Rolling Galileons: Evolving Braiding Strength for Viable Dark Energy

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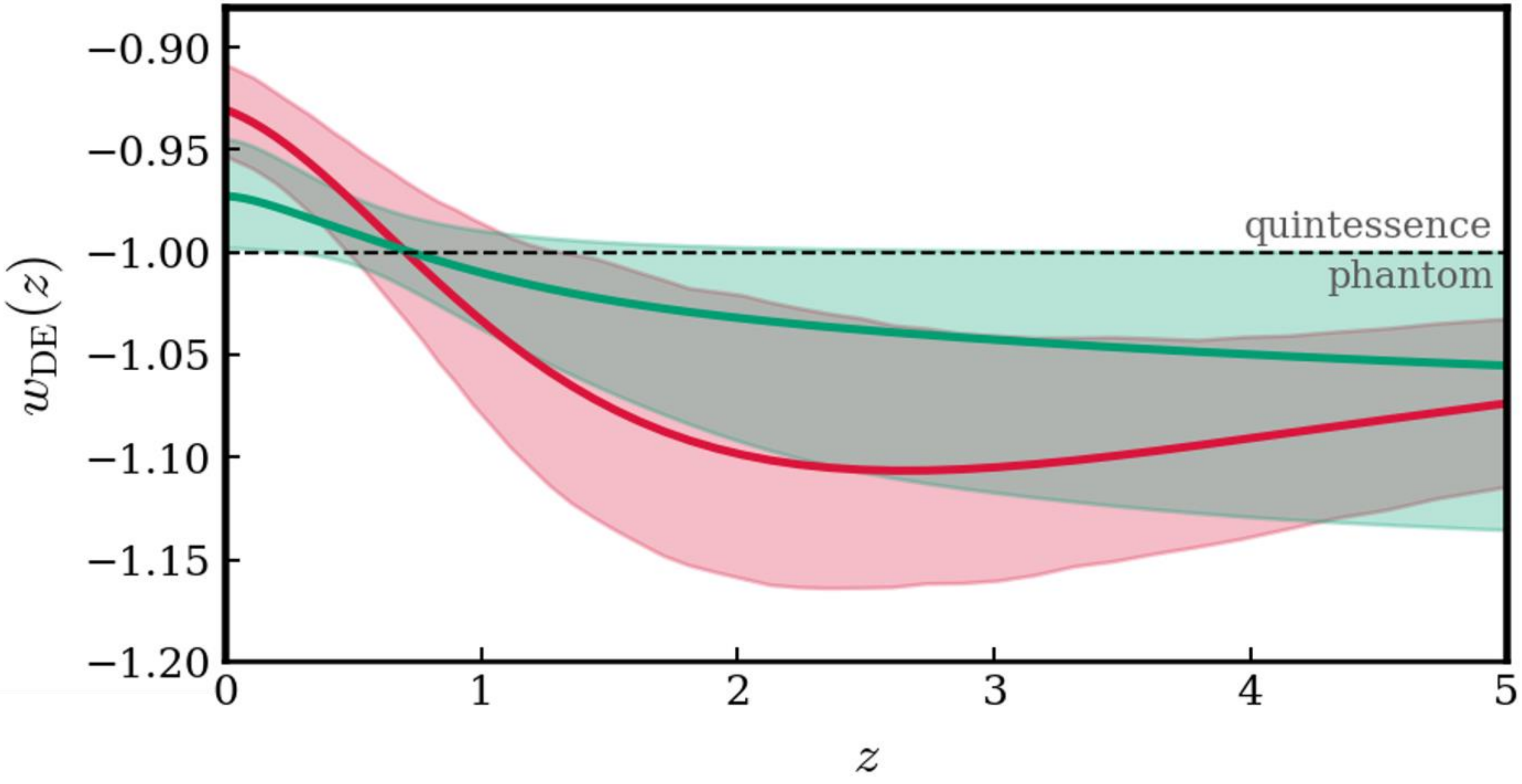
Overview

- Motivation and Scalar-Tensor Theories
- Rolling Galileons and their Phenomenology
- Confrontation with Data

Motivation for Dynamical Dark Energy

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu}^{(\mathrm{matter})} + T_{\mu\nu}^{(\mathrm{DE})} \right)$$

$$w_{\mathrm{DE}} = \frac{\rho_{\mathrm{DE}}}{p_{\mathrm{DE}}}$$



Theoretical Background

$$X := -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Luminal Horndeski Gravity

$$\mathcal{L} = G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\Box\phi$$

Non-Minimal Coupling ($G_3 = 0$)

$$\mathcal{L} = G_4(\phi)R + K(\phi, X)$$

can cross the phantom divide

hard to reconcile growth with distance

Kinetic Gravity Braiding ($G_4 = \frac{1}{2}$)

$$\mathcal{L} = \frac{1}{2}R + K(\phi, X) - G_3(\phi, X)\Box\phi$$

can cross the phantom divide

under question...

k-essence ($G_3 = 0$, $G_4 = \frac{1}{2}$)

$$\mathcal{L} = \frac{1}{2}R + K(\phi, X)$$

cannot cross the phantom divide

Rolling Galileons

$$\mathcal{L} = \frac{1}{2}R - \Lambda + \mathcal{L}_\phi$$

General KGB: $\mathcal{L}_\phi = K(\phi, X) - G_3(\phi, X)\Box\phi$

simplest model

Cubic Galileon: $\mathcal{L}_\phi = -k_0 X - X\Box\phi$

Rolling Galileons

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Cubic Galileon: $\mathcal{L}_\phi = -k_0 X - X\Box\phi$

promote coupling to function of field

$$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$$

Rolling Galileons

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promote coupling to function of field

$$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$$

Recover the Cubic Galileon:

$$k(\phi) = k_0$$

$$q(\phi) = 0$$

Asymptotically Cubic Galileons:

$$0 < \phi_{\text{ini}} \ll 1$$

$$k(\phi \to 0) \to k_0 \qquad q(\phi \to 0) \to 0$$

Tessa's talk;
K. Naidoo, et. al. 2606.20794

Phenomenological Requirements

1. Late-time phantom crossing $(z < 1)$

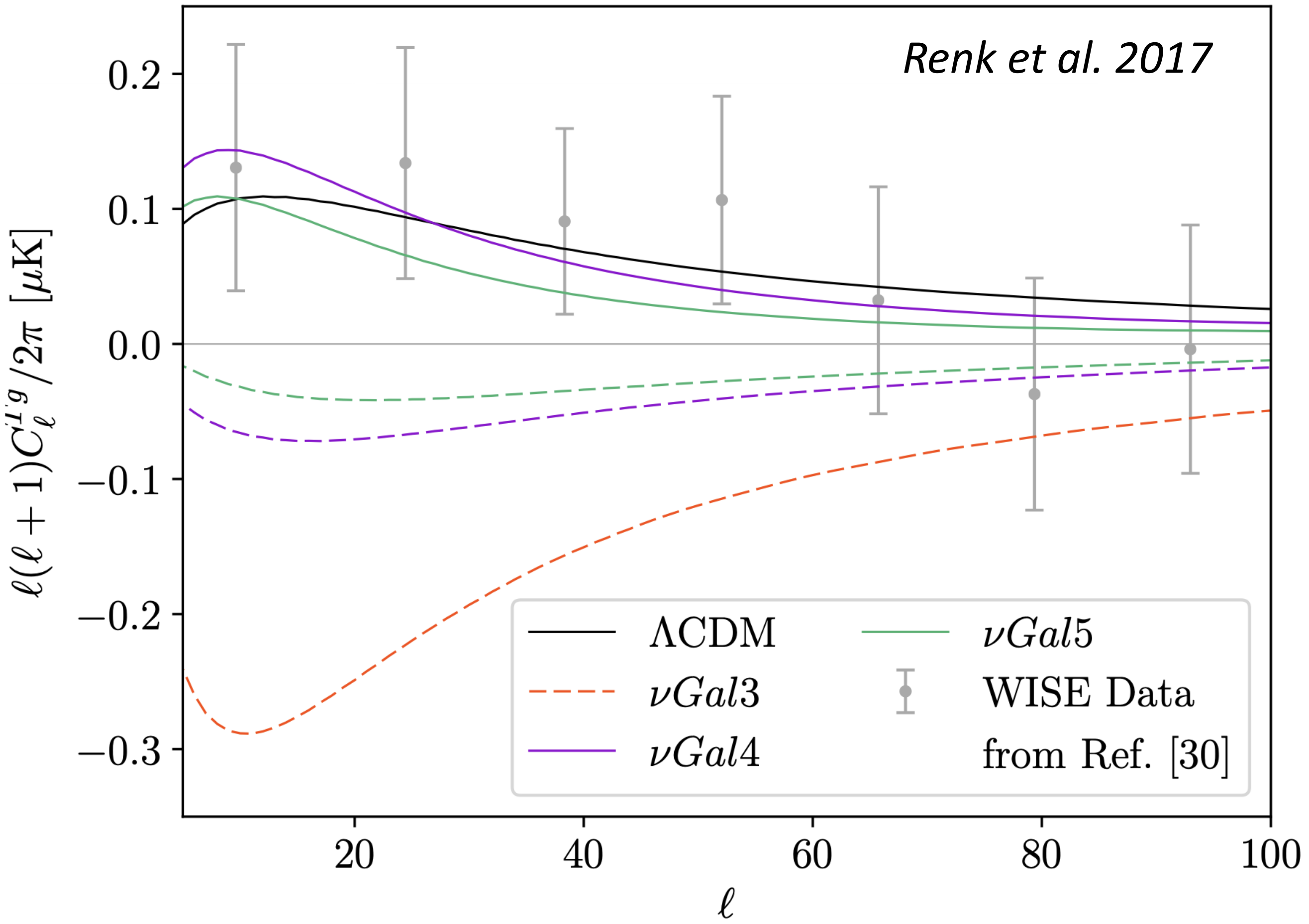
Phenomenological Requirements

2. Positive ISW signature

$$\mathcal{I}(a) \coloneqq 1 - f_1(a) - \zeta(a) > 0$$

Linear growth rate

Rate of change of Σ (lensing)



Phenomenological Requirements

3. Healthy screening in voids

$$\frac{F_\phi}{F_N} = 2\mathcal{C}\frac{1}{\chi}\left[\sqrt{1+\chi} - 1\right]$$

$$\chi = \mathcal{B}(a)^2 \frac{A_2^2}{\left(A_0 + A_2^2\right)^2}\delta_m$$

spherical mass perturbation

Phenomenological Requirements

3. Healthy screening in voids

spherical mass perturbation

$$\left.\frac{F_\phi}{F_N}\right|_{\text{Voids}} = -2\mathcal{C}\frac{1}{|\chi|}\left[\sqrt{1-|\chi|}-1\right] \qquad \chi = \mathcal{B}(a)^2\frac{A_2^2}{\left(A_0+A_2^2\right)^2}\delta_m$$

$$\underbrace{\delta_m < 0}_{\text{in voids}} \implies \frac{\chi}{\delta_m} \leq 1$$

Phenomenological Requirements

$$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$$

1. Late-time phantom crossing $(z < 1)$

2. Positive ISW signature $\mathcal{I}(a) \coloneqq 1 - f_1(a) - \zeta(a) > 0$

3. Healthy screening in voids $\frac{\chi}{\delta_m} \leq 1$

$$\dot{\phi} > 0 : \qquad k, q > 0, \quad k_\phi < 0, \quad q_\phi < 0$$

Simple Rolling Functions

$$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$$

$$\dot{\phi} > 0 : \qquad k, q > 0, \quad k_\phi < 0, \quad q_\phi < 0$$

$$k(\phi) = \frac{k_0}{\sqrt{\phi}} \qquad q(\phi) = \frac{q_0}{\phi}$$

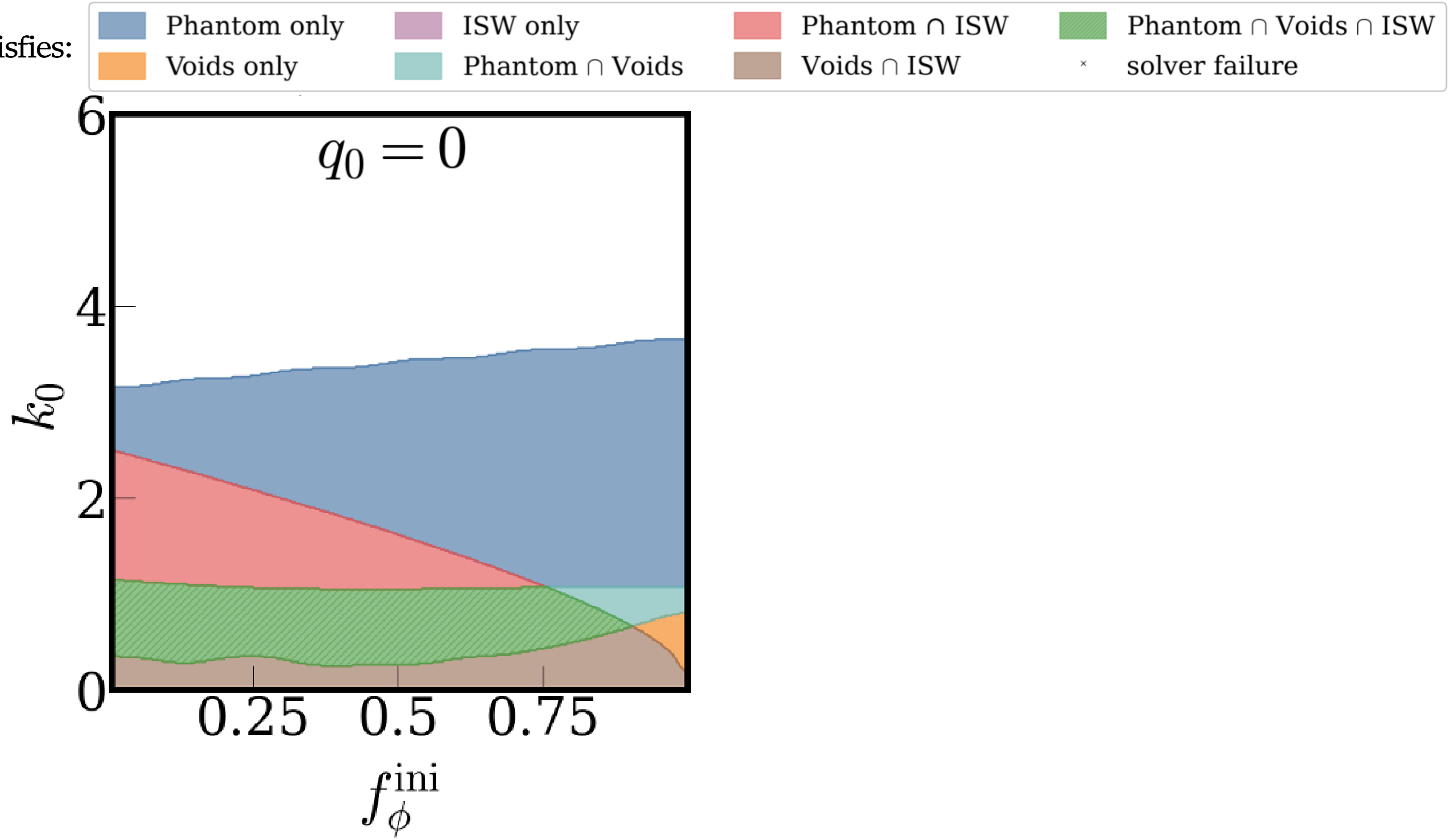
satisfies analytical
rolling conditions

$$\{k_0, q_0, \phi_{\rm ini}, f_\phi^{\rm ini}\} \qquad f_\phi(z) = \frac{\Omega_\phi(z)}{\Omega_\phi(z) + \Omega_\Lambda(z)}$$

Visualising the Parameter Space

$$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$$

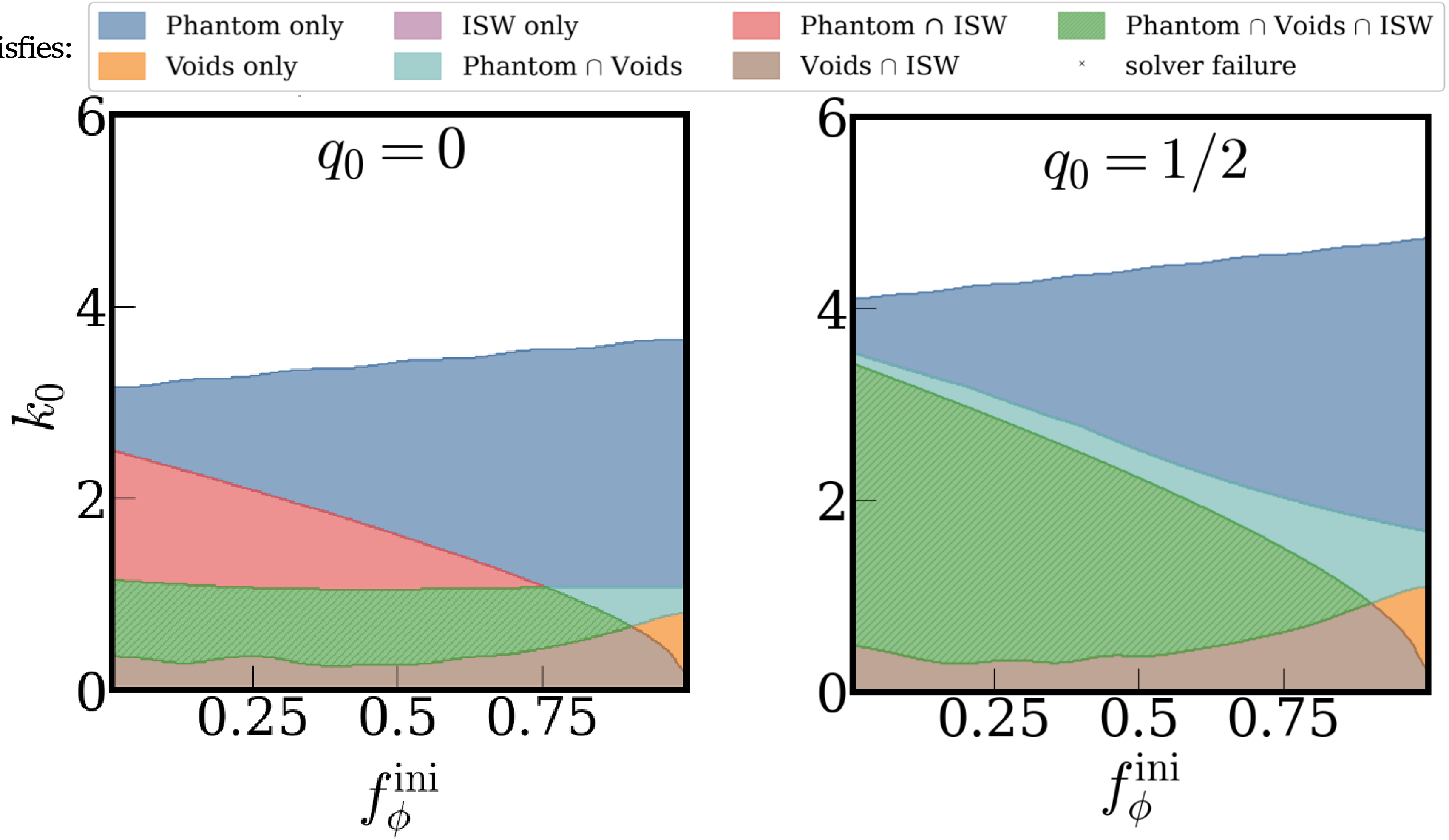
$\phi_{\rm ini} = 0.1$ satisfies:



Visualising the Parameter Space

$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$

$\phi_{\rm ini} = 0.1$ satisfies:



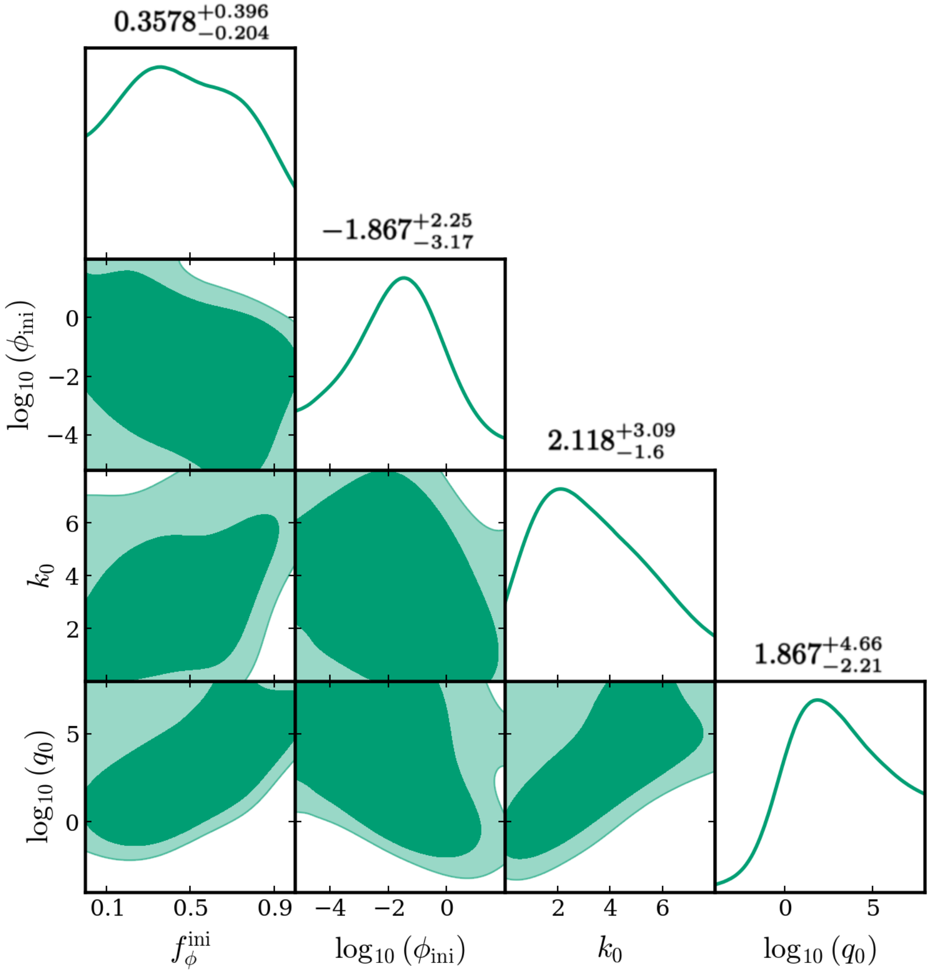
Fitting the Model to the Data

$$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$$

CMB+BAO+SN+[ISW]

k–q model:

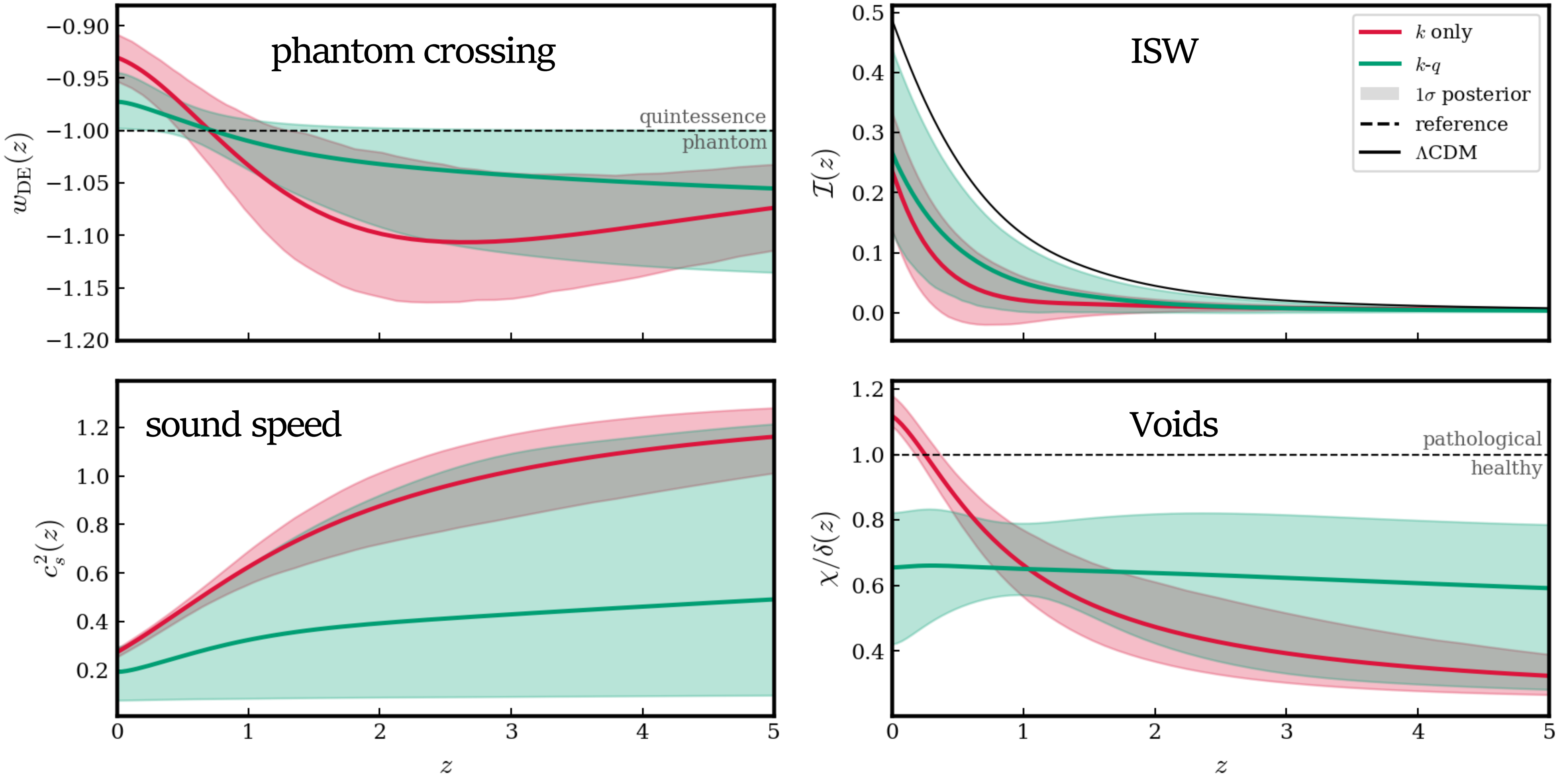
$$k(\phi) = \frac{k_0}{\sqrt{\phi}} \qquad q(\phi) = \frac{q_0}{\phi}$$



Parameter	k–q
H_0	$67.88^{+0.504}_{-0.549}$
$100\omega_b$	2.236 ± 0.014
$100\omega_c$	1.1796 ± 0.090

Posterior Predictive Viability

$$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$$



k-only model:

$$k(\phi) = \frac{k_0}{\sqrt{\phi}} \qquad q(\phi) = 0$$

k–q model:

$$k(\phi) = \frac{k_0}{\sqrt{\phi}} \qquad q(\phi) = \frac{q_0}{\phi}$$

Parameter	k-only	k–q
$\Delta\chi^2$	−11.2	−11.7

Conclusions

- Rolling Galileons: a minimal rolling extension of the cubic Galileon

$$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$$

- Imposed three phenomenological conditions
  1. Late-time phantom crossing
  2. Positive ISW Signature
  3. Healthy Screening in Voids

$$\dot{\phi} > 0: \qquad k, q > 0, \quad k_\phi < 0, \quad q_\phi < 0$$

- Found that data resides in viable region

$$k(\phi) = \frac{k_0}{\sqrt{\phi}} \qquad q(\phi) = \frac{q_0}{\phi} \qquad \Delta\chi^2 = -11.7$$

Reinforcements

"Rolling Galileons"
by ChatGPT

Simple Rolling Ansatz

$$\mathcal{L}_\phi = -k(\phi)X + q(\phi)X^2 - X\Box\phi$$

$$\mathcal{L}_\varphi = -\mathcal{X} - g_0\mathcal{X}\Box\varphi$$

$$\mathcal{L}_\varphi = -\mathcal{X} - g(\varphi)\mathcal{X}\Box\varphi + h_0\mathcal{X}^2$$

$$g(\varphi) = g_1\varphi$$

$$\mathcal{L}_\varphi = -\mathcal{X} - g_1\varphi\mathcal{X}\Box\varphi + h_1\mathcal{X}^2$$

$$\varphi \to \varphi(\phi)$$

such that we normalise the braiding term

$$q_0 = h_1 k_0^2$$

$$\mathcal{L}_\phi = -\frac{k_0}{\sqrt{\phi}}X - X\Box\phi + \frac{q_0}{\phi}X^2.$$

Closure of Rolling Galileons

$$\mathcal{L}_\varphi = -A(\varphi)\mathcal{X} - B(\varphi)\mathcal{X}\Box\varphi + C(\varphi)\mathcal{X}^2, \qquad (11)$$

where A, B, and C are arbitrary functions of φ. Under a general field redefinition

$$\varphi \to \varphi(\phi), \qquad (12)$$

one has

$$\mathcal{X} \to \varphi_\phi^2 X, \qquad (13)$$
$$\Box\varphi \to \varphi_\phi \Box\phi - 2\varphi_{\phi\phi} X. \qquad (14)$$

Substituting (13)-(14) into (11), the Lagrangian retains the same operator form,

$$\mathcal{L}_\phi = -\widetilde{A}(\phi)X - \widetilde{B}(\phi)X\Box\phi + \widetilde{C}(\phi)X^2, \qquad (15)$$

with the transformed coefficient functions

$$\widetilde{A}(\phi) = A(\varphi(\phi))\varphi_\phi^2, \qquad \widetilde{B}(\phi) = B(\varphi(\phi))\varphi_\phi^3, \qquad (16)$$
$$\widetilde{C}(\phi) = C(\varphi(\phi))\varphi_\phi^4 + 2B(\varphi(\phi))\varphi_\phi^2\varphi_{\phi\phi}. \qquad (17)$$

$$\widetilde{k} := \frac{\widetilde{A}}{\widetilde{B}^{2/3}} = \frac{A\varphi_\phi^2}{B^{2/3}(\varphi_\phi^3)^{2/3}} = \frac{A}{B^{2/3}} = k.$$

$$\begin{aligned}\widetilde{D} &:= \widetilde{C} - \frac{2}{3}\widetilde{B}_\phi \\ &= \left(C\varphi_\phi^4 + 2B\varphi_\phi^2\varphi_{\phi\phi}\right) - \frac{2}{3}\left(B_\varphi\varphi_\phi^4 + 3B\varphi_\phi^2\varphi_{\phi\phi}\right) \\ &= \left(C - \frac{2}{3}B_\varphi\right)\varphi_\phi^4 = D\varphi_\phi^4. \qquad (25)\end{aligned}$$

The inhomogeneous terms cancel and $\widetilde{D}$ transforms homogeneously as φ_ϕ^4.

We may therefore form an invariant by dividing through by an object with the same scaling. Since

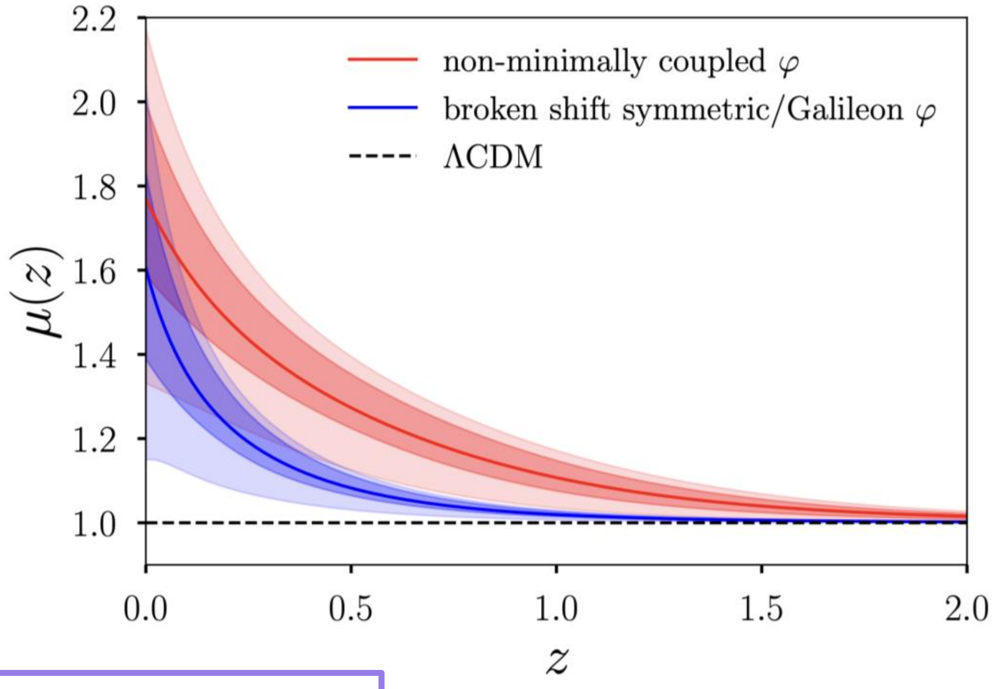
$$\widetilde{B}^{4/3} = B^{4/3}\varphi_\phi^4, \qquad (26)$$

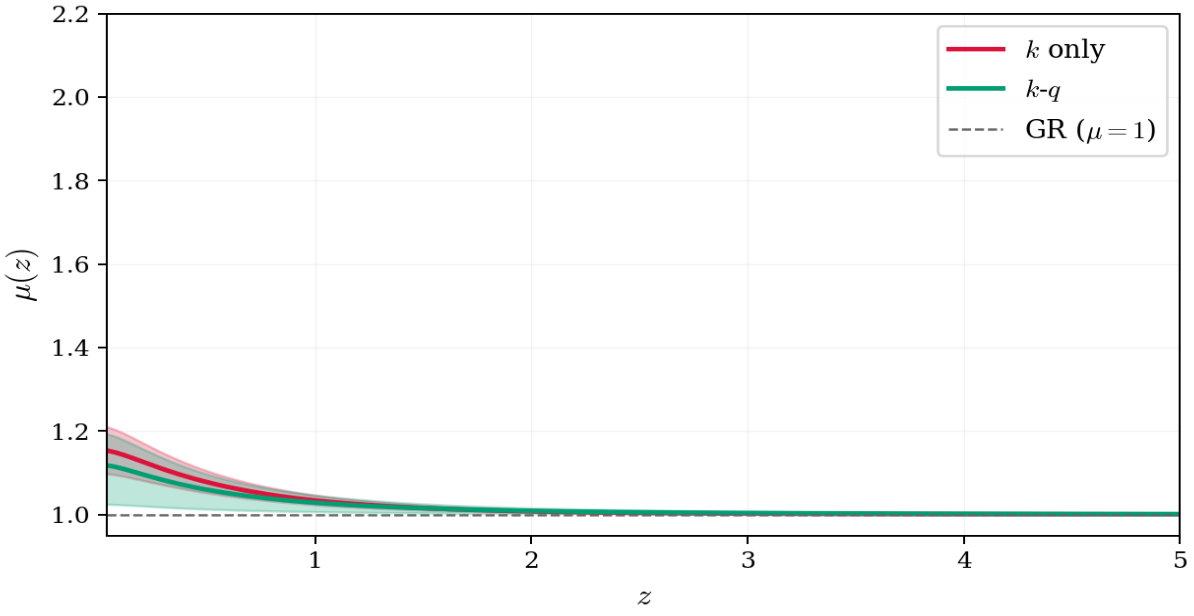
a natural choice is

$$\widetilde{q}(\phi) := \frac{\widetilde{C}(\phi) - \frac{2}{3}\widetilde{B}_\phi(\phi)}{\widetilde{B}(\phi)^{4/3}} = q(\varphi(\phi)). \qquad (27)$$

Growth

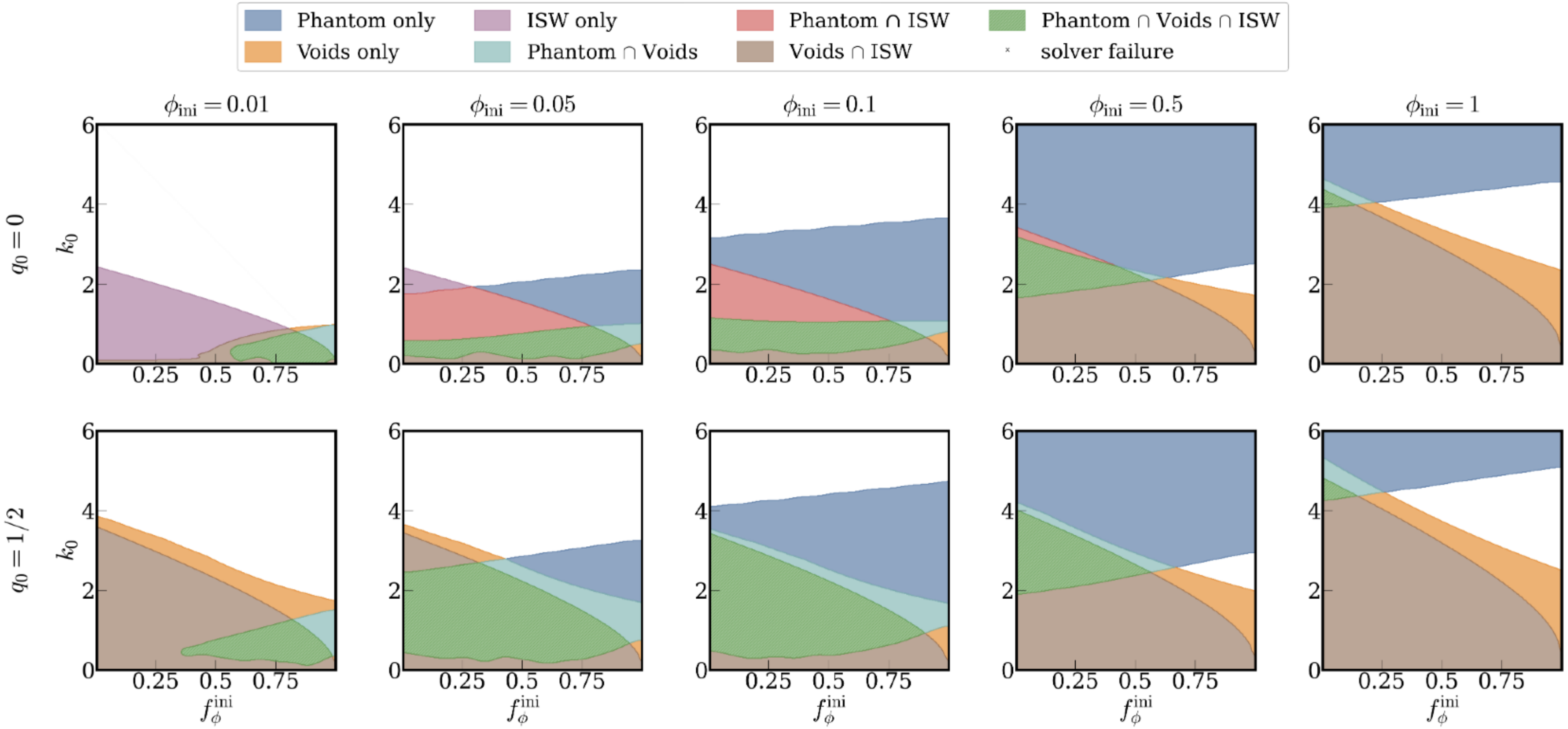
W. Wolf, et. Al. 2026



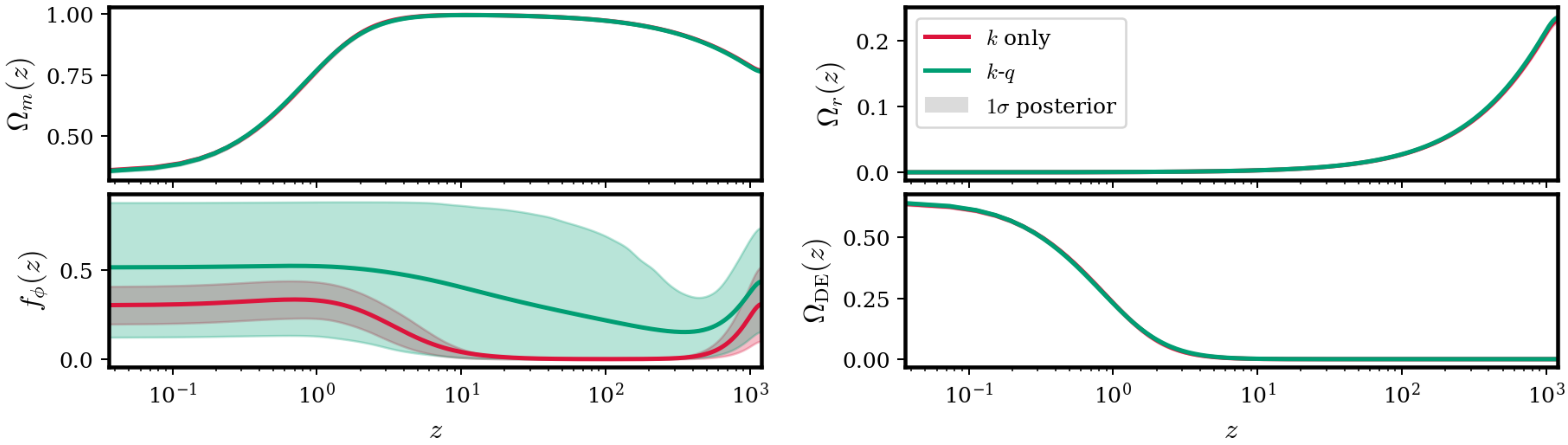


$$\mathcal{L}_\phi = X - V(\phi) - X\Box\phi$$

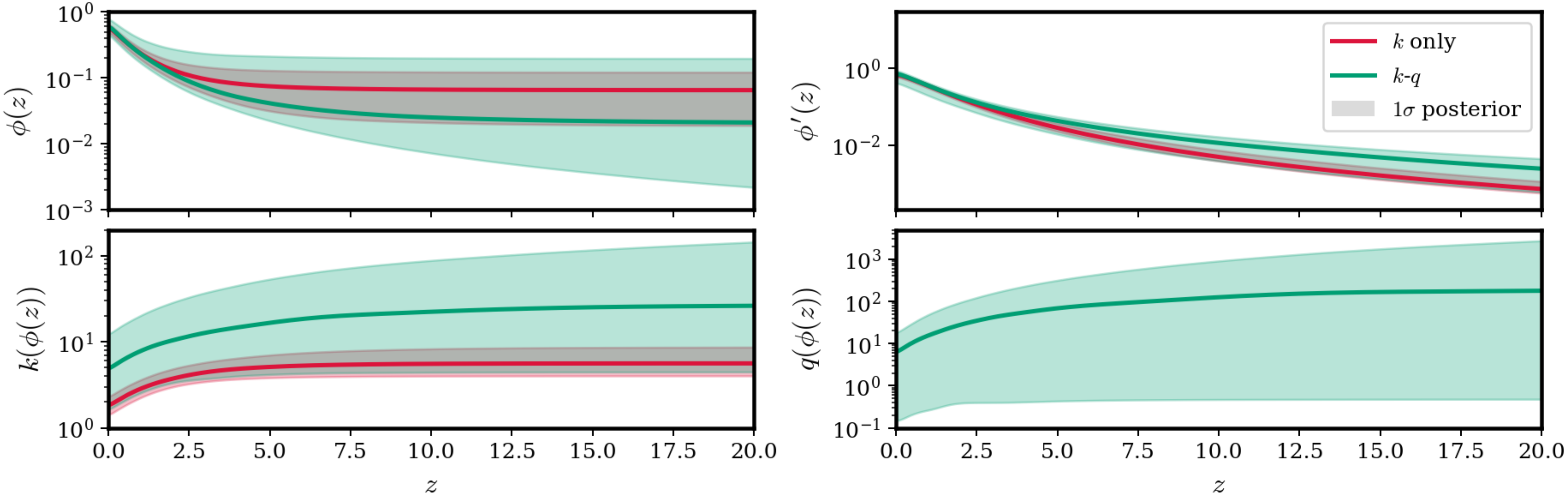
More Carpets



Background Quantities



phi Related Quantities



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$$X := -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Kinetic Gravity Braiding

$$\mathcal{L} = \frac{1}{2}R - \Lambda + \mathcal{L}_\phi, \qquad \mathcal{L}_\phi = K(\phi, X) - G_3(\phi, X)\Box\phi$$

Shift Symmetric

$$\mathcal{L}_\phi = K(X) - G_3(X)\Box\phi$$

requires non-standard kinetic structure

e.g.

$$\mathcal{L}_\phi = \frac{X}{\sqrt{1-2X}} - X\Box\phi$$

Cubic Galileon

$$\mathcal{L}_\phi = X - X\Box\phi$$

eternally phantom

Broken Shift Symmetry

with a Potential

e.g.

$$\mathcal{L}_\phi = X - V(\phi) - X\Box\phi$$

adverse gravitational effects

Rolling Galileons

this talk!