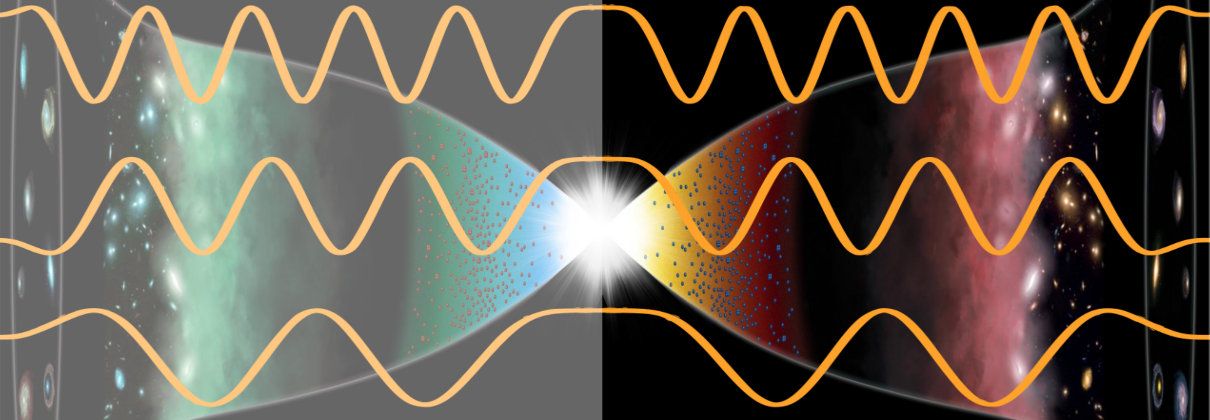
CPT-Symmetric Kähler-Dirac Fermions:

Implication of the Standard Model and Cosmology



Wei-Ning Deng, University of Cambridge (`wnd22@cam.ac.uk`)

Pascos, University of Sheffield (2026)

arXiv:2511.11548

Outline

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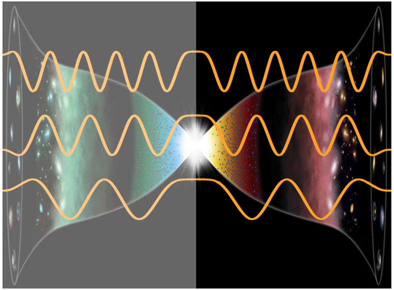
Part 2: CPT-symmetric Kähler-Dirac fermions

Part 3: Implications in the Standard Model and Cosmology

Future directions: Lattice KD, 4D String theory



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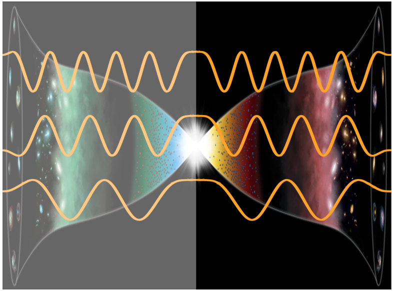

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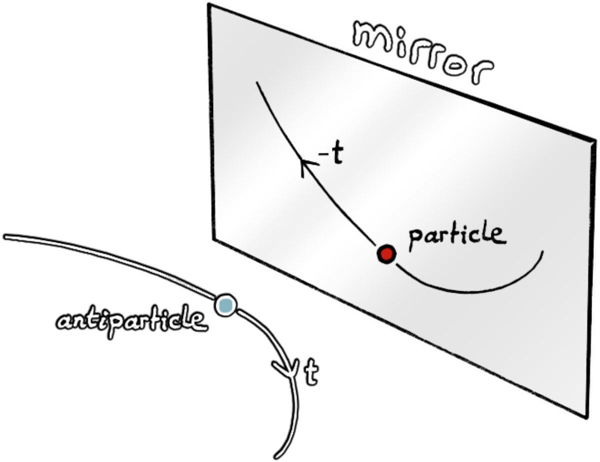
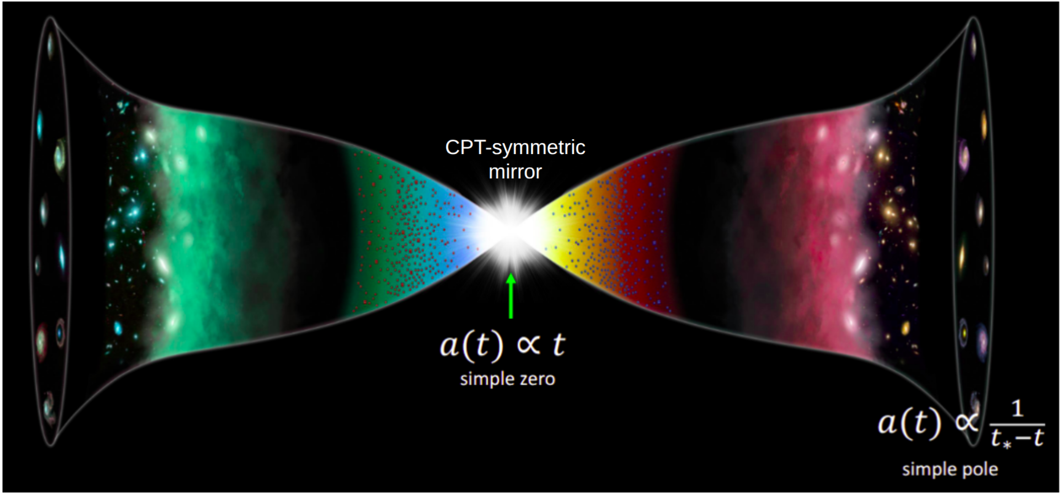
QFT– physics is **locally** CPT-symmetric

Question: Is there a universe that is **globally** CPT-symmetrical to our universe?

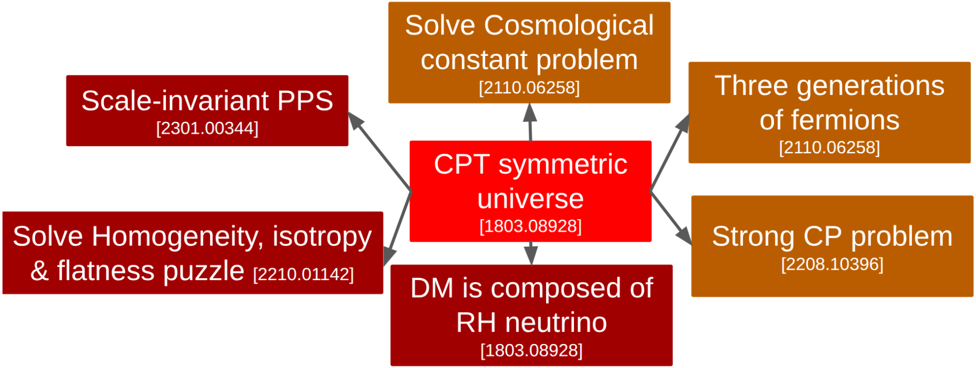


CPT-symmetric universe



CPT-symmetry universe can solve lots of problems

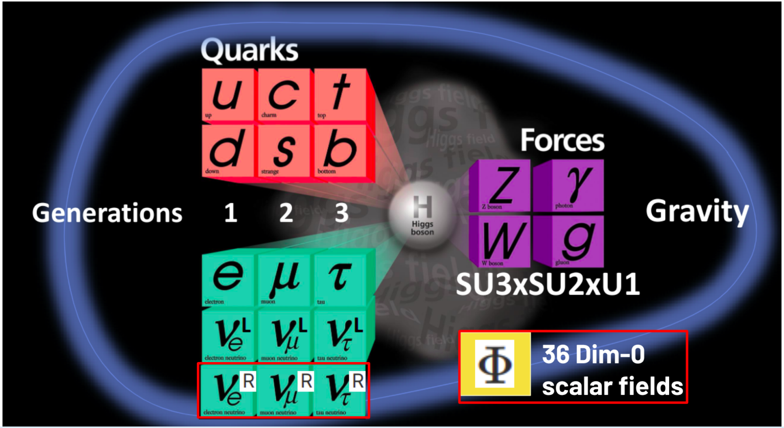
(Proposed by Latham Boyle and Neil Turok)



Extended SM solve Cosmological Constant (CC) problem

Problem: assume cosmological constant Λ coming from QFT vacumm energy $\Rightarrow$ $\Lambda \sim M_P^4$ (120 orders of magnitude larger than observed).

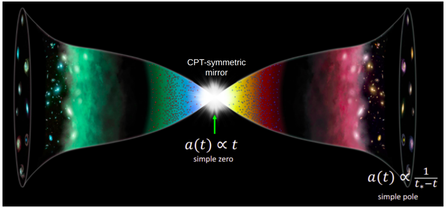
Solution: Extended SM with 3 ν_R + 36 dim-0 scalars + composite Higgs $\Rightarrow$ solve CC problem [arXiv:2509.09346, 2110.06258].

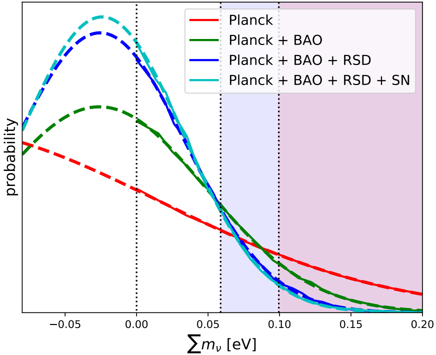


Right-handed neutrino as dark matter, explain Baryon asymmetry

CPT-symmetric universe predicts:

- Seesaw mechanism explains small ν_L masses by introducing heavy ν_R
- $\mathbb{Z}_2$ symmetry stabilises *one* $\nu_R \Rightarrow$ **lightest neutrino is massless** ($\sum m_\nu \approx 0.06$ or $0.10\,\mathrm{eV}$)
- The stable ν_R is produced via metric time-dependence when $H \sim M$ (analogous to Hawking radiation)
- ν_R dark matter mass: $M_{\nu_R} \approx 5 \times 10^8\,\mathrm{GeV}$ [arXiv: 1803.08928]
- The other two ν_R's decay asymmetrically (CP violation) and transfer to **baryon asymmetry** via Spaleron processes.





Dim-0 scalar field seeds scale-invariant PPS

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\,\varphi\Delta_4\varphi$$

$$\langle\delta\varphi_i(\eta,\mathbf{x})\,\delta\varphi_j(\eta,\mathbf{x}')\rangle = \delta_{ij}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\,\frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}{4ak^3}$$

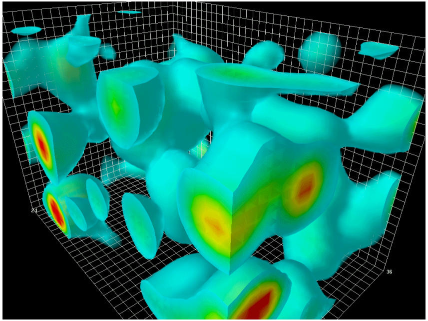
$$\mathcal{P}_{\mathcal{R}} = \frac{3^2 5^3}{7(2\pi)^4}\left(\frac{c_\beta^{SM}}{\mathcal{N}_{eff}}\right)^2$$

Predicted: $A = 12.9 \pm 4.5 \times 10^{-10}$,

$n_s \approx 1 - 7\alpha_3(M_P)/\pi = 0.958$

Observed (Planck): $A = 21 \pm 0.3 \times 10^{-10}$,

$n_s = 0.9587 \pm 0.0056$



Quantum fluctuations of dim-0 scalars seed scale-invariant PPS

Strong CP problem: solved by Big Bang as a CT mirror

[arXiv: 2208.10396] Boyle, Teuscher & Turok (2022)

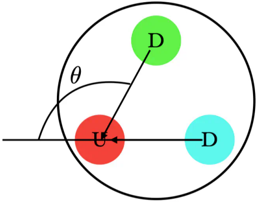
The strong CP problem:
QCD allows a CP-violating term
$\bar{\theta}\,\frac{g_s^2}{32\pi^2}\,G_{\mu\nu}\tilde{G}^{\mu\nu}$
but neutron EDM demands $|\bar{\theta}| < 10^{-10}$.
Why so small?

Embed SM in left-right symmetric model (LRSM):
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

Big Bang $=$ CT mirror: fields swap $L \leftrightarrow R$ and complex-conjugate across the Bang.

How this solves strong CP

S_{LRSM} invariant under CT
$\Rightarrow$ Yukawa matrices **hermitian**: $Y = Y^\dagger$
$\Rightarrow$ No overall Yukawa phase
$\Rightarrow$ $\theta G\tilde{G}$ not regenerated by chiral anomaly
Strong CP solved, without an axion!



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Part 2: **CPT-symmetric Kähler-Dirac fermions**

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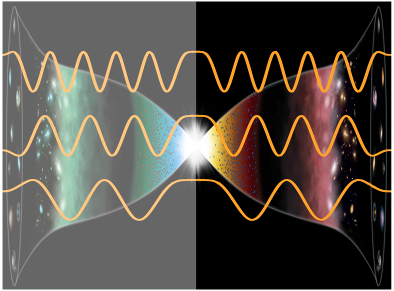

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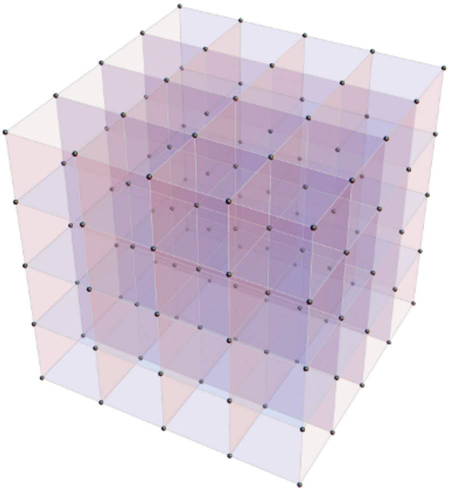
Fermion doubling problem on the lattice

Wilson's lattice gauge theory works beautifully for bosons:

- 0-forms (scalars) $\to$ vertices
- 1-forms (gauge fields) $\to$ links
- 2-forms (field strengths) $\to$ faces

Problem with spinors: no natural p-form interpretation $\Rightarrow$ placed on vertices $\Rightarrow$ extra doublers in the continuum limit

Chiral theory $\xrightarrow{\text{lattice}}$ non-chiral theory (Nielsen–Ninomiya, 1981)



Kähler-Dirac (KD) fermions

Key idea: KD fermions are p-forms – they live naturally on any cell complex.

KD field Φ is a *polyform*: sum of p-forms $(p = 0, \ldots, 4)$ with 16 complex components.

KD action and equation:

$$S_{KD} = \langle \bar{\Phi} | (K - m) \Phi \rangle, \qquad (d - d^\dagger - m)\Phi = 0$$

where $K = d - d^\dagger$, $\quad -K^2 = \Box$ (wave operator on forms).

Replacing $dx^\mu \to \gamma^\mu$ assembles the 16 components into a 4×4 matrix Ψ:

$$(i\gamma^\mu \partial_\mu - m)\Psi = 0 \quad \Rightarrow \quad S_{KD} = \int d^4x \, \mathrm{Tr}\big[\bar{\Psi}(i\partial\!\!\!/ - m)\Psi\big]$$

with $\bar{\Psi} \equiv \gamma^0 \Psi^\dagger \gamma^0$ (extra γ^0 on the left!).

Symmetry: $\Psi' = U_\Lambda \Psi V$, where $(U_\Lambda, V) \in \mathsf{Spin}(3,1) \times U(2,2)$. When restrict $V = U_\Lambda^{-1} \in SU(2,2)$, behave like forms.

The wrong-sign problem in Lorentzian signature

Diagonalise the extra γ^0: $\gamma^0 = \Omega\Lambda\Omega^T$, $\Lambda = \mathrm{diag}(+1, +1, -1, -1)$.

In terms of $\Psi' \equiv \Psi\Omega$:

$$S_{KD} = \int d^4x \; \mathrm{Tr}\left[\Lambda\, \Psi'^{\dagger}\gamma^0(i\partial\!\!\!/ - m)\Psi'\right]$$

- Columns 1–2 of Ψ': right sign $\Rightarrow$ positive norm
- Columns 3–4 of Ψ': wrong sign $\Rightarrow$ negative norm states (ghosts)!

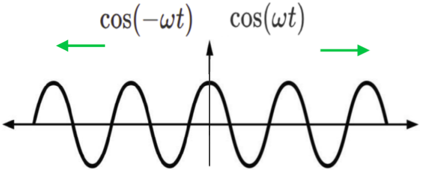
Resolution (Donoghue & Menezes) : a wrong-sign action $=$ reversal of causality arrow $=$ PT reversal [1908.04170].

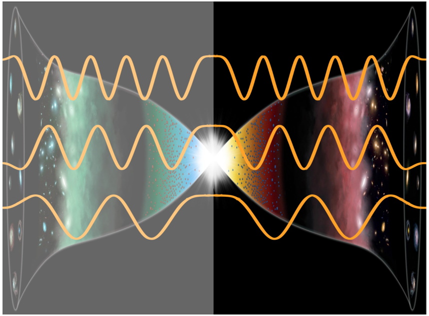
$\Rightarrow$ the two pairs of columns live on **two sheets** of spacetime, related by PT symmetry (or, equivalently, $i \leftrightarrow -i$).

Two-sheeted spacetime

- Forward sheet ($\Lambda_{ii} = +1$, cols 1–2): standard quantum mechanics with e^{iS}, $[x, p] = +i$
- Backward sheet ($\Lambda_{ii} = -1$, cols 3–4): uses e^{-iS}, $[x, p] = -i$
- The two sheets are PT **mirrors** of each other
- Positive energy, positive norm on *both* sheets

"Our" Big Bang is the junction of the two sheets – exactly the CPT-symmetric universe picture!





Physical explanation of the movie Tenet?

If movie does not play: [watch online]

KD-Majorana condition

Problem: What are the particles in the backward sheet?

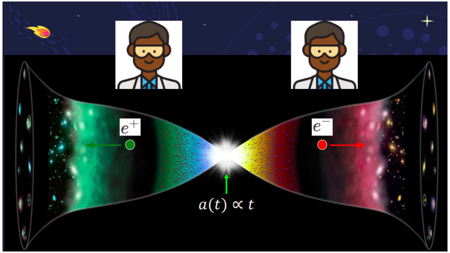
Solution: Physics should be invariant under charge conjugation $\Rightarrow$ require KD-Majorana condition

$$\Psi^c \equiv (i\gamma^2)\Psi^*(i\gamma^2) = \Psi$$

In terms of Ψ', the four columns are forced to take the form:

$$\Psi' = \left(\psi_1,\ \psi_2,\ -\psi_2^c,\ \psi_1^c\right)$$

- A LH particle on the forward sheet is *always* paired with a RH anti-particle on the backward sheet
- Physical meaning: CPT symmetry – every particle has a mirror (anti-)partner on the other sheet
- No unphysical doubling of the SM



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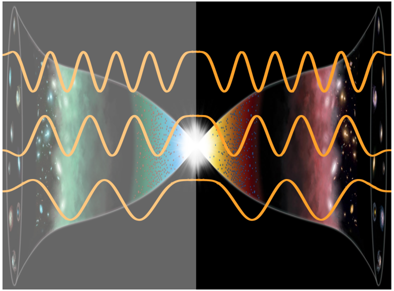

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Relation to the Standard Model

One SM generation (including ν_R) $=$ 16 Weyl spinors $=$ 4 KD-Majorana fields.

Following Pati-Salam: allocate to four copies $\Psi'_{A=0,1,2,3}$ ([S. Catterall, 2209.03828])

$$\Psi'_0 = \chi_0 + \chi_0^c, \quad \chi_0 = \begin{pmatrix} \nu_L & e_L \\ \nu_R & e_R \end{pmatrix} \quad \text{(leptons)}$$

$$\Psi'_i = \chi_i + \chi_i^c, \quad \chi_i = \begin{pmatrix} u_L^i & d_L^i \\ u_R^i & d_R^i \end{pmatrix} \quad \text{(quarks, color } i = 1, 2, 3\text{)}$$

Fermion kinetic terms unify:

$$\mathcal{L}_{\mathsf{KD}} = \mathrm{Tr}\left[\bar{\Psi}_A (i\partial\!\!\!/) \Psi_A\right], \quad A = 0, \ldots, 3$$

Gauging $SU(3) \subset SU(4)$ and $SU(2) \times U(1) \subset U(2,2)$ (the internal symmetry) reproduces *all* SM fermion kinetic terms.

Yukawa couplings unify in KD formalism

Standard Yukawa Lagrangian (4 separate terms):

$$\tilde{h}^T \bar{q}_L u_R y_u + h^T \bar{q}_L d_R y_d + \tilde{h}^T \bar{\ell}_L \nu_R y_\nu + h^T \bar{\ell}_L e_R y_e + \text{h.c.}$$

KD formulation (single compact expression):

$$\mathcal{L}_Y = \mathrm{Tr}\Big[\mathcal{H}^T \, \widehat{\Psi}'^{\,i}_{A,L} \, \Psi'^{\,j}_{A,R} \, \mathcal{Y}^{ij}_A\Big] + \text{h.c.}$$

where $\mathcal{H}$ is the *KD Higgs field* and $\mathcal{Y}^{ij}_A$ the *KD Yukawa matrix*.

- Expanding for leptons reproduces exactly the two lepton Yukawa terms
- Forward sheet $\leftrightarrow$ particles; backward sheet $\leftrightarrow$ antiparticles
- All four SM Yukawa sectors unified in one line

Connection to the CPT-symmetric universe

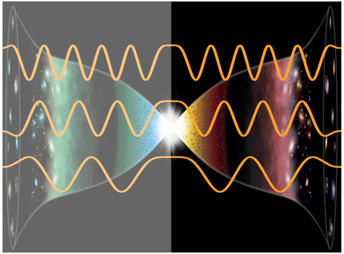
Question: *Why* does the Big Bang act as a CPT mirror?

The CPT-symmetric universe was motivated by theoretical and observational reasons (explain lots of problems elegantly).

KD answer: Two sheeted is required for solving negative norm problem. The KD-Majorana condition *requires* every particle to have a mirror (anti-)partner on the other sheet.

The Big Bang is precisely the junction where the two spacetime sheets meet.

$$\underbrace{\text{KD-Majorana}}_{\text{field theory}} \longleftrightarrow \underbrace{\text{CPT mirror at bang}}_{\text{cosmology}}$$



Black mirror: an alternative black holes theory

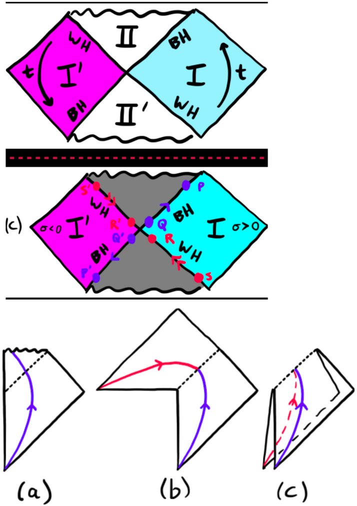
Standard picture: black hole has an interior.

- Problem: Hidden from Observation? Curvature singularities? Information paradox? Violates all global symmetries (CPT, B-L)?

Black mirror proposal [Tzanavaris, Latham [arXiv: 2412.09558]]

- The interior *does not exist*
- The horizon mirrors our exterior to the "other" exterior (anti-universe)
- Infalling particles *annihilate* with anti-partners from the anti-universe at the horizon

KD-Majorana condition: explains why every infalling particle is paired with a mirror anti-partner.



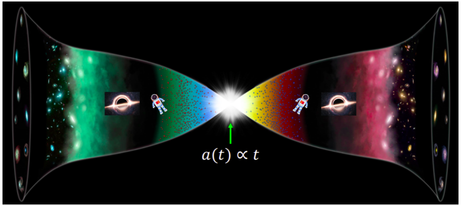
Theoretical description of Tenet?!

Same as CPT-symmetric universe:

1. There is *you* and *anti-you* in the universe and anti-universe
2. Time goes backward in the anti-universe
3. The doors connecting universe and anti-universe are **black holes**

Difference:

1. You and anti-you **annihilate** at the transition door
2. If viewed from our universe: you come out of a **white hole** and find that time goes backward. However, you *cannot change anything* – only re-experience the past



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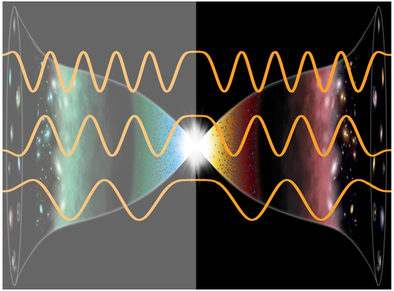

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Future direction 1: KD fermions on the lattice

Boyle, Vaibhav (in preparation)

Problem: chirality in Euclidean signature

Barrett [arXiv: 2403.18428]: chiral theory in Euclidean space requires dim $= 2 \bmod 8$ $(2, 10, 18, 26, \ldots)$

Solution: double the Hilbert space (Barrett [arXiv: 2403.18428])

Double the Hilbert space $\mathcal{H}' = \mathcal{H} \oplus \mathcal{H}$, with charge conjugation J interchanging the two copies: $S_{\text{chiral}} = \frac{1}{2}\langle J'\Psi_L, K'\Psi_L\rangle$

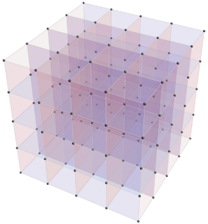
't Hooft anomaly $\Rightarrow$ Pati-Salam

Cancelling the $\mathbb{Z}_4$ chiral anomaly on the lattice requires **4 copies** of reduced KD fermions $\Rightarrow$ gauge group $SU(4) \times SU(2)_L \times SU(2)_R$

Two-sheeted spacetime is not just cosmology!

$\mathcal{H}' = \mathcal{H} \oplus \mathcal{H} \equiv$ universe + anti-universe

Required to simulate a **chiral gauge theory** on the lattice



Nielsen-Ninomiya avoided: equal $n_L = n_R$ in Pati-Salam

Chiral anomaly preserved on the lattice

Future direction 2: Kähler-Dirac string theory in two-sheeted 4D

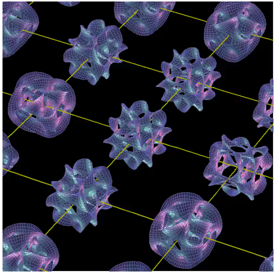
1. **String theory as a candidate for quantum gravity**

   Feynman vertices are smoothed into 2D worldsheet intersections ⇒ no point-like UV divergences

2. **The dimension problem**

   Conformal invariance of the 2D worldsheet fixes the target spacetime dimension:

   $$\text{bosonic string} \;\Rightarrow\; 26\text{D}, \quad \text{superstring} \;\Rightarrow\; 10\text{D}$$



These extra dimensions are unobserved and require compactification.

3. **Open question**

   Can we construct a ghost-free superstring theory by placing Kähler-Dirac fields on a **two-sheeted 4D spacetime**?

   Note: strings in 3D space admit **knots & braids** – absent in $d \geq 4$ spatial dims

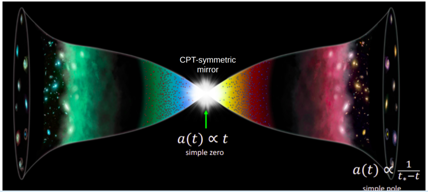
Summary

Part 1: Introduction

- CPT-symmetric universe solves CC, strong CP, baryonic asymmetry, DM, PPS problems
- Extended SM: 3 ν_R + 36 dim-0 scalars; Higgs composite
- ν_R DM ($M \approx 5 \times 10^8$ GeV); lightest ν massless

Future directions

- KD on lattice $\Rightarrow$ chiral SM (lattice consistency)
- KD string theory in 4D $\Rightarrow$ ghost-free superstring theory



Part 2: Kähler-Dirac fermions

- KD fermions = polyforms; live naturally on the lattice
- Wrong-sign columns $\Rightarrow$ two-sheeted spacetime (PT mirrors)
- KD-Majorana: every particle paired with mirror anti-partner

Part 3: SM and cosmology

- Unifies all SM fermion kinetic + Yukawa terms
- SU(3)xSU(2)xU(1) emergy from SU(4)xU(2,2) internal symmetry
- Explains *why* the Big Bang is a CPT mirror, Black mirror

Thank you!

`wnd22@cam.ac.uk`, `arXiv:2511.11548`
`https://weiningdeng.com`

Analytic solution of Friedmann equation in conformal time

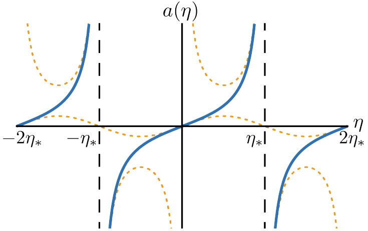
Metric:

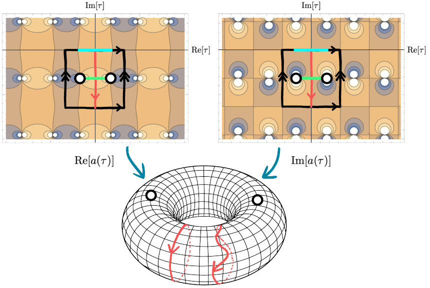
$$ds^2 = a(t)^2\left(-dt^2 + \gamma_{ij}\, dx^i dx^j\right)$$

Friedmann equation (Planck units):

$$3\dot{a}^2 = \underbrace{r}_{\text{rad}} + \underbrace{\mu a}_{\text{matter}} - \underbrace{3\kappa a^2}_{\text{curvature}} + \underbrace{\lambda a^4}_{\Lambda}$$

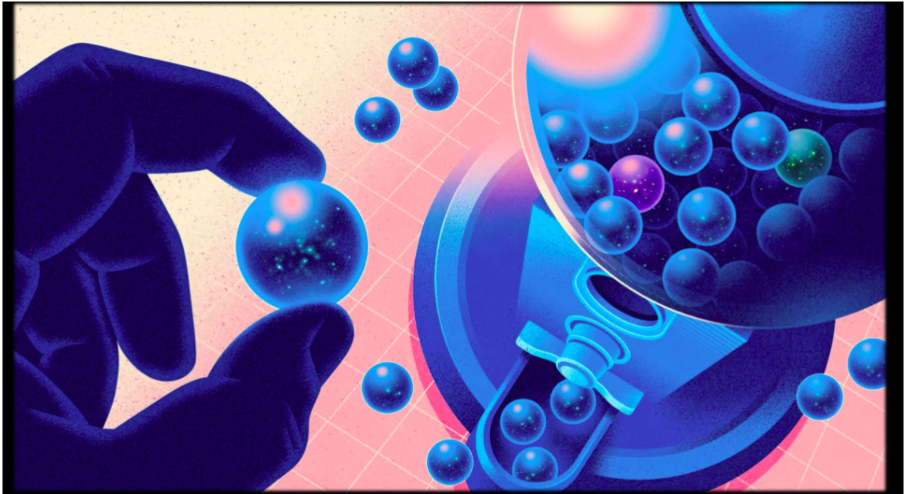
General solution (Jacobi elliptic function):
$a(t)$ is doubly periodic in the complex t-plane.
The imaginary time period and the action computed over a period determine T_H and the gravitational entropy S_g. (thermodynamics selects the most probable universe).





Euclidean torus: imaginary-time instanton

Flat, isotropic, homogeneous universe is most probable



Quanta Magazine, Nov 17, 2022; *WIRED*, Jan 22, 2023

Gravitational UV fixed points

Problem: coupling QFT to gravity $\Rightarrow$ UV divergences in $\langle T_{\mu\nu}\rangle$, implying wildly curved spacetime on short scales.

ERGE approach: identify UV fixed points where *all* gravitational β-functions vanish.

Boyle, Turok, Vaibhav, arXiv:2509.09346

Beta functions:

$$\beta_{\Lambda/G} \propto n_0 - 2n_{1/2} + 2n_1 + 2n_0' = 0$$
$$\beta_i\,(\text{four-deriv. terms}) = 0, \quad i = 1,2,3,4$$

All β-functions vanish iff:

$$\boxed{n_{1/2} = 4n_1, \quad n_0' = 3n_1, \quad n_0 = 0}$$

Conformally coupled fields:

- n_1: gauge fields
- $n_{1/2}$: Weyl/Majorana fermions
- n_0: Klein-Gordon scalars
  standard 2-derivative
- n_0': Fradkin-Tseytlin (FT) scalars
  4-derivative conformally coupled

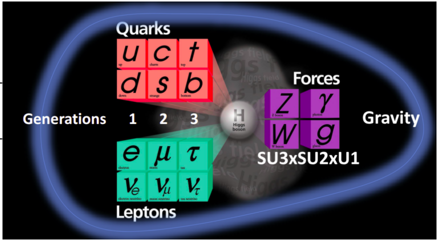
At the fixed point:

- $G(k) \sim k^{-2}$ in UV ("softening")
- $\langle T_{\mu\nu}\rangle$ and all $\langle TT\rangle$ UV-finite
- Cosmological constant problem solved

Standard Model at the gravitational fixed point

SM gauge group $SU(3) \times SU(2) \times U(1)$:
$n_1 = 8 + 3 + 1 = 12$ gauge fields

	Required	SM (incl. ν_R)
Fermions $n_{1/2}$	$4 \times 12 = 48$	$3 \times 16 = \mathbf{48}$ ✓
KG scalars n_0	0	Higgs $\to$ composite
FT scalars n'_0	$3 \times 12 = 36$	0 (to predict!)



IR: positive G + arbitrarily small Λ

arXiv:2509.09346

Predictions:

1. **3 generations +** ν_R**:** $n_{1/2} = 48$ requires right-handed neutrinos
2. **Composite Higgs:** $n_0 = 0$ forbids fundamental KG scalars
3. **36 FT (dim-0) scalars** also generate scale-invariant primordial spectrum!

$SU(3) \times SU(2) \times U(1)$ from KD formalism

Step 1: $SU(4)$ from four KD copies

$$\mathcal{L}_{\text{KD}} = \mathrm{Tr}[\Lambda \Psi_A'^{\dagger} \gamma^0 (i\partial\!\!\!/) \Psi_A'], A = 0, 1, 2, 3$$

has $SU(4)$ permuting the four copies.
Gauge $SU(3) \subset SU(4)$ $\Rightarrow$ QCD color force.

Step 2: $SU(2) \times U(1)$ from the internal V

To prevent mixing of the two sheets, we require an internal symmetry $V = U(2) \times U(2) \subset U(2,2)$, which the first/second $U(2)$ acting on the universe/antiverse respectively. Furthermore, $U(2) \cong SU(2) \times U(1)$. Restricting to one sheet $\Rightarrow$ $\boxed{SU(3) \times SU(2) \times U(1)}$

Step 3: covariant derivative

$$D_\mu \Psi_L'^i =$$
$$(\partial_\mu + ig_3 G_{\mu j}^{\ i}) \Psi_L'^j + ig_2 \Psi_L'^i \mathcal{W}_\mu + ig_1 \tfrac{2}{3} \Psi_L'^i \mathcal{B}_\mu$$

$$\mathcal{W}_\mu = \begin{pmatrix} W_\mu & 0 \\ 0 & \sigma^2 W_\mu^* \sigma^2 \end{pmatrix}, \quad \mathcal{B}_\mu = \begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu^* \end{pmatrix}$$

All four SM fermion sectors:

Field	remove $\mathcal{W}_\mu$	remove G_μ
$\Psi_L'^i$ (quark L)		
$\Psi_R'^i$ (quark R)	✓	
$\Psi_L'^0$ (lepton L)		✓
$\Psi_R'^0$ (lepton R)	✓	✓

Replacing $\partial\!\!\!/ \to D\!\!\!\!/$ in $\mathcal{L}_{\text{KD}}$ reproduces *all* SM fermion kinetic terms with gauge interactions.

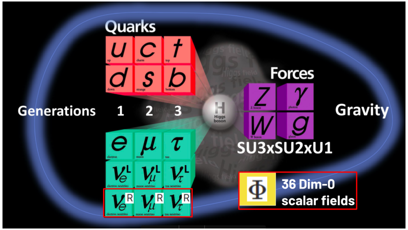
Future direction 3: composite Higgs from dim-0 scalars

Recall: anomaly cancellation predicts $n_{s,1} = 0$ – *no fundamental dimension-1 scalars*.

$\Rightarrow$ **The Higgs must be composite.**

The challenge:

- Build a viable composite Higgs model from the collective dynamics of the **36 dim-0 scalar fields**
- Constrained by LHC data: no new physics at the TeV scale (κ_V, κ_F within 5% of SM)
- Must reproduce Higgs mass, couplings, and EW symmetry breaking without a fine-tuned hierarchy



Getting this right would unify the origin of cosmic structure and mass in a single framework.

Future direction 4: quadratic gravity as architect of the PPS

A deeper possibility: the scale-invariant power spectrum may not require any new matter fields at all.

Quadratic gravity (Stelle, 1977):

- Renormalizable theory of quantum gravity:
  $\mathcal{L} = \frac{2}{\kappa^2} R + \frac{1}{6 f_0^2} R^2 - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}$
- The ghost problem in quadratic gravity $\Rightarrow$ resolved by the Donoghue–Menezes PT reversal [arXiv: 2112.01974].
- Initial results (Boyle): quadratic gravity at the Big Bang can *mimic* the dim-0 scalar effect $\Rightarrow$ scale-invariant PPS without any new matter fields!
- Problem: need new explanation for cosmological constant problem.

Problem: dim-0 scalars vs. quadratic gravity. Which one is the true architect of the scale-invariant PPS?