Probing Energy Correlators in Heavy States

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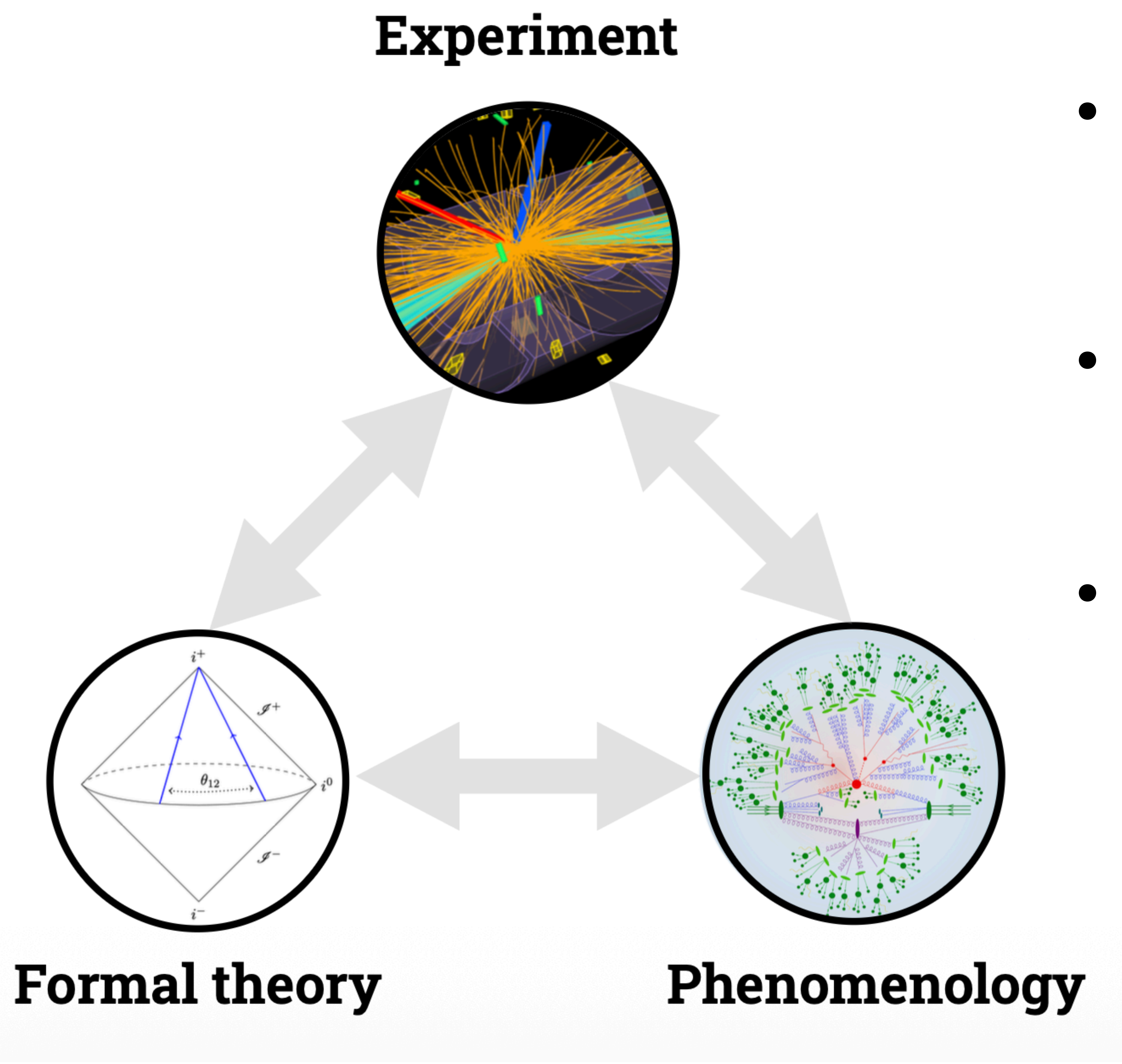
Based on work to appear with Alba Grassi, Cristoforo Iossa and Alexander Zhiboedov

Introduction

One Observable,Three Communities



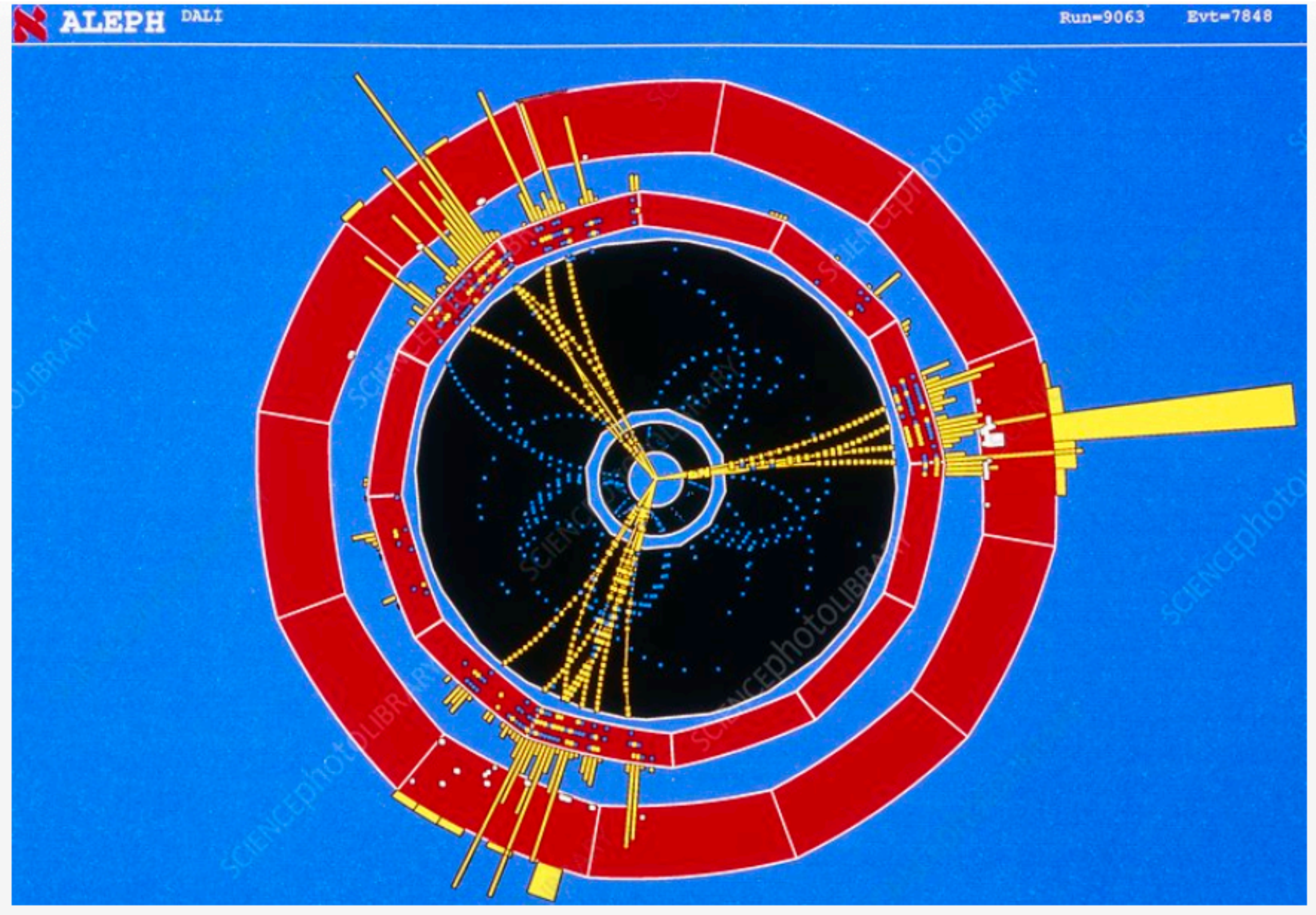
- Experiment: directly measurable energy flow in e^+e^-, pp, and heavy-ion collisions
- Formal theory: conformal colliders, light-ray operators, positivity
- Phenomenology: factorization and precision calculations, physics of jets

[Moult, Zhu 25]

Energy Flow

Energy is positive and conserved, its flow is one of the simplest universal probes of dynamics.

Today we are interested in energy correlators:



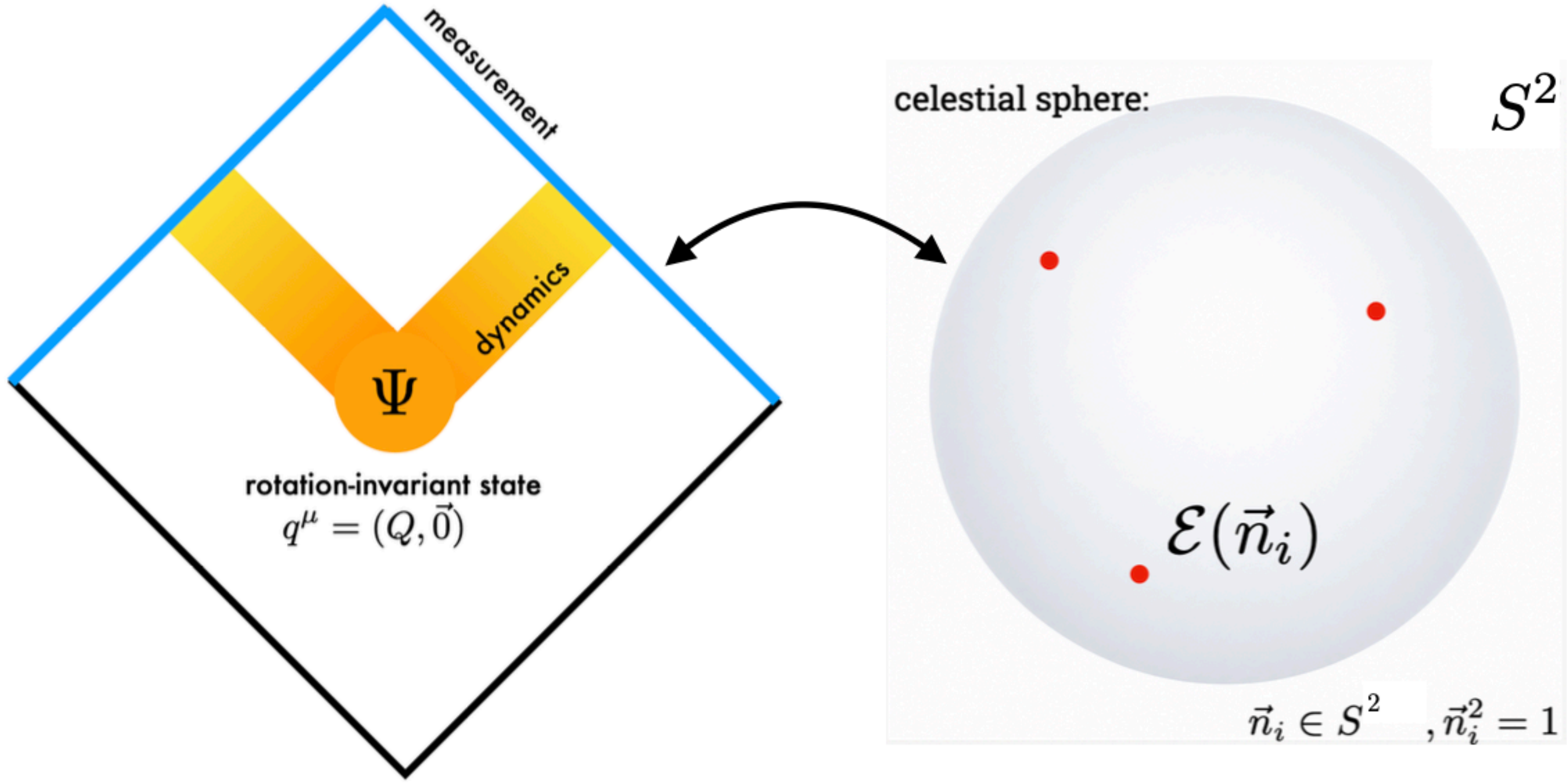
$$EEC(z) \equiv \left\langle \sum_{i,j} E_i E_j \delta(z - z_{ij}) \right\rangle_{events}$$

- Averaging over events/final states
- Weighting pairs by energy $E_i E_j$
- Bin by angular separation $z_{ij} = (1 - \cos\theta)/2$

[Basham, Brown, Ellis, Love 78]

Detectors

The energy flux operator is an idealized detector placed at infinity:



$$\mathscr{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_{t_0}^{\infty} dt\, n^i T_{0i}(t, r\vec{n}) \ , \quad \partial_\mu T^{\mu\nu} = 0$$

[Sveshnikov, Tkachov, Korchemsky, Sterman, Hofman, Maldacena]

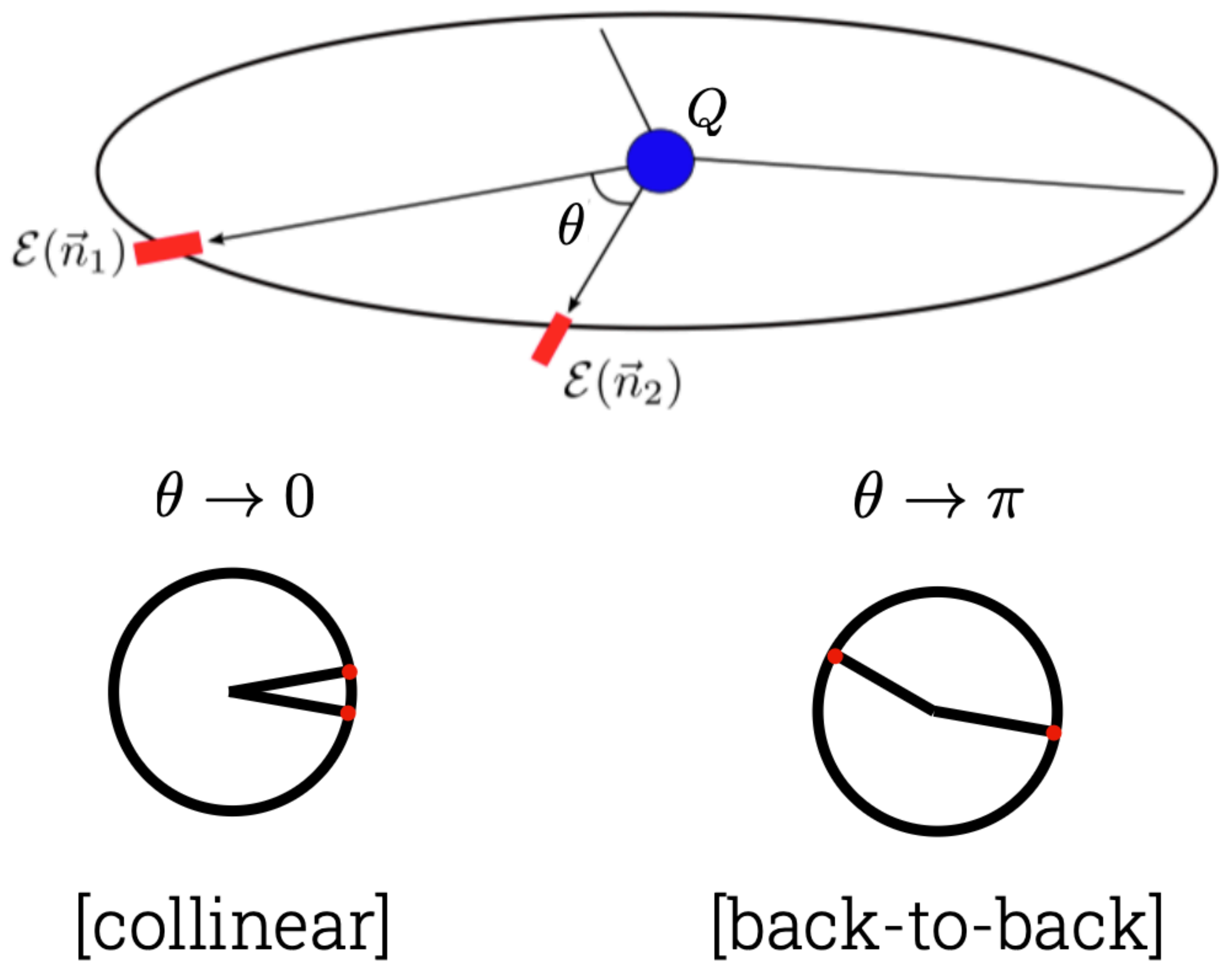
To compute energy correlators we should consider:

$$\langle \mathscr{E}(\vec{n}_1)\ldots\mathscr{E}(\vec{n}_k)\rangle \equiv \langle \psi | \, \mathscr{E}(\vec{n}_1)\ldots\mathscr{E}(\vec{n}_k) \, | \psi \rangle$$

Local operators prepare the state $|\psi\rangle$, detectors $\mathscr{E}(\vec{n})$ measure what reaches infinity.

Energy-Energy Correlators

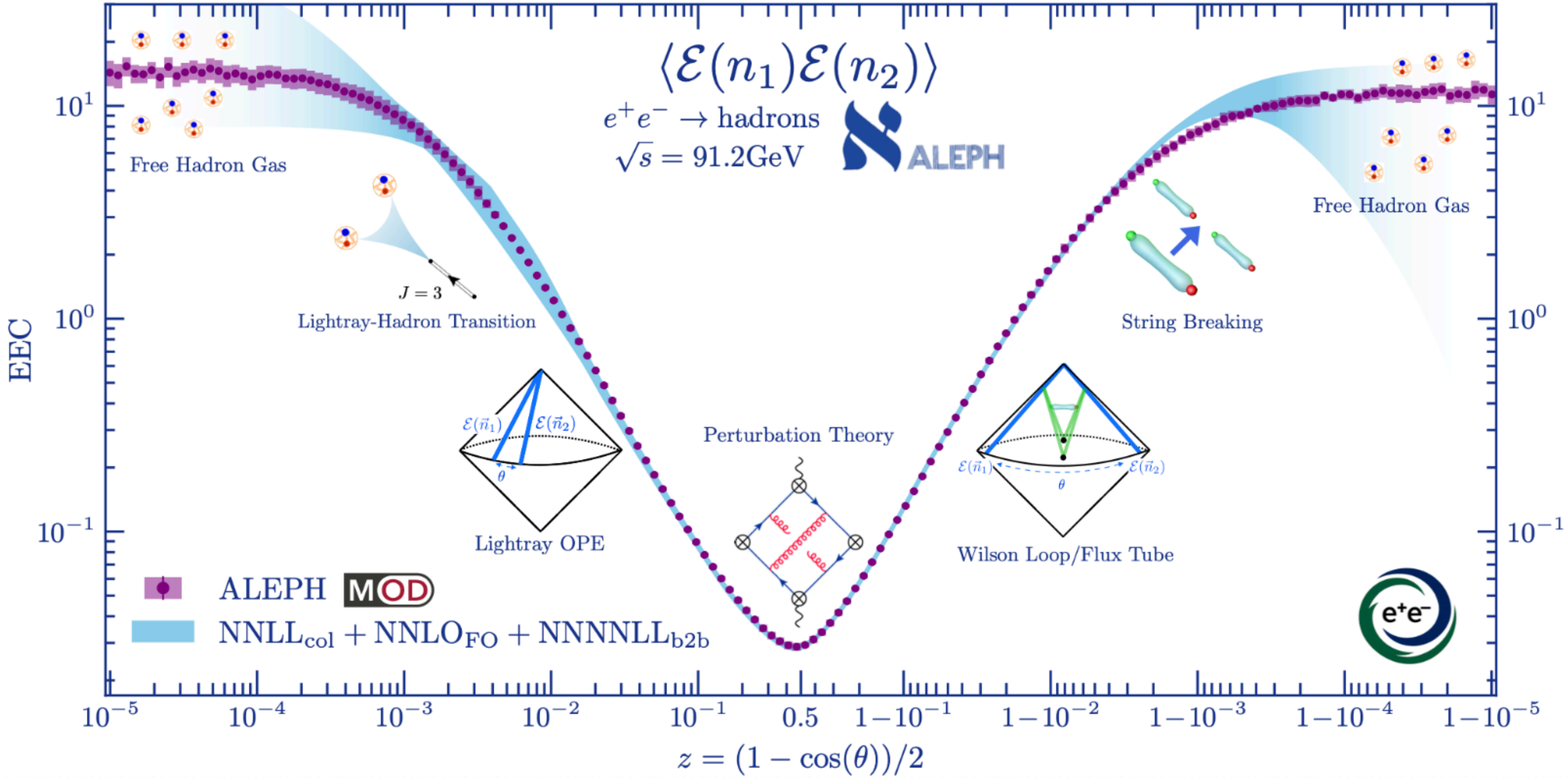
Placing two detectors $\mathscr{E}(\vec{n}_1), \mathscr{E}(\vec{n}_2)$ separated by an angle θ reveals rich dynamics:



$$z = \frac{1 - \cos\theta}{2}$$

- $0 \leq z \leq 1$
- $z \to 0$, collinear limit, light-ray operators
- $z \to 1$, back-to-back limit, soft physics, null Wilson lines

EEC in QCD



[Bossi et al. 25, ALEPH archival data]
[Jaarsma,Li,Moult,Waalwijn,Zhu 25]

$\mathcal{N} = 4$ SYM: A Theoretical Lab

Goal: calculate the same observable with far better analytical control

Energy correlators are best understood for states close to the vacuum: few-particle states, jets, and other perturbative excitations.

[See however: Chicherin, Korchemsky, Sokatchev, Zhiboedov, 23]
[Cuomo, Firat, Nardi, Ricci, 25]

But many physical systems are not small perturbations of the vacuum: finite density, finite temperature, heavy-ion matter

It is believed that energy correlators may become useful probes of quark-gluon plasma dynamics in heavy-ion collisions.

[See for example: Barata, Kuzmin, Moult, Sadofyev, Silva 26]

We use $\mathcal{N} = 4$ SYM to give a very explicit and calculable CFT version of such heavy states

Heavy States in $\mathcal{N} = 4$ SYM

Heavy state EECs start from states created from a local heavy operator:

$$|H(q)\rangle = \int d^4x \; e^{iqx} O_H(x)\,|0\rangle$$

We work in $SU(N)$, $\mathcal{N} = 4$ SYM. The theory has six scalar fields Φ^I , $I = 1,\ldots,6$ charged under the $SO(6)_R$ R-symmetry. We introduce a polarization Y^I , $Y^2 = 0$ and define:

$$O_K(x, Y) \sim Y_{I_1}\ldots Y_{I_K}\mathrm{Tr}(\Phi^{I_1}\ldots\Phi^{I_K})$$

Here O_H will be a “canonical” heavy operator built out of m identical blocks of length K:

$$O_K^m(x, Y) \sim O_K(x, Y)^m \;, \qquad \Delta_H = mK$$

In the limit $m \to \infty$, O_K^m becomes the heavy insertion creating the state $|H\rangle$.

A New ’t Hooft Limit

More precisely, the useful limit is:

$$m \to \infty \qquad\qquad g_{YM}^2 \to 0 \qquad\qquad \lambda = m g_{YM}^2 \quad \text{fixed}\,.$$

This is a large-charge ’t Hooft limit, but it is NOT the usual planar gauge-theory ’t Hooft limit:

N is fixed, Δ_H is the large parameter

The coupling gets weaker as the charge grows, but the collective interaction remains finite

[Grassi, Komargodski, LT 19]

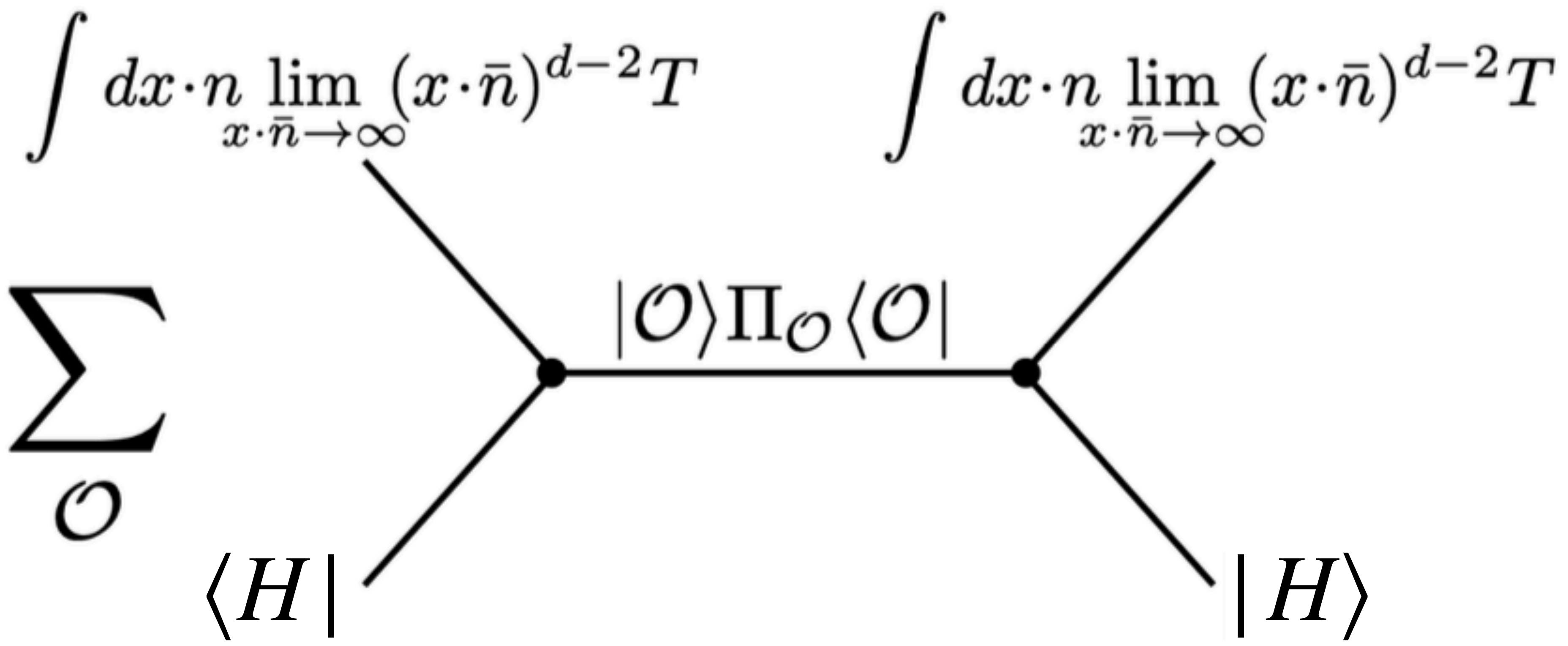
[Caetano, Komatsu, Wang, 23]

From CFT to EEC: Source Detector OPE

[Kologlu,Kravchuk,Simmons-Duffin,Zhiboedov19]

EEC can be expressed through CFT data: conformal dimensions and OPE coefficients

$$\langle \mathcal{E}\mathcal{E} \rangle = \sum_{\mathcal{O}}$$



Energy detectors are light-ray integrals of $T_{\mu\nu}$. In $\mathcal{N}=4$ SYM, $T_{\mu\nu}$ is a descendant of the same susy multiplet whose bottom component is the “light” 1/2-BPS scalar $O_2 \sim \mathrm{Tr}(\Phi^2)$

[Korchemsky, Sokatchev 15]

$$\langle O_H O_2 O_2 O_H \rangle \xrightarrow{\text{SUSY + Light-Ray}} \langle O_H \mathcal{E}\mathcal{E} O_H \rangle$$

Useful shortcut: compute with scalar half-BPS correlators, then take the detector/light-ray limit.

Computational Setup

In the rest frame $q^\mu = (Q, \vec{0})$, the expectation value:

$$\langle \mathscr{E}(\vec{n}) \rangle_H = \frac{Q}{4\pi}$$

We thus introduce normalized energy flow operators $\hat{\mathscr{E}}(\vec{n}) = \dfrac{\mathscr{E}(\vec{n})}{\langle \mathscr{E}(\vec{n}) \rangle}$ and compute:

$$\left\langle \hat{\mathscr{E}}(\vec{n}_1)\hat{\mathscr{E}}(\vec{n}_2) \right\rangle_H = 1 + \left\langle \hat{\mathscr{E}}(\vec{n}_1)\hat{\mathscr{E}}(\vec{n}_2) \right\rangle_{H,c}$$

The connected contribution is the interesting part: it is not fixed by symmetry and can encode real dynamics of the state. In particular we have that:

$$\left\langle \hat{\mathscr{E}}(\vec{n}_1)\hat{\mathscr{E}}(\vec{n}_2) \right\rangle_{H,c} = \mathscr{F}_{\Delta_H}(z)$$

We can determine the behavior of $\mathscr{F}_{\Delta_H}(z)$ exactly in λ!

EEC from Heavy-Light CFT Data

Using the source-detector OPE we can write the EEC as a sum over exchanged operators:

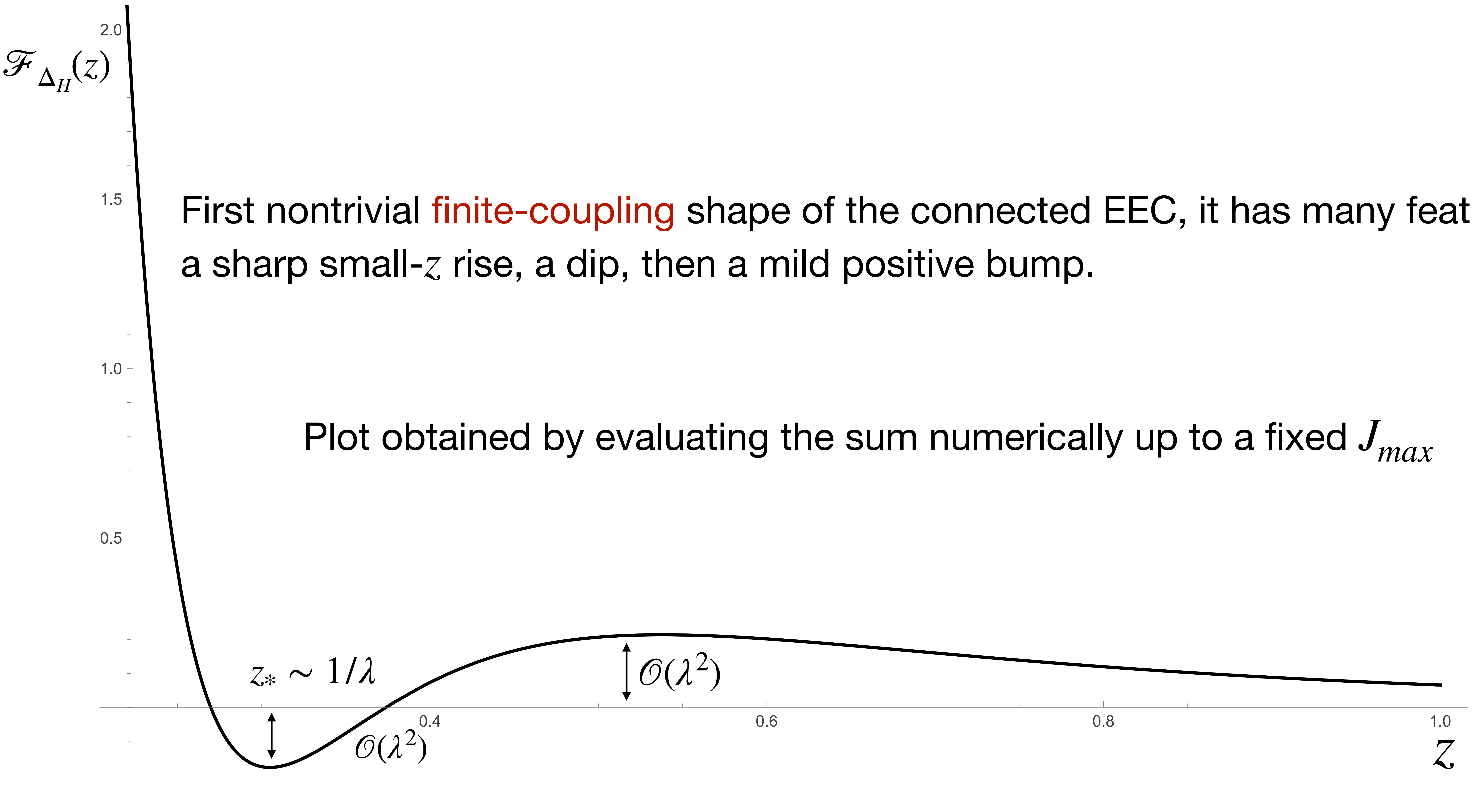
$$\mathcal{F}_{\Delta_H}(z) = \frac{1}{\Delta_H} \sum_{s=1}^{K} \nu_s \sum_{J \geq 0} \sum_{n \geq 0} \sum_{r=1}^{J+1} C^{(s)}_{J,n,r} S_{J,\gamma_{HL}}(z)$$

- ν_s is a multiplicity label, J: spin, n : excitation level
- $C^{(s)}_{J,n,r}$ is the Heavy-Light OPE 3-point function
- $S_{J,\gamma_{HL}}(z)$: detector superblock, giving the angular contribution of a given exchanged multiplet, depends on γ_{HL} appearing in $\Delta_{exch} = \Delta_H + \gamma_{HL}$.
- Both $C^{(s)}_{J,n,r}$ and γ_{HL} are known and depend explicitly on λ [Brown, Galvagno, Grassi, Iossa, Wen, 25]

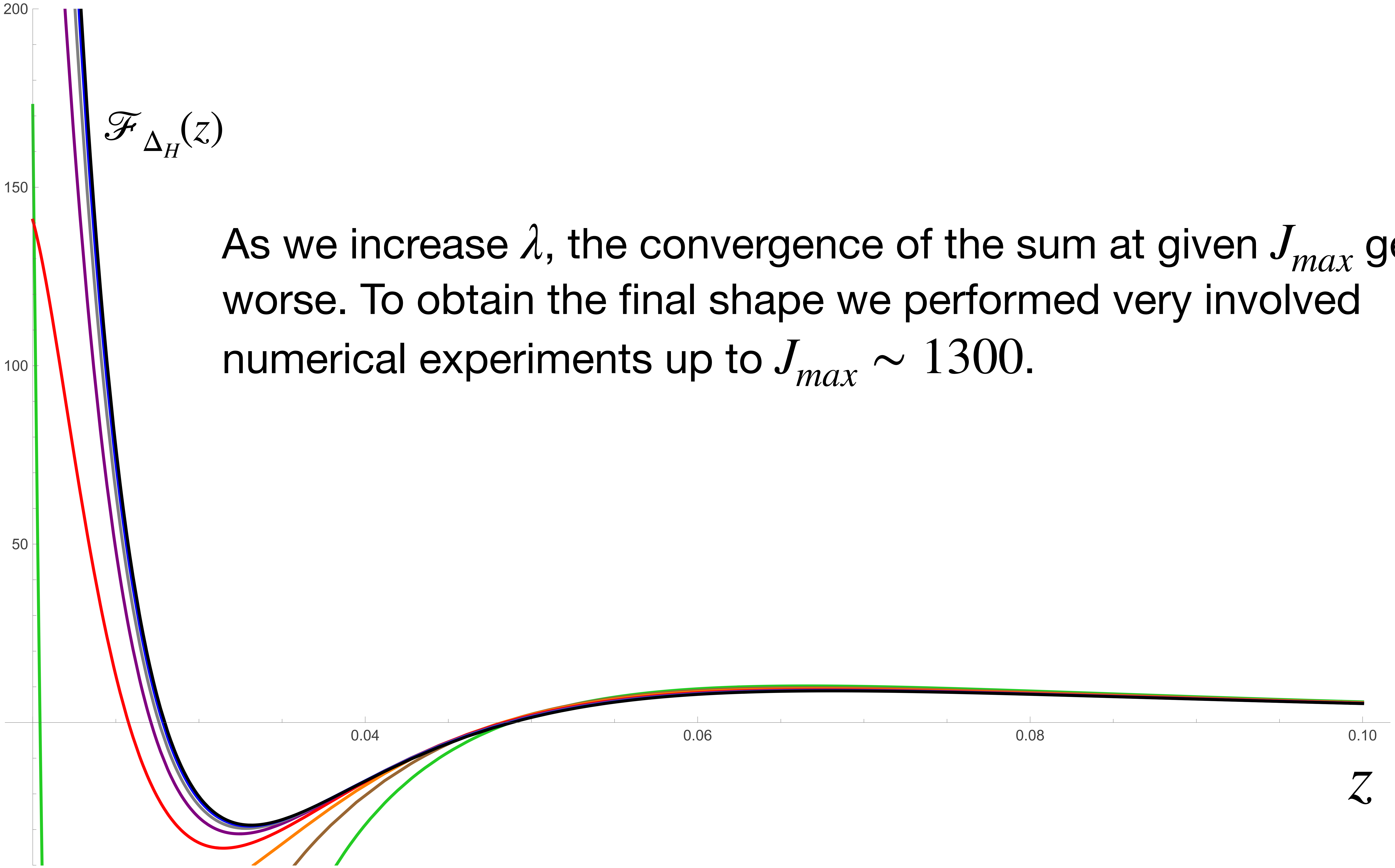
$\lambda = 1$

First nontrivial finite-coupling shape of the connected EEC, it has many features: a sharp small-z rise, a dip, then a mild positive bump.

Plot obtained by evaluating the sum numerically up to a fixed J_{max}

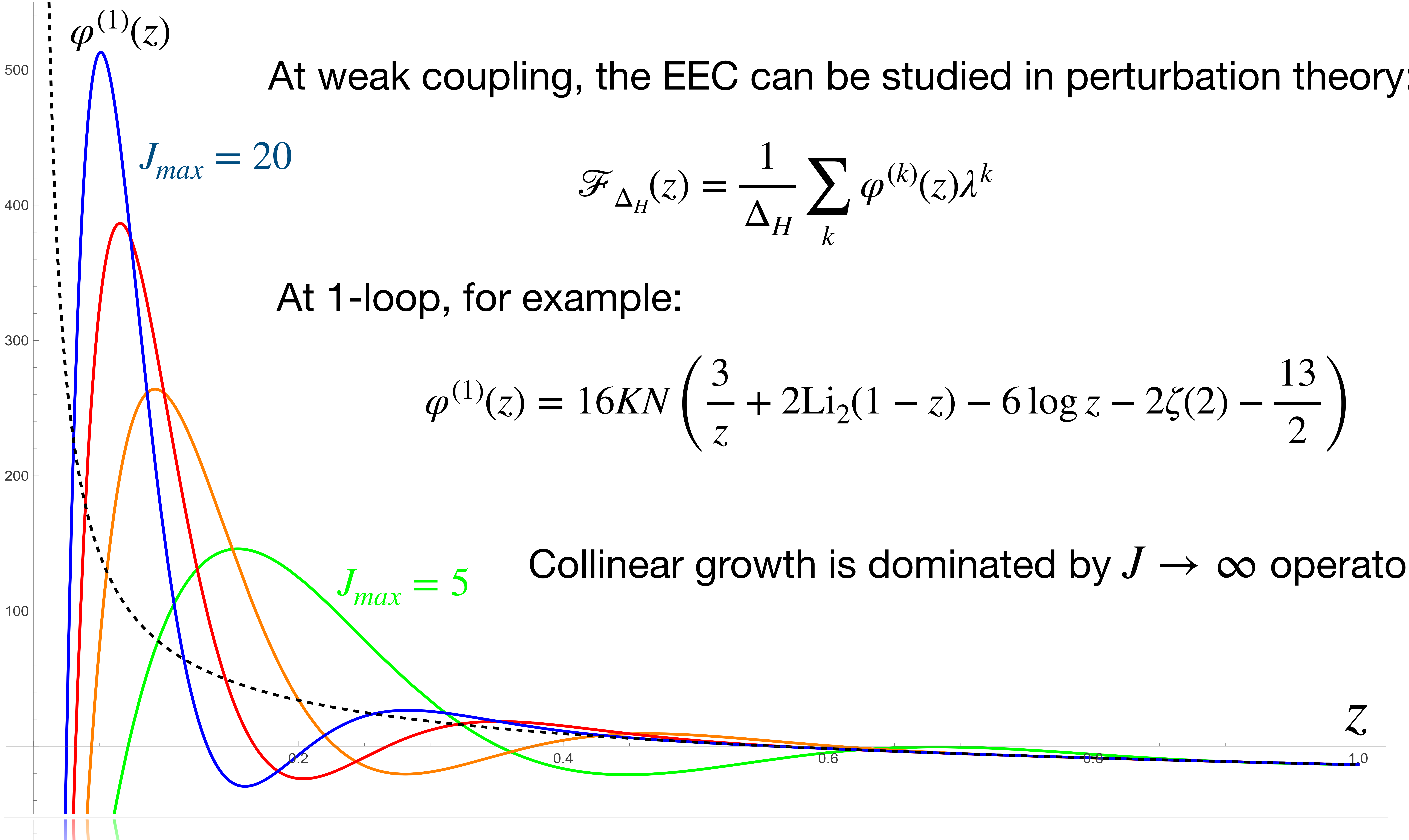


$\lambda = 10$



As we increase λ, the convergence of the sum at given J_{max} gets worse. To obtain the final shape we performed very involved numerical experiments up to $J_{max} \sim 1300$.

Small λ



At weak coupling, the EEC can be studied in perturbation theory:

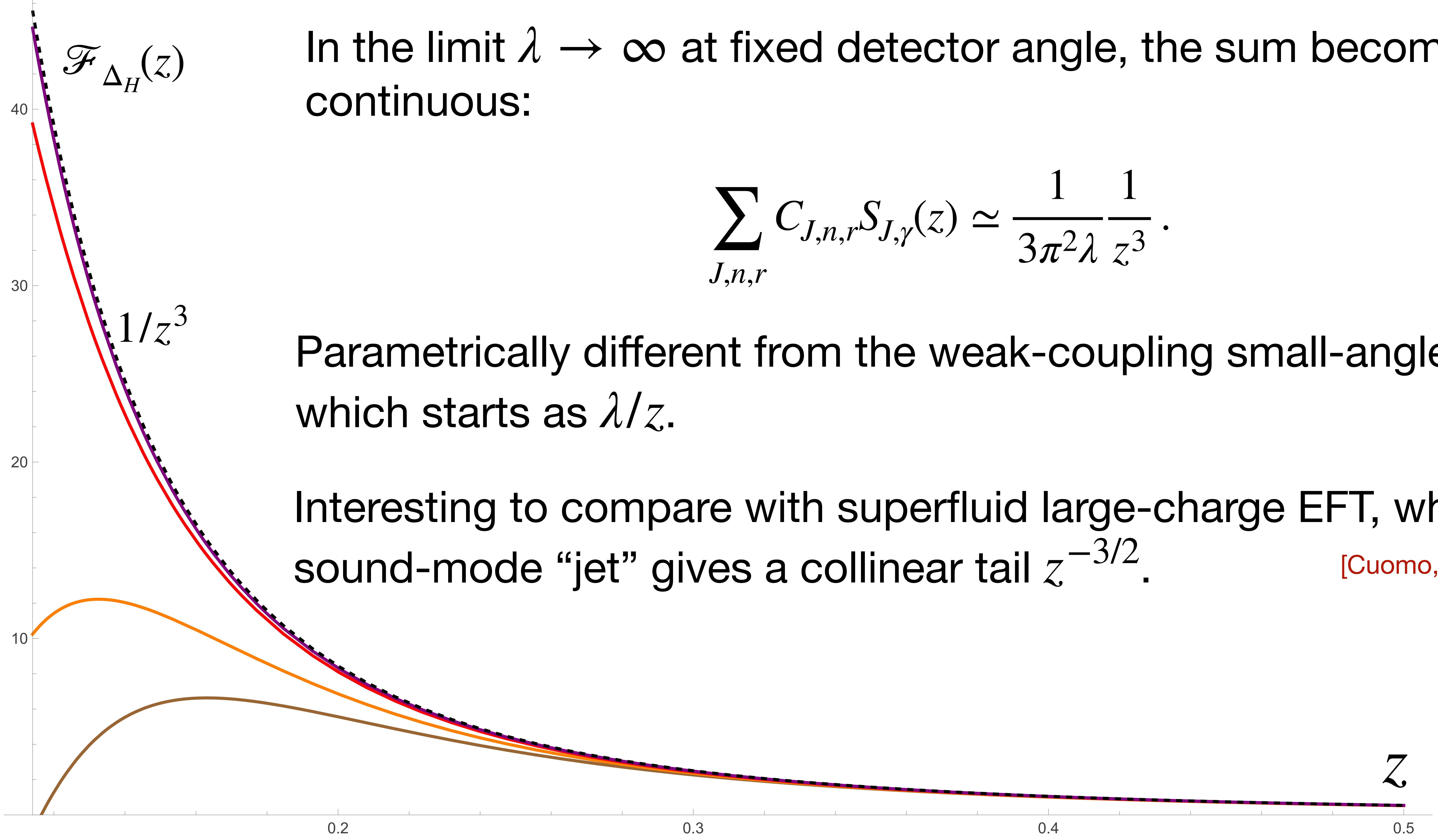
$$\mathcal{F}_{\Delta_H}(z) = \frac{1}{\Delta_H} \sum_k \varphi^{(k)}(z) \lambda^k$$

At 1-loop, for example:

$$\varphi^{(1)}(z) = 16KN \left(\frac{3}{z} + 2\mathrm{Li}_2(1-z) - 6 \log z - 2\zeta(2) - \frac{13}{2} \right)$$

Collinear growth is dominated by $J \to \infty$ operators.

Large λ



In the limit $\lambda \to \infty$ at fixed detector angle, the sum becomes effectively continuous:

$$\sum_{J,n,r} C_{J,n,r} S_{J,\gamma}(z) \simeq \frac{1}{3\pi^2\lambda}\frac{1}{z^3}\,.$$

Parametrically different from the weak-coupling small-angle behavior, which starts as λ/z.

Interesting to compare with superfluid large-charge EFT, where the sound-mode “jet” gives a collinear tail $z^{-3/2}$. [Cuomo, Firat, Nardi, Ricci, 25]

Summary

Energy correlators are a common language between collider physics and formal theory.

We computed the connected EEC in heavy large-charge states of $\mathcal{N} = 4$ SYM.

The answer is controlled by heavy-light CFT data and becomes highly structured as the coupling is varied.

This allows us to explore questions that are hard in QCD: how energy correlators respond to strong coupling, high density, and collective dynamics.

Outlook

Lots of open questions, come talk to me if you are interested!

Thanks!

Backup: Large Charge Data

$$C^{(s)}_{J,n,r} = \frac{(r+n)(J+2+n-r)}{\sqrt{(r+n)^2+4\lambda}\sqrt{(J+2+n-r)^2+4\lambda}}$$

$$\gamma_{HL} = \sqrt{(r+n)^2+4\lambda} + \sqrt{(J+2+n-r)^2+4\lambda} - 2$$

Backup: Multipole Expansion

Resolving the identity in the source-detector OPE gives energy multipoles:

$$\langle \mathscr{E}(\vec{n}_1)\mathscr{E}(\vec{n}_2)\rangle = \left(\frac{Q}{4\pi}\right)^2 \left[1 + \sum_{J=2}^{\infty} c_J P_J(\cos\theta)\right]$$

- energy multipoles $c_J \geq 0$ are constrained by unitarity
- energy/momentum conservation fixes $c_0 = 1,\, c_1 = 0$
- point-wise positivity and unitarity give complementary constraints.