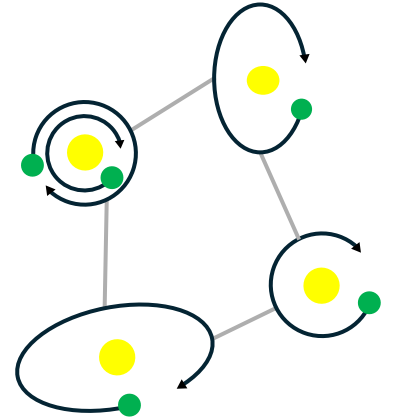
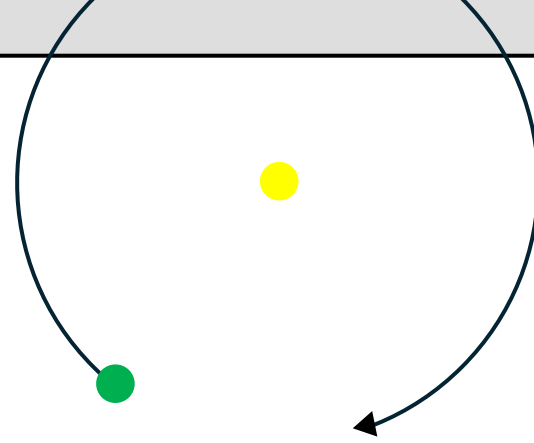




University of Sheffield



Axion-dilaton interactions in the dark sector

Adam Smith

Philippe Brax



Cliff Burgess



Carsten van de Bruck



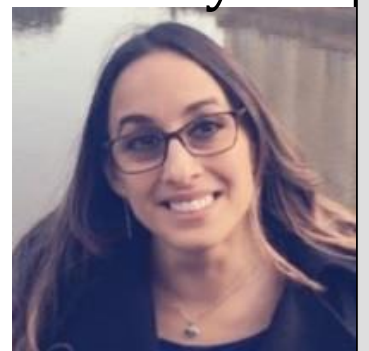
Anne Davis



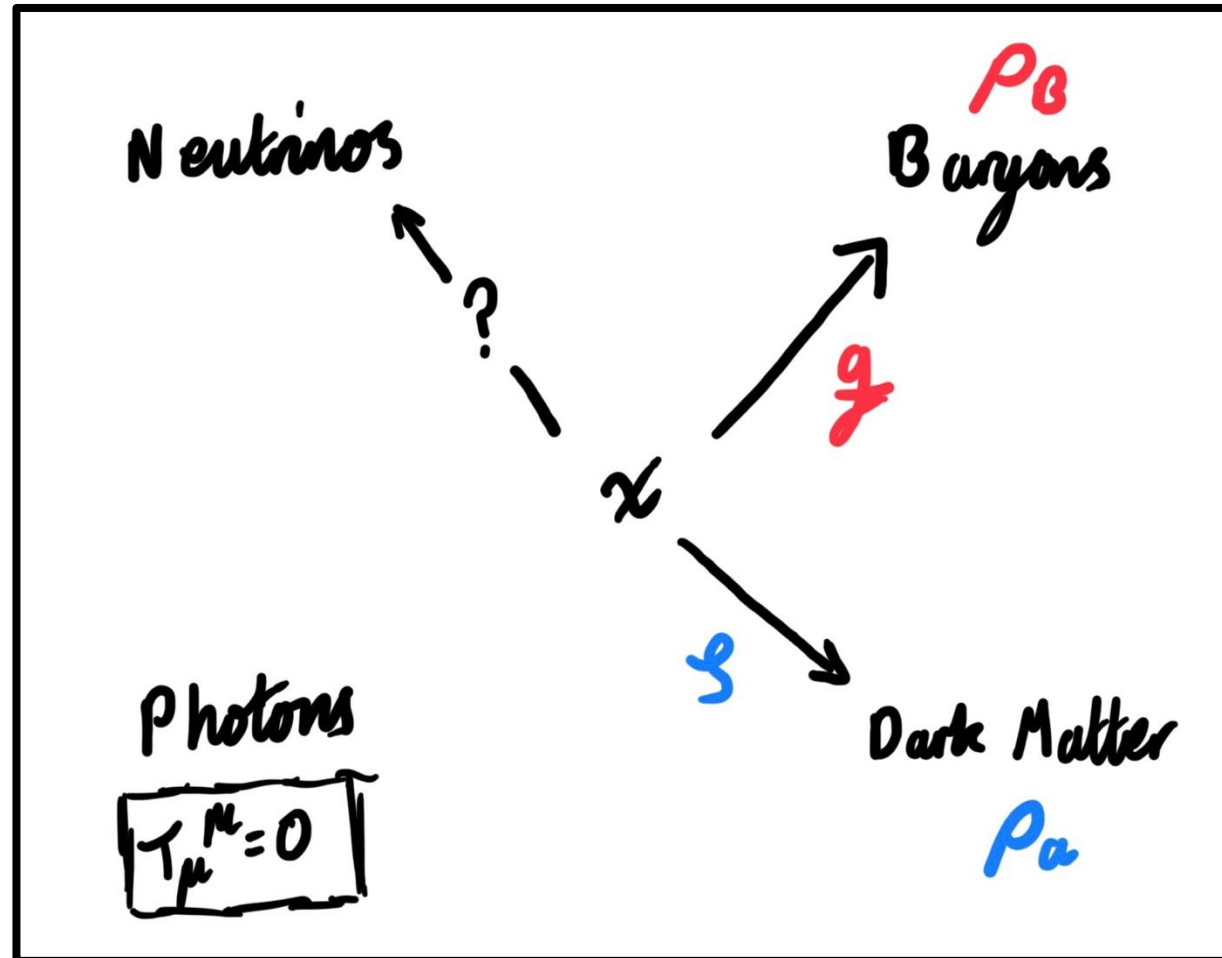
Eleonora Di Valentino



Maria Mylova



Minimal Axion-Dilaton Models



Axion-Dilaton Class

Consequence of the axion's shift symmetry

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2 \sqrt{-g} \left[(\partial\chi)^2 + W^2(\chi)(\partial\mathbf{a})^2 \right]$$

Axion-Dilaton Class

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2 \sqrt{-g} \left[(\partial\chi)^2 + W^2(\chi)(\partial\mathbf{a})^2 \right]$$

Fundamental theory

- Combine into complex fields in extra-dimensional UV completions

$$\Phi = \frac{1}{2} (e^{\zeta\chi} + i\mathbf{a})$$

Dilaton \Leftrightarrow Volume modulus

Axion-Dilaton Class

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2 \sqrt{-g} \left[(\partial\chi)^2 + W^2(\chi)(\partial\mathbf{a})^2 \right]$$

Fundamental theory

- Combine into complex fields in extra-dimensional UV completions

$$\Phi = \frac{1}{2} (e^{\zeta\chi} + i\mathbf{a})$$

Dilaton \Leftrightarrow Volume modulus

$$W = W_0 e^{-\zeta\chi}$$

Axion-Dilaton Class

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2 \sqrt{-g} \left[(\partial\chi)^2 + W^2(\chi)(\partial\mathbf{a})^2 \right]$$

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Dilaton \Leftrightarrow Volume modulus

$$W = W_0 e^{-\zeta\chi}$$

Cosmology

- Dilaton is naturally light, DE scalar candidate

$$V = U e^{-\lambda\chi}$$

Axion-Dilaton Class

$$\mathcal{L}_{axio-dilaton} = -\frac{1}{2}M_p^2 \sqrt{-g} \left[(\partial\chi)^2 + W^2(\chi)(\partial\mathbf{a})^2 \right]$$

Fundamental theory

- Combine into complex fields in extra-dimensional UV completions

$$\Phi = \frac{1}{2} (e^{\zeta\chi} + i\mathbf{a})$$

Dilaton \Leftrightarrow Volume modulus

$$W = W_0 e^{-\zeta\chi}$$

Cosmology

- Dilaton is naturally light, DE scalar candidate

$$V = U e^{-\lambda\chi}$$

- Axions have wide range of uses, e.g. CDM

$$V(a) = \frac{m_a^2}{2} (a - a_+)^2 + \dots$$

The Late-Time Theory:

$$S = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} M_p^2 \left[R + \underbrace{\partial_\mu \chi \partial^\mu \chi}_{\text{Dilaton Kinetic Term}} + \underbrace{W^2(\chi) \partial_\mu \mathbf{a} \partial^\mu \mathbf{a}}_{\text{Axion Kinetic Term}} \right] + \underbrace{V(\chi, \mathbf{a})}_{\text{Axio-dilaton Potential}} \right\} + \mathcal{L}_m(\Psi, \tilde{g})$$

The dilaton couples to matter as a pseudo-Brans-Dicke scalar:

$$\tilde{g}_{\mu\nu} := C^2(\chi) g_{\mu\nu}$$

$$C(\chi) = e^{\mathbf{g}\chi}$$

The Late-Time Theory:

$$S = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} M_p^2 \left[R + \underbrace{\partial_\mu \chi \partial^\mu \chi}_{\text{Dilaton Kinetic Term}} + \underbrace{W^2(\chi) \partial_\mu \mathbf{a} \partial^\mu \mathbf{a}}_{\text{Axion Kinetic Term}} \right] + \underbrace{V(\chi, \mathbf{a})}_{\text{Axio-dilaton Potential}} \right\} + \mathcal{L}_m(\Psi, \tilde{g})$$

The dilaton couples to matter as a pseudo-Brans-Dicke scalar:

$$\tilde{g}_{\mu\nu} := C^2(\chi) g_{\mu\nu} \quad \frac{1}{\sqrt{-g}} \frac{\partial \mathcal{L}_m}{\partial \chi} = \mathbf{g} \rho_m$$

$$C(\chi) = e^{\mathbf{g}\chi}$$

$$m_B(\chi) = m e^{\mathbf{g}\chi}$$

The Late-Time Theory:

$$S = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} M_p^2 \left[R + \underbrace{\partial_\mu \chi \partial^\mu \chi}_{\text{Dilaton Kinetic Term}} + \underbrace{W^2(\chi) \partial_\mu \mathbf{a} \partial^\mu \mathbf{a}}_{\text{Axion Kinetic Term}} \right] + \underbrace{V(\chi, \mathbf{a})}_{\text{Axio-dilaton Potential}} \right\} + \mathcal{L}_m(\Psi, \tilde{g})$$

Everything dilaton related is a runaway exponential

$$m_B(\chi) = m e^{\mathbf{g}\chi}$$

$$V(\mathbf{a}) = \frac{m_a^2}{2} (\mathbf{a} - \mathbf{a}_+)^2 + \dots$$

$$W = W_0 e^{-\zeta \chi}$$

$$V = U(\chi) e^{-\lambda \chi} + V(\mathbf{a})$$

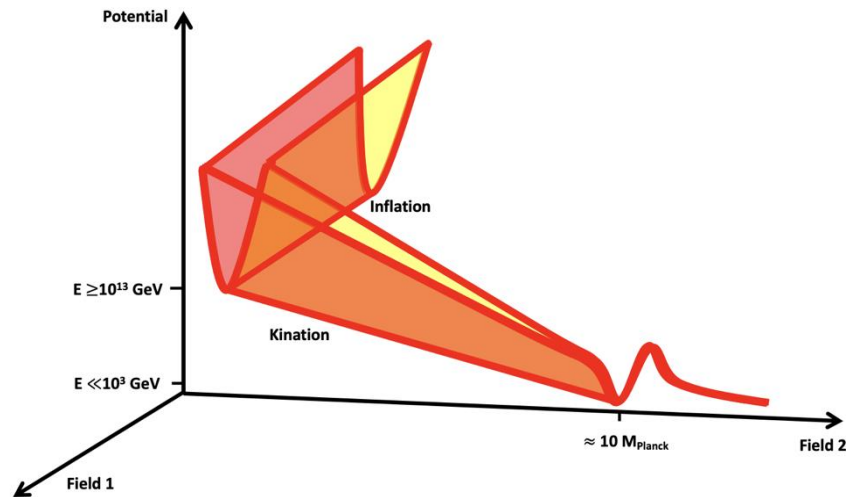
$$U(\chi) = U_0 \left[1 - u_1 \chi + \frac{u_2}{2} \chi^2 \right]$$

Potential Motivations

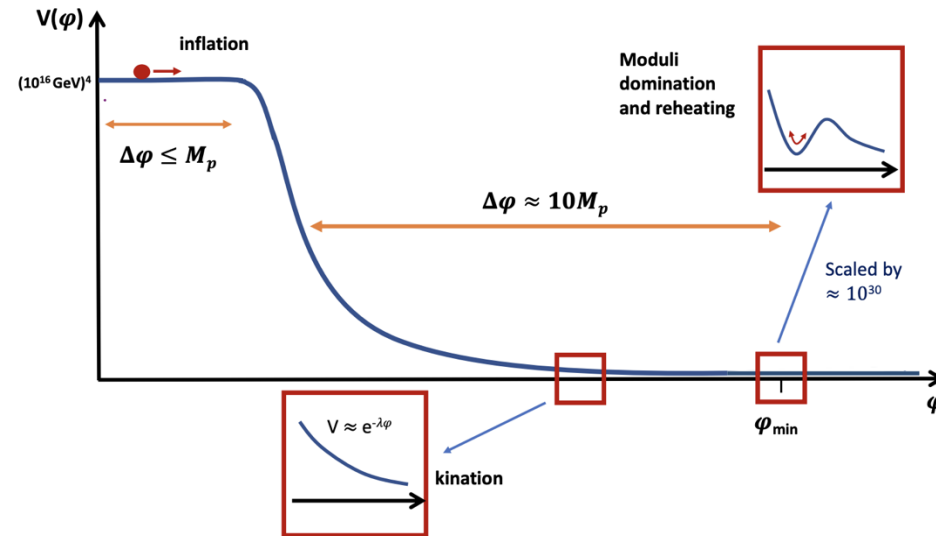
Albrecht-Skordis Potential (2001) [0107573](#)

$$V = U(\chi)e^{-\lambda\chi} + V(\mathbf{a})$$

$$U(\chi) = U_0 \left[1 - u_1\chi + \frac{u_2}{2}\chi^2 \right]$$



Cicoli et al (2023) [2303.04819](#)

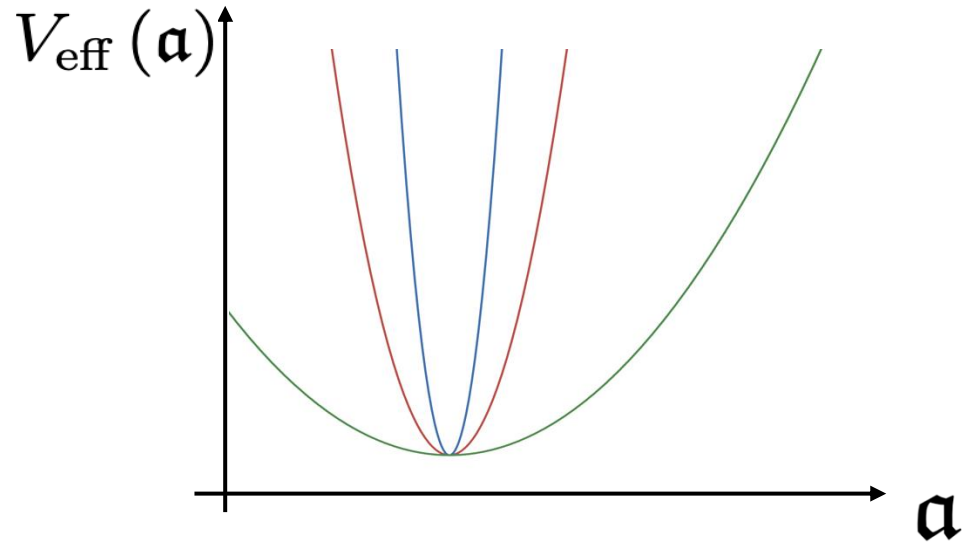


Apers et al (2024) [2401.04064](#)

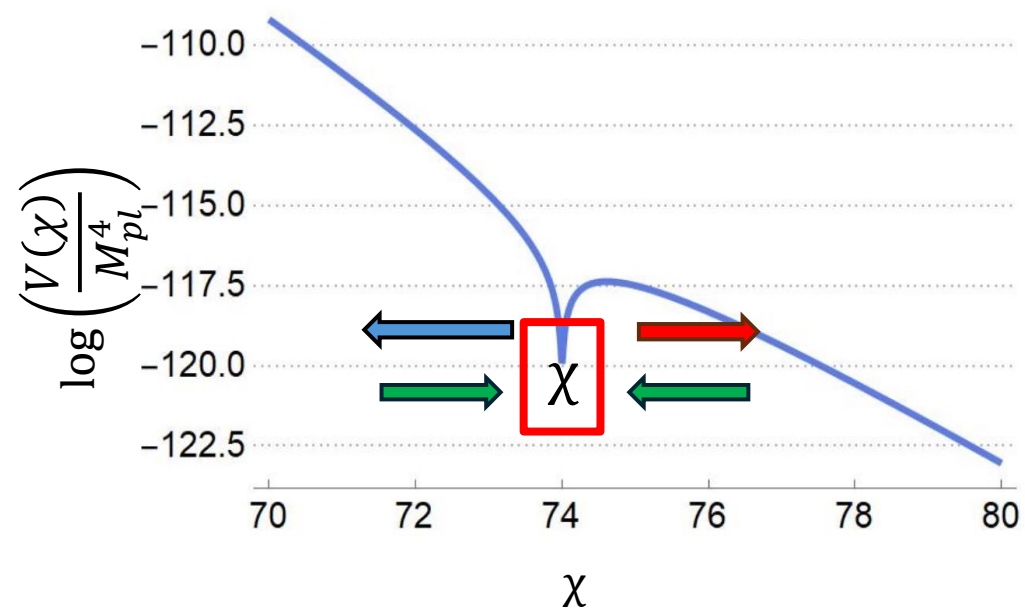
Axion

$$W = W_0 e^{-\zeta \chi}$$

Dilaton

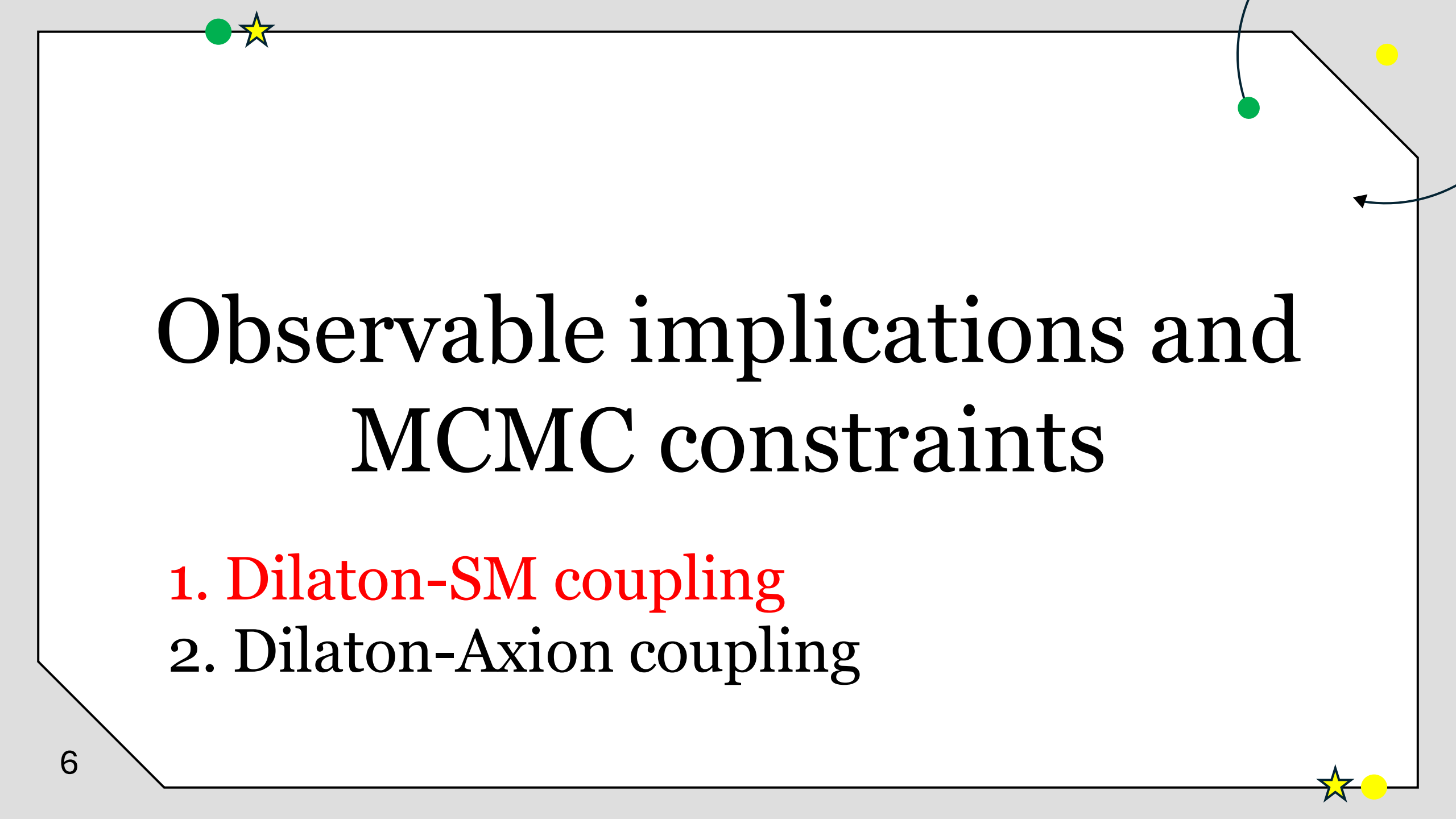


$$\bar{\chi}'' + 2\mathcal{H}\bar{\chi}' + \frac{a^2}{M_p^2} V_{,\chi} = \frac{a^2}{M_p^2} \left(g\bar{\rho}_B - \zeta\bar{\rho}_{ax} \right)$$



$$\bar{\rho}_{ax} = \frac{Cm(t)}{a^3}$$

$$m^2(t) = \frac{m_a^2}{W^2(\bar{\chi})}$$



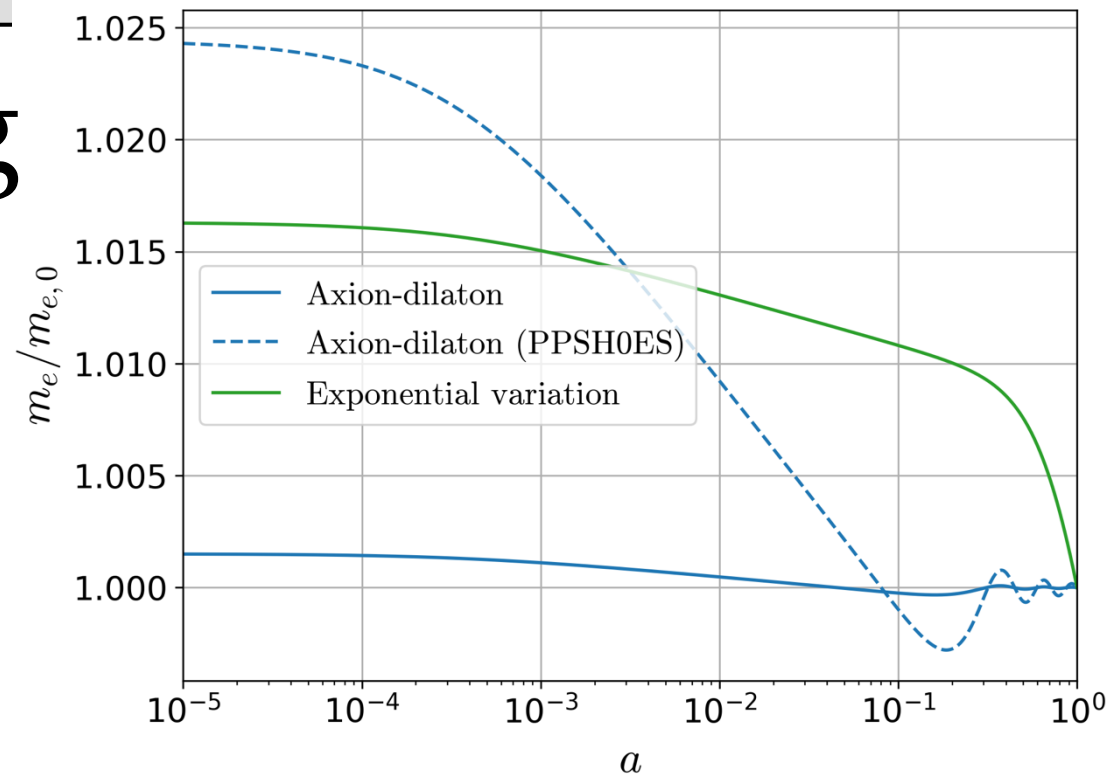
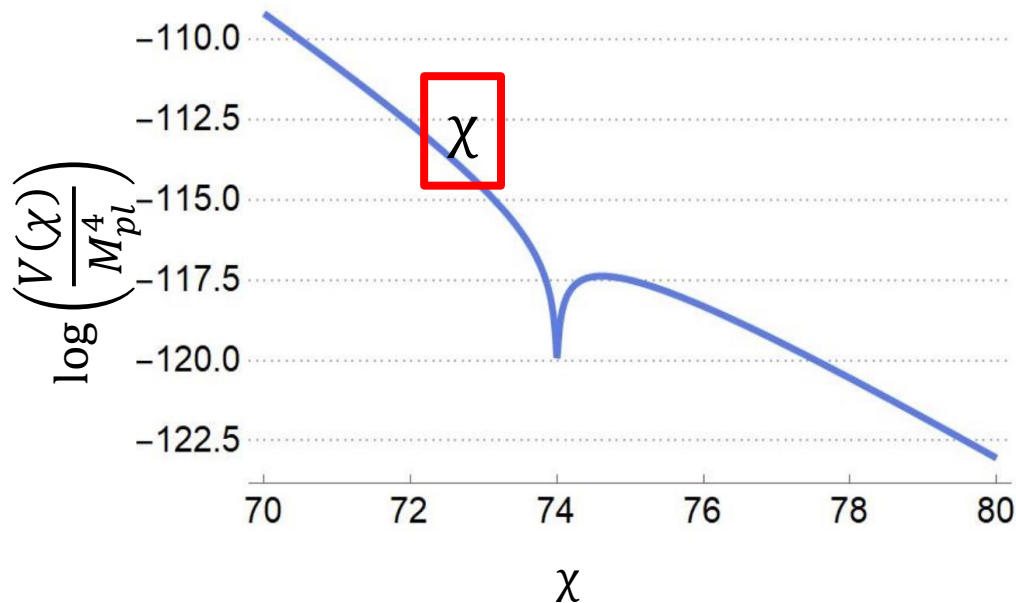
Observable implications and MCMC constraints

1. Dilaton-SM coupling
2. Dilaton-Axion coupling

1. Dilaton-SM Coupling

MCMC results using:

- Planck 2018 + ACT Dr6 lensing
- Desi Dr2 BAO
- Pantheon +



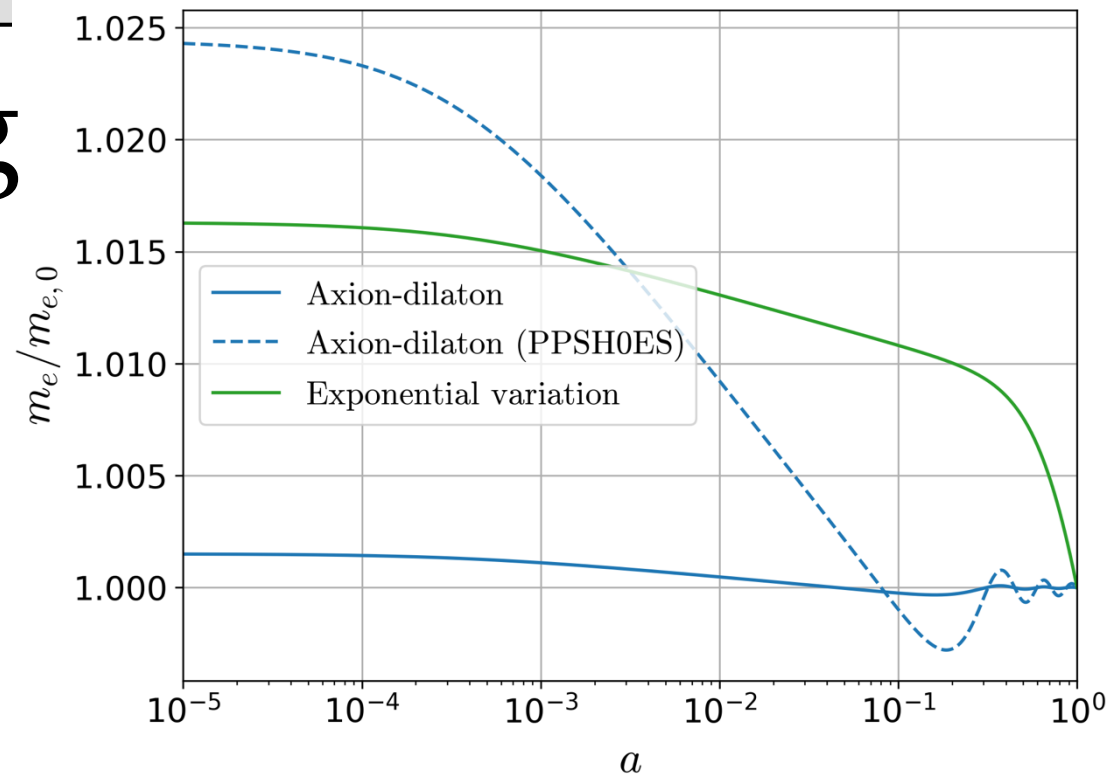
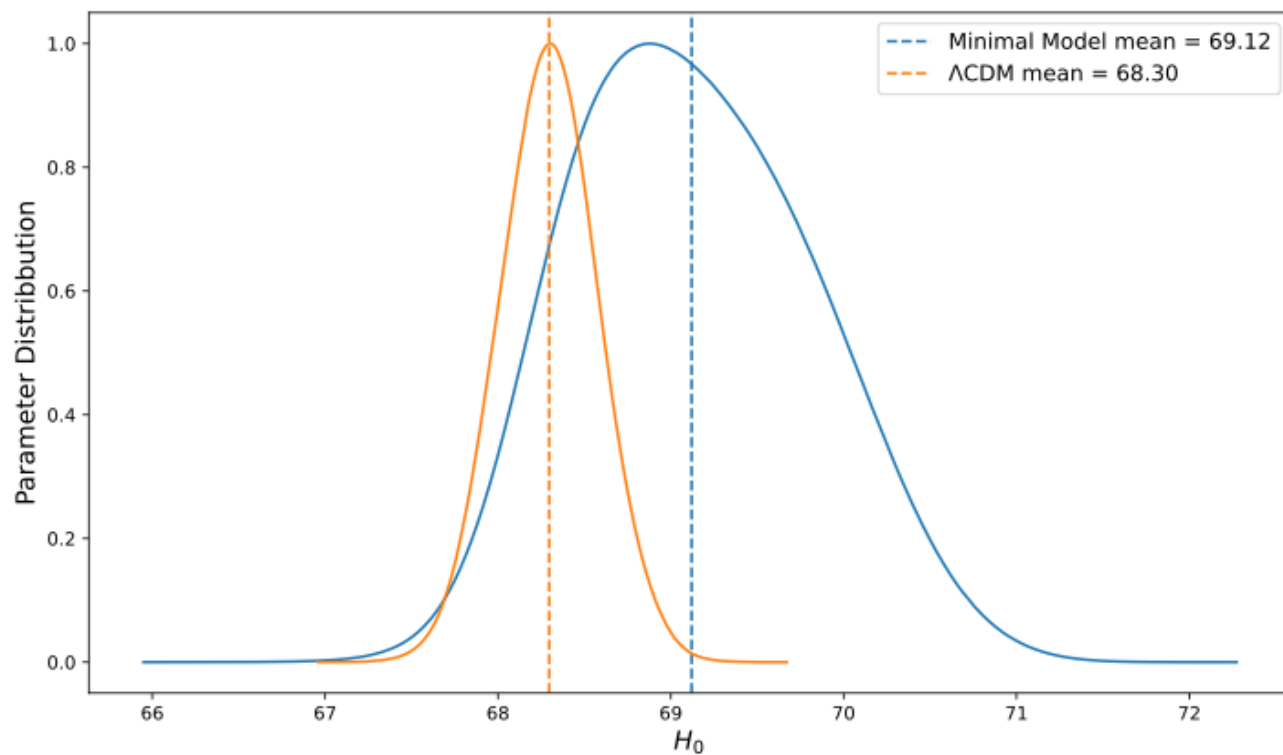
Best-fitting cosmology



1. Dilaton-SM Coupling

MCMC results using:

- Planck 2018 + ACT Dr6 lensing
- Desi Dr2 BAO
- Pantheon +



χ_i

$$W = W_0 e^{-\zeta \chi}$$

$$m_B(\chi) = m e^{g \chi}$$

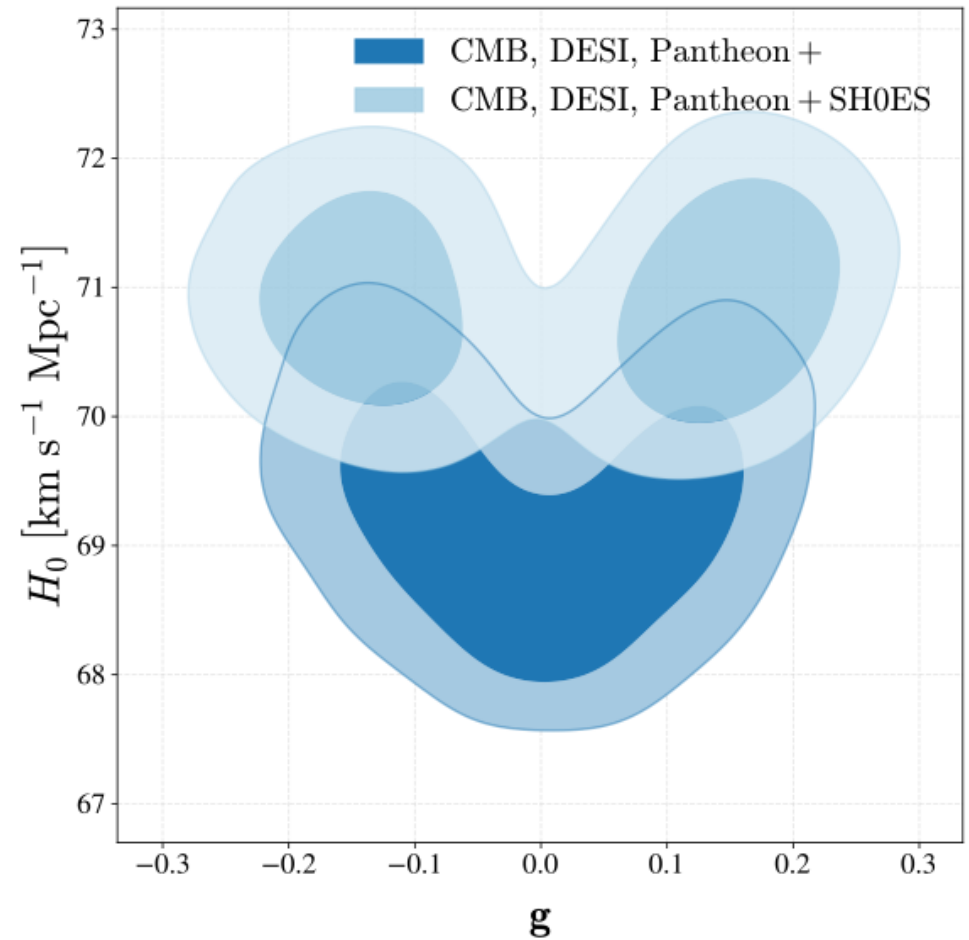


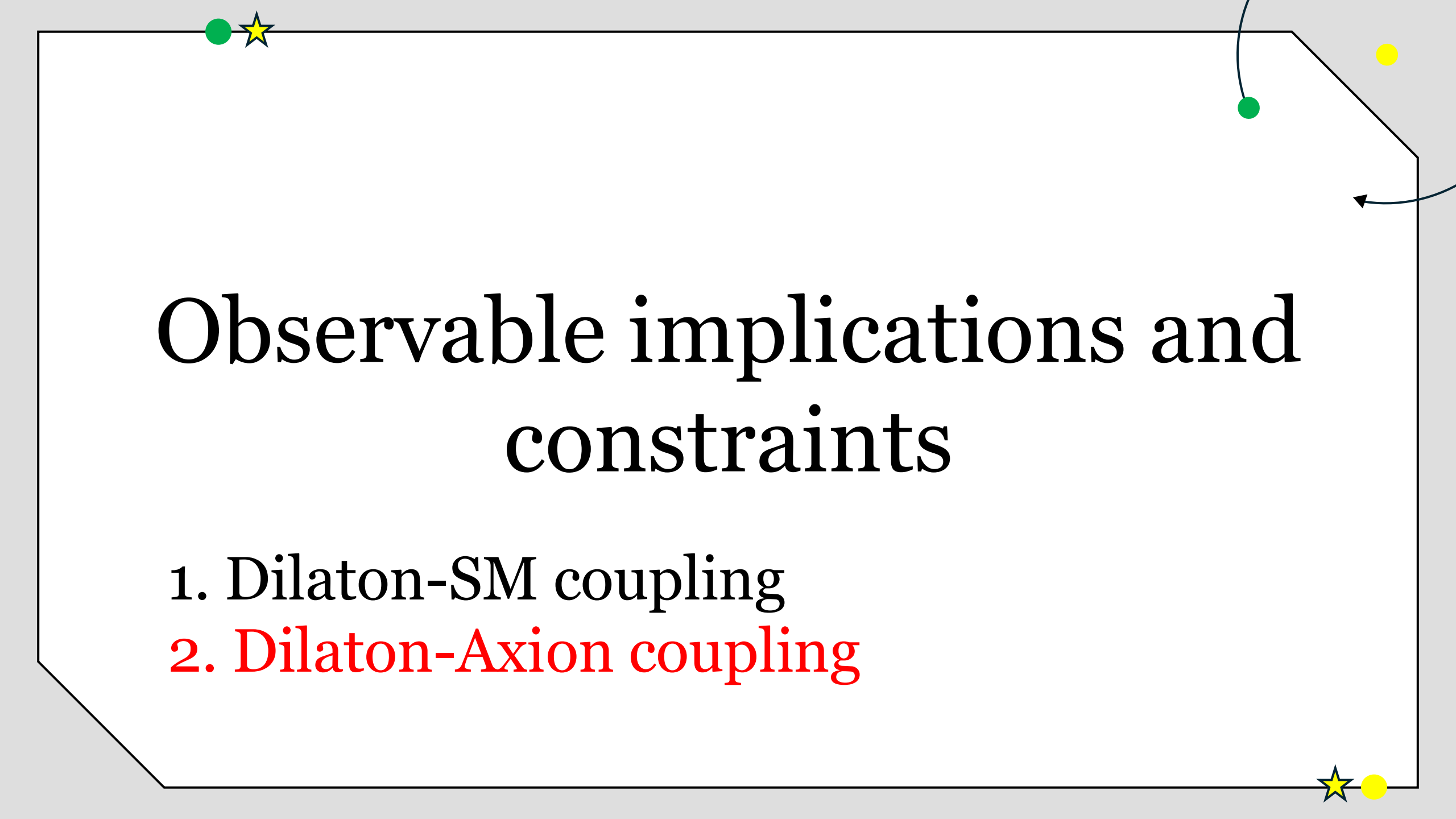


Hubble Tension

- Alleviating the Hubble tension requires couplings preferred non-zero
- Grossly violates local physics constraints (unless screened)

$$m_B(\chi) = m e^{g\chi}$$



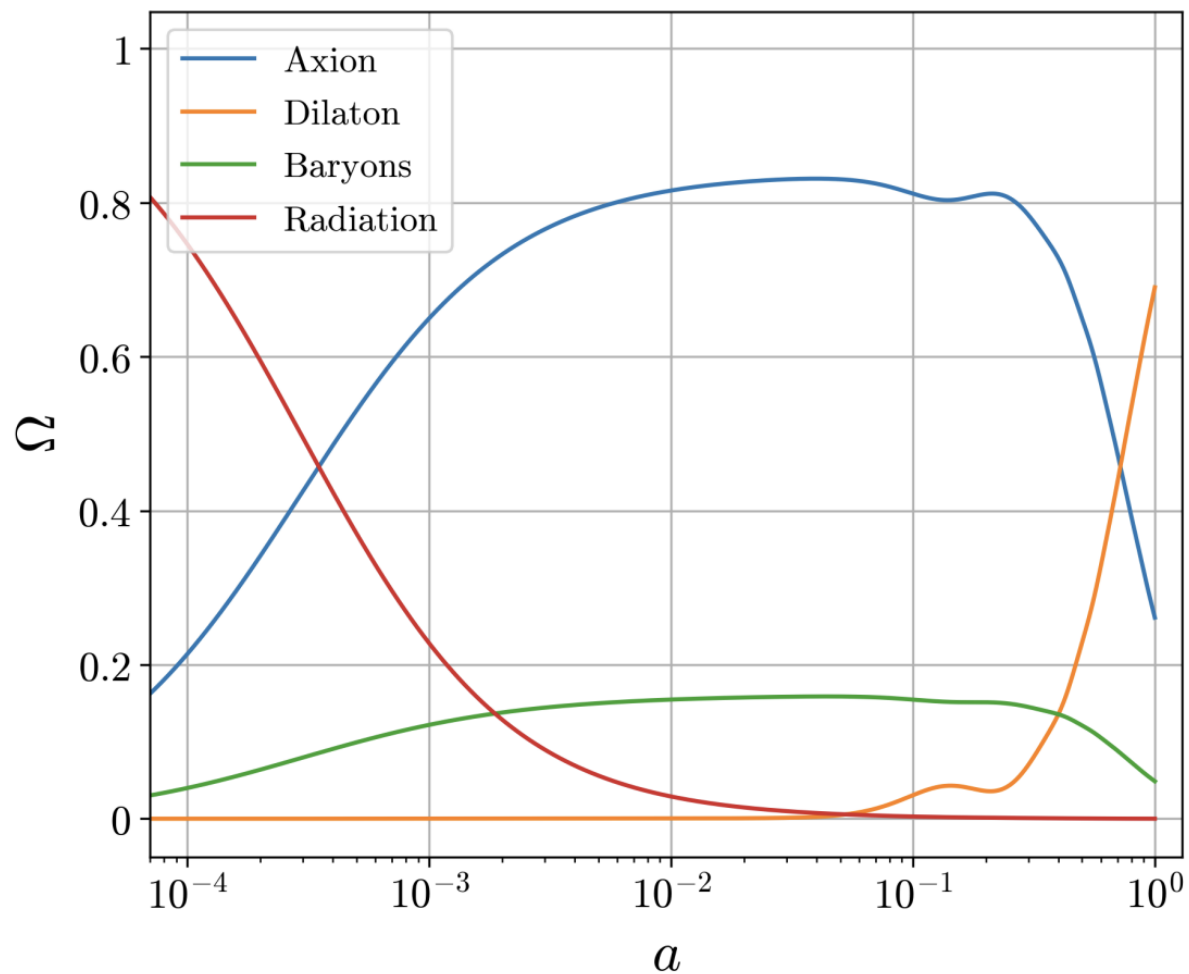
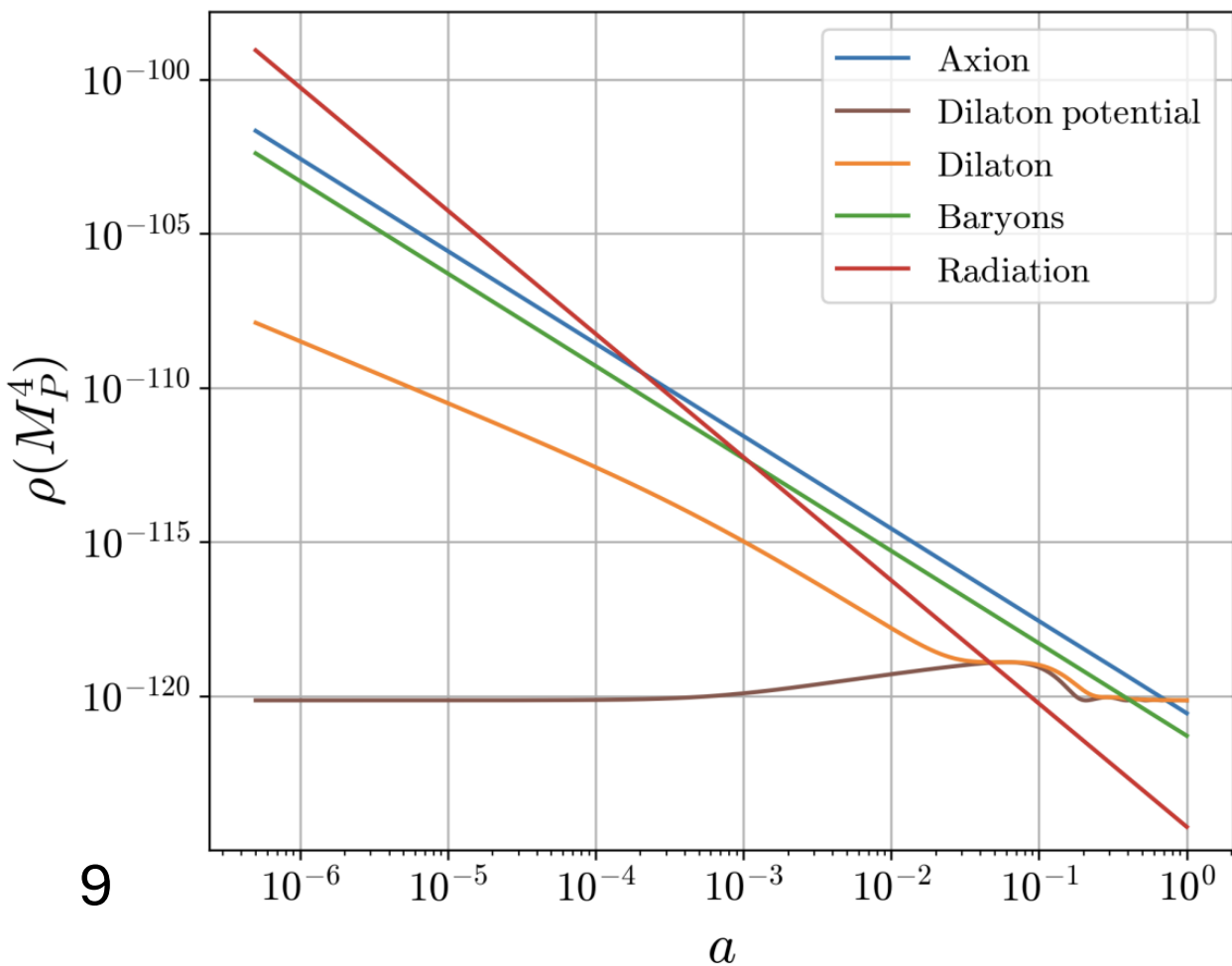


Observable implications and constraints

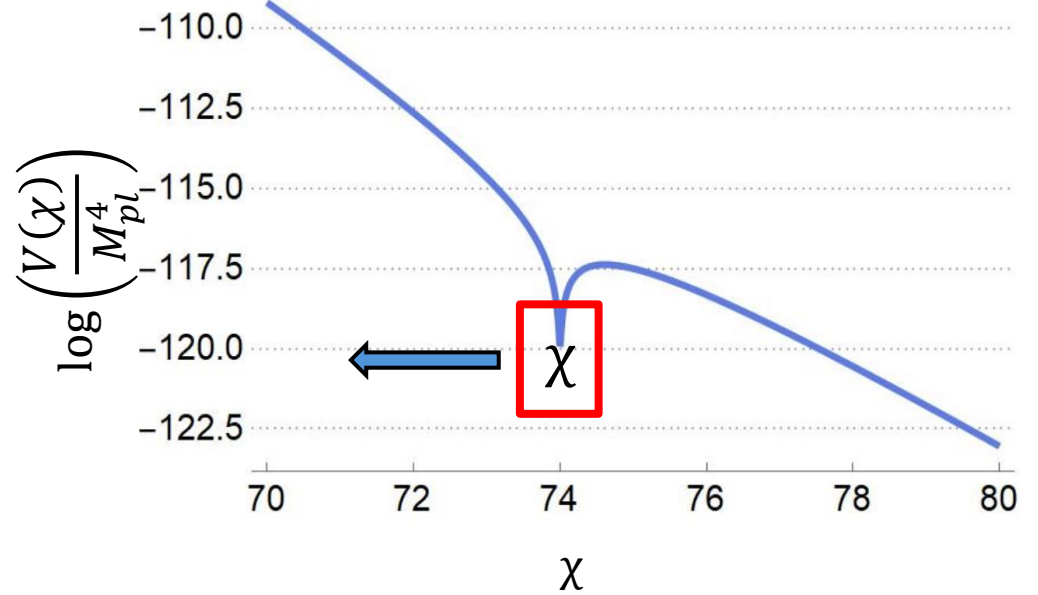
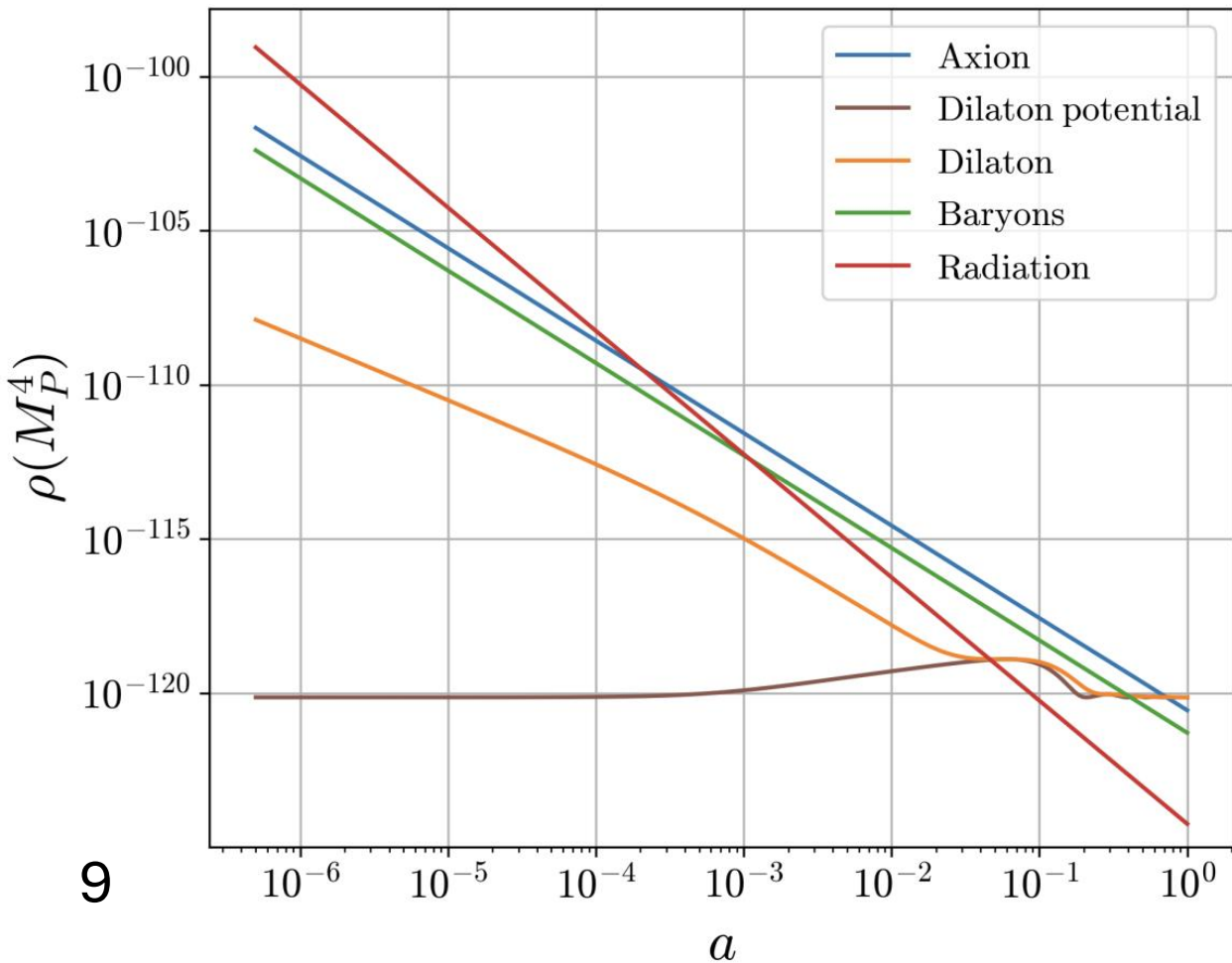
1. Dilaton-SM coupling
2. Dilaton-Axion coupling

2. Dilaton-Axion Coupling

$$\zeta = 0.1 \quad \text{and} \quad \mathbf{g} = -10^{-3}$$

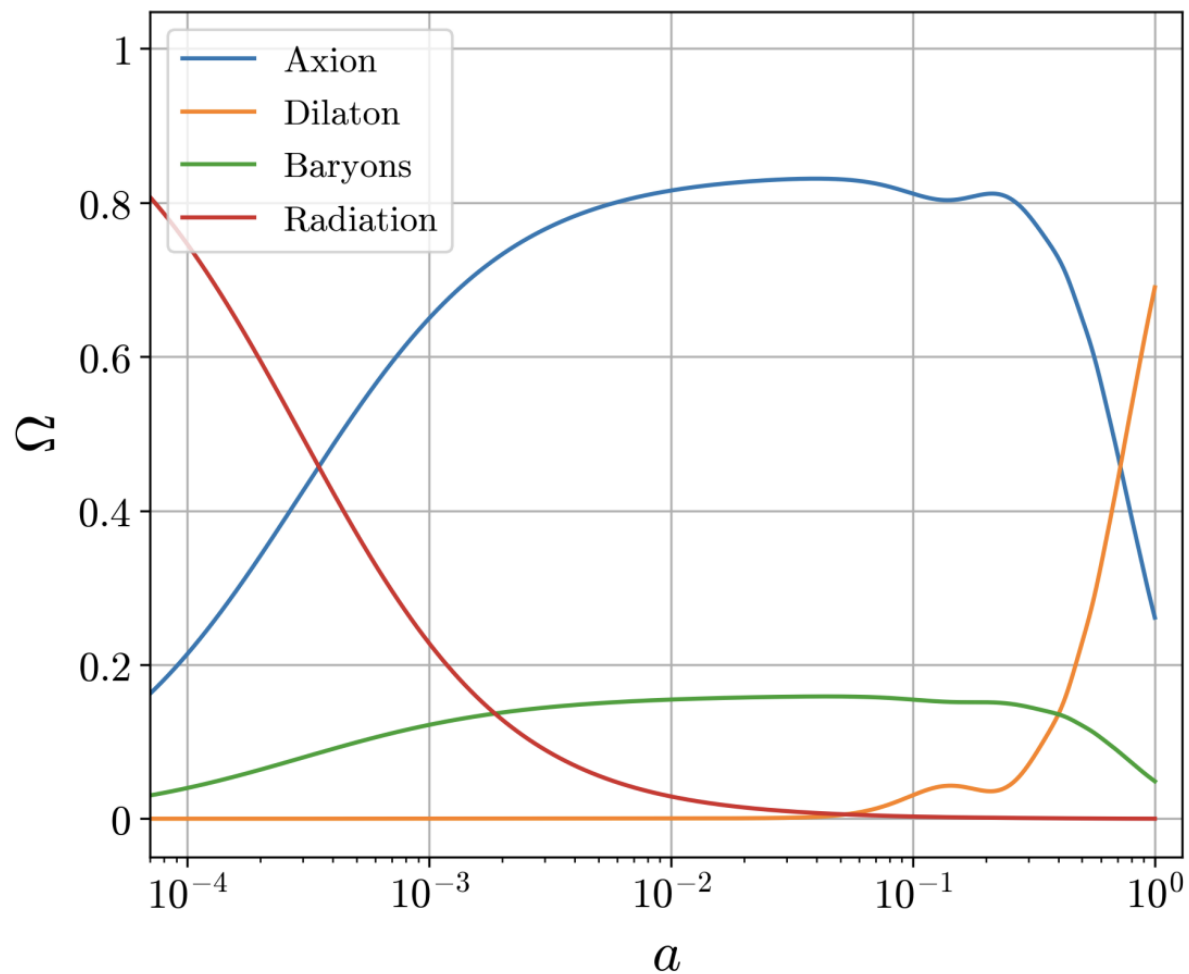
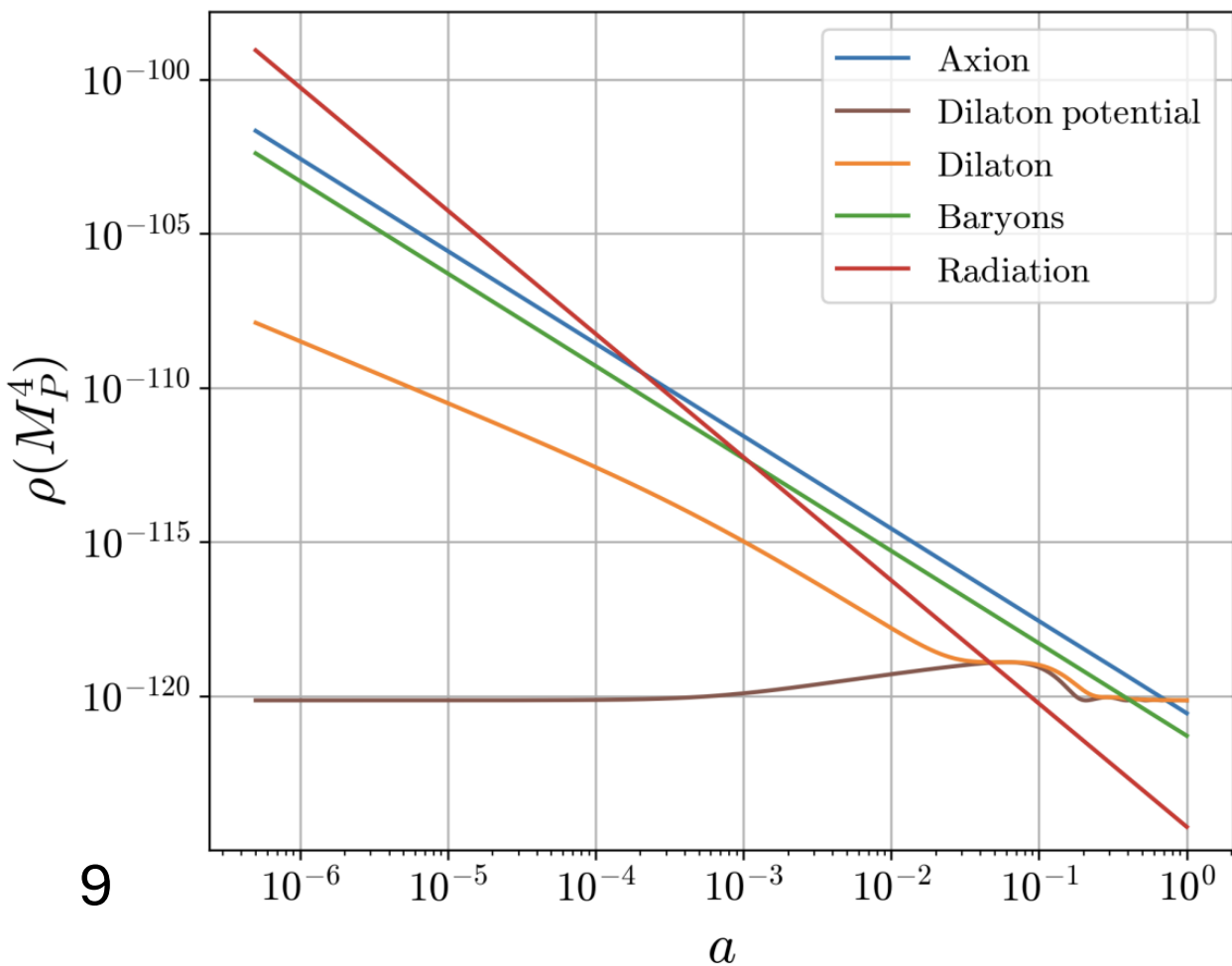


2. Dilaton-Axion Coupling

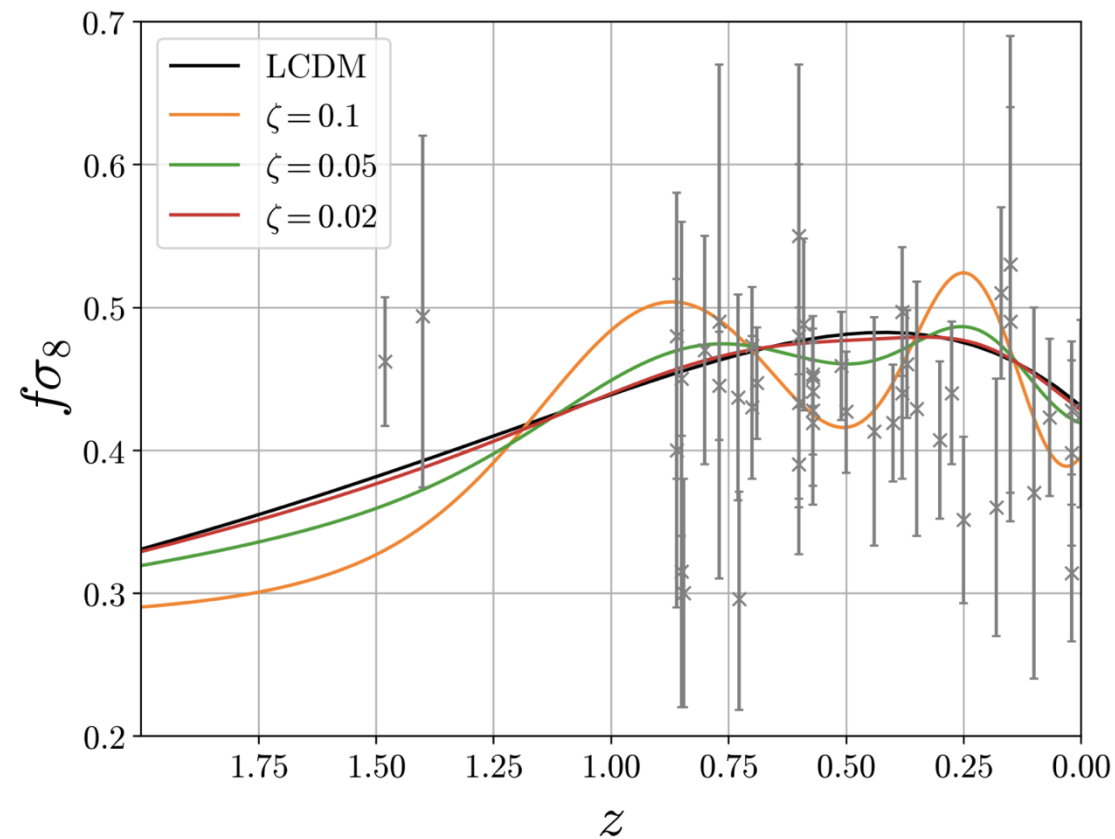
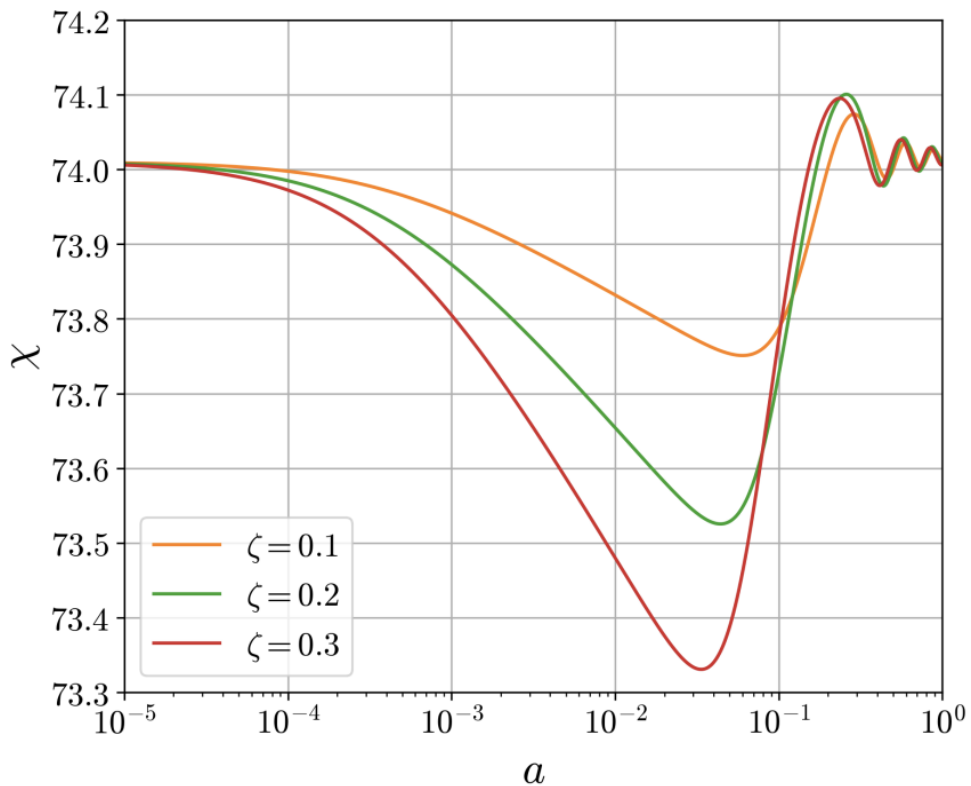


2. Dilaton-Axion Coupling

$$\zeta = 0.1 \quad \text{and} \quad \mathbf{g} = -10^{-3}$$



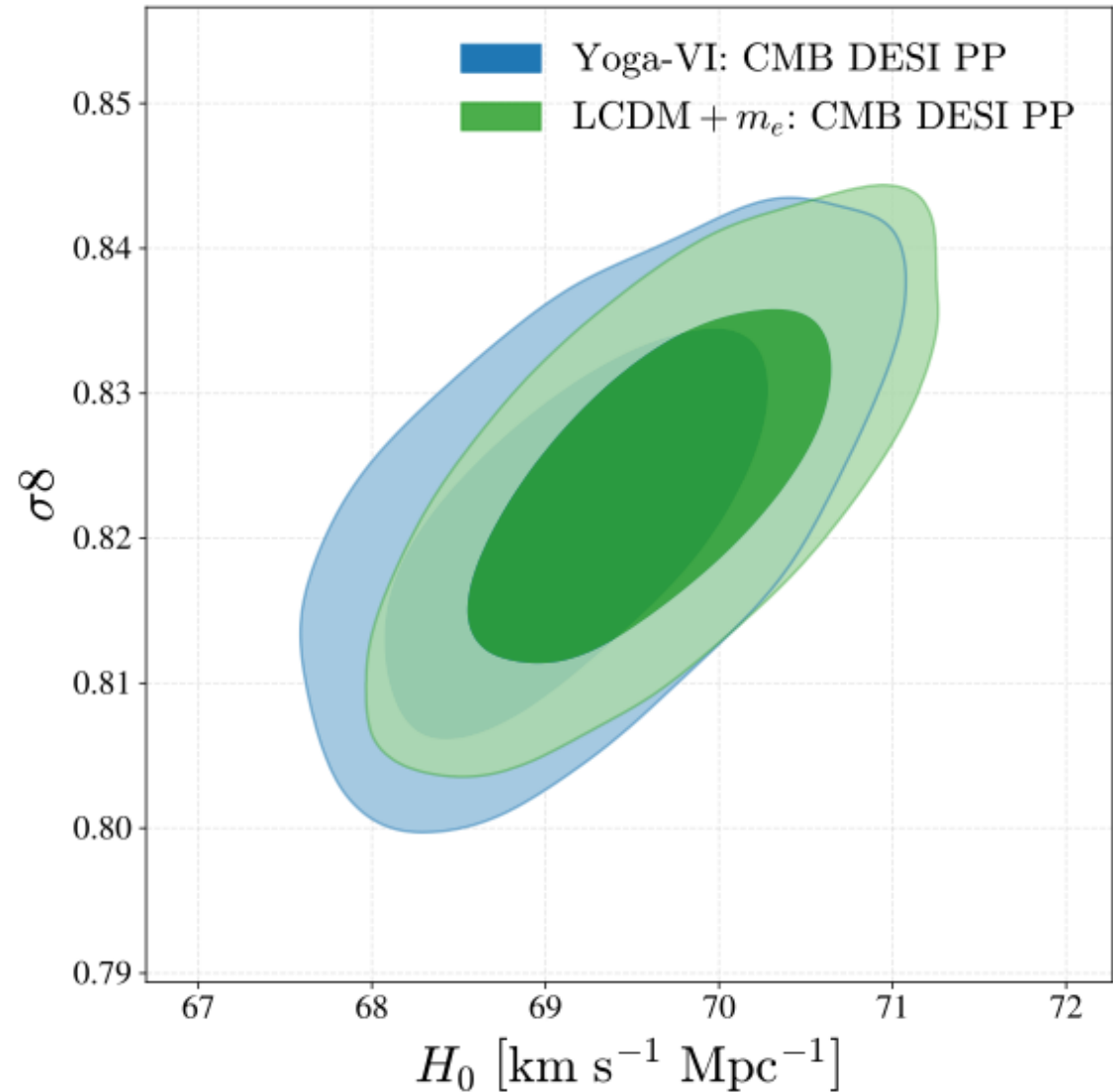
Structure Growth



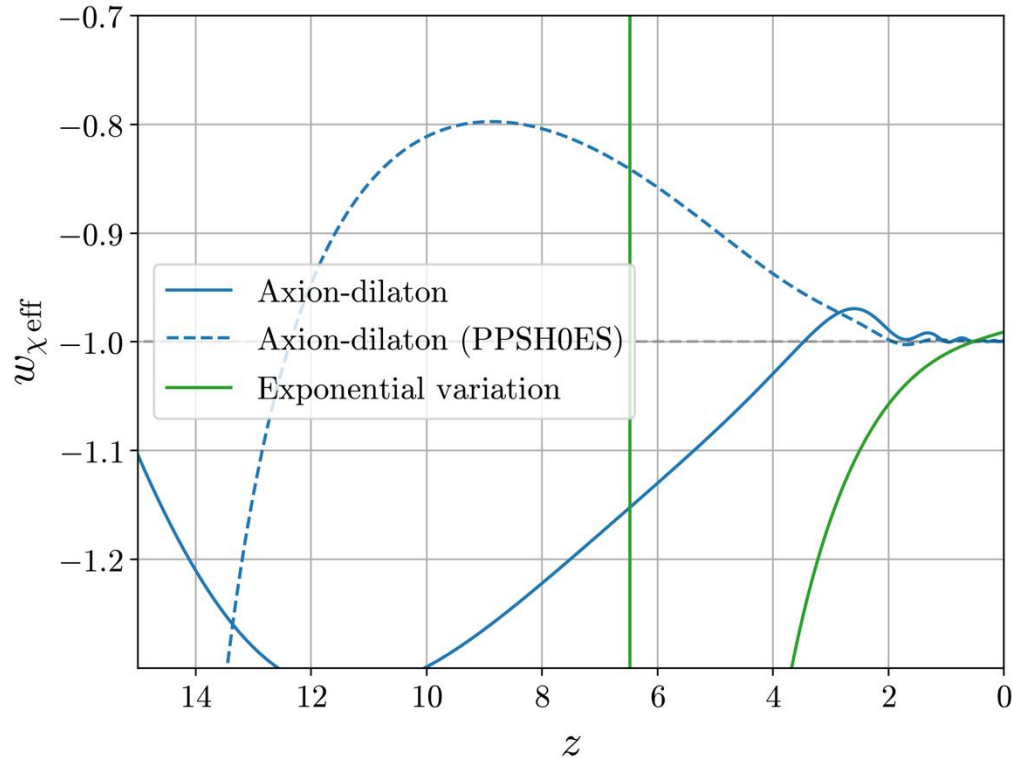
$$f\sigma_8 = \frac{\sigma_8(z, k_{\sigma 8})}{\mathcal{H}} \frac{\delta'_m(z, k_{\sigma 8})}{\delta_m(z, k_{\sigma 8})}$$

Structure Growth

- Models trace the same degeneracy line with H_0 as standard varying electron mass extensions.
- Don't expect these models to alleviate problems with σ_8



The Effective Equation of State



Best-fitting cosmology when combining CMB
DESI Pantheon +

- The preference for a phantom equation of state from DESI results assumes matter species evolve $\propto 1/a^3$
- This preference remains in the coupled quintessence cases studied here.

$$\rho_{\chi}^{\text{eff}} = \rho_{\chi} + \left[\frac{W(\chi_0)}{W(\chi)} - 1 \right] \frac{\rho_{\text{ax}0}}{a^3}$$

$$\omega_{\chi}^{\text{eff}} = \frac{\omega_{\chi}(\chi)}{1 + [e^{\zeta(\chi - \chi_0)} - 1] \frac{\rho_{\text{ax}0}}{a^3 \rho_{\chi}}}$$





Fitness Tests (With SHoES calibration)

Model	H_0	g	ζ	$\Delta\chi^2$
Yoga VI	70.85 ± 0.58 (71.11)	$0.03^{+0.17}_{-0.20}$ (-0.214)	-0.012 ± 0.060 (0.082)	-19.7
EXP	70.79 ± 0.59 (71.03)	0.000 ± 0.160 (0.161)	-0.003 ± 0.062 (-0.058)	-18.9
Λ CDM+ m_e	70.97 ± 0.57 (71.34)	–	–	-19.2
$w_0w_a+m_e$	70.51 ± 0.73 (70.37)	–	–	-19.4

- Any model with varying electron mass gets massively preferred
- Not possible to distinguish between them with current data



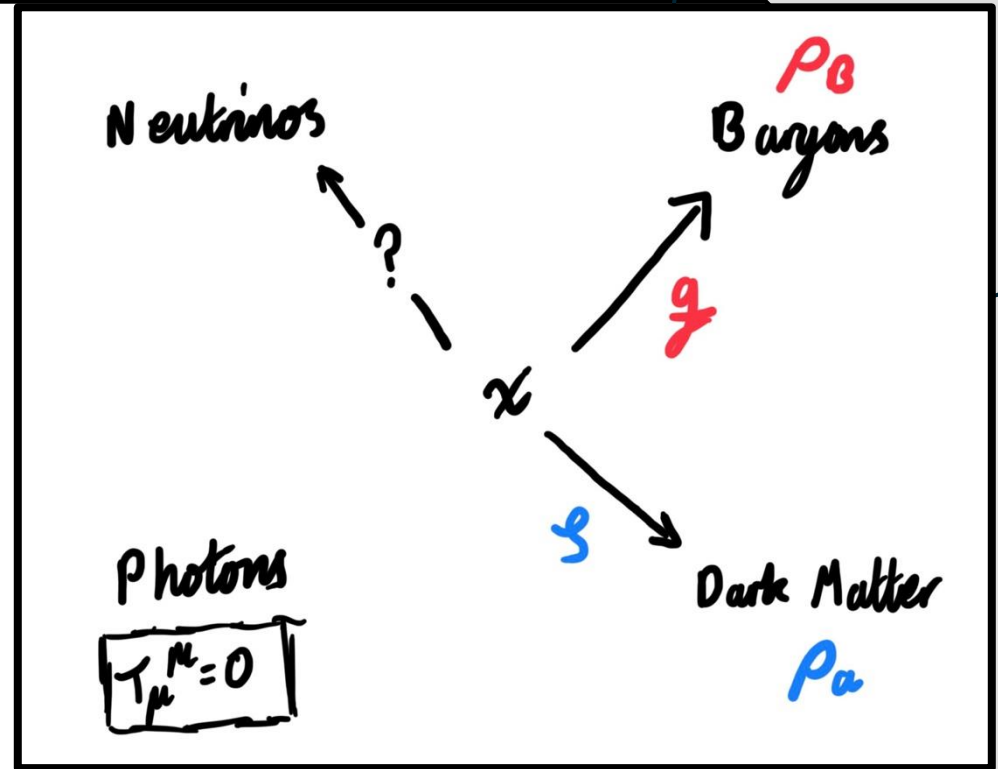


Conclusion

- Axion can play the role of dark matter
- Dilaton can play the role of dark energy

Interactions between them can give:

- i. Oscillations in structure growth
- ii. Deviations in the CMB
- iii. Particle masses to oscillate at late times



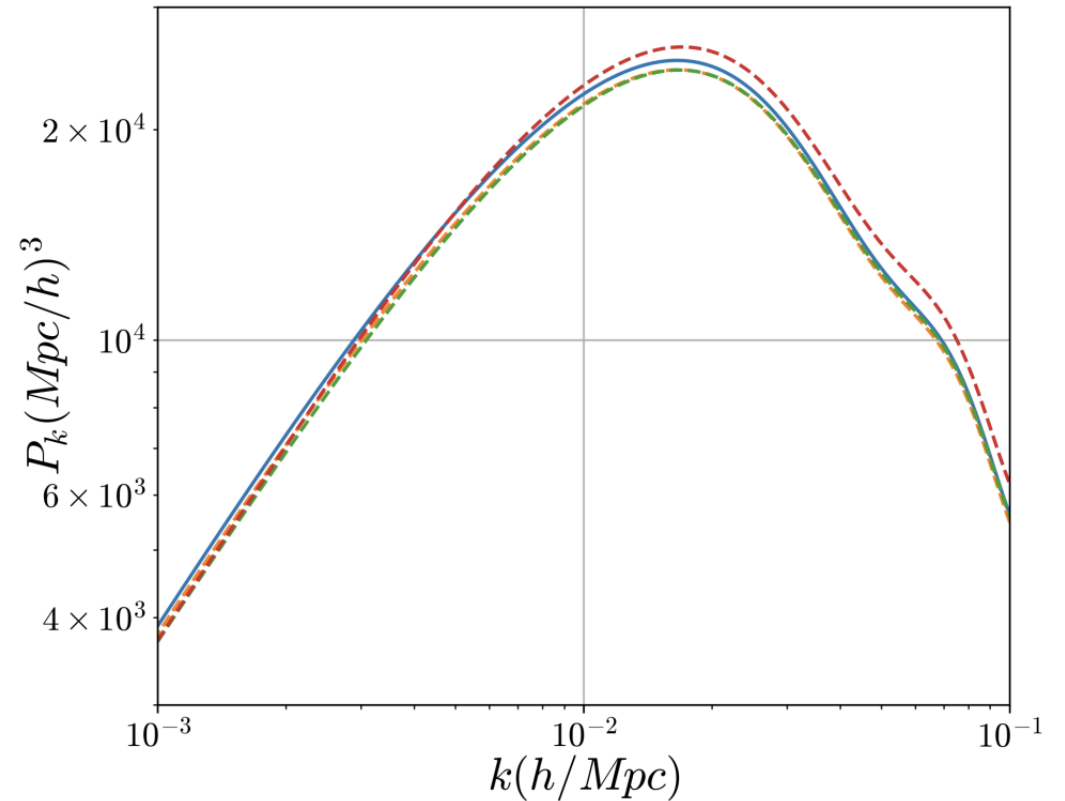
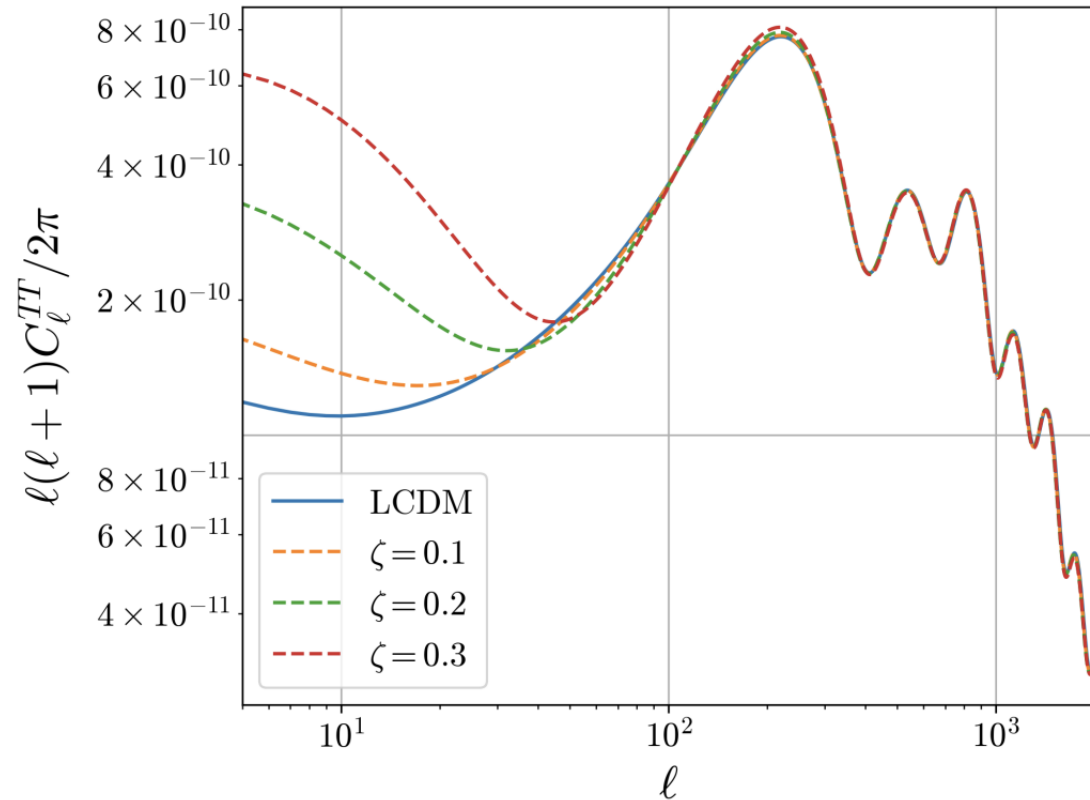
Models like this can reproduce the results from phenomenological parametrizations very well, but are un-screened and violate solar system gravity tests



Backup Slides



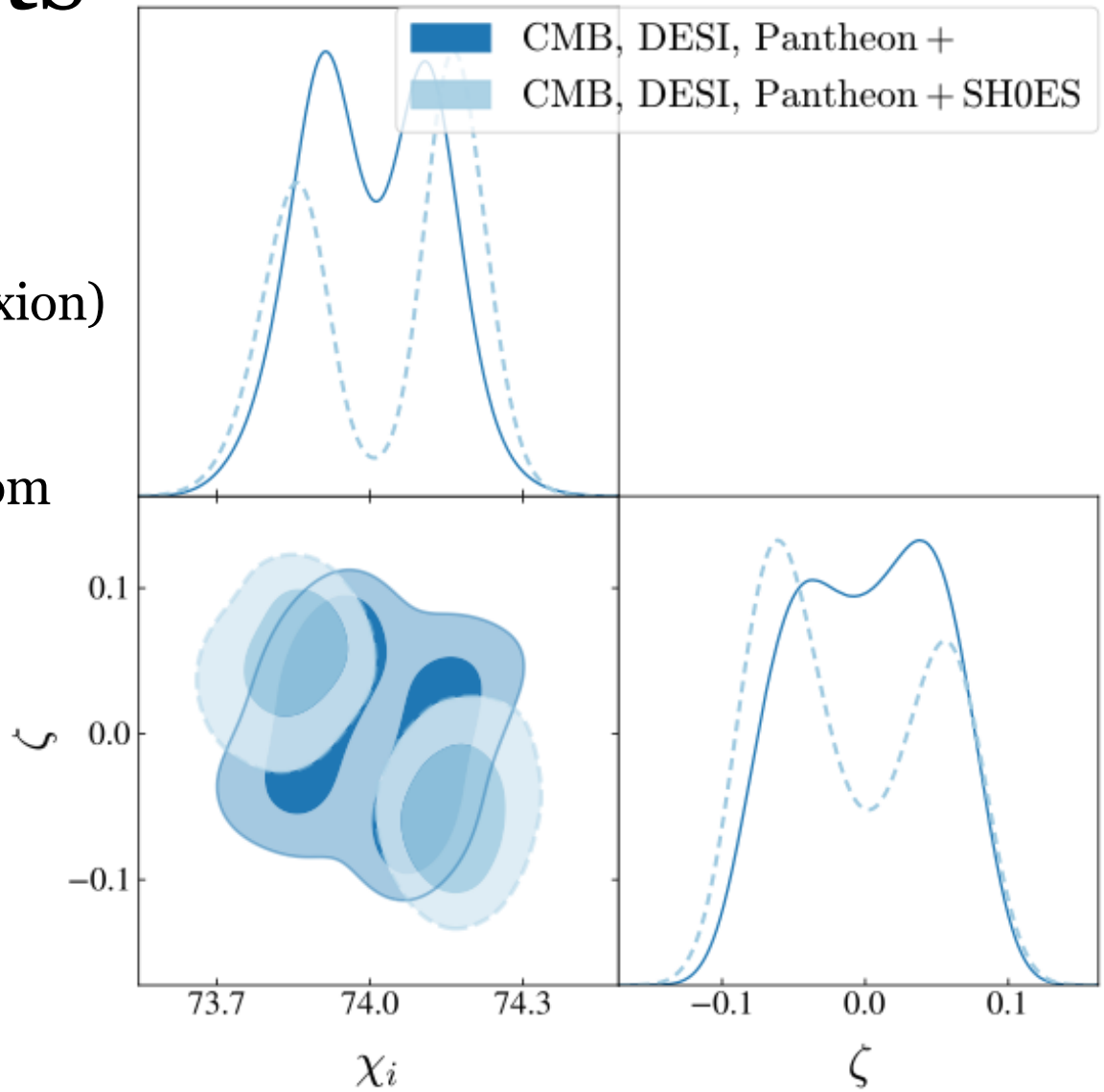
Power Spectra of the CMB



Coupling Constraints

- Late time ISW effect is constraining:
 - Dark energy (dilaton) – Dark matter (axion) coupling to $\zeta < 0.1$
 - Initial condition of dilaton $< 0.2 M_{pl}$ from local minimum

$$W = W_0 e^{-\zeta \chi}$$

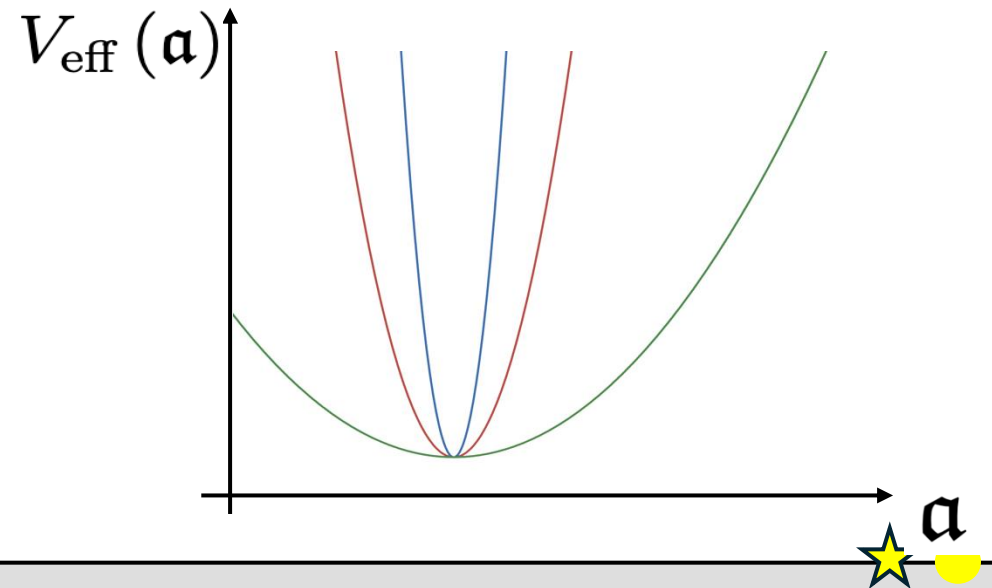


Axion Phenomenology

$$\mathbf{a} = \bar{\mathbf{a}}(\bar{\rho}) + \frac{1}{\sqrt{2}} \left(e^{-i \int_0^t dt \mathbf{m}(t)} \psi + e^{i \int_0^t dt \mathbf{m}(t)} \psi^* \right), \quad \text{where} \quad \mathbf{m}^2(t) = \frac{m_a^2}{W^2(\bar{\chi})}$$

$$V(a) = \frac{m_a^2}{2} (a - a_+)^2 + \dots$$

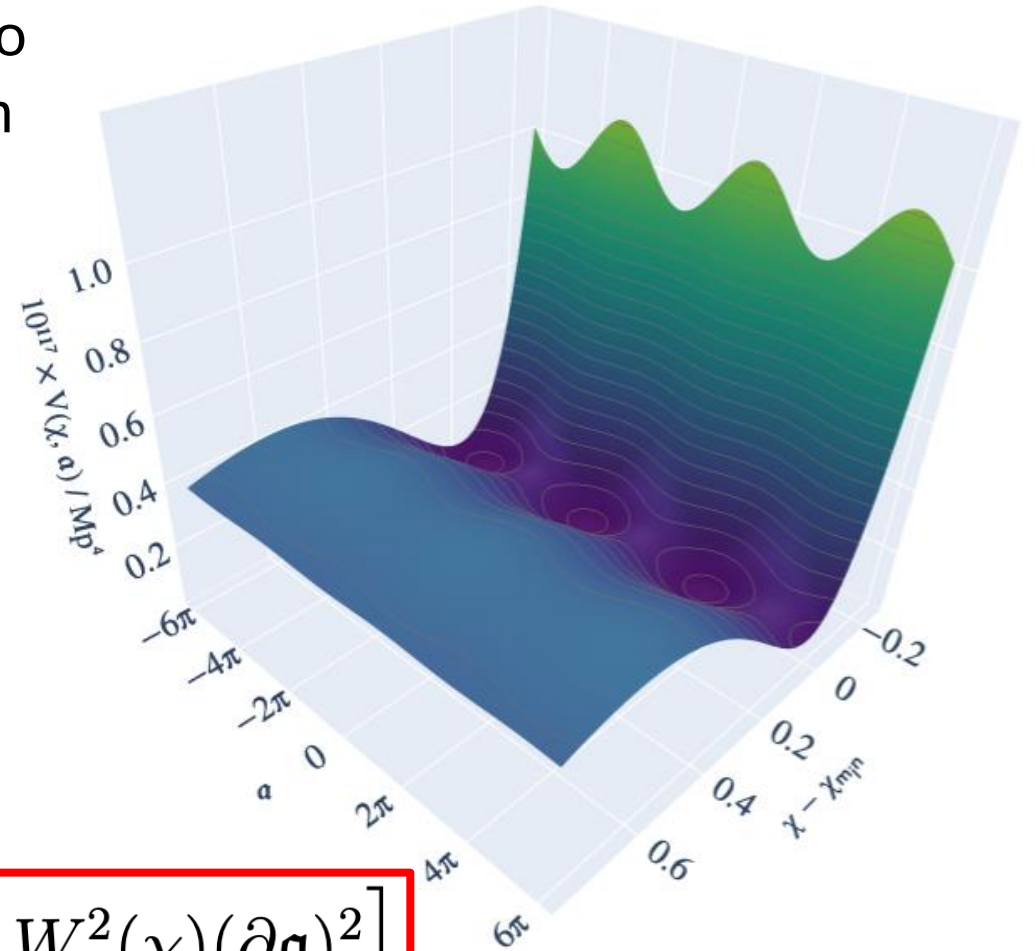
$$\bar{\rho}_a = \frac{C m(t)}{a^3}$$



Yoga Dilatons

arXiv:2111.07286: **Yoga Dark Energy** [C.P. Burgess](#), [Danielle Dineen](#), [F. Quevedo](#)

- Special class of dilatons that are designed to address the cosmological constant problem
- Approximate scale invariance supplied by the dilaton
- Supersymmetric gravity sector (No Standard Model SUSY)
- Relaxation framework



$$\mathcal{L}_{\text{axio-dilaton}} = -\frac{1}{2} M_p^2 \sqrt{-g} \left[(\partial\chi)^2 + W^2(\chi) (\partial\mathbf{a})^2 \right]$$



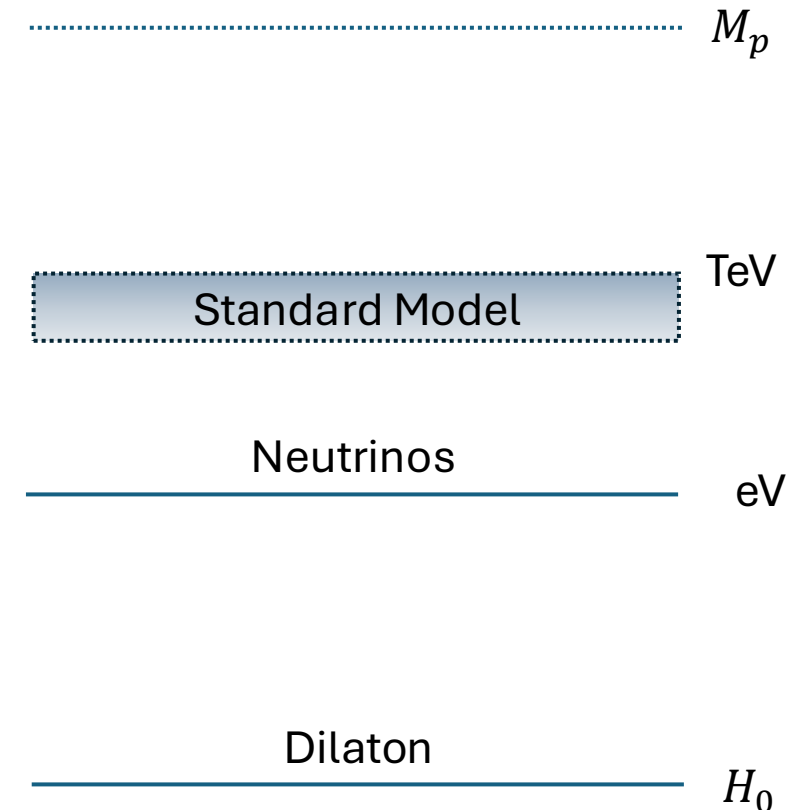
Yoga Dilatons

arXiv:2111.07286: **Yoga Dark Energy** [C.P. Burgess](#), [Danielle Dineen](#), [F. Quevedo](#)

- In these models, **all** scales track the dilaton

$$\begin{aligned} m_{\text{ew}} &\propto \sqrt{e^{-\zeta\chi}} \\ m_\nu &\propto e^{-\zeta\chi} \\ m_\chi &\propto e^{-2\zeta\chi} \end{aligned}$$

- Every** field's vacuum potential gets suppressed as dilaton rolls down to the dark energy scale



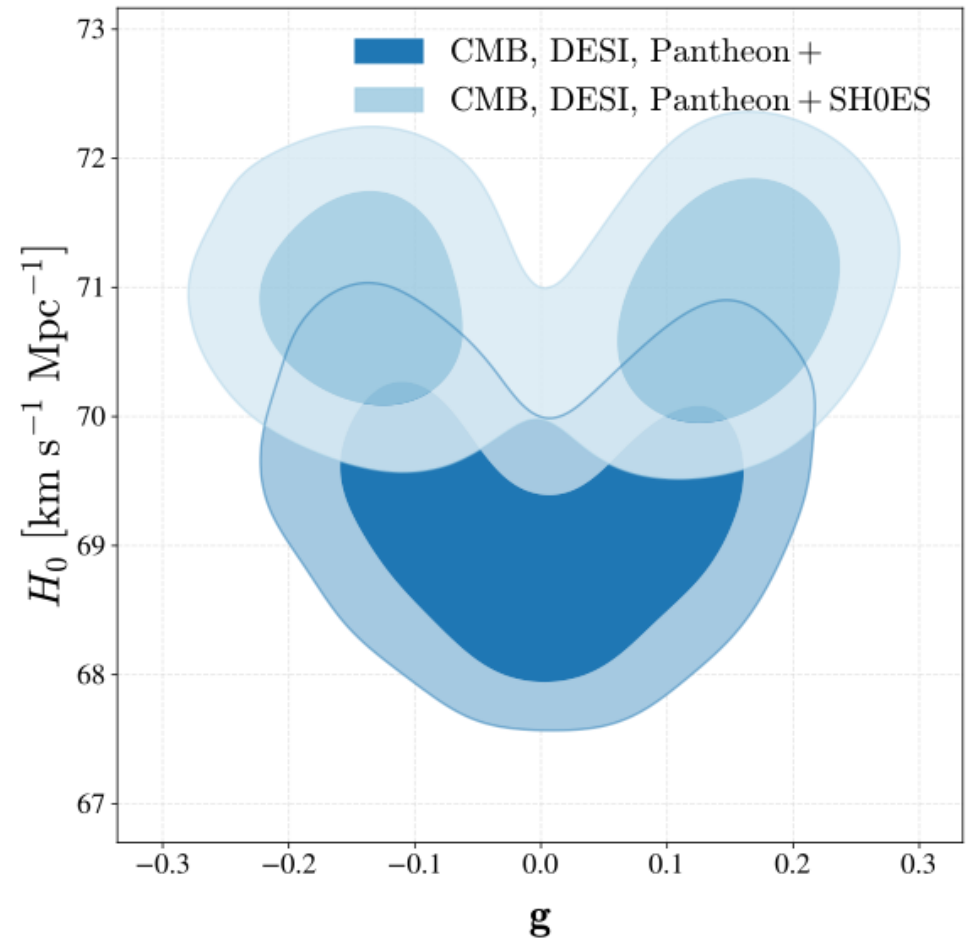


Yoga Dilatons

$$g = -1/\sqrt{6} \approx -0.4$$

Off by a factor of 2!

$$m_B(\chi) = m e^{g\chi}$$



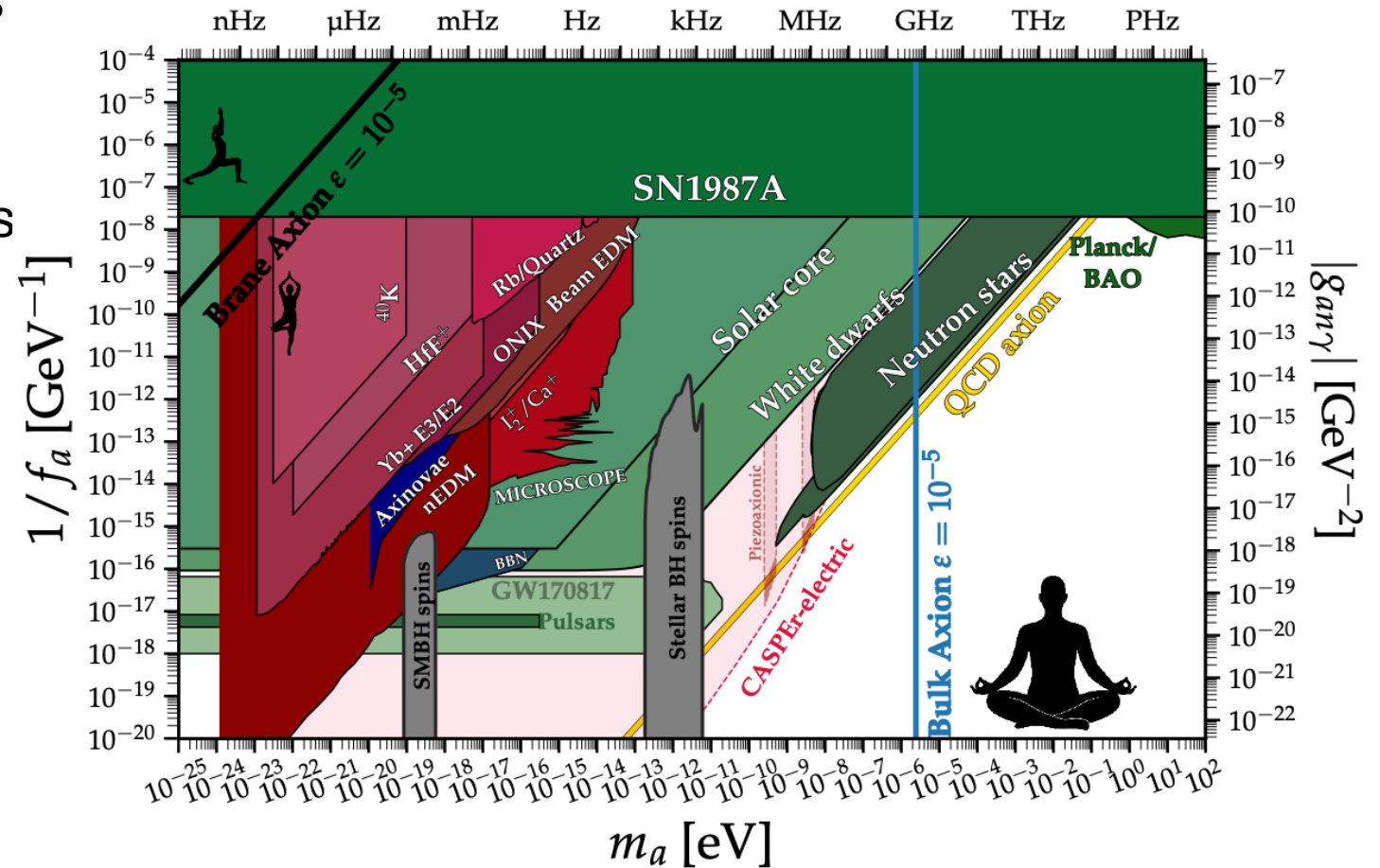
QCD Axions in Yoga Models

- QCD Axions don't work in this setup.
- Scalar potential of axions gets suppressed by the dilaton.

$$V_{\text{vac}}(\mathbf{a}, \tau) = -\varepsilon^2 V_0 [1 - \cos(\mathbf{a})] e^{-\lambda\chi}$$

- Matter coupling dominates.

- Higgs & collider physics work in these models!





Particle Mass Variation

$$m_i(z) = A(\chi) m_i^{(0)} \quad \Lambda_{\text{QCD}}(z) = A(\chi) \Lambda_{\text{QCD}}^{(0)}$$

- Cosmology accommodates up to 10% shifts

- Astrophysics has no constraint

Epoch / probe	z range	Observable	Mass dependence	Constraint
BBN	$z \sim 10^9 - 10^7$	$Y_p, D/H, {}^3\text{He}, {}^7\text{Li}$	$\Delta m_{np}, Q_{n\beta}, B_D$	$ \Delta \ln A _{\text{BBN}} \lesssim 10^{-2}$
CMB (recombination)	$z \simeq 1100$	Peaks, damping tail	m_e via E_H, σ_T	$ \Delta \ln A _{z \simeq 1100} \lesssim 2 \times 10^{-2}$
CMB (drag)	$z \simeq z_d \sim 10^3$	Sound horizon $r_s(z_d)$	m_B, m_e via $R = 3\rho_B/4\rho_\gamma$	$ \Delta \ln A _{\text{drag}} \lesssim (6-14) \times 10^{-3}$
CMB (post-rec.)	$10^4 \gtrsim z \gtrsim 10^2$	$\tau, \text{kSZ}, \mu/y$ distortions	m_e through $\sigma_T, \text{ionisation}$	$ \Delta \ln A _{\text{post}} \lesssim \mathcal{O}(10^{-1})$
BAO / LSS	$z \lesssim 3$	$D_H/r_s, D_M/r_s, f\sigma_8$	Ω_B, Ω_m via $A(\chi)$	$ \Delta \ln A _{z < 3} \lesssim \text{few} \times 10^{-2}$
21 cm	$z \sim 10 - 200$	Global signal, $P_{21}(k)$	$\mu, \text{residual } x_e(m_e)$	$ \Delta \mu/\mu \sim 10^{-3}$ (forecast, non-univ.)
Astrophysics	$z \lesssim 1$	Stellar tracks, SNe Ia	Environment-dependent $A(\chi)$ (screened)	None if universal; strong if broken
Spectroscopy	$0 \lesssim z \lesssim 4.5$	Atomic / molecular lines	μ (inversion, vib/rot)	$ \Delta \mu/\mu \lesssim 3 \times 10^{-8}$ (non-univ.)
Geochemical	$z \simeq 0$	Resonances, decay Q	m_q/Λ_{QCD} vs. m_e	$10^{-7} - 10^{-6}$ (non-univ.)
Laboratory	$z \simeq 0$	Clocks, EP tests	$\dot{\mu}/\mu, \dot{\alpha}/\alpha$	$< 10^{-17} \text{ yr}^{-1}; \eta \lesssim 10^{-13}$

- Only thing that can constrain these models are gravity tests





SPT data Included

Model	Dataset	H_0	\mathbf{g}	ζ	χ_i	$R - 1$	$\Delta\chi^2$
Yoga-VI	CMB-B DESI PP	69.19 ± 0.70 (69.13)	0.003 ± 0.095 (-0.052)	0.002 ± 0.050 (-0.003)	74.020 ± 0.150 (73.844)	0.075	-7.2
	CMB-B DESI	69.42 ± 0.71 (69.50)	-0.012 ± 0.098 (-0.037)	-0.001 ± 0.048 (-0.032)	$73.98^{+0.21}_{-0.18}$ (73.814)	0.228	-6.8
EXP	CMB-B DESI PP	69.16 ± 0.68 (69.99)	0.002 ± 0.093 (0.104)	-0.001 ± 0.044 (-0.022)	–	0.165	-7.3
	CMB-B DESI	69.38 ± 0.71 (70.16)	$-0.013^{+0.090}_{-0.10}$ (-0.164)	0.001 ± 0.045 (0.071)	–	0.596	-6.2
	CMB-B	$67.47^{+0.68}_{-1.2}$ (67.78)	-0.004 ± 0.074 (0.041)	0.001 ± 0.048 (-0.002)	–	0.039	–
w0-wa + me	CMB-B DESI PP	68.40 ± 0.84 (68.51)	–	–	–	0.007	-12.3
	CMB-B DESI	$64.2^{+1.9}_{-2.6}$ (63.78)	–	–	–	0.041	-10.5
w0-wa	CMB-B DESI PP	67.65 ± 0.59 (67.52)	–	–	–	0.010	-9.6
Λ CDM	CMB-B DESI PP	68.04 ± 0.26 (68.25)	–	–	–	0.007	0.0
	CMB-B DESI	68.13 ± 0.26 (68.23)	–	–	–	0.006	0.0

TABLE IV: Posterior means with quoted 1σ marginal uncertainties and best-fit values in parentheses for all models fit to the CMB-B dataset combinations including SPT-3G. Columns list the inferred Hubble constant, the dilaton coupling \mathbf{g} , the axion CDM kinetic coupling ζ , the initial dilaton value χ_i when present, and the change in best-fit $\Delta\chi^2$ relative to the corresponding Λ CDM run.



Coupling just base datasets

Model	Dataset	H_0 [km s ⁻¹ Mpc ⁻¹]	\mathbf{g}	ζ	χ_i	$R - 1$	$\Delta\chi^2$
Yoga-VI	CMB-A DESI PP	$69.18^{+0.63}_{-0.81}$ (69.71)	0.00 ± 0.10 (0.14)	0.002 ± 0.052 (-0.061)	74.00 ± 0.13 (74.10)	0.071	-2.1
	CMB-A DESI	$69.38^{+0.68}_{-0.83}$ (69.61)	0.00 ± 0.10 (0.15)	0.000 ± 0.052 (-0.060)	74.00 ± 0.14 (74.09)	0.034	-3.5
(no- m_e)	CMB-A DESI PP	68.50 ± 0.35 (68.51)	0.006 ± 0.083 (-0.009)	-0.001 ± 0.054 (-0.050)	$74.01^{+0.18}_{-0.16}$ (73.88)	0.016	-2.1
Yoga	CMB-A DESI PP	68.42 ± 0.35 (68.33)	-0.038 ± 0.086 (-0.055)	$0.020^{+0.077}_{-0.094}$ (0.032)	–	0.010	-0.7
	CMB-A DESI	68.53 ± 0.36 (68.54)	-0.041 ± 0.088 (-0.078)	$0.023^{+0.076}_{-0.097}$ (0.050)	–	0.025	-1.8
EXP	CMB-A DESI PP	$69.10^{+0.64}_{-0.76}$ (68.81)	-0.003 ± 0.099 (0.127)	0.005 ± 0.049 (-0.058)	–	0.082	-2.7
	CMB-A DESI	$69.35^{+0.69}_{-0.78}$ (69.96)	$0.004^{+0.12}_{-0.098}$ (-0.167)	-0.001 ± 0.049 (0.077)	–	0.057	-3.8
	CMB-A	$68.03^{+0.78}_{-1.6}$ (67.43)	0.000 ± 0.089 (-0.013)	0.001 ± 0.051 (0.036)	–	0.051	-1.1
w0-wa + me	CMB-A DESI PP	68.26 ± 0.87 (68.29)	–	–	–	0.008	-8.2
	CMB-A DESI	$63.9^{+1.9}_{-2.7}$ (64.78)	–	–	–	0.006	-8.6
w0-wa	CMB-A DESI PP	67.64 ± 0.60 (67.71)	–	–	–	0.012	-6.9
Λ CDM+me	CMB-A DESI PP	69.62 ± 0.69 (69.91)	–	–	–	0.007	-2.6
	CMB-A DESI	69.85 ± 0.69 (70.12)	–	–	–	0.006	-4.2
Λ CDM	CMB-A DESI PP	68.30 ± 0.28 (68.03)	–	–	–	0.021	0.0
	CMB-A DESI	68.39 ± 0.29 (68.53)	–	–	–	0.009	0.0
	CMB-A	67.30 ± 0.54 (67.27)	–	–	–	0.007	0.0

TABLE III: Posterior means with 1σ confidence intervals, best-fit values in parentheses, and $\Delta\chi^2$ relative to the corresponding Λ CDM run for each dataset combination. The table summarises how the different axio-dilaton (Yoga and EXP), w_0 - w_a , and varying- m_e models shift H_0 , the scalar couplings (\mathbf{g}, ζ), and the overall goodness of fit relative to Λ CDM across the CMB-A, CMB-A DESI, and CMB-A DESI PP datasets.

