Counting AdS Vacua

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based on 2512.04151
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Introduction

- A furious debate has been raging about accelerating cosmology in string theory
  - Notably: do $\mathrm{dS}_4 \times M_6$ solutions exist?

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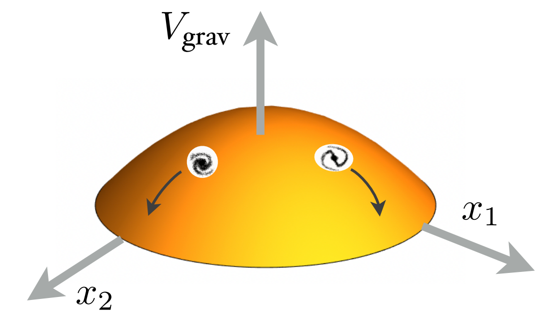

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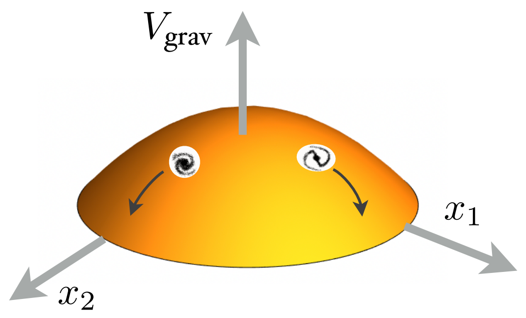

  - O-planes, string effects can violate it, but it's difficult to control them.

    Many models, but none universally agreed to be correct.

[Kachru, Kallosh, Linde, Trivedi '02, Balasubramanian, Berglund, Conlon, Quevedo '05; Saltman, Silverstein '04; Burgess, Kallosh, Quevedo '03; Danielsson, Haque, Shiu, Van Riet '09; ...; Córdova, De Luca, AT '18; De Luca, Silverstein, Torroba '21]

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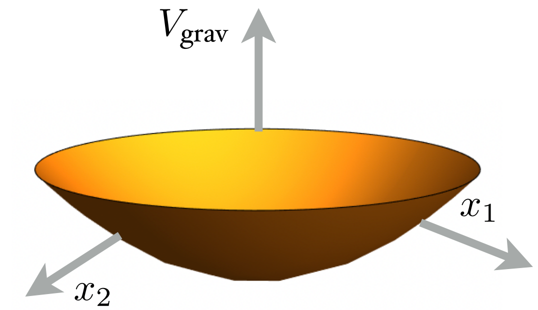

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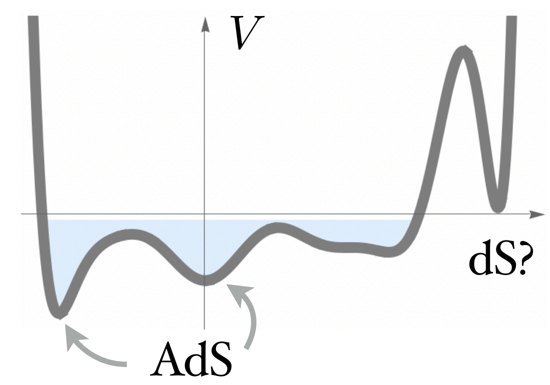
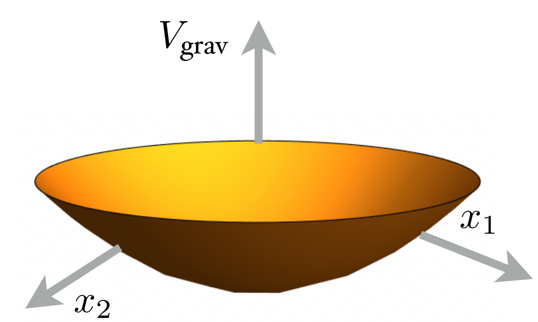
cf. Grothendieck's *rising sea*





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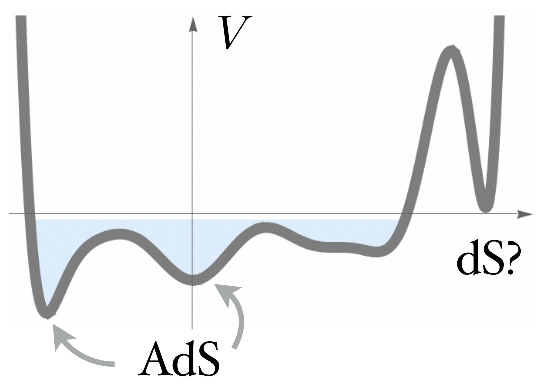

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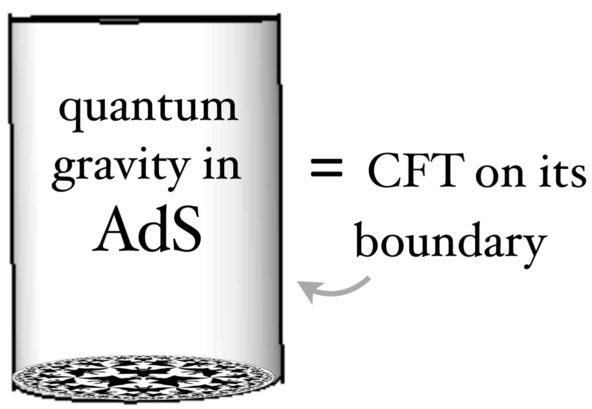
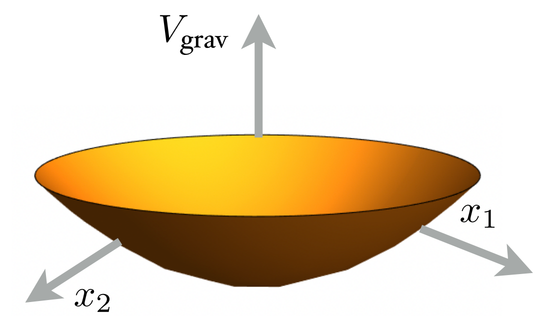
- Bonus: *holography*

[Maldacena '98]





- Over the years, many classification results for $\mathrm{AdS}_d \times M_{10-d}$
  - G-structures, generalized (complex) geometry...
  - For example: AdS_7, AdS_6 susy solutions fully classified.
  - Lower dimensions and susy: sometimes reduced to PDEs

- It might be time to take stock of these general results and extract general lessons.
  - Today: can we count AdS solutions?

Plan

- Counting with cutoffs
- Types of AdS solutions
- Moduli and tachyons
- Outlook

Counting with cutoffs

Consider a mass cut-off μ.

$N(\mu) \equiv$ number of AdS$_d$ vacua with

- no massive particles with $m^2 < \mu^2$
- no massive p-branes with $T < \mu^{p+1}$
  {tension}

We're interested in asymptotics as $\hat{\mu} \equiv \frac{\mu}{M_d} \to 0.$

d-dim. Planck mass

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- Variants: restrict to vacua with a certain amount of susy, global symmetry, etc.

- often: $\mu \sim m_{\text{KK}} \sim \frac{1}{L_{\text{AdS}}} \sim M_d \, \hat{c}^{-\frac{1}{d-2}}$

central charge
of dual CFT

- Warm-up: AdS4 with maximal supersymmetry. ⇨ $\mathrm{AdS}_4 \times S^7$ in $d = 11$ su.gra.

$$\mathrm{d}s^2_{11} = R^2 \left(\tfrac{1}{4}\mathrm{d}s^2_{\mathrm{AdS}_4} + \mathrm{d}s^2_{S^7}\right) \,, \qquad G_4 = \tfrac{3}{8} R^3 \mathrm{vol}_{\mathrm{AdS}_4} \,, \qquad (M_{11} R)^6 = 32\pi^2 n$$

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$$\Rightarrow \quad N(\hat{\mu}) = \#\{n | n^{-3/4} \gtrsim \hat{\mu}\} \sim \hat{\mu}^{-4/3}$$

- in terms of CFT$_3$s: $c \sim n^{3/2}$ $\qquad N(\hat{c}) = \#\{n | n^{3/2} < \hat{c}\} \sim \hat{c}^{2/3}$

The big picture

[complete susy classifications]

	M-theory	IIA	IIB
AdS_7	S^4/Γ	∞: $S^2 \curvearrowright I$	$\emptyset$
AdS_6	$\emptyset$	one	∞: $S^2 \curvearrowright D$

[Apruzzi, Fazzi, Rosa, AT '13; Brandhuber, Oz '99;
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[Maldacena, Nuñez '00; Gaiotto, Maldacena '09; Bah, Beem, Bobev, Wecht '12; Gauntlett, Martelli, Sparks, Waldram '04,'05; Apruzzi, Fazzi, Passias, AT '15 Gabella, Gauntlett, Palti, Sparks, Waldram '09...]

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	M-theory	IIA	IIB	
$\mathcal{N} \geq 4$	S^7/Γ	?	$S^2 \times S^2 \curvearrowright$ disk, annulus	[Assel, Bachas, Estes, Gomis '11, '12]
$\mathcal{N} = 3$	3-Sasaki	top. S^6	S-quotient	[Pang, Rong '15; Guarino, Varela '15; Assel, AT '18]
$\mathcal{N} = 2$	SE_7 top. S^7 top. $S^4 \curvearrowright H_3/\Gamma$	top. $S^2 \curvearrowright \mathrm{KE}_4$ $\mathbb{CP}^3$ top. S^6	S-folds	[Corrado, Pilch, Warner '01; Gabella, Martelli, Passias, Sparks '12; Halmagyi, Pilch, Warner '12; Petrini, Zaffaroni '09; Guarino, Jafferis, Varela '15; Passias, Prins, AT, '18; Guarino, Sterckx, Trigiante '20...]
$\mathcal{N} = 1$	weak G_2 top. S^7	nearly-Kähler coset/twistor top. $S^3 \curvearrowright H_3/\Gamma$ approx. CY + O6 top. S^6	top. $S^5 \times S^1$ S-folds	[Behrndt, Cvetic '04; AT '07; Rota, AT '15; Arav, Cheung, Gauntlett, Roberts, Rosen '20]

- Let's try to make sense of this mess.

I. *Tame* solutions

similar to $\mathrm{AdS}_4 \times S^7$; more integers, inequalities (flux quanta, orbifold numbers...)

$$N(\hat{\mu}) = \{n_i \,|\, n_1^{-a_1} \dots n_k^{-a_k} > \hat{\mu}\} \sim \hat{\mu}^{-1/\min\{a_i\}}$$

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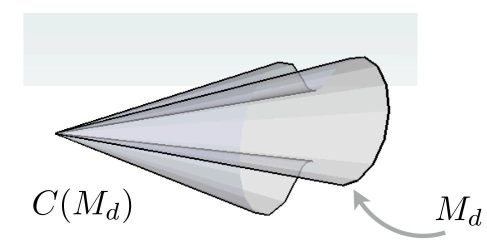
II. *Geometrical proliferation*

such as $\mathrm{AdS}_4 \times \mathrm{SE}_7$, $\mathrm{AdS}_5 \times \mathrm{SE}_5$

SE=*Sasaki–Einstein*:

$\mathrm{Cone}(\mathrm{SE}_{d-1}) = \mathrm{CY}_d$



many methods: cosets; solutions of ODEs;
toric methods; K-stability

[Witten '84, Castellani, D'Auria, Fré '84...; Gauntlett, Martelli, Sparks, Waldram '04...; Futaki, Ono, Wang '09; Collins, Szekelyhidi '15]

families: volume homogeneous fn. of $p_1, \ldots, p_k \in \mathbb{Z}$, bounds on p_i/p_k

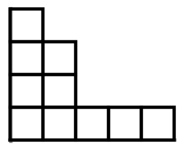
[max. known $k = 3$]

$$N(\hat\mu) = \hat\mu^{-k/3} \sim \hat c^{k}$$

III. ***Brane*** *proliferation*

Solutions with many brane configurations.

Usually characterized by partitions (~ Young diagrams) of a flux integer



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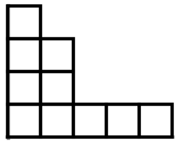

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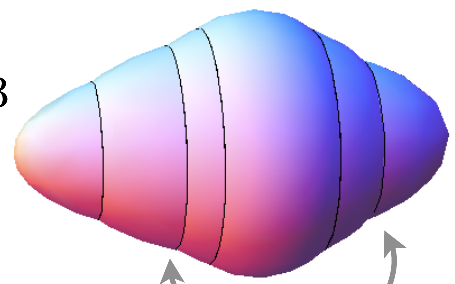
in IIA

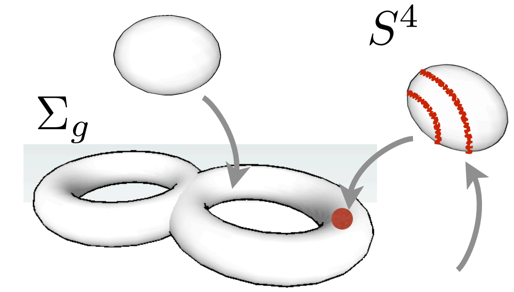


fully explicit
[Apruzzi, Fazzi, Rosa, AT '13...]

D8-branes:
two partitions

total boxes $< n \equiv$ internal flux

$\mathrm{AdS}_5 \times M_6$
in 11d sugra



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[Maldacena, Nuñez '00;
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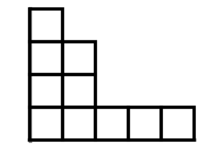

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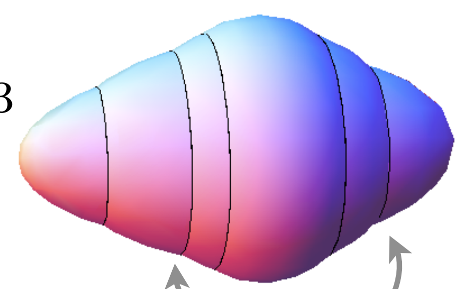

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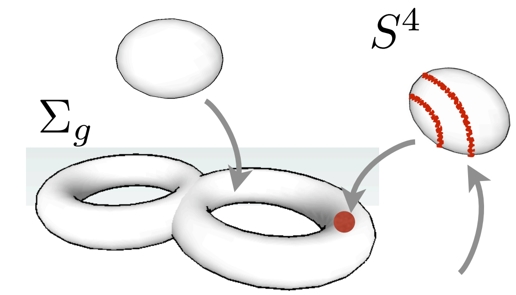

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[Hardy, Ramanujan 1918]

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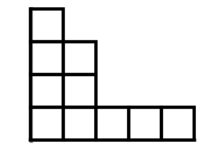

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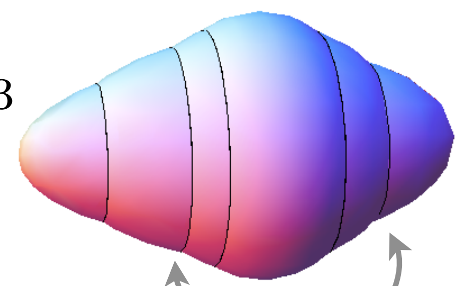

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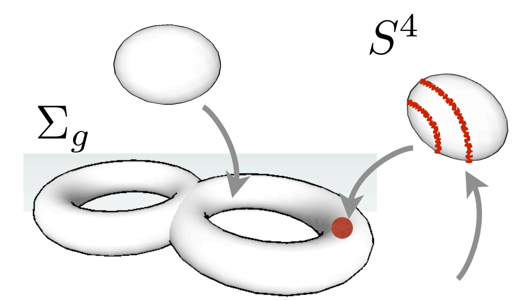

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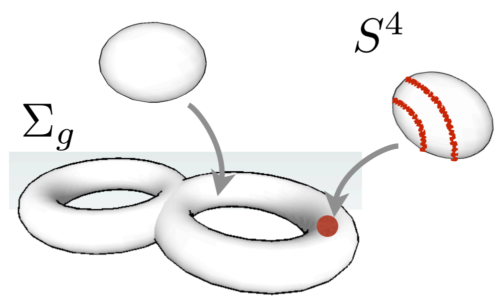
- Known analogues for AdS_6, AdS_4; many more should exist

[D'Hoker, Gutperle, Karch, Uhlemann '15; Assel, Bachas, Estes, Gomis '11, '12]

Moduli and tachyons

But wait a second: what about *moduli*?

- For ex.: in the AdS_5 solutions, deformations of Σ_g



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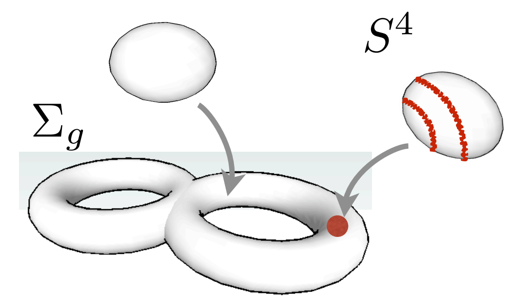

- Maybe:

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- no massive p-branes with $T < \mu^{p+1}$

weighted by Vol(moduli space)

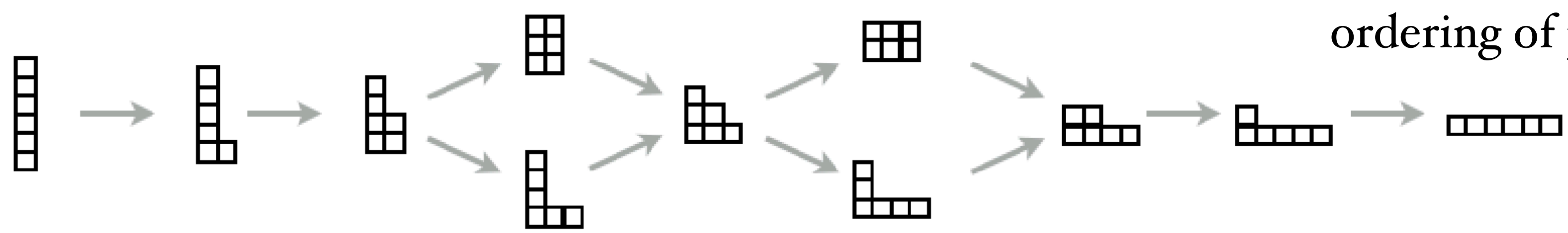
in metric where $S_d \supset M_d^3 \int \mathrm{d}^d x(\sqrt{g_d} R_d - \partial(\text{moduli})^2)$

But wait a second: what about *tachyons*?

- All examples with brane proliferations have many.

- Dual to relevant operators whose *vev* trigger RG flows

The RG flows follow the natural ordering of partitions.



- Similar prescription:

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weighted by Vol(moduli space)

$\times$Vol(field theory space with tachyons)

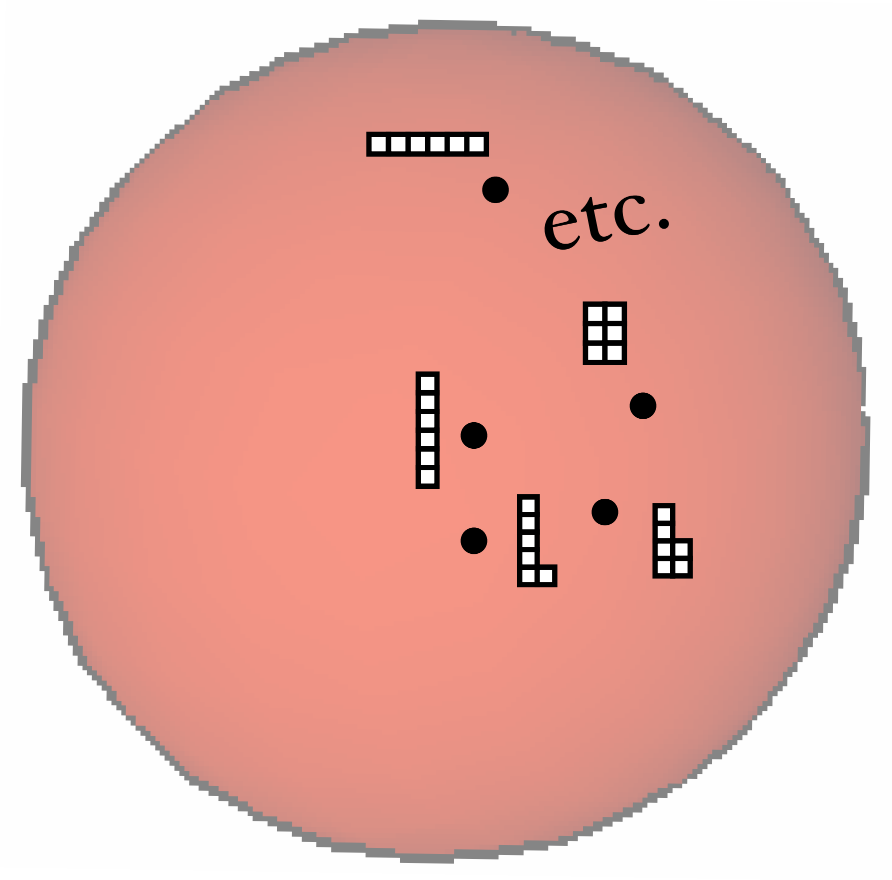
- For AdS7: there is a 7d sugra with vacua labeled by two partitions.

[De Luca, Gnecchi, Lo Monaco, AT '18]

a *gauged supergravity*; not quite a consistent truncation!

  - Potential is multidimensional and complicated; approx. tachyonic regions as balls in field space

  - Actually a relatively small ball contains all vacua.



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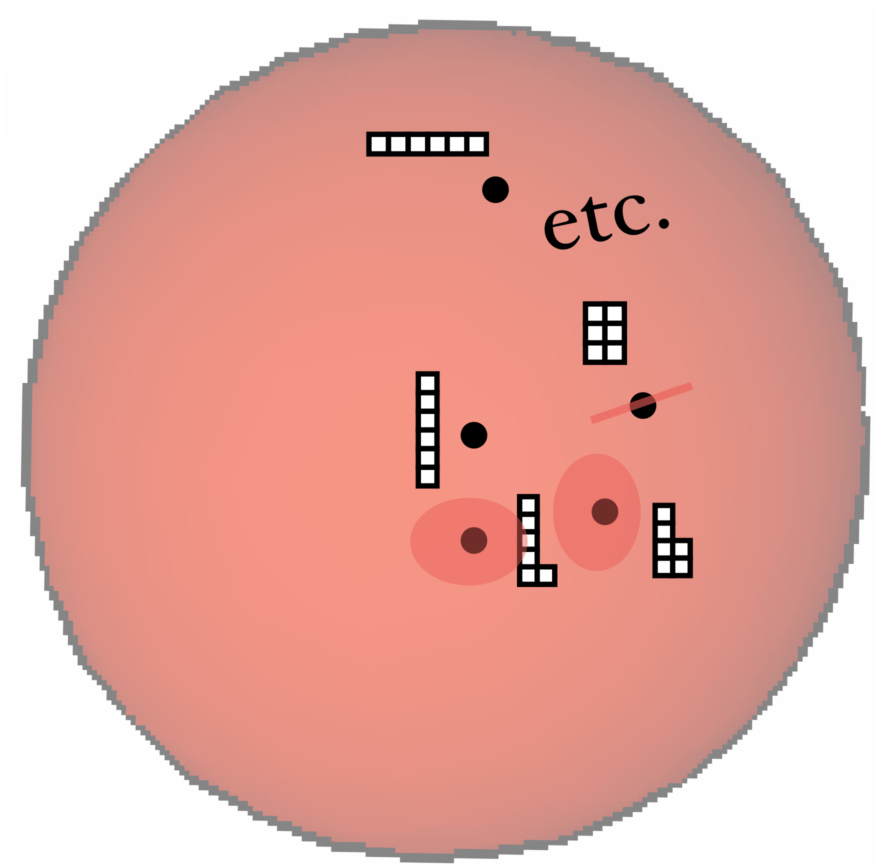

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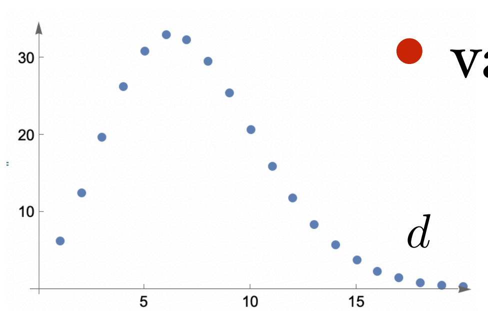
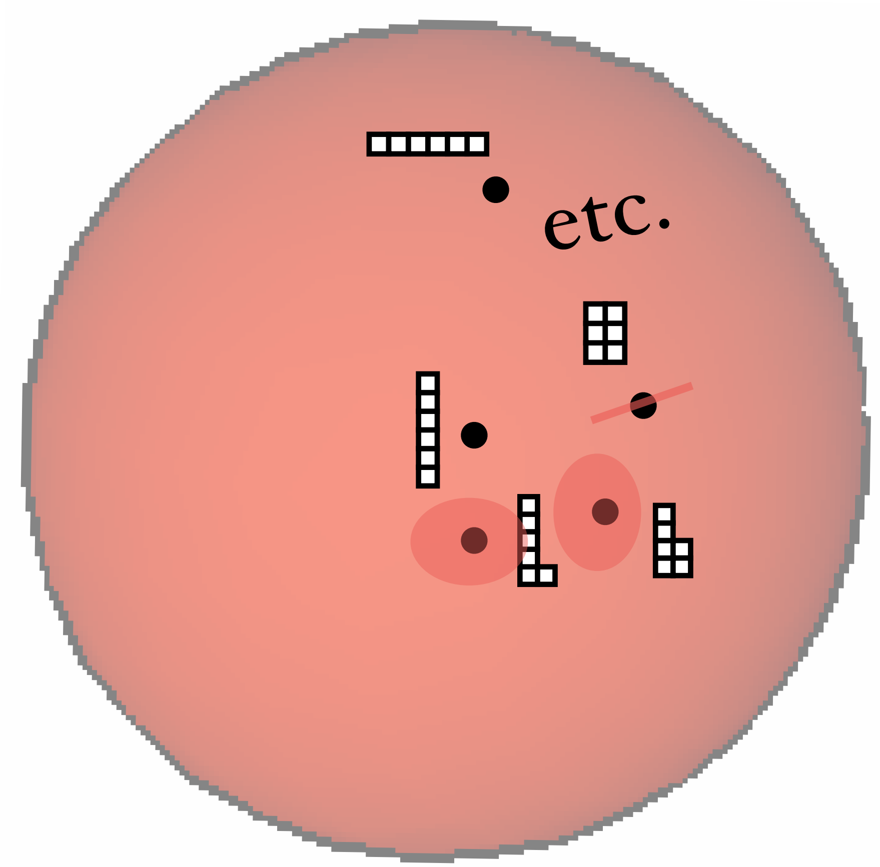
$\mathrm{Vol}(S^d)$



  - vacua with fewer tachyons are most important



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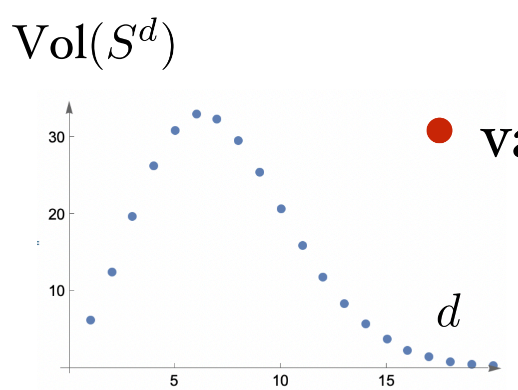
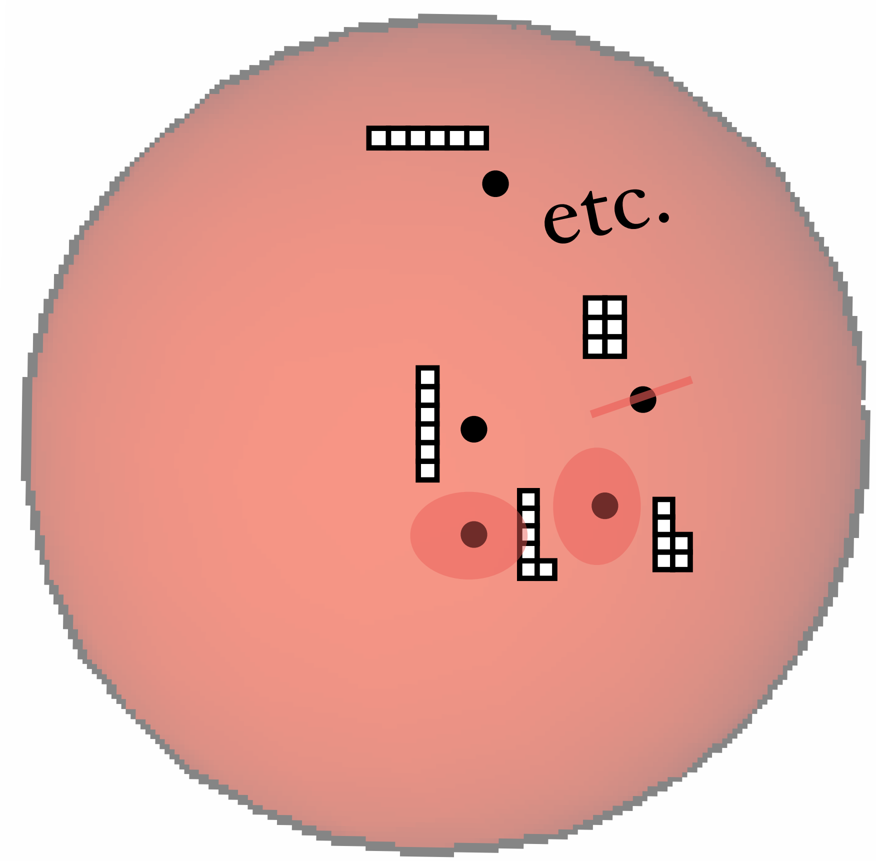
  - Actually a relatively small ball contains all vacua.

  - But: we should weigh each vacuum with its ball



  - vacua with fewer tachyons are most important



  - We get again a power law!

$$N(\hat{\mu}) \sim \hat{\mu}^{35/6} \cancel{\exp\left(2\pi\sqrt{\tfrac{2}{3}}\hat{\mu}^{-5/4}\right)} \quad \longrightarrow \quad N(\hat{\mu}) \sim \hat{\mu}^{-5/3}$$

- We find it plausible that this mechanism works for the other cases with brane proliferation

- We don't have the appropriate reduced gauged supergravity theories

$$N(\hat{\mu}) \sim \hat{\mu}^{35/6} \cancel{\exp\left(2\pi\sqrt{\tfrac{2}{3}}\hat{\mu}^{-5/4}\right)} \longrightarrow N(\hat{\mu}) \sim \hat{\mu}^{-5/3}$$

Conclusions

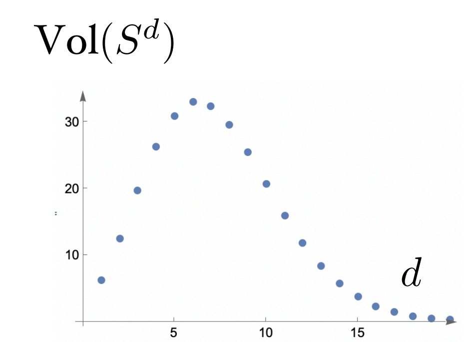
- $N(\hat{\mu})$ diverges exponentially as $\hat{\mu} \to 0$ $\qquad \hat{\mu} = \frac{\mu}{M_d} = \frac{\text{cutoff}}{\text{Planck mass}}$

  ... dominated by brane proliferation

- It gets softened to a power-law if we weight by *field-theory volume*

- Implications for understanding the *typical* holographic CFT

- If a similar story holds for [even approximately] dS solutions:



  - 'typical' solution has few light moduli

  - distribution of vacua would be softened: $N_{\rm dS}(\Lambda) \sim \exp\left(\frac{3\pi}{G\Lambda}\right) \sim e^{\rm dS} \to \Lambda^{-b}$?