

# Entropic Observables for Symmetry Breaking

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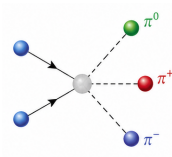
23 June 2026, Sheffield (UK)



NP-QFT

# Symmetry Breaking

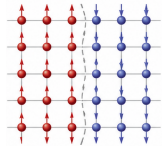
It explains numerous phenomena in physics:



Pion Lagrangian



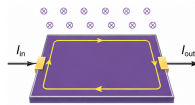
Superfluids



Paramagnetic /  
ferromagnetic  
phase transitions



Hydrodynamics



Fractional quantum  
Hall effect

★ Standard approach to detect **symmetry breaking**:

local order parameters  $\mathcal{O}(x)$

$$\langle \psi | \mathcal{O}(x) | \psi \rangle \neq 0 \quad \Rightarrow \quad \text{symmetry is broken}$$

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- ★ Quantum-information or entropic approach:

[Vaccaro, Anselmi, Wiseman, Jacobs 05; Gour, Marvian, Spekkens 09; Chitambar, Gour 18]

[Casini, Huerta, Magan, Pontello 19 & 20]

[Ares, Murciano, Calabrese 22 & 25]

**Entanglement asymmetry**  $\Delta S[\rho_B]$

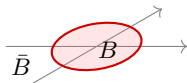
- $\Delta S[\rho_B] > 0$  if and only if the symmetry is broken
- Quantifies the amount of symmetry breaking
- Depends on a subsystem  $B$  (probes the RG flow)
- Applicable to ground states  $|0\rangle$   
as well as to excited (or out-of-equilibrium) states (study temporal evolution)

## Definition of Entanglement Asymmetry $\Delta S[\rho]$

[Ares, Murciano, Calabrese 22]

★ Take a state  $\rho$ :

- Could be pure,  $\rho = |\psi\rangle\langle\psi|$ , or mixed
- Could be a state of the full system:  $\rho = \rho_{\text{full}}$   
or the reduced density matrix on a subregion  $B$ :  $\rho = \text{Tr}_{\bar{B}} \rho_{\text{full}}$

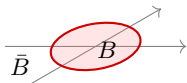


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★ Suppose the system has a **symmetry group**, e.g.  $G = U(1)$

Construct a *symmetrized density matrix* by averaging over adjoint action of  $G$ :

$$\rho_S = \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{-i\alpha Q_B} \rho e^{i\alpha Q_B}$$

Extends to non-Abelian Lie groups & discrete groups

$$\rho_S = \frac{1}{|G|} \sum_g U_g \rho U_{g^{-1}}$$

★ Compare  $\rho$  with  $\rho_S$ :

Entanglement asymmetry  $\Delta S$  is defined using relative entropy

$$\Delta S[\rho] \equiv S(\rho \parallel \rho_S) = \text{Tr} \left[ \rho (\log \rho - \log \rho_S) \right] \stackrel{\text{here}}{=} S[\rho_S] - S[\rho]$$

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Features:

- $\Delta S \geq 0$ , and  $\Delta S = 0$  if and only if  $\rho = \rho_S$
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★ Rényi asymmetries  $\Delta S_n = \frac{1}{1-n} \log \frac{\text{Tr}(\rho_S^n)}{\text{Tr}(\rho^n)}$  then  $\Delta S = \lim_{n \rightarrow 1} \Delta S_n$

Asymmetry has been computed in a variety of quantum systems:

spin chains, 2d free fermions, 2d integrable systems,  
matrix product states, random quantum circuits, . . .

Mainly in 2d (= 1+1d) systems, often numerically.

[Alba, Ares, Banerjee, Bertini, Caceffo, Calabrese, Capizzi, Chalas, Chen, Collura, Das, De Luca, Dubail, Ferro, Fossati, Klich, Klobas, Li, Liu, Mazzoni, Murciano, Piroli, Russotto, Rylands, Sengupta, Turkeshi, Vernier, Vitale, Yamashika, Yin, Yu, Zhang, . . .]

Here I will focus on QFT and CFT approaches.

Asymmetry *quantifies* symmetry breaking:

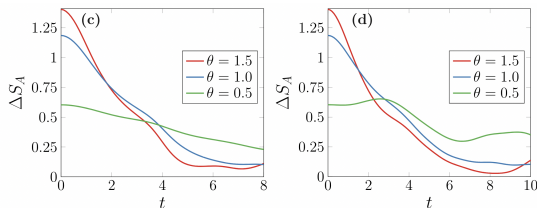
Temporal evolution of out-of-equilibrium states & relaxation of symmetry breaking

Asymmetry *quantifies* symmetry breaking:

Temporal evolution of out-of-equilibrium states & relaxation of symmetry breaking

★ Quantum Mpemba effect in spin chains  
(in closed quantum systems)

[Mpemba, Osborne 69]  
[Ares, Murciano, Calabrese 22]



periodic and open  
boundary conditions

images from Ares, Murciano, Calabrese,  
Nature Commun. 14 (2023) 2036

This effect has been *experimentally measured*:

[Joshi *et al.* 24]

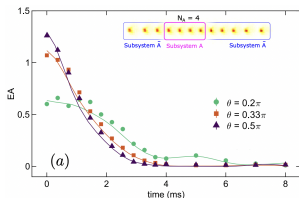


image from Joshi *et al.*  
Phys. Rev. Lett. 133 (2024) 010402

Similar effects in 3d and 4d?

Study in CFTs

[FB, Godet, Singh 24]

★ We choose a simple class of excited states  $|\psi\rangle$ .

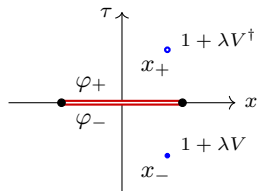
★ We choose a simple class of excited states  $|\psi\rangle$ .

● States with a Euclidean path-integral preparation. (a sort of quench)

Select a scalar primary  $V$  of dimension  $\Delta$ ,  
insert at  $x_-$  in the lower half-plane (Euclidean past)

$$|\psi\rangle = (1 + \lambda V(x_-)) |0\rangle$$

Reduced density matrix:  $\langle \varphi_- | \rho | \varphi_+ \rangle =$



★ These states have a natural description in holography

[Skenderis, van Rees 09][Botta-Cantcheff, Martínez, Silva 15]

[Faulkner, Haehl, Hijano, Parrikar, Rabideau, Van Ramsdonk 17]

## Quadratic Expansion of Relative Entropy

★ In any dimension  $d \geq 2$ , deform the vacuum  $|0\rangle\langle 0|$  with small  $\lambda$ :

$$\rho = \sigma + \lambda \sigma (V + V^\dagger) + \mathcal{O}(\lambda^2) \quad \text{where} \quad \sigma = \text{Tr}_{\bar{B}} |0\rangle\langle 0| \simeq \rho_S$$

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Integral representation of relative entropy around  $\sigma$  at  $\mathcal{O}(\lambda^2)$ :

(Fisher information metric  $F_\sigma(\delta\rho, \delta\rho)$  around  $\sigma$ )

$$S(\rho \parallel \rho_S) = \frac{\lambda^2}{4} \int_{\mathbb{R}} \frac{ds}{1 + \cosh(s)} \text{Tr} \left[ \delta\rho \sigma^{-\frac{1}{2} - \frac{is}{2\pi}} \delta\rho \sigma^{-\frac{1}{2} + \frac{is}{2\pi}} \right] + \mathcal{O}(\lambda^4)$$

$\sigma = e^{-K}$  generates modular flow  $\Rightarrow$  integrated 2-point function

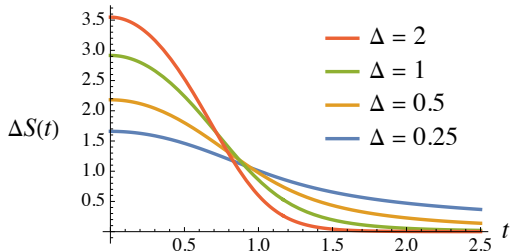
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- Different out-of-equilibrium states can relax at different rates.



We observe a **quantum Mpemba effect** in any CFT, in any dimension.

Rényi asymmetries: 
$$\Delta S_n[\rho] = \frac{1}{1-n} \log \frac{\text{Tr}(\rho_S^n)}{\text{Tr}(\rho^n)}$$

Traces are integrals of **charged moments**:

$$\text{Tr}(\rho_S^n) = \int \frac{d\alpha_1 \cdots d\alpha_{n-1}}{(2\pi)^{n-1}} \underbrace{\text{Tr} \left[ \rho e^{i\alpha_1 Q_B} \rho e^{i\alpha_2 Q_B} \cdots \rho e^{-i(\alpha_1 + \cdots + \alpha_{n-1}) Q_B} \right]}_{X_n(B, \vec{\alpha})}$$

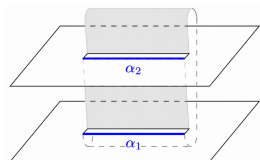
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- ★ Charged moments  $X_n$  computed by **partition functions on replicated geometries** with insertion of **topological symmetry** (or twist) **defects**

[Fossati, Ares, Dubail, Calabrese 24]



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We can predict  $\Delta S$  by performing the computation in Effective Field Theory (EFT)

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- ★ Discrete symmetry breaking in a gapped phase

Finite group  $G$  broken to  $H$ :

[Capizzi, Mazzone 23]

[Capizzi, Vitale 23]

[FB, Calabrese, Fossati, Singh, Venuti 25]

$$\Delta S[\rho] = S[\rho_S] - S[\rho] \simeq \log(|G|/|H|) = \log n_{\text{vacua}}$$

for large subregion size  $R \gg \mu^{-1}$  or whole system

★ Breaking of discrete **noninvertible symmetries** ← generalized symmetries  
[Gaiotto, Kapustin, Seiberg, Willett 14]

Noninvertible (or categorical) symmetries in 2d: **fusion algebras** instead of group laws

$$\mathcal{L}_a \times \mathcal{L}_b = \sum_c N_{ab}^c \mathcal{L}_c$$

Familiar from Verlinde lines in 2d RCFT's

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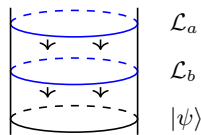
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Familiar from Verlinde lines in 2d RCFT's

- The physical setup determines a Hilbert space  $\mathcal{H}$  — which depends on spatial manifold, subregion  $B$ , boundary conditions, ...

Space-like topological operators act on  $\mathcal{H}$  as  
(finite-dimensional) semisimple  **$C^*$ -algebra  $\mathcal{A}$**   
(from unitarity):

$$\mathcal{X}_a \times \mathcal{X}_b = \sum_c T_{ab}^c \mathcal{X}_c$$



- How to symmetrize with respect to  $\mathcal{A}$ ?

★ Goal: construct a **symmetrizer**  $S : \rho \rightarrow \rho_S$  [FB, Calabrese, Fossati, Singh, Venuti 25]

- Linear projector to operators that commute with  $\mathcal{A}$ :

$$\mathcal{X}_a \rho_S = \rho_S \mathcal{X}_a \quad \forall \mathcal{X}_a \in \mathcal{A}$$

- Trace preserving:  $\text{Tr } \rho_S = \text{Tr } \rho$

- If  $\rho$  is density matrix,  $\rho_S$  is density matrix:  $\rho_S^\dagger = \rho_S, \quad \rho_S \geq 0$ .

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- If  $\rho$  is density matrix,  $\rho_S$  is density matrix:  $\rho_S^\dagger = \rho_S, \quad \rho_S \geq 0$ .

This fixes a **unique solution**:

$$\rho_S = \sum_{ab} (K^{-1})^{ab} \mathcal{X}_a \rho \mathcal{X}_b$$

$$K_{ab} = \sum_{mn} T_{an}^m T_{bn}^m \quad \text{non-degenerate bilinear pairing}$$

- $S$  is a quantum channel

- EFT for a gapped phase:

2d TQFT with local operators and action of the Fusion Category  $\mathcal{C}$

[Huang, Lin, Seifnashri 21; Bhardwaj, Bottini, Pajer, Schäfer-Nameki 23]

It describes Hilbert space of zero-energy states  $\mathcal{H} = \mathbb{C}^N$

- ★ Such TQFTs are classified by Module Categories  $\mathcal{M}$  over  $\mathcal{C}$ :  
possible patterns of symmetry breaking

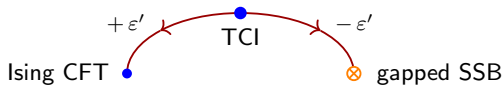
Vacua  $|v_i\rangle$ :

(1-1 with simple objects in the module category)

- Satisfy cluster decomposition:  $\langle v_i | \mathcal{O}_\mu \mathcal{O}_\nu | v_i \rangle = \langle v_i | \mathcal{O}_\mu | v_i \rangle \langle v_i | \mathcal{O}_\nu | v_i \rangle$
- Transform as non-negative integer-valued matrix (NIM) representation:

$$\mathcal{L}_a |v_i\rangle = \sum_j n_{ij}^{(a)} |v_j\rangle$$

★ E.g.: Tricritical Ising deformed by  $\varepsilon'_{(\frac{3}{5}, \frac{3}{5})} = \phi_{1,3}$  [Huse 84; Chang, Lin, Shao, Wang, Yin 18]



The “**Ising fusion category**  $\mathcal{C}$ ” is the symmetry of the Ising CFT

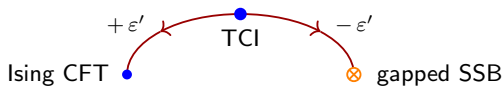
Symmetry  
elements:

$$\{ \mathcal{L}_1, \mathcal{L}_\eta, \mathcal{L}_{\text{KW}} \}$$

Fusion  
algebra:

$$\begin{aligned} \mathcal{L}_\eta \times \mathcal{L}_\eta &= \mathcal{L}_1 \\ \mathcal{L}_\eta \times \mathcal{L}_{\text{KW}} &= \mathcal{L}_{\text{KW}} \\ \mathcal{L}_{\text{KW}} \times \mathcal{L}_{\text{KW}} &= \mathcal{L}_1 + \mathcal{L}_\eta \end{aligned}$$

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Symmetry elements:	$\{ \mathcal{L}_1, \mathcal{L}_\eta, \mathcal{L}_{\text{KW}} \}$	Fusion algebra:	$\mathcal{L}_\eta \times \mathcal{L}_\eta = \mathcal{L}_1$ $\mathcal{L}_\eta \times \mathcal{L}_{\text{KW}} = \mathcal{L}_{\text{KW}}$ $\mathcal{L}_{\text{KW}} \times \mathcal{L}_{\text{KW}} = \mathcal{L}_1 + \mathcal{L}_\eta$
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Spontaneous breaking of Ising symmetry  $\rightarrow$  3 vacua  $|+\rangle, |-\rangle, |f\rangle$

$$\Delta S[|f\rangle] = \log 2$$

$$\Delta S[|\pm\rangle] = \log \sqrt{8}$$

Physically inequivalent vacua upon spontaneous breaking!

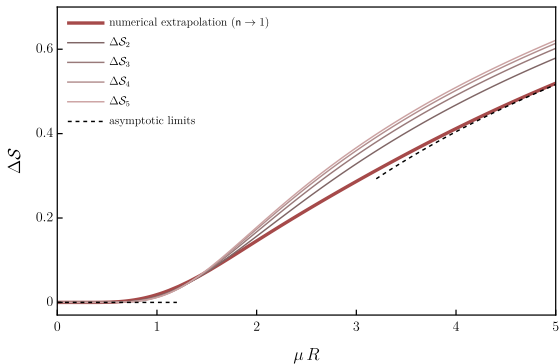
[Huse 84; Aguilera-Damia, Argurio, FB, Benvenuti, Copetti, Tizzano 23]

[Bhardwaj, Bottini, Pajer, Schäfer-Nameki 23]

What about **spontaneous breaking** of **continuous** symmetries?

In infinite volume, asymmetry grows logarithmically with the subsystem size  $R$

★ Result:  $\Delta S[\rho_A] \simeq \frac{d-2}{2} \log(\mu R)$  for  $R \gg \mu^{-1}$  and  $U(1)$



Free compact boson of target radius  $\mu^{(d-2)/2}$

[FB, García-Valdecasas, Vitouladitis 25]

★ “Page curve” of entanglement asymmetry

[Ares, Murciano, Piroli, Calabrese 23]

[Russotto, Ares, Calabrese 24]

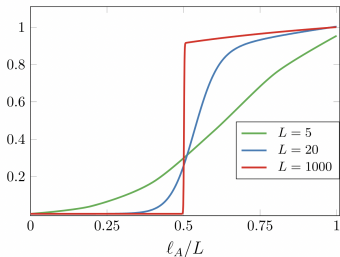


image from Ares, Murciano, Piroli, Calabrese,  
Phys. Rev. D 110 (2024) L061901

Here second Rényi asymmetry w.r.t. a  $U(1)$  symmetry

- Page model is of black hole evaporation...

# Conclusions

**Entanglement asymmetry** is a quantum-information observable well suited to detect and *quantify symmetry breaking* in QFT, both in ground states and in out-of-equilibrium states.

Many questions:

- Phase transitions
- Topological order
- Weak vs Strong breaking in mixed states and hydrodynamics
- Detect anomalies using asymmetry
- Entanglement asymmetry in gravitational theories and relation to black-hole physics and holography

[WIP with O.Mamroud, N.Talwar, M.Tortora, P.Veltman]