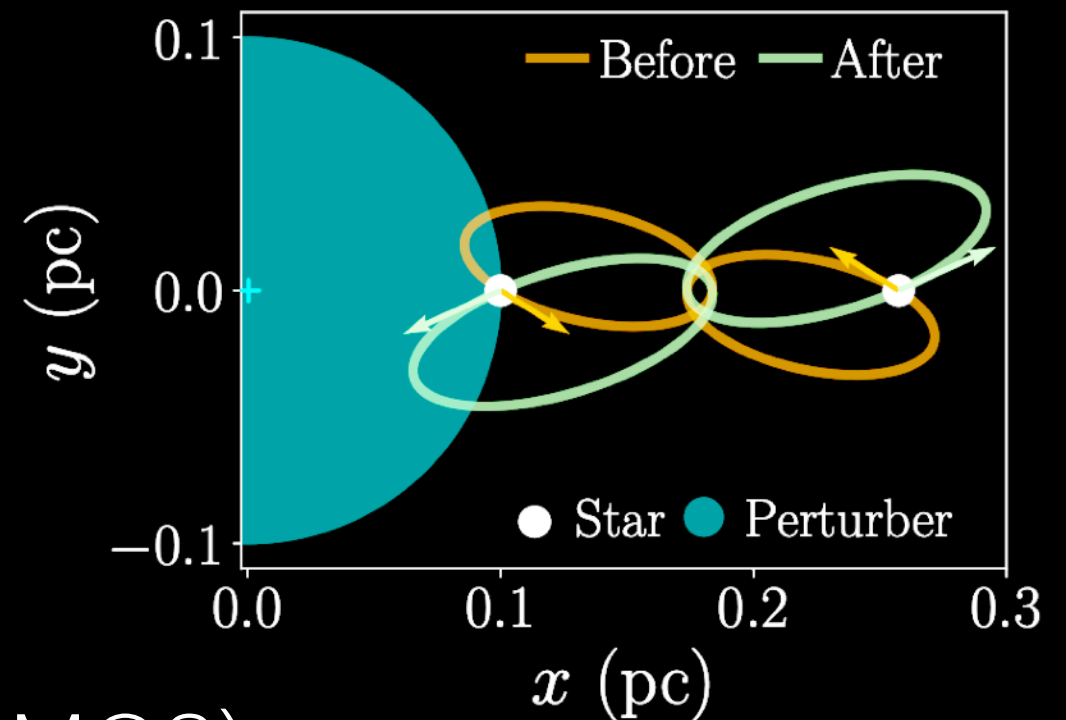
$$\rightarrow R_{\text{UCMH}} \lesssim R_{\text{binary}} \sim 0.1 \text{ pc}$$

Ramirez and Buckley, MNRAS, arXiv: 2209.08100

- Ultrafaint dwarf **heating** (Keck/DEIMOS)

$$\rightarrow R_{\text{UCMH}} \lesssim R_{1/2} \sim 30 \text{ pc}$$

Graham and Ramani, PRD, arXiv: 2404.01378



Gravitational probes

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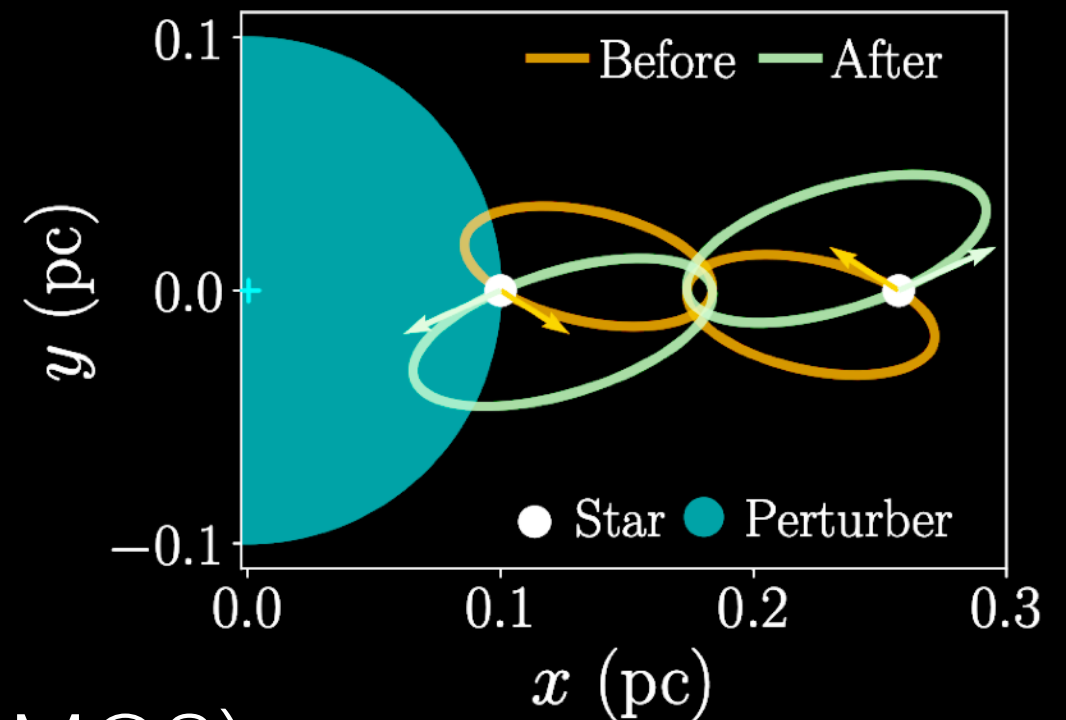
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Graham and Ramani, PRD, arXiv: 2404.01378

- Heating of the CMB $\rightarrow R_{\text{UCMH}} \lesssim \min(R_{\text{Bondi}}, r_{\text{cool}})$
- Microlensing in special circumstances ($R_{\text{UCMH}} \lesssim R_E$)
 1. Near a galaxy cluster caustic
 2. Supernova microlensing



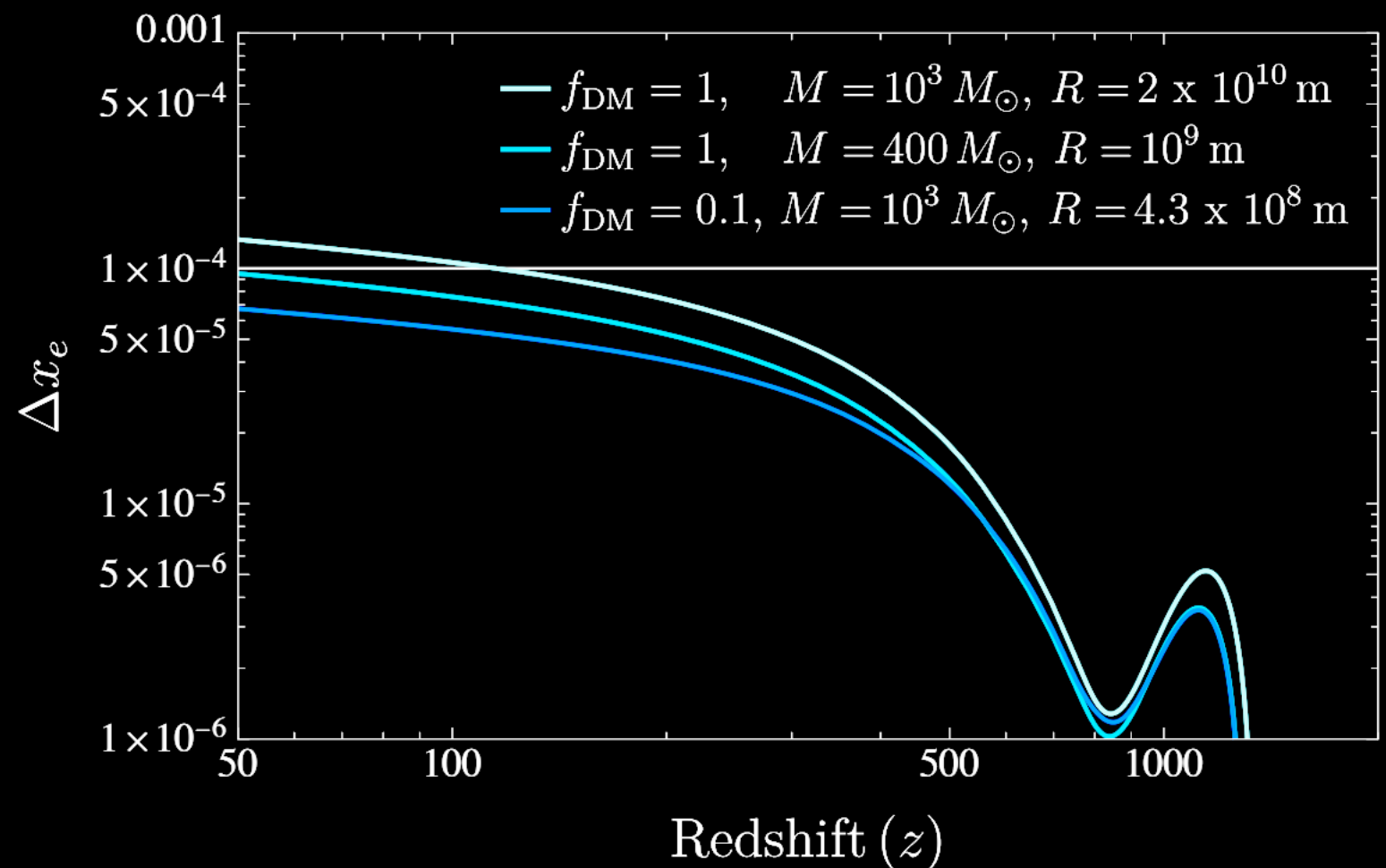
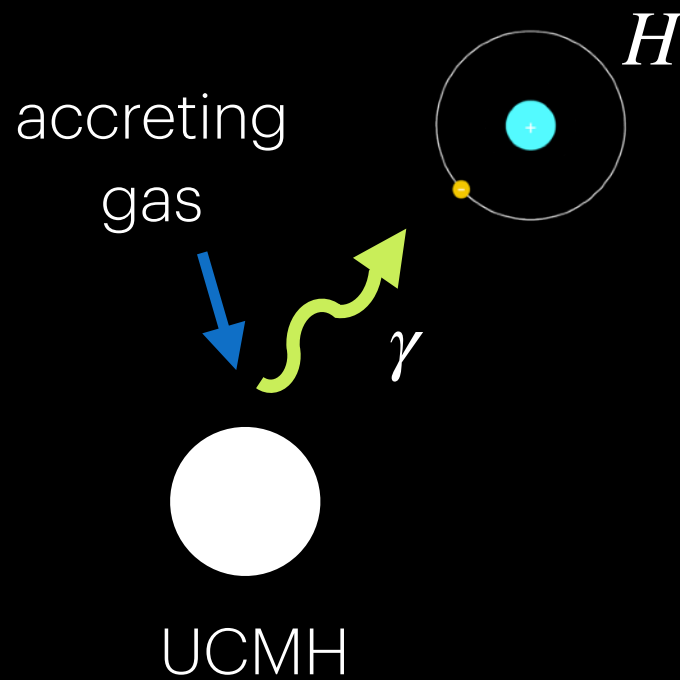
See also Tamara Davis' talk on Wednesday

Gravitational probes: CMB accretion

Bai, Long, Lu, JCAP, arXiv:2003.13182

DC, Sevillano Muñoz, JCAP arXiv:2403.13072

Baryons fall into the object $\rightarrow$ radiate photons $\rightarrow$ energy deposited
 $\rightarrow$ hydrogen more ionised $\rightarrow$ CMB anisotropies modified



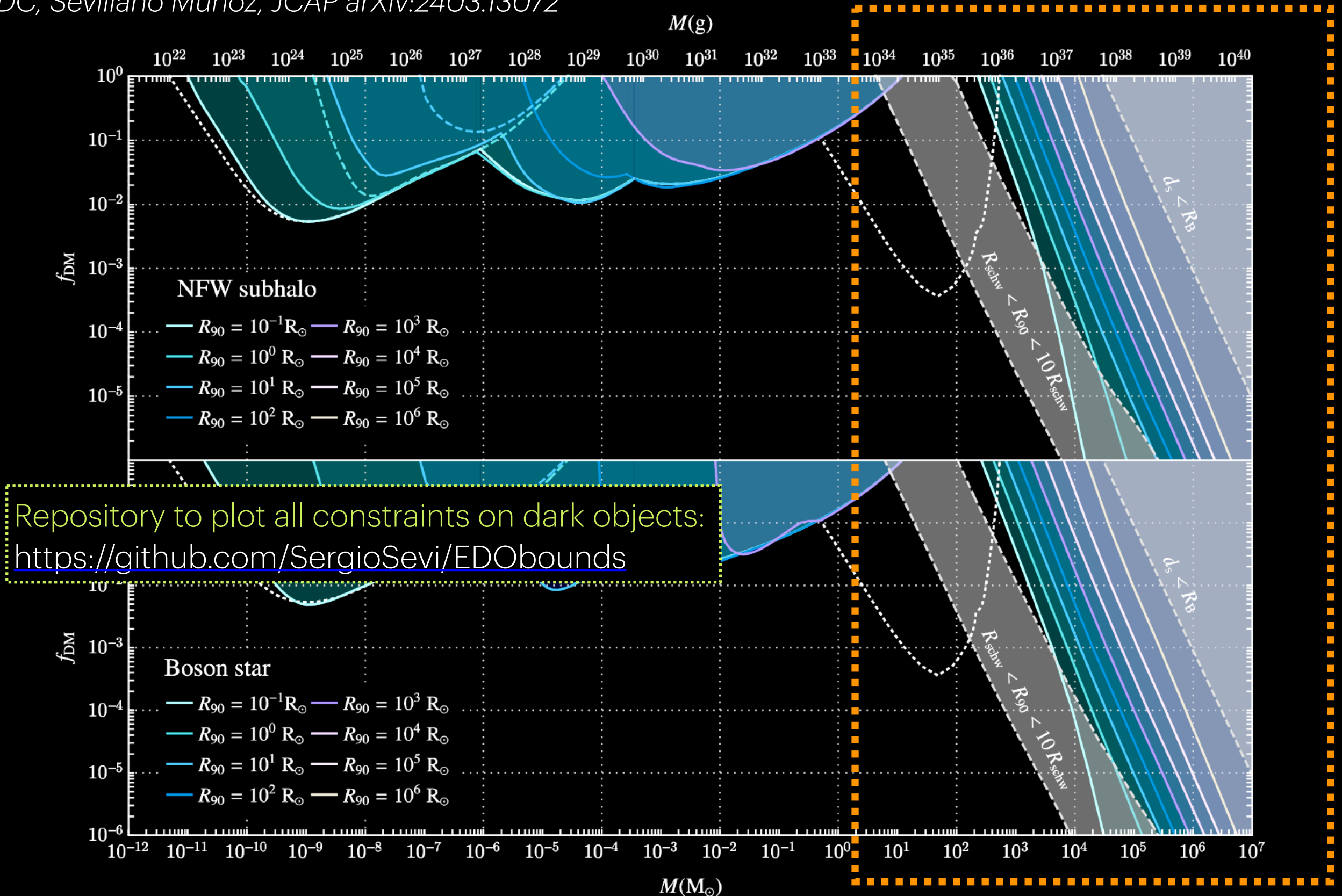
Relevant for $R_{\text{UCMH}} \lesssim \min(R_{\text{Bondi}}, r_{\text{cool}})$

UCMH gravitational potential dominates over gas pressure $\rightarrow$ infalling gas can lose thermal energy

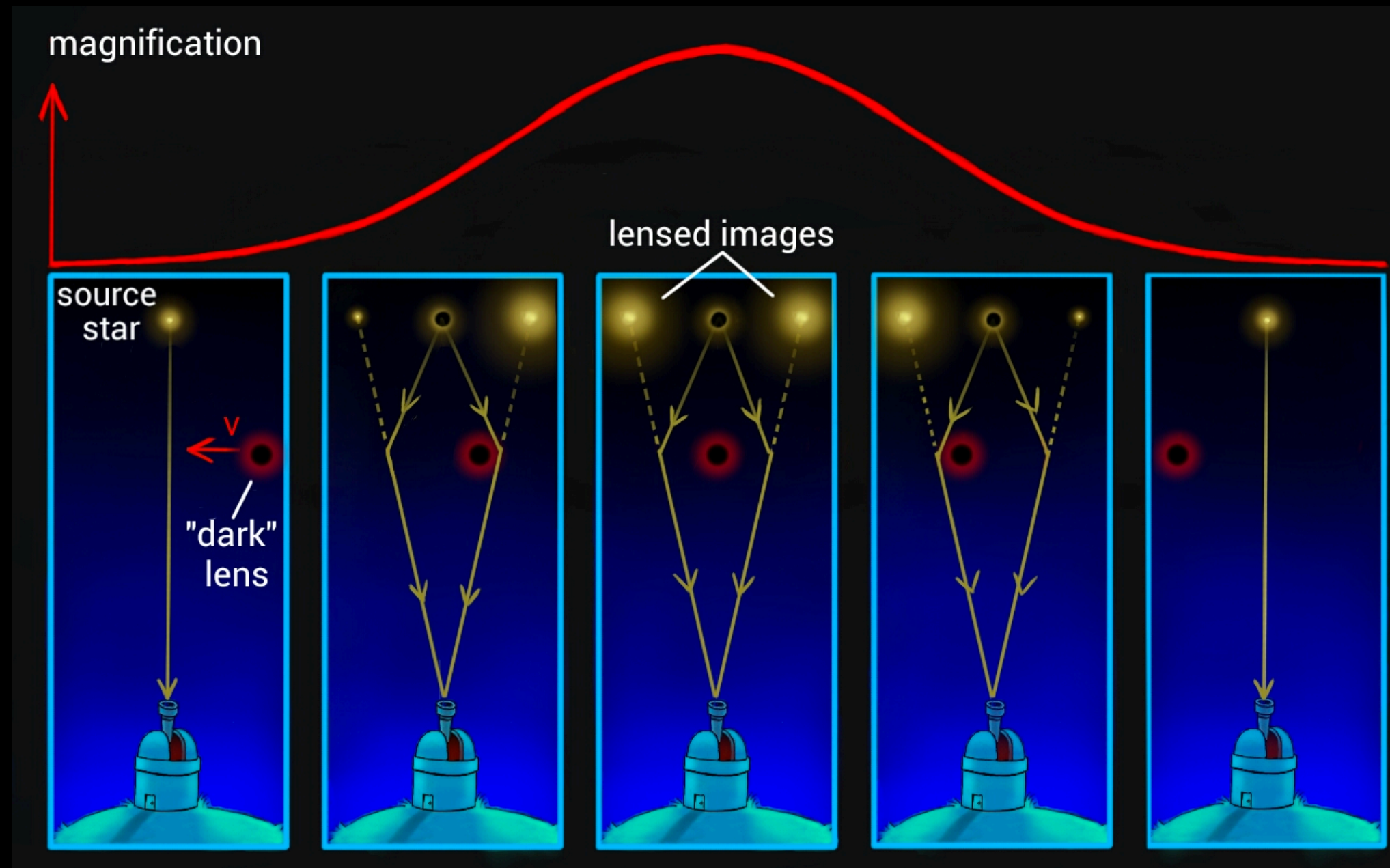
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Microlensing: things to know



Microlensing: things to know



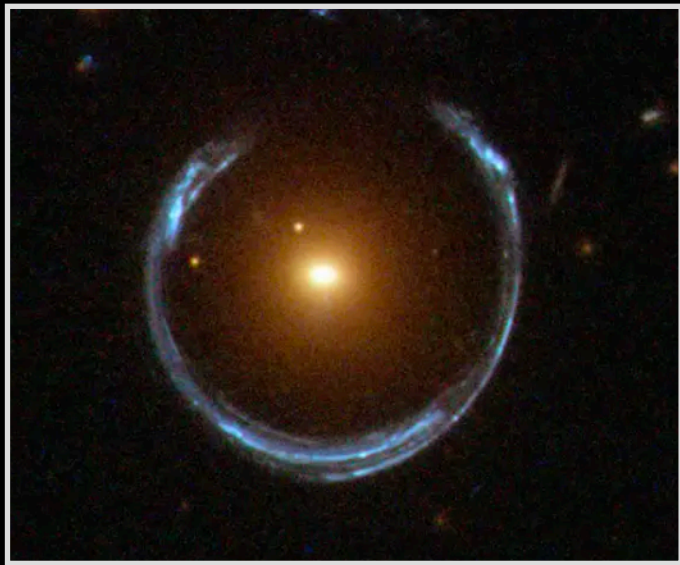
The Einstein radius

Detectible magnification for
impact parameter $\lesssim R_E$

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}}$$

$$R_E = \theta_E D_L \sim 4\text{AU} \sqrt{\frac{M}{M_\odot}} \sqrt{\frac{D_S}{8\text{kpc}}}$$

Microlensing: things to know



The Einstein radius

Detectable magnification for impact parameter $\lesssim R_E$

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Caustics

Focussing of light rays, creating large magnification along certain curves

Source plane

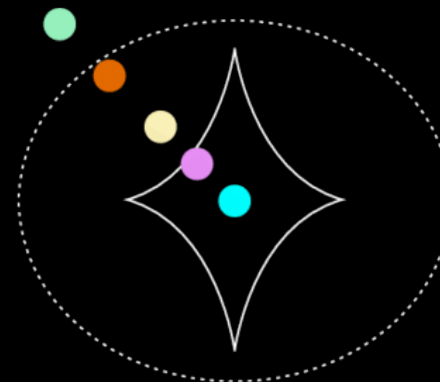
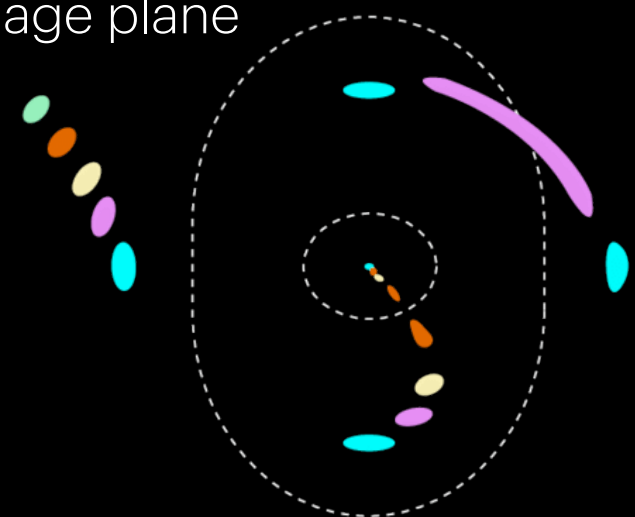
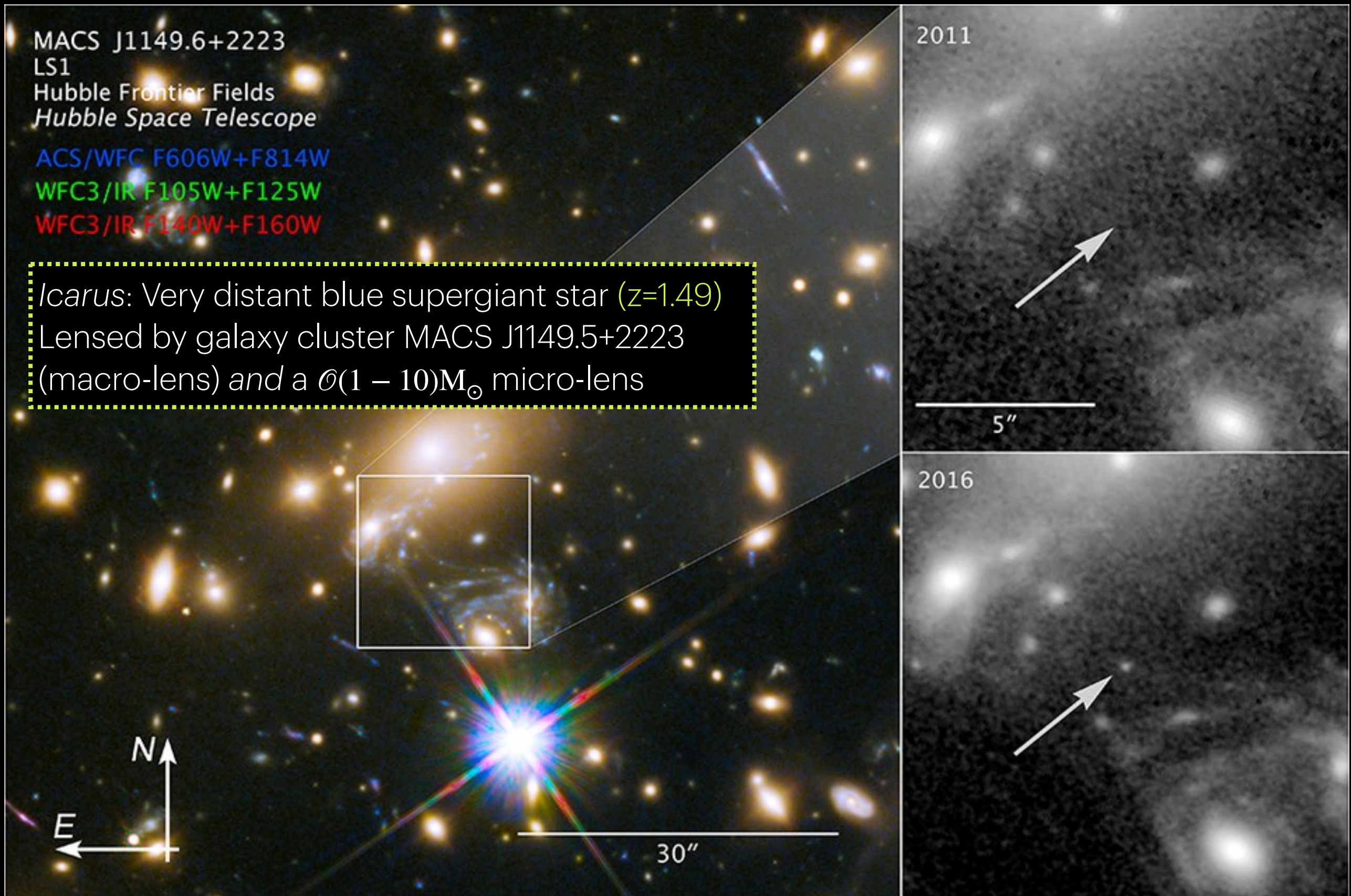


Image plane

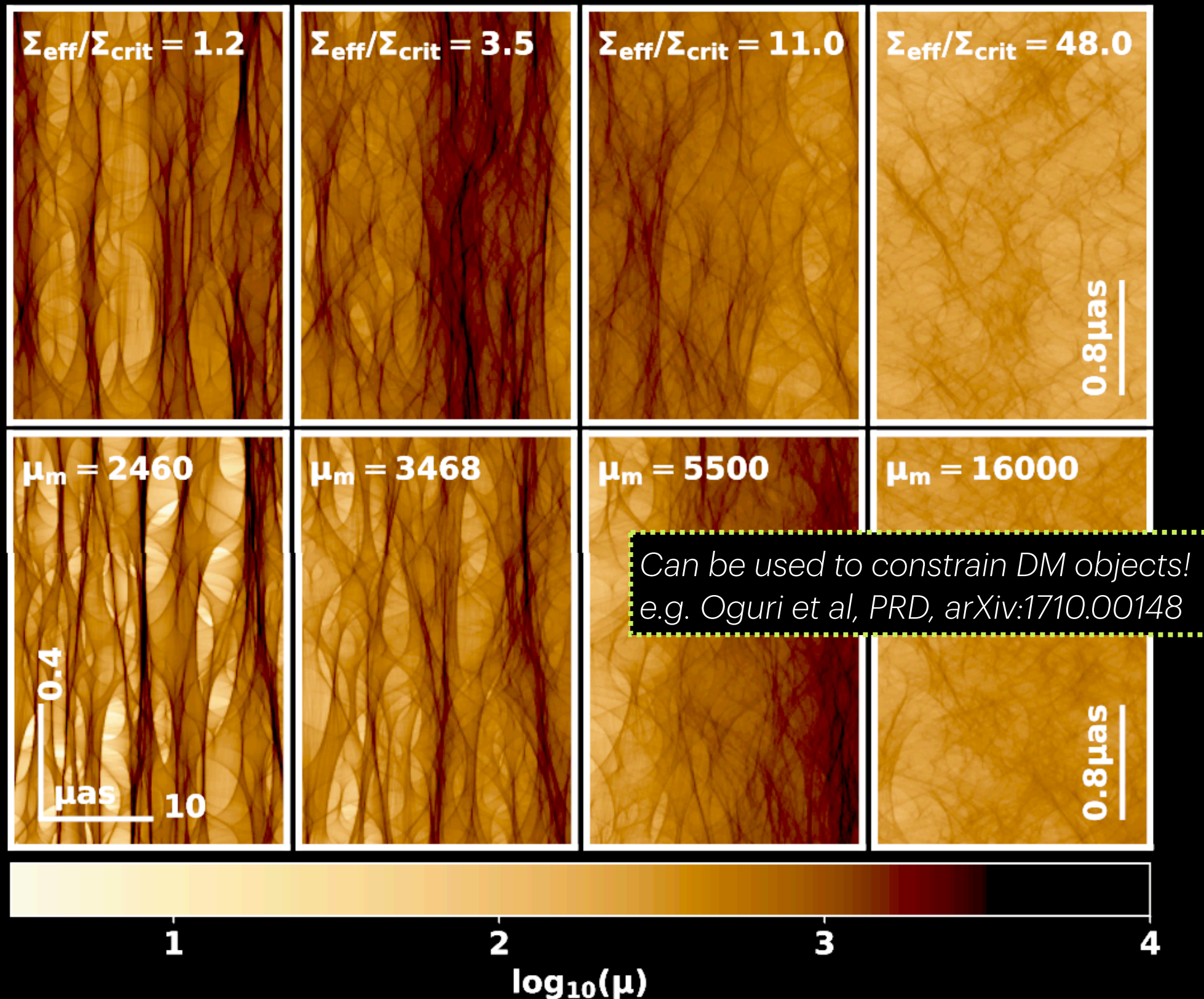


Caustic crossings in giant arcs



Caustic crossings in giant arcs

More microlenses 



Caustic crossings in giant arcs

In a macro-lens background, Einstein radius of a microlens effectively boosted by $\sqrt{\mu_t}$

*e.g. Diego et al., ApJ,
arXiv:1706.10281*

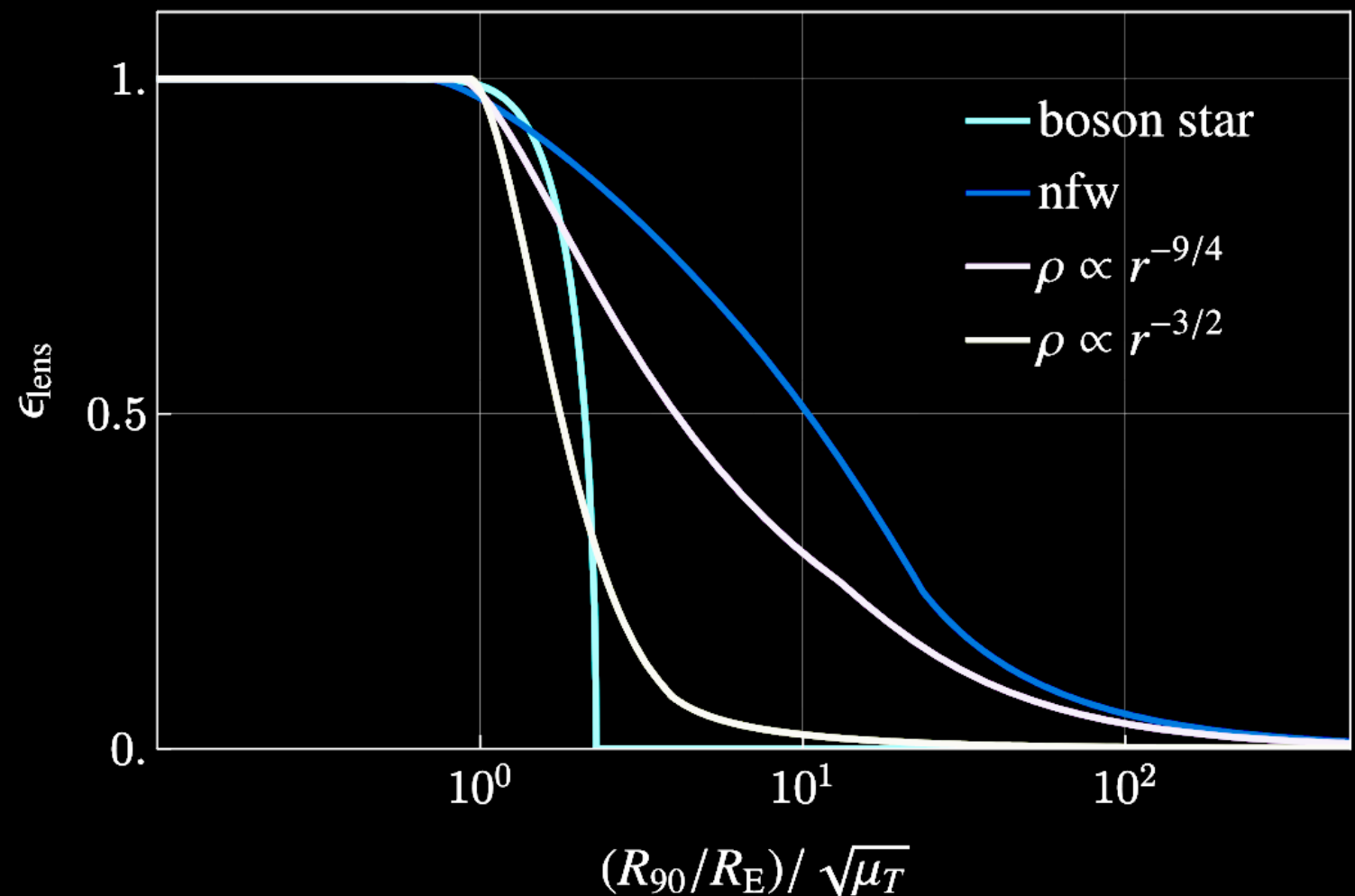
→ extended lens efficiency = $\frac{\text{effective Einstein radius near CC}}{\text{point-like Einstein radius near CC}}$

Caustic crossings in giant arcs

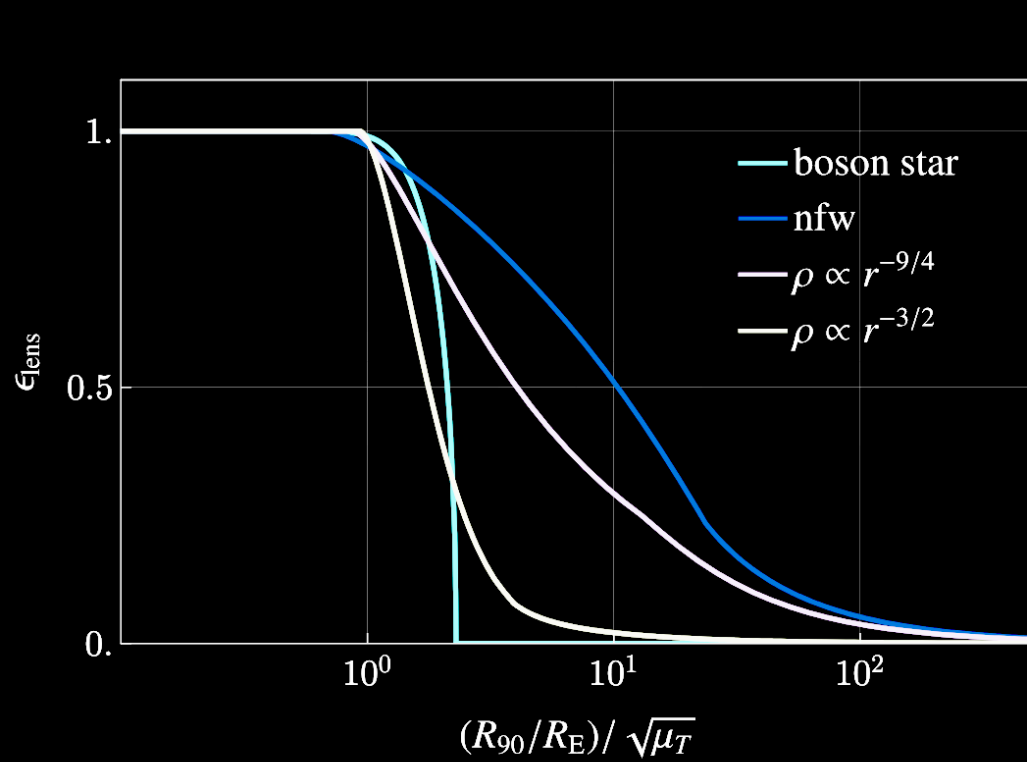
In a macro-lens background, Einstein radius of a microlens effectively
boosted by $\sqrt{\mu_t}$

e.g. Diego et al., ApJ,
arXiv:1706.10281

→ extended lens efficiency = $\frac{\text{effective Einstein radius near CC}}{\text{point-like Einstein radius near CC}}$

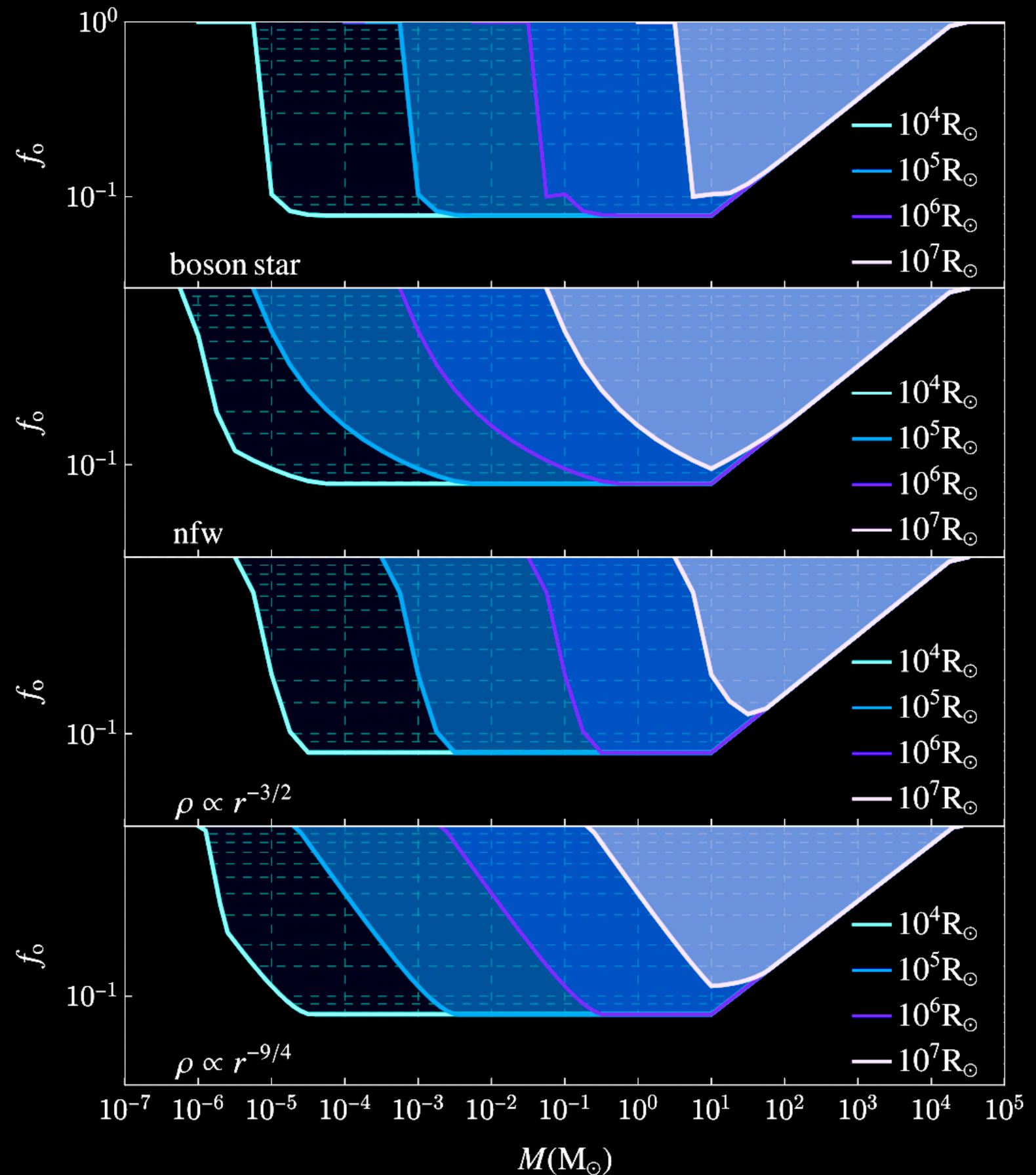


Caustic crossings in giant arcs



Sensitive to much more extended EDOs than galactic microlensing

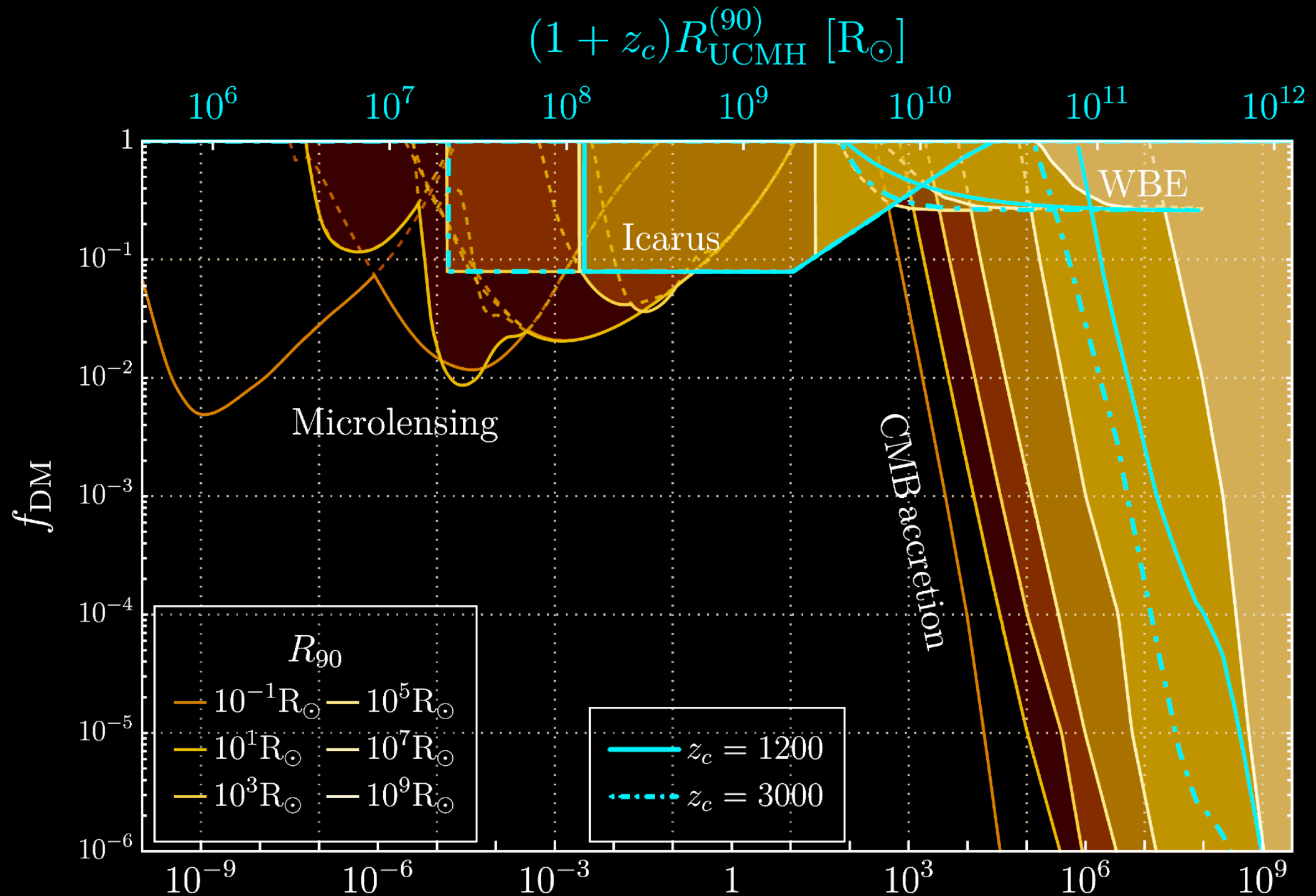
$$R_E \sim 7 \text{pc} \sqrt{\frac{M}{M_\odot}} \sqrt{\frac{D_S}{\text{Gpc}}} \sqrt{\frac{\mu_t}{10^3}}$$



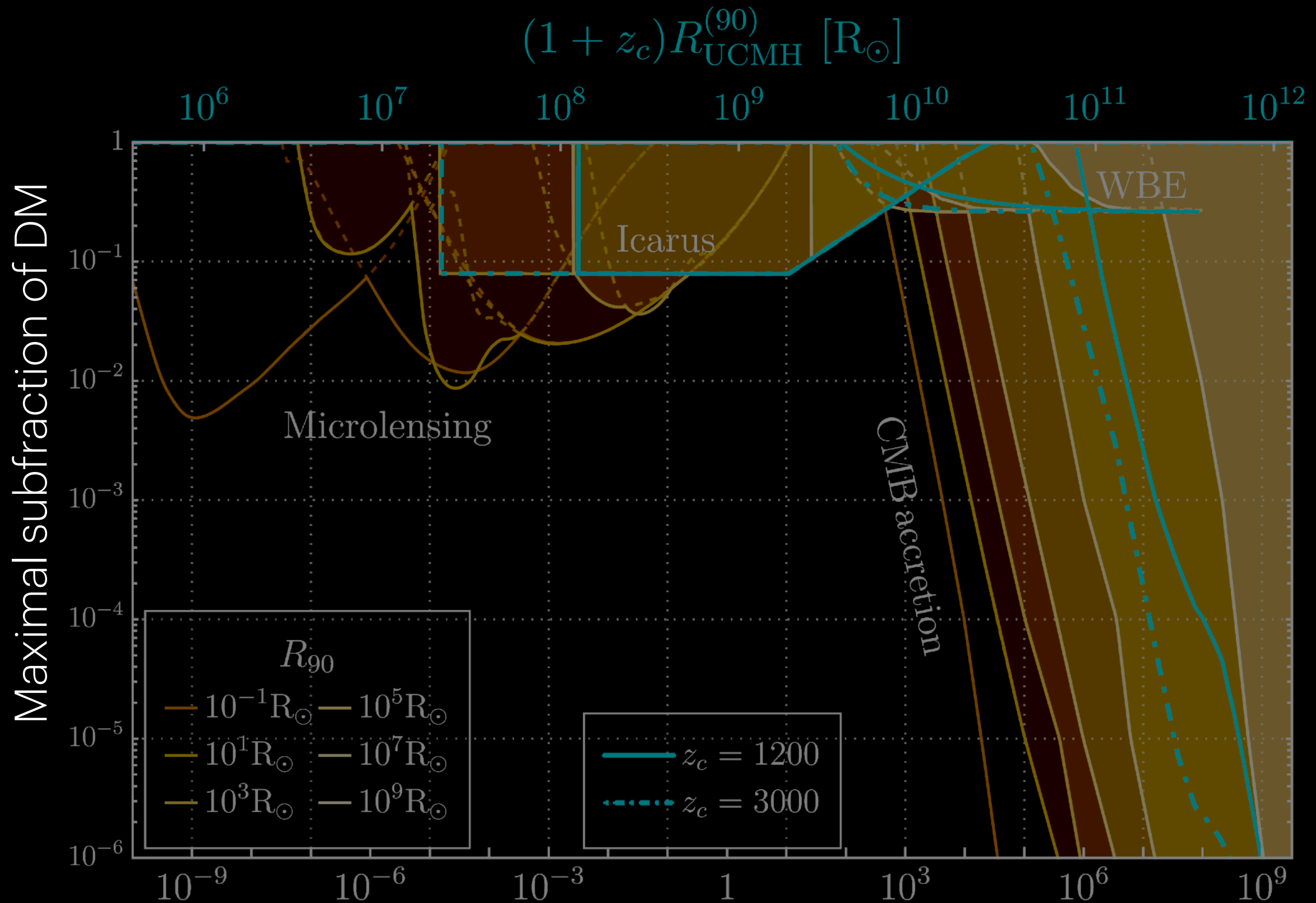
The small-scale power spectrum



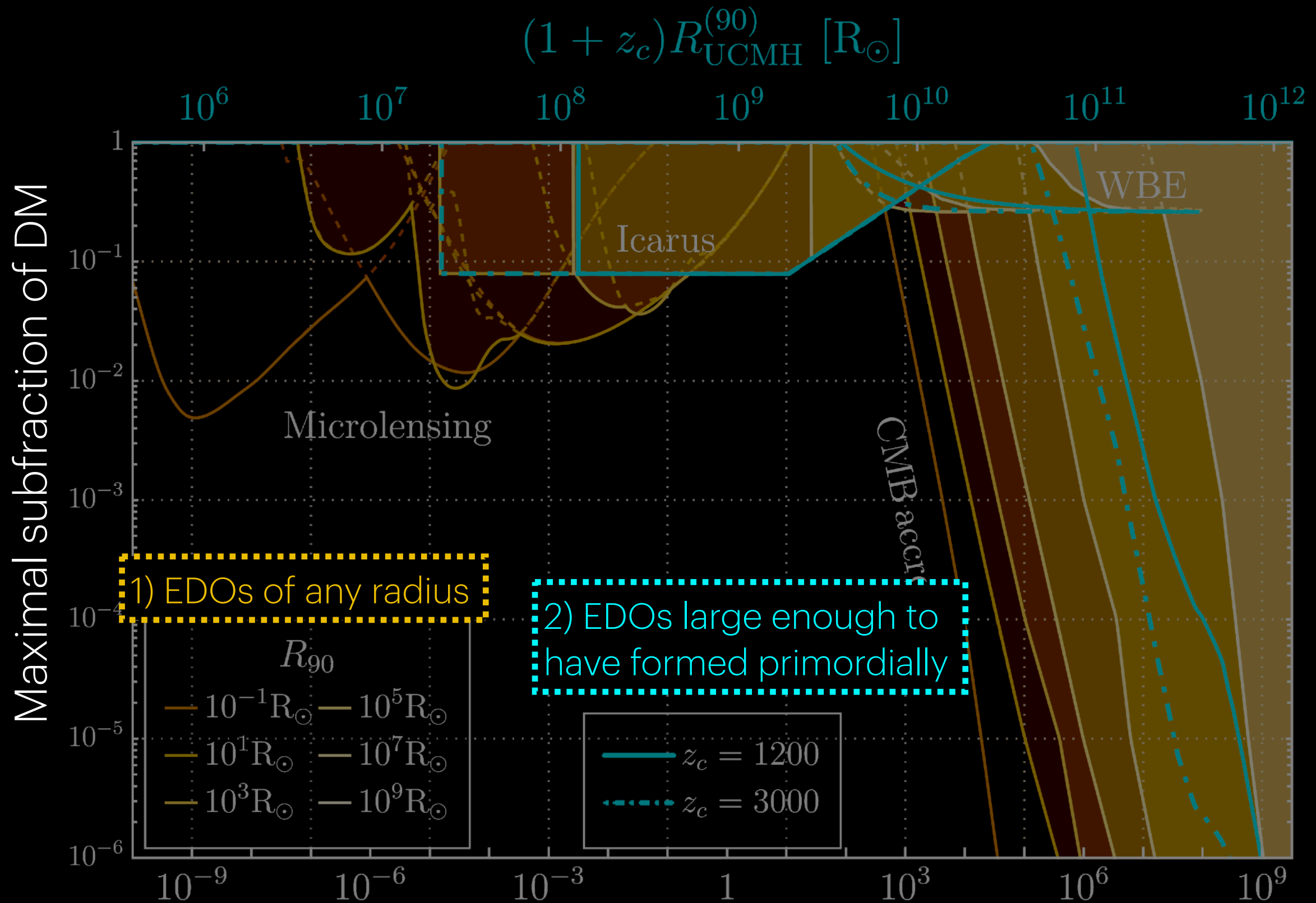
The primordial power spectrum



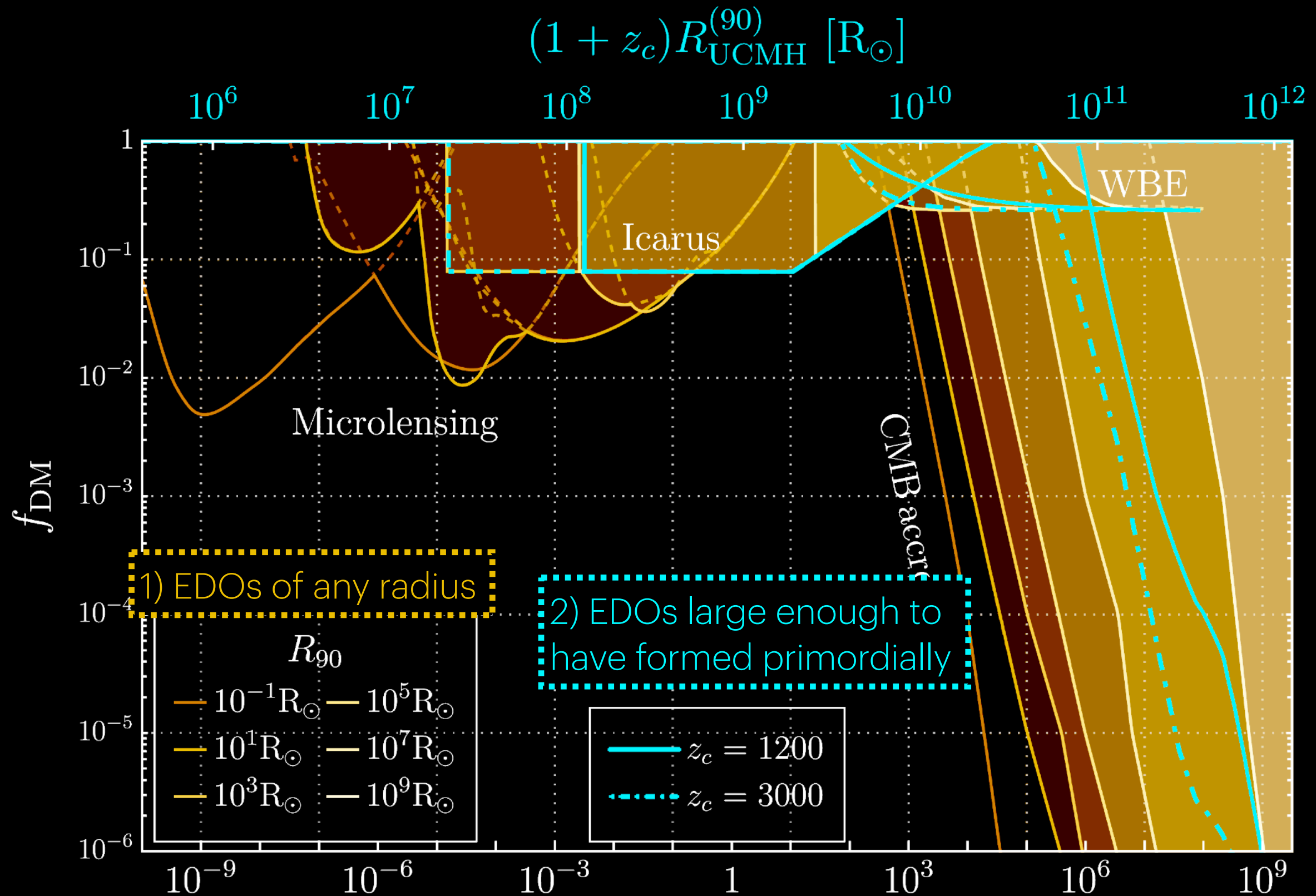
The primordial power spectrum



The primordial power spectrum



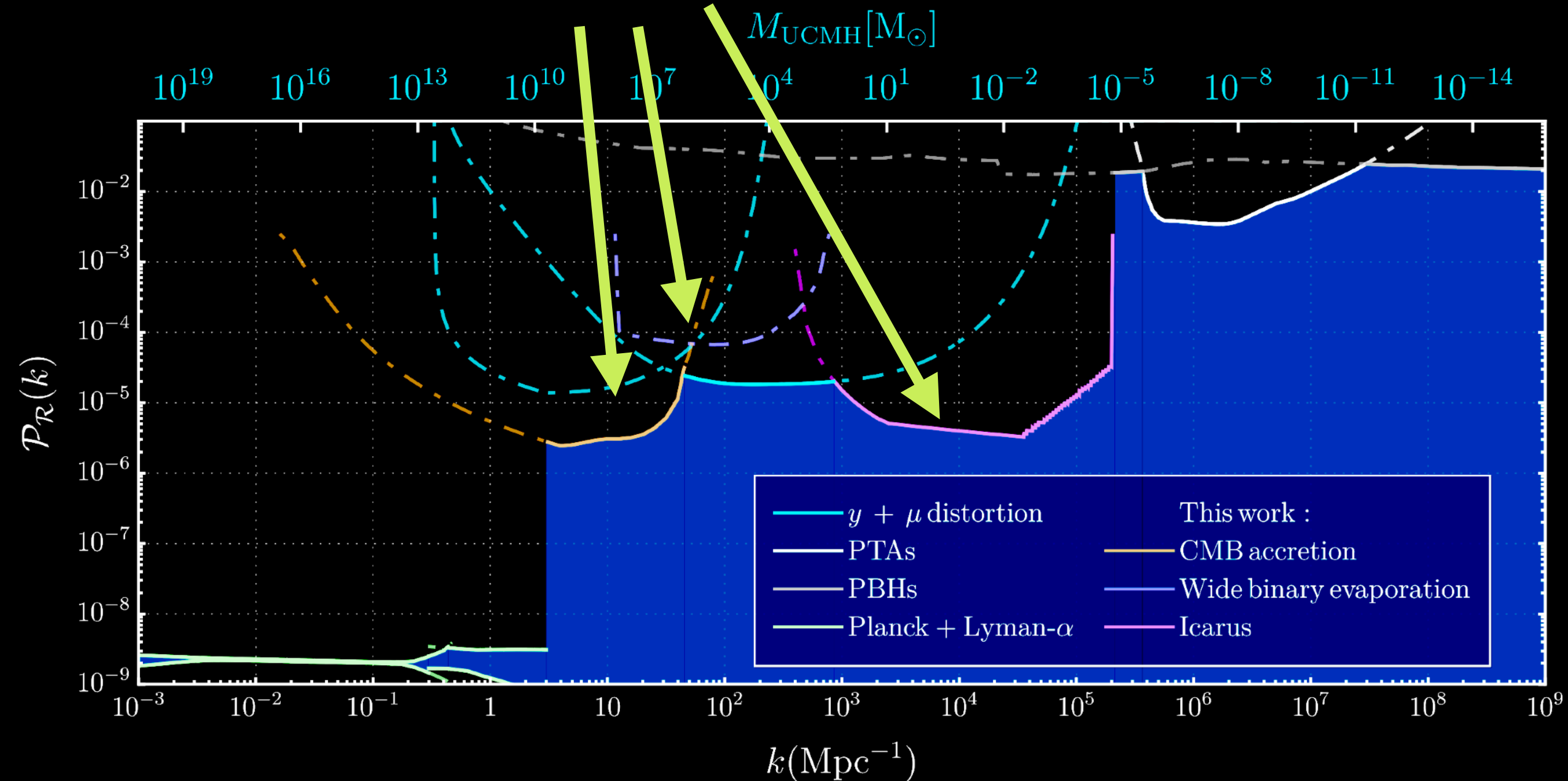
The primordial power spectrum



The primordial power spectrum

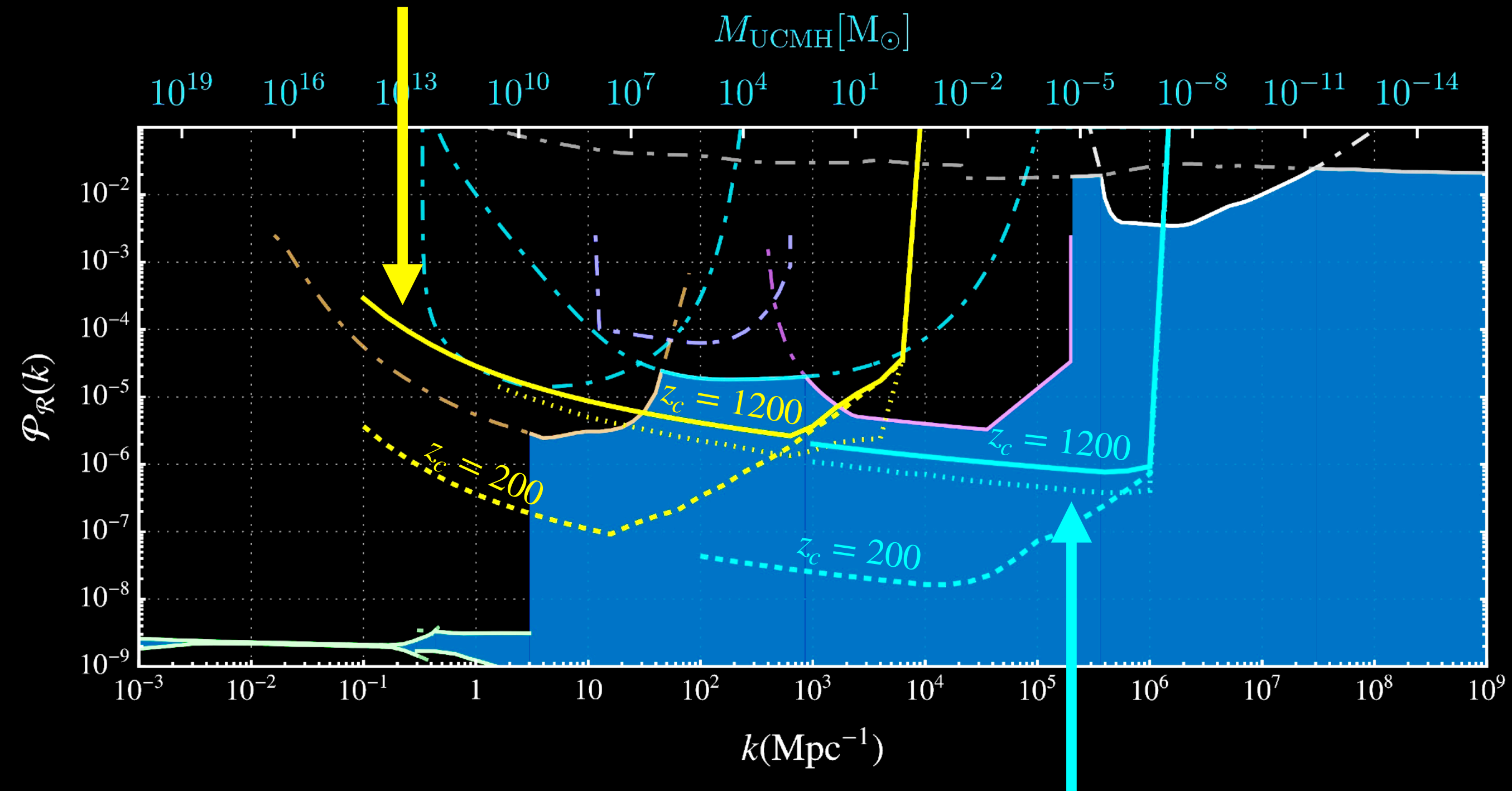
$z_c \geq 1200$, $\mathcal{P}_{\mathcal{R}}(k)$ flat
between $k_R/3 < k < 3k_R$

New constraints $z_c \geq 1200$



The primordial power spectrum

SN microlensing* with LSST, assuming $f_{\text{DM}} \lesssim 10^{-2}$

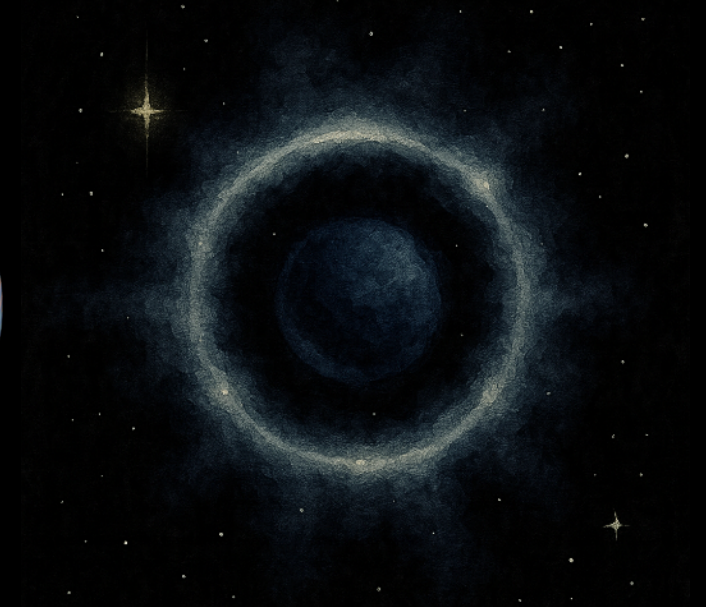


Future CC microlensing event $\mu_t \geq 300$

* See Shah et al. (DES), MNRAS (2024),
Zumalacarregui & Seljak (JLA + Union 2.1), PRL (2018)

To conclude,

- The primordial / matter power spectrum and sub halo mass function are interesting but hard to probe on small scales!
- Enhancements produce cosmic relics: primordial black holes or ultracompact minihalos
- Non-observations of these relics can then be used to constrain the curvature power spectrum on otherwise inaccessible scales



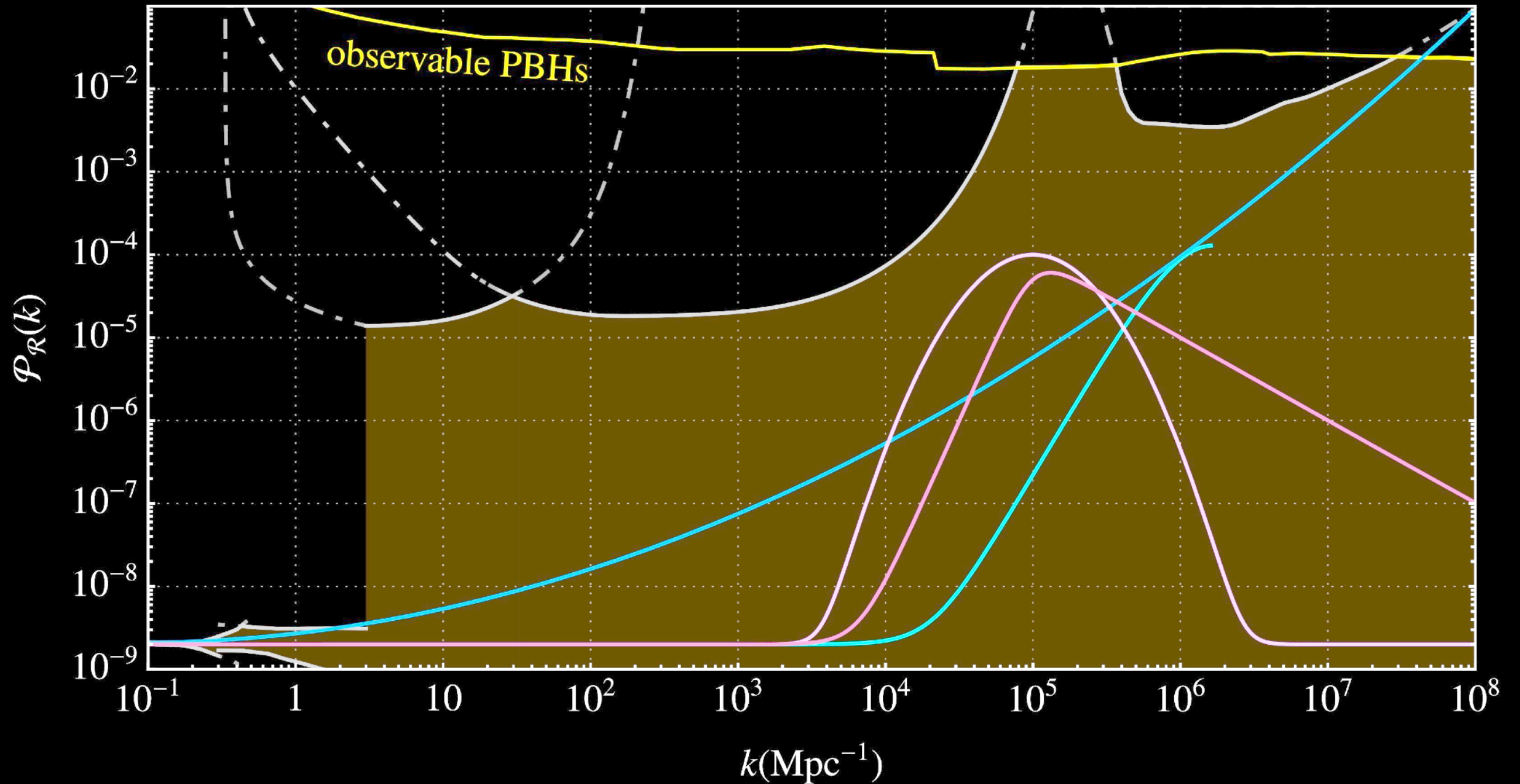
Thank you!

...ask me anything you like!

djuna.l.croon@durham.ac.uk | djunacroon.com



Cosmology's small scale frontier



Cosmology's small scale frontier

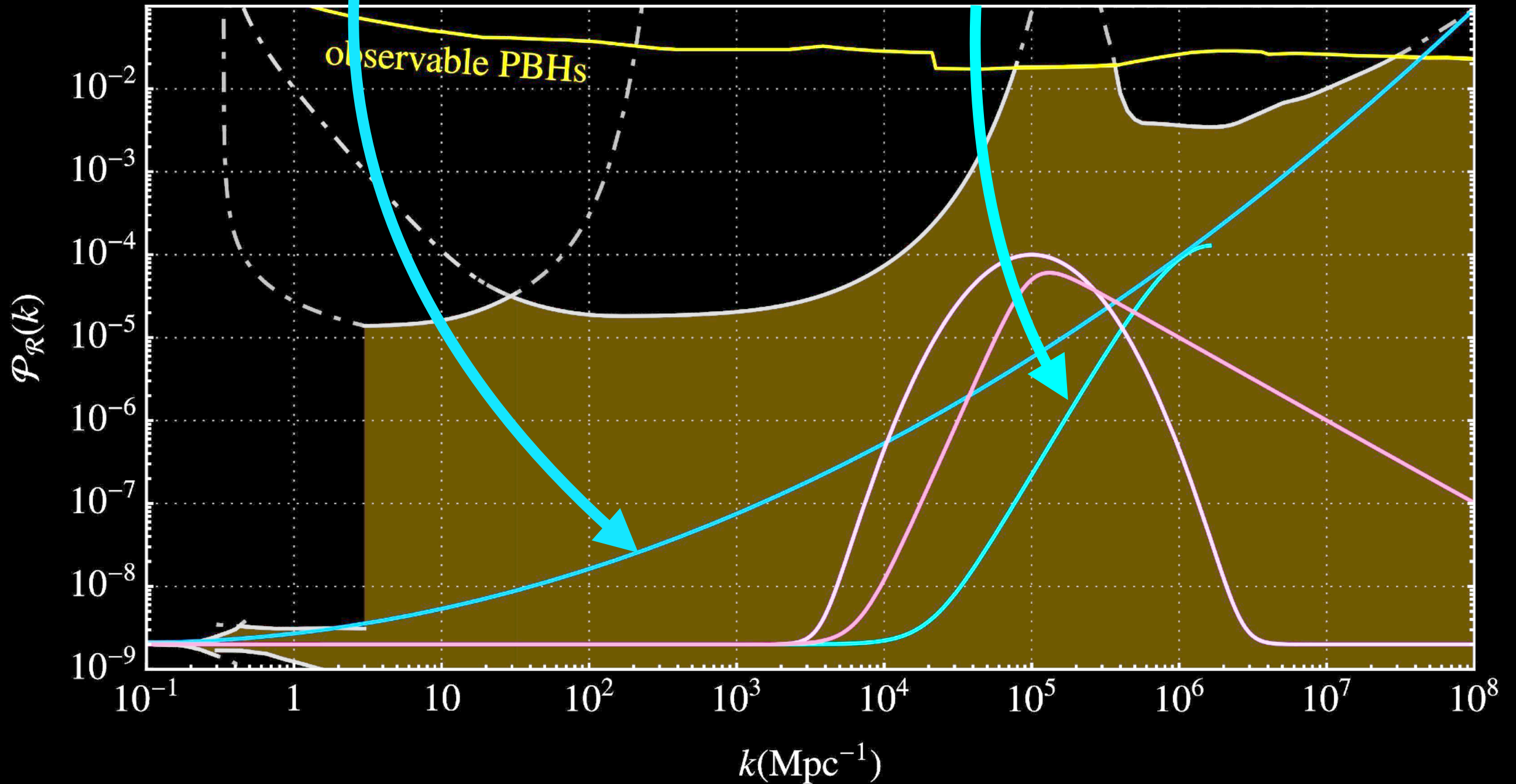
Running spectral index

$$n_s(k) = n_s + 0.08 \ln(k/k_*)$$

First order phase transition

$$\alpha = 0.5; \beta/H = 5; T_r = 100 \text{ MeV}$$

Wang, Balazs, Ding, Tian, PRD, arXiv:2601.14412



Cosmology's small scale frontier

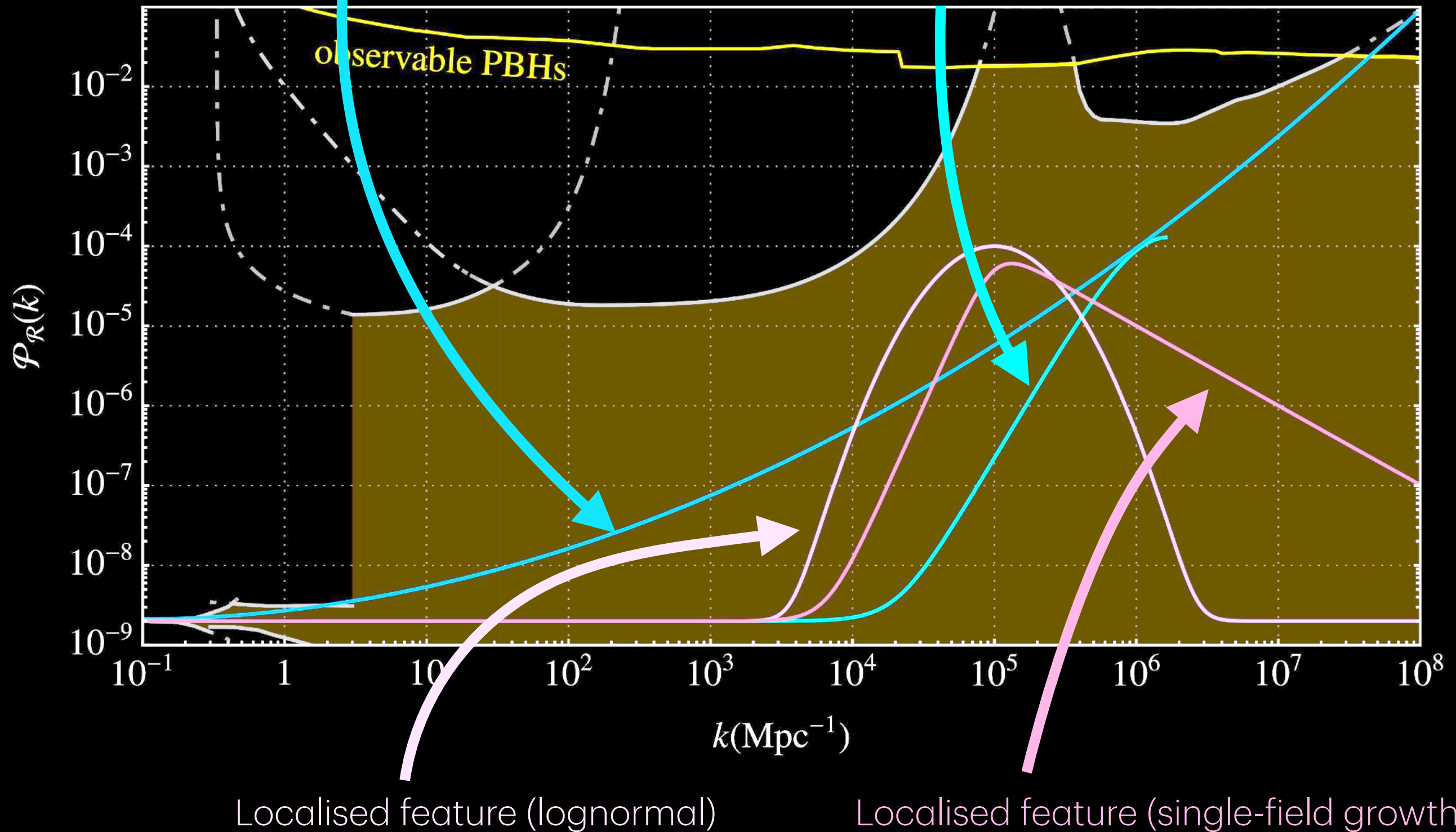
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Wang, Balazs, Ding, Tian, PRD, arXiv:2601.14412



Caustic crossings in giant arcs

$$u_1 = \frac{\tau_1}{\mu_r} - \frac{\tau_1}{\tau^2}$$

$$u_2 = \frac{\tau_2}{\mu_t} - \frac{\tau_2}{\tau^2}$$

tangential and radial
macro-magnifications

Oguri et al, PRD,
arXiv:1710.00148



$$u_1 = \frac{\tau_1}{\mu_r} - \frac{\tau_1}{\tau^2} m(\tau)$$

$$u_2 = \frac{\tau_2}{\mu_t} - \frac{\tau_2}{\tau^2} m(\tau)$$

DC, Crossey, Diego, Kavenagh,
Palencia, PRD, arXiv:2511.20761

$\propto$ Local convergence



$\propto$ Mean convergence



$\propto$ Tangential shear



$$\mu^{-1} = \left| \left(\frac{1}{\mu_r} - \frac{m'(\tau)}{2\tau} \right) \left(\frac{1}{\mu_t} - \frac{m'(\tau)}{2\tau} \right) - \left(\frac{m'(\tau)}{2\tau} - \frac{m(\tau)}{\tau^2} \right)^2 + \left(\frac{1}{\mu_r} - \frac{1}{\mu_t} \right) \left(\frac{m'(\tau)}{2\tau} - \frac{m(\tau)}{\tau^2} \right) \cos 2\phi \right|$$

$\left\{ \begin{array}{l} m(\tau) = 0 \rightarrow \text{recover macro-lens only result } \mu^{-1} = (\mu_t \mu_r)^{-1} \\ m(\tau) = 1 \rightarrow \text{recover point-like result} \end{array} \right.$