Generalized Symmetries in QFT and Quantum Matter

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What is a Global Symmetry?

Textbook view: a symmetry is a group G acting on local operators, with a conserved Noether current $\partial^\mu j_\mu = 0$ and charge $Q = \int_{M_{d-1}} \star j$.

Modern view: the charge is a topological operator $U_g(M_{d-1})$ – correlators are invariant under deformations of M_{d-1}:

$$\text{conservation} \iff \text{topological invariance of } U_g(M_{d-1})\,.$$

It acts by linking: surrounding a charged operator $\mathcal{O}$ measures its charge,

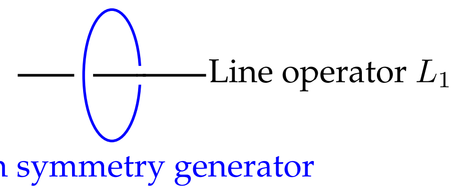
$$U_g(S^{d-1})\,\mathcal{O}(x) = g[\mathcal{O}]\,\mathcal{O}(x)\,.$$

Once a symmetry is just **a topological operator**, we can relax the following assumptions:

- no need for a current $\Rightarrow$ include finite symmetries
- the operator need not have codimension one $\Rightarrow$ higher-form symmetries
- operators of different dimension can mix $\Rightarrow$ higher-groups
- the operator need not be invertible $\Rightarrow$ non-invertible / categorical symmetries

New Structures: Generalized Symmetries

\# **Higher-form symmetries:** [Gaiotto, Kapustin, Seiberg, Willett]
Example: Line operators charged under 1-form symmetry, e.g. center symmetry $\mathbb{Z}_N$ of $SU(N)$.



Physics: Confinement = 1-form symmetry preserving phase.

\# **Higher-groups**: p-form symmetries of different degree are not independent – they fuse into a single structure (e.g. 2-groups), detected by characteristic mixed transformations.

\# **Non-invertible / categorical symmetries in higher dim** [Kaidi, Ohmori, Zheng][Choi, Cordova, Hsin, Lam, Shao][Bhardwaj, Bottini, SSN, Tiwari]: defects with no inverse, obeying a fusion algebra

$$a \otimes b = \bigoplus_c N_{ab}^c \, c \,, \qquad N_{ab}^c \in \mathbb{Z}_{\geq 0} \,,$$

so the symmetry is a (higher) fusion category, not a group.

All of these appear generically – especially in $d \geq 4$ QFT and on the lattice.

What type of non-invertible symmetries exist?

What theories realize them?

Duality Defects

Self-dualities give rise to non-invertible symmetries.

- 1+1d Kramers-Wannier (KW) duality symmetries:
  Critical Ising CFT has a $\mathbb{Z}_2$ spin flip symmetry η and

$$N \otimes N = 1 \oplus \eta$$

  This originates from the KW duality $g \to 1/g$ of the transverse field Ising chain

$$H = -\sum_j \sigma_j^z \sigma_{j+1}^z - g \sum_j \sigma_j^x \, .$$

  At $g = 1$, this becomes a non-invertible symmetry.

- 3+1d theories: [Kaidi, Ohmori, Zheng][Choi, Cordova, Hsin, Lam, Shao] using self-duality symmetries D

$$\text{QFT} \cong \text{QFT}/D \qquad \Rightarrow \qquad \mathcal{N}_3 \otimes \mathcal{N}_3^\dagger = \mathcal{C} = \text{condensation defect}$$

  condensation defect = gauge a higher form symmetry on a subspace

Self-duality defects exist in any even spacetime dimensions, $d = 2n$, where gauging an $(n-1)$-form symmetry gives back an $(n-1)$-form symmetry.

- 6d (2,0) theories:

  [Lawrie, Yu, Zhang][Apruzzi, SSN, Warman][Bonetti, del Zotto, Minasian]

  Self-duality from **Green-Schwarz (GS) automorphisms**, i.e. automorphisms of lattice of BPS string charges

  $$\text{GS}: \qquad \Lambda_{\text{BPS}} \to \Lambda_{\text{BPS}}$$

  combined with

  - stacking a 2-form symmetry SPT $\exp(i\pi \int_{M_6} C_3 \cup C_3)$
  - gauging the 2-form symmetry, i.e. summing over background fields C_3

  result in non-invertible G-ality defects, where G= group formed by the GS-automorphisms.

Gauging Outer Automorphisms

Any outer automorphism can be gauged to give rise to a non-invertible symmetry [Bhardwaj, Bottini, SSN, Tiwari]

Example: $O(2)$ gauge theory as $U(1)/\mathbb{Z}_2^{\text{cc}}$, gauging charge conjugation.

There is a 1-form symmetry generated by $D_\alpha := e^{i\alpha \int *F}$.

Charge conjugation maps $*F \to - * F$ and so

$$\mathbb{Z}_2^{\text{cc}} : \; D_\alpha \to D_{-\alpha}$$

The invariant combination is

$$D_\alpha^{\text{inv}} = D_\alpha \oplus D_{-\alpha}$$

which has non-invertible fusion * is

$$D_\alpha^{\text{inv}} \otimes D_\alpha^{\text{inv}} = 1 \oplus D_{2\alpha}^{\text{inv}}$$

* this depends on α and also should include condensation defects on the RHS.

ABJ Anomalies

Any ABJ anomaly – usually viewed as a non-symmetry – can be reinterpreted as a non-invertible symmetry. [Choi, Lam, Shao][Cordova, Ohmori]

Example: 4d QED with massless charge 1 Dirac fermion

$$\mathcal{L}_{\text{QED}+\Psi} = \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}\left(\partial_\mu - iA_\mu\right)\gamma^\mu \Psi$$

the axial current $j_\mu = \frac{1}{2}\bar{\Psi}\gamma_5\gamma_\mu\Psi$ is not conserved due to the ABJ anomaly

$$d \star j = \frac{1}{8\pi^2} F \wedge F$$

Define an operator dressed by 3d Topological QFT that has opposite anomaly

$$\mathcal{N}_{\frac{1}{N}}(M_3) = \int [Da] \exp\left(\int_{M_3} \frac{2\pi i}{N} \star j + \frac{iN}{4\pi} ada + \frac{i}{2\pi} adA\right) .$$

It is topological, but satisfies non-invertible fusion

$$\mathcal{N}_{\frac{1}{N}} \times \mathcal{N}^\dagger_{\frac{1}{N}} = \mathcal{C} = \text{condensation defect for 1-form symmetry}$$

Topological Phases of Matter

2+1-dimensional topologically order (TO) is a **gapped** quantum many-body phase which is not an SSB phase for 0-form symmetry and which exhibits **long-range quantum entanglement**. I.e. it cannot be transformed into a product state by a finite depth circuit.

(2+1)d topological phases of matter have a characterization in terms of **anyons**, i.e. **topological line operators**:

- non-invertible composition $a \otimes b = \oplus_c N_{ab}^c c$
- braiding.

Simplest examples: Finite G gauge theory (or Dijkgraaf Witten theory) in 2+1d. Abelian G has invertible fusion, non-abelian has non-invertible fusion.

Applications:

- 2+1d topological order: quantum hall states, e.g. Ising TO, $SU(2)_k$ CS theory etc.
- Topological quantum computing: either using braiding of anyons or surface code.

What are the representations of generalized symmetries?

Selection rules?

Things we expect from global symmetries:

- Charges are "Generalized Charges", which correspond to representations of the non-invertible symmetry.
- Selection Rules and Anomalies. 't Hooft anomalies and generalized charges for non-invertible symmetries constrain IR dynamics.
- Phases of Matter. Generalized symmetries control gapped and gapless phases **beyond** Landau

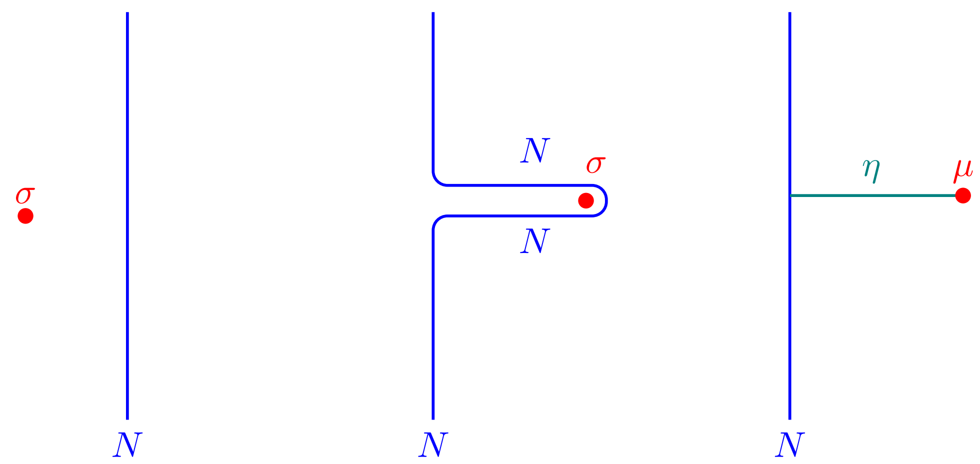
Unifying tool: **the SymTFT** A $(d+1)$-dimensional topological field theory packages the symmetry, its charges, anomalies, gaugings, and phases in a single object; close in spirit to "holography".

The full physical scope of generalized symmetries is only being explored. In particular **non-invertible symmetries**.

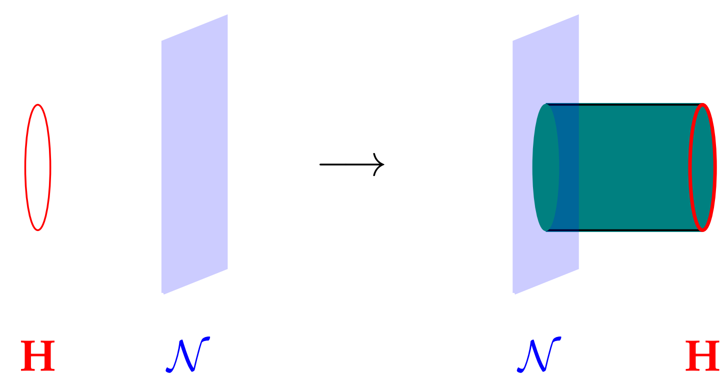
Generalized Charges

They map genuine operators to non-genuine, i.e. endpoints of extended (topological) defects, or: order to disorder operators.

Example: Ising CFT $N^2 = 1 + \eta$, act on spin operator σ (1/16 primary):



Example: Witten effect. 4d $SO(3)$ SYM: 't Hooft loop gets flux attachment



General representation theory: [Bhardwaj, SSN]^2[Bartsch, Bullimore, Ferrari, Pearson]

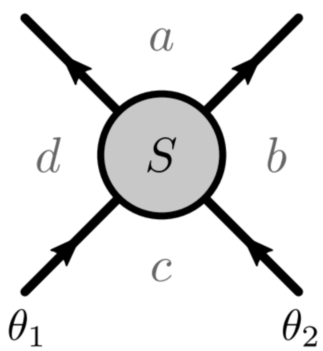
Selection Rules: Modified Crossing Relations

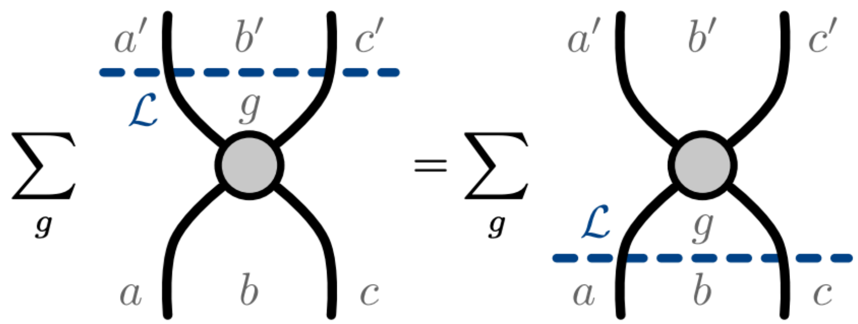
Non-invertible symmetries lead to modified crossing relations for S-matrices!

Example: (1+1)d CFTs have non-invertible symmetries, generated by lines $\mathcal{L}$

Relevant, integrable deformations can preserve some of $\mathcal{L}$.
$\Rightarrow$ IR are gapped vacua. $\mathcal{L}$ constrains S-matrix of kinks through Ward ids:

$S^{ab}_{dc}(\theta) =$





[Copetti, Lucia Cordova, Komatsu] showed: crossing incompatible with symmetry/integrability/unitarity. Consistency implies modified crossing

$$S^{ab}_{dc}(\theta) = \sqrt{\frac{d_a d_c}{d_b d_d}}\, S^{bc}_{ad}(i\pi - \theta)\,, \qquad d_a = \langle \mathcal{L}_a \rangle \tag{1}$$

Modified crossing 2+1d see: [Mehta, Minwalla, Patel, Prakash, Sharma]

Recent applications: **fermion-monopoles** as scattering off non-invertibe defects [van Beest et al][Antinucci et al][Arias-Tamargo et al][$\cdots$]

Application: Classifying Symmetric Phases

Non-invertible Symmetries lead to new IR phases, and new second order Phase Transitions!

Landau paradigm:
A 2nd order phase transition is a symmetry breaking transition for a group G.

- Gapped Phases: G spontaneously broken (SSB) to subgroup H. Phase has $|G/H|$ vacua, which are acted upon by the broken symmetry.
- Phase transitions:
  Unbroken symmetry group $H_i \subset G$ in each gapped phase, then there is a transition if $H_1 \subset H_2$
- Order Parameters:
  field transforming trivially in H_1, non-trivially in H_2.

Categorical Landau paradigm: [Bhardwaj, Bottini, Pajer, SSN][Bhardwaj, Pajer, SSN, Warman]:

$\mathcal{S}$ be a non-invertible symmetry, e.g. 1+1d fusion category symmetries.
Equally applicable approach in any dimension.

- Gapped Phases:
  Using the so-called SymTFT we can propose a full classification of gapped phases.

- Phase Transitions:
  We furthermore can give necessary conditions for a symmetric second order phase transition:

$$\boxed{\mathcal{S} \text{ Gapped Phase}} \quad \longleftarrow \quad \boxed{\text{CFT}} \quad \longrightarrow \quad \boxed{\mathcal{S} \text{ Gapped Phase'}}$$

- Order Parameters:
  generalized charges, which can be read off from the SymTFT approach.

$\Rightarrow$ **Categorical Landau Paradigm** [Bhardwaj, Bottini, Pajer, SSN]

Systematic approach to classification, and new phases

Applications to Particle Physics*

* So far not too many sharp applications here. Probably the applications to particle physics have so far been the least surprising.

Generalized Symmetries: Applications to the Standard Model

The global form of the SM gauge group is not fixed by the Lie algebra:

$$G_{\text{SM}} = \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_n}, \qquad n \in \{1, 2, 3, 6\}.$$

The quotient $\mathbb{Z}_n$ is an **electric 1-form symmetry** acting on Wilson lines [Tong '17].

- The six allowed forms agree on all local physics, but differ in their spectrum of line operators and magnetic monopoles.
- Distinguishing them directly is in principle possible, but probably not practical

Beyond 1-form symmetries:

- Magnetic 1-form symmetries organize the monopole sector.
- Anomalies of these higher symmetries give nonperturbative constraints on matter content, hypercharge quantization and global consistency of the SM.

[Tong][Anber, Poppitz][Hsin, Gomis][...]

ABJ-Non-Invertible Symmetries in Particle Physics

Classical $U(1)$ symmetries of the SM broken only by the ABJ anomaly become **non-invertible**, not absent: dress the current with a 3d TQFT

$$\mathcal{N}_{1/N}(M_3) = \int [Da] \, \exp\left(\int_{M_3} \frac{2\pi i}{N} \star j + \frac{iN}{4\pi} a \, da + \frac{i}{2\pi} a \, dA \right) .$$

- Chiral / axial symmetry of QED and QCD: exact as a non-invertible symmetry, with $\mathcal{N} \times \mathcal{N}^\dagger = \mathcal{C}$ [Choi, Cordova, Hsin, Lam, Shao][Cordova, Ohmori].
- $\pi^0 \to \gamma\gamma$: pion decay is controlled by the non-invertible chiral symmetry, giving a selection rule on the coupling.
- $B + L$: non-invertibly conserved – electroweak instantons spoil invertibility, but a non-invertible defect survives [Choi, Lam, Shao].
- Model building: Z' with non-invertible chiral symmetry $\Rightarrow$ exponentially small breaking, relevant for neutrino masses [Cordova, Hong, Koren, Ohmori].

Quark-Lepton Color-Flavor Unification

[Delgado, Koren '26]

- A flavor-based unification: gauge quark color-flavor $SU(9)$ together with lepton flavor $SU(3)$ into

$$SU(12) \times SU(2)_L \times U(1)_R \,.$$

- A single Yukawa shared by up-type quarks and neutrinos; no new fermions.
- The SM emerges in the IR as a **quotient of the SM gauge group**

$$G_{\text{SM}} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y \times \mathbb{Z}_{18}^X}{\mathbb{Z}_3 \times \Gamma \times \mathbb{Z}_3}\,, \qquad \Gamma \in \{1, \mathbb{Z}_2\}\,.$$

Has good properties in terms of proton stability etc, and a rich structure of higher group symmetries to pull this off.

Applications to hep-ph: probably needs further exploration to really give striking implications.

How to systematically study

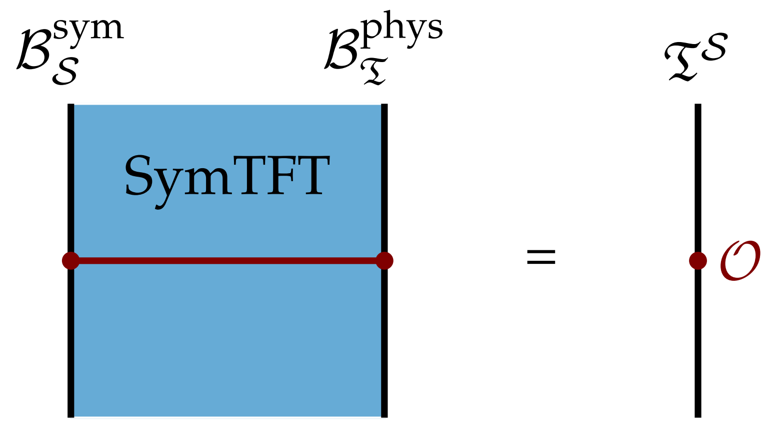
Generalized/Non-Invertible Symmetries?

SymTFT

Symmetry TFT (SymTFT) Sandwich

[Ji, Wen][Gaiotto, Kulp][Apruzzi, Bonetti, Garcia-E, Hosseini, SSN][Freed, Moore, Teleman]

Let $\mathfrak{T}$ be a QFT with finite symmetry $\mathcal{S}$ in d dimensions. The SymTFT is a $d+1$ dimensional TQFT obtained by gauging $\mathcal{S}$ in $(d+1)$ dims:



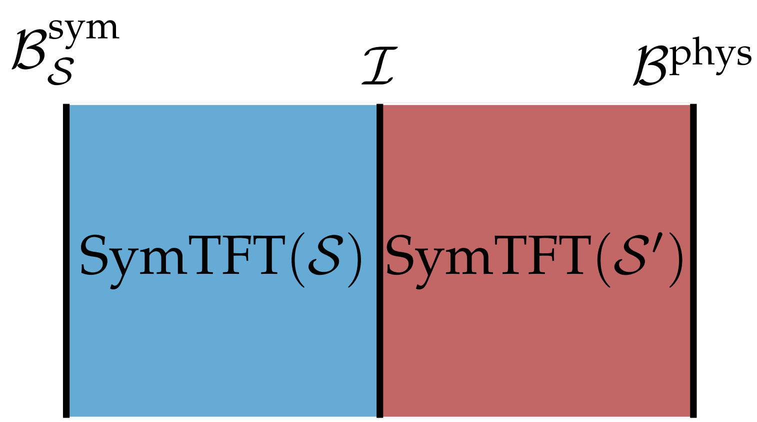
For G a group, this is the Dijkgraaf Witten theory for G.

- Topological defects of the SymTFT: In $d+1=3$: anyons
- $\mathcal{B}^{\text{sym}}_{\mathcal{S}}$ = Symmetry boundary, gapped, realizes symmetry
- $\mathcal{B}^{\text{phys}}_{\mathfrak{T}}$ = Physical boundary, encodes dynamics
- Order paramters are generalized charges (anyons) that end on $\mathcal{B}^{\text{phys}}$. If they also end on $\mathcal{B}^{\text{sym}}$ then these are local order parameters.

SymTFT has natural origin in holography, e.g. topological couplings in supergravity, and in fact in CM is referred to as ”topological holography”

Interfaces and SymTFT Club Sandwiches

We can study topological interfaces from the SymTFT of $\mathcal{S}$ to other TQFTs (or topological orders), by condensing anyons:



Studying this configurations, allows us to

- categorical Landau phases, phase transitions for pure and mixed states
- anomalies of categorical symmetries [Antinucci, Copetti, Gai, SSN]
- derive new constant depth implementations of non-Clifford gates. [Hsin, Kobayashi, Zhu][Warman, SSN]

Example: $\mathbb{Z}_2$ SymTFT

$\mathcal{L}(\phi)$ with $\mathbb{Z}_2$ symmetry, e.g. $\phi \to -\phi$. Then couple the theory to a $\mathbb{Z}_2$ background (flat) fields a. Lets first consider $U(1)$ valued fields:

$$\mathcal{L}(\phi) = |d\phi|^2 + V(\phi) \to \mathcal{L}(\phi, a) = |(d+a)\phi|^2 + V(\phi)\,.$$

Then gauge a in 2+1d: introduce a "Lagrange multiplier that imposes flatness", or dual gauge field b, and **BF-action**:

$$S_{\text{SymTFT}} = \frac{2i}{2\pi} \int_{M_2 \times I} \mathbf{bda} + \int_{M_2^{\text{phys}}} \mathcal{L}(\phi, a)$$

Impose Dirichlet boundary condition on symmetry boundary:

$$a|_{\mathcal{B}^{\text{sym}}} = 0$$

Wilson lines of a and b correspond to the e and m anyons:

$$e = e^{i \oint a}\,, \qquad m = e^{i \oint b}\,.$$

Properties:

$$e^2 = 1\,, \qquad m^2 = 1\,,$$

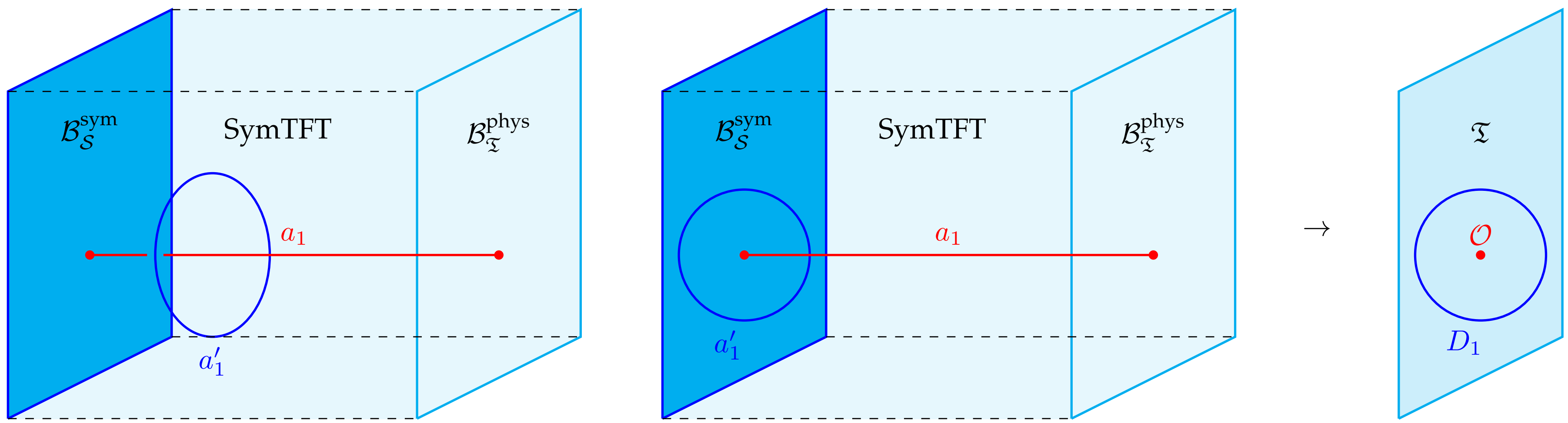
but they also link non-trivally

$$e_\gamma m_{\gamma'} = e^{\frac{2\pi i}{2} \text{Link}(\gamma, \gamma')} m_{\gamma'} e_\gamma\,.$$

SymTFT Application 1: Generalized Charges

The topological defects a of the SymTFT are the generalized charges of $\mathcal{S}$: they classify the allowed multiplets of (extended) charged operators [Bhardwaj, SSN].

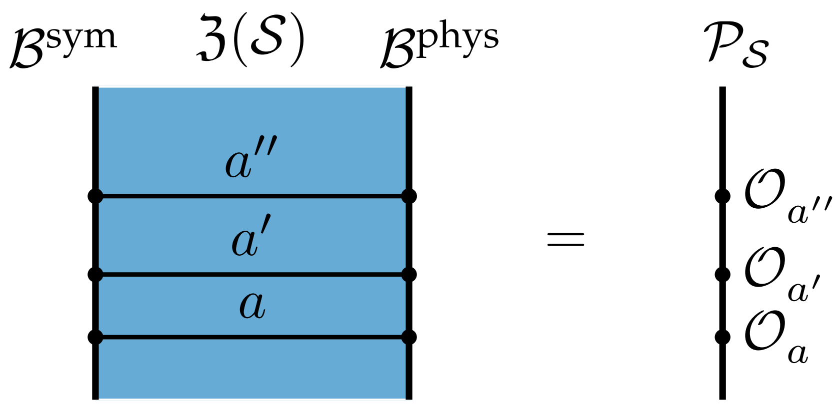
For example: consider a 1+1d theory, then the SymTFT is theory of anyons a_1



- An anyons a stretched across the sandwich. A defect a' on $\mathcal{B}^{\text{sym}}$ measures the charge by **linking** with a.
- For non-invertible $\mathcal{S}$ a single multiplet generically contains both **genuine and non-genuine** operators.
- $\mathbb{Z}_2$: $a|^{\text{sym}}_{\mathcal{B}} = 0$ means e anyons can end. m project and give rise to $\mathbb{Z}_2$ symmetry generators.

SymTFT Application 2: Phases of Matter in (1+1)d

Construct $\mathcal{S}$ symmetric phases systematically via SymTFT [Bhardwaj, Bottini, Pajer, SSN]. Fix $\mathcal{B}^{\text{sym}}_{\mathcal{S}}$. Then any **gapped $\mathcal{B}^{\text{phys}}$** results in an $\mathcal{S}$-symmetric gapped phase $\mathcal{P}_{\mathcal{S}}$:

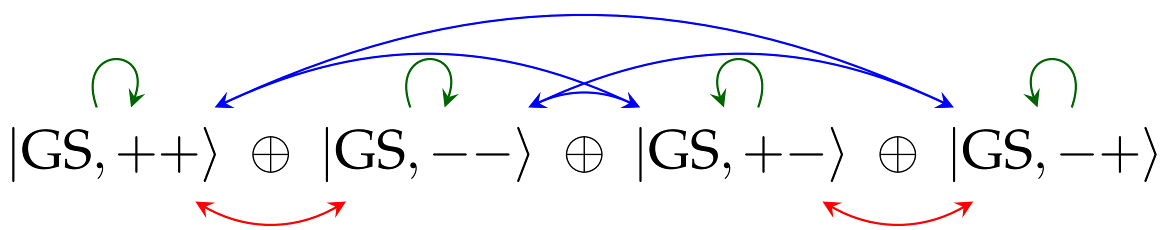


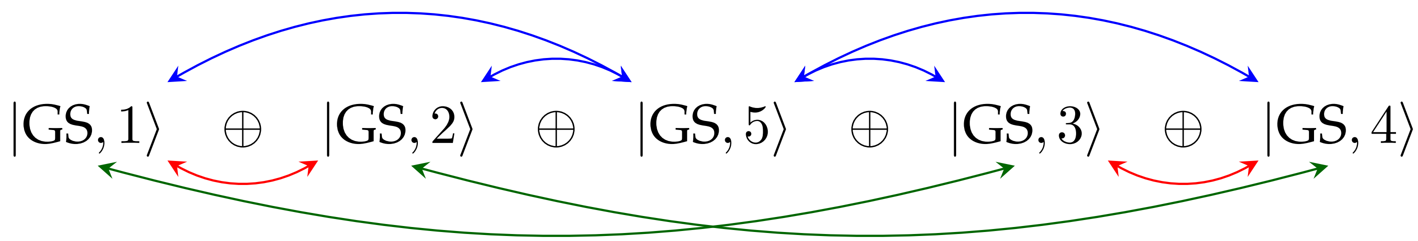
- The order parameters are precisely the generalized charges a that condense (get a vev) on $\mathcal{B}^{\text{phys}}$ – spontaneous breaking is rephrased as ending on both physical and symmetry boundaries.
- Example: Ising symmetry has one gapped phase (as it only has one Dirichlet gapped BC) with three ground states.

Gapless phases & transitions: club sandwich – producing $\mathcal{S}$-symmetric CFTs interpolating between gapped phases [Bhardwaj, Pajer, SSN, Warman].

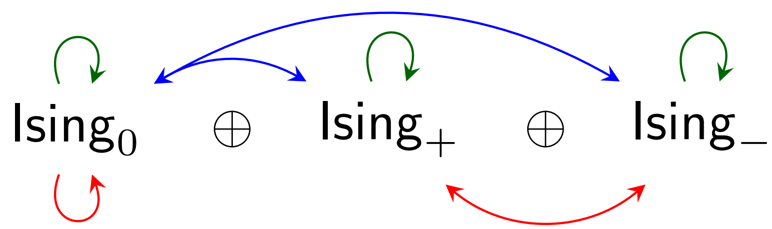
Example: Rep(D_4) Transition: Ising igSSB

Rep(D_4) is a non-invertible symmetry with SymTFT D_4 gauge theory. The SymTFT predicts there are among others two gappes phases – $\mathbb{Z}_2 \times \mathbb{Z}_2$ SSB and the Rep(D_4) SSB, where the red/green are 1d irreps and blue 2d irrep:



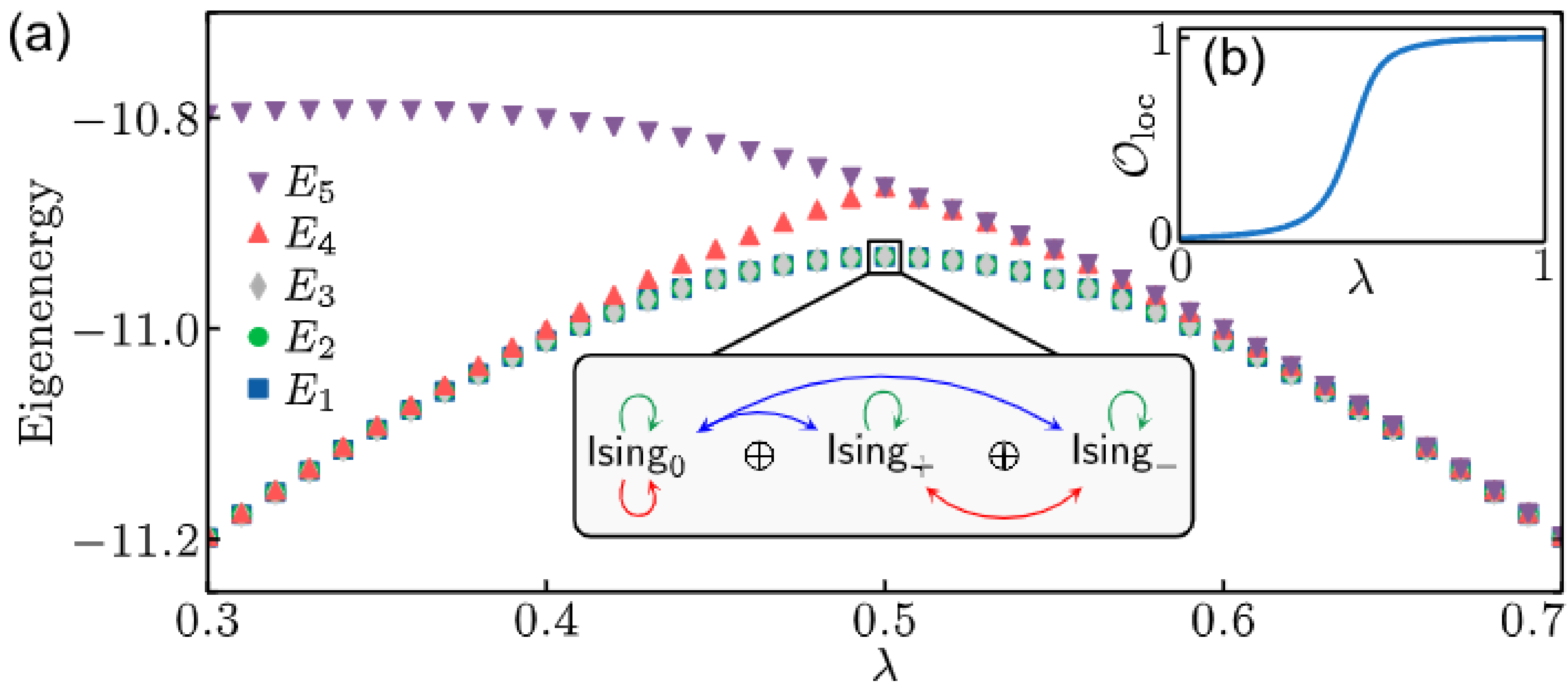


The club sandwich predicts that there is a gapless phase



igSSB: gapless phase with 3 vacua, that can only be gapped by further SSBing.

$H(\lambda)$ is the linear interpolation between a $\mathbb{Z}_2 \times \mathbb{Z}_2$ SSB and the Rep(D_4) SSB



The critical point is an Ising igSSB:

3 copies of the Ising CFT related by spontaneously broken Ising category.

[Alison Warman, Fan Yang, Apoorv Tiwari, Hannes Pichler, SSN]

Near future implementation of such non-invertible phases/phase transitions with Rydberg atoms [wip].

SymTFT sheds new light on old settings

[Warman, Gai, SSN (2605.31601)]

Beyond Landau:
For groups: $(\mathbb{Z}_2 \times \mathbb{Z}_2, \omega)$:

$$\begin{array}{ccc} & (\mathbb{Z}_2^a \times \mathbb{Z}_2^b, \omega) & \\ \swarrow & & \searrow \\ \mathbb{Z}_2^a & \not< & \mathbb{Z}_2^b \end{array}$$

These are called deconfined quantum critical points (DQCPs). But gauging one of the $\mathbb{Z}_2$ yields a Landau transition

$$\begin{array}{ccc} & \mathbb{Z}_4 & \\ \swarrow & & \searrow \\ \mathbb{Z}_4 & > & 1 \end{array}$$

Question: can there be **intrinsically non-Landau transitions**, i.e. they are not SSB transitions even after gauging the symmetry.

Yes, in fact with finite group symmetries [Warman, Gai, SSN].

Definition: Twin phases of a symmetry $\mathcal{S}$:
inequivalent gapped phases whose order parameters belong to the same generalized charge of $\mathcal{S}$.

In the SymTFT they are twin gapped boundaries $\mathcal{B}^{\text{phys}} = L_1, L_2$, which have the same content in terms of topological defects but different composition rules. The associated gapped phases have distinct order parameters:

$$\mathcal{O}_a^{(1)} \neq \mathcal{O}_a^{(2)} \,, \qquad \text{same generalized charge } a \,.$$

Main result: a stable direct transition between twin phases is never an SSB transition, **even after (partially) gauging $\mathcal{S}$**. An **intrinsically beyond-Landau** transition.

Example: $G = GL(2, \mathbb{F}_3) \cong Q_8 \rtimes S_3$ with anomaly $\omega \in H^3(G, U(1))$

$$\begin{array}{ccc} & GL(2,\mathbb{F}_3)^\omega & \\ \swarrow & & \searrow \\ S_3^{(1)} & \not\leftrightarrow & S_3^{(2)} \end{array} \qquad \text{with} \qquad \begin{array}{c} S_3^{(1)} \cong S_3^{(2)} \\ S_3^{(1)} \not\sim S_3^{(2)} \end{array}$$

The mixed anomaly between the symmetries $S_3^{(1)} \cong S_3^{(2)}$ preserved in the two phases – drives a stable second-order transition between the twins.

SymTFT Application 3: Higher Dimensions & Spacetime

Phases in (2+1)d.

The SymTFT is now (3+1)d and the symmetry is a fusion 2-category. Gapped phases again follow from pairs $(\mathcal{B}^{\text{sym}}, \mathcal{B}^{\text{phys}})$, but the structure is far richer [Bhardwaj, Pajer, SSN, Tiwari, Warman][Rui Wen, Potter][Rui Wen], due to non-trivial topological order in (2+1)d.

Continuous Symmetries SymTFT.

For continuous symmetries, the bulk is NOT a G gauge theory with dynamical gauging, but flat gauging, i.e. BF-theory for G. For abelian groups [Antinucci, Bennini][Brennan, Sun][Apruzzi, Bedogna, Dondi][Bonetti, del Zotto, Minasian]]

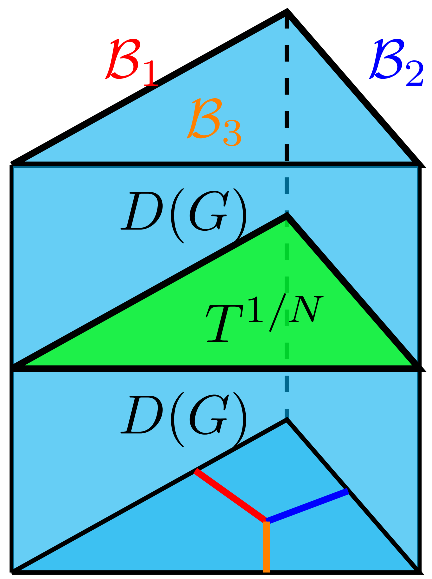
Spacetime SymTFT.

The same sandwich logic extends from internal to continuous spacetime symmetries, with $\mathfrak{Z}$ a $(d+1)$-dimensional BF theory for the spacetime symmetry group [Apruzzi, Dondi, Garcia-Etxebarria, Lam, SSN (2509.07965)].

- Conformal symmetry and its spontaneous/explicit breaking are treated on the same footing as internal symmetries;
- Can be thought of as a topological truncation of gravity and holography.

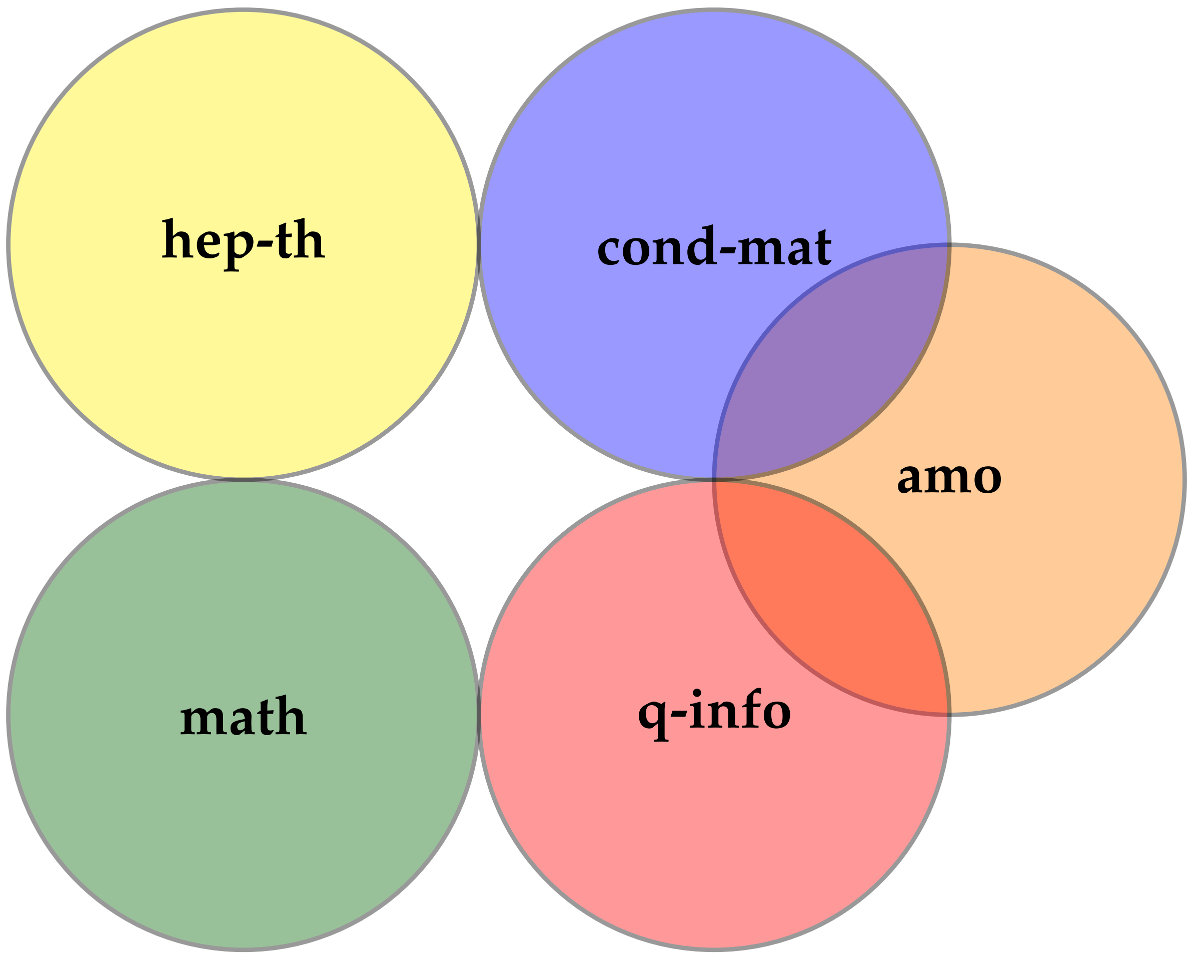
Quantum Information Theory.

Topological Codes: Universal gate sets implemented transversally in 2D using non-abelian quantum doubles $D(G)$, aka SymTFTs for non-abelian finite G group symmetries. [Warman, SSN, 12/2025]



Surprising fact: only recently non-abelian codes have been studied and we can show that they enable universal quantum computing in 2d spatial dimensions. (in fact this collapses the Bravyi-Koenig bound to 2d entirely by tuning G to D_{4N}).

Main development:
The synergy between hep-th, with cond-mat, math, and even quantum info and amo, has become invaluable.



”Synergy by Symmetries”