THE ROYAL SOCIETY

Quantum Number Theory & Cosmic Millicharged Background

PASCOS 2026

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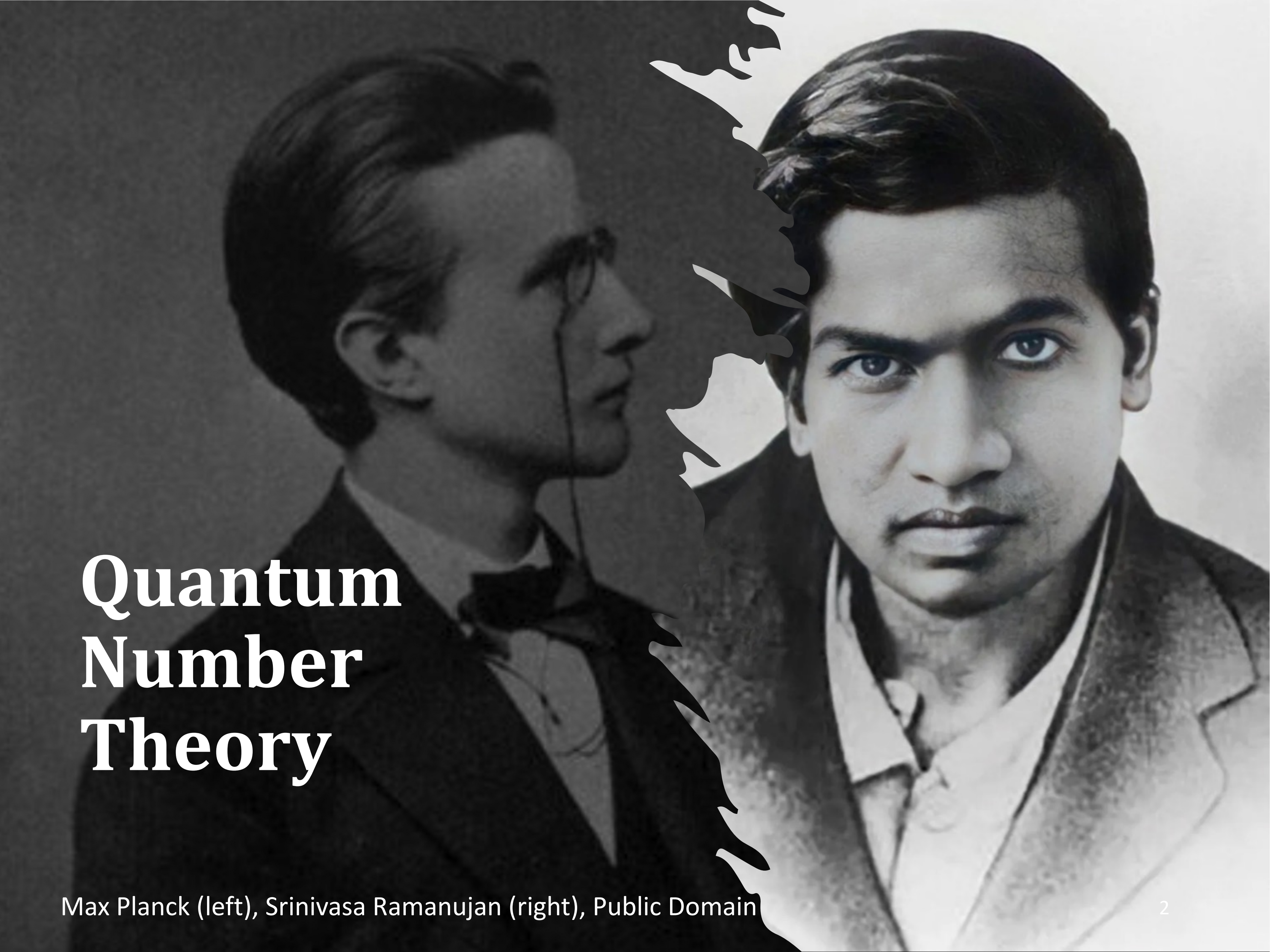
The University of Sheffield



Quantum Number Theory

Max Planck (left), Srinivasa Ramanujan (right), Public Domain

Theoretical Motivations of Millicharged Particles

Millicharged particle (mCP) is a particle χ with {mass, electric charge} =$\{m_\chi, \epsilon e\}$

Q_χ is the mCP electric charge, $\epsilon = Q_\chi / e$
(q_χ)

1. Long-standing questions: *Is electric charge quantized? To what unit? Why? Connections to monopoles*

2. **A generic feature of string theory**, Wen, Witten, *NPB* 1985

3. **A product of Grand Unification Theories (GUTs)**, Holdom, *PLB* 1986

4. **A simple example leading to Quantum Number Theory** (2603.12320)

5. **Finding mCP can be a strong probe of early-universe reheating cosmology**
Gan, **Tsai**, *JHEP* 2025, 2308.07951, **Cosmic Millicharged Particles**

- **"Theory Review of Millicharged & Fractionally Charged Particles"**
Planned review for Progress in Particle and Nuclear Physics (PPNP)

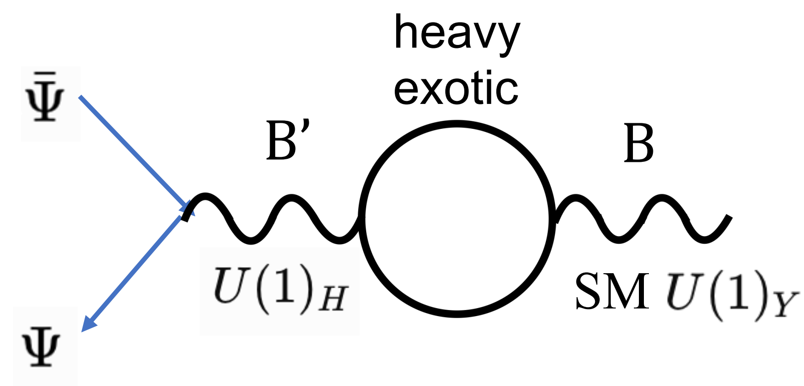
Types of Millicharged Particles (mCP)

"Pure" mCP

- Theoretical implication of mCP with a **small rational or irrational charge**, NO dark photon needed
- **Prediction of string theories**
- **Indirect test of GUTs models**

Effective "Kinetic-Mixing" mCP
(focus on this case)

- **Predicted by GUTs.**



Choose a proper basis: **massless dark photon B' decouples from SM**

$$\mathcal{L}_{\rm MCP} = i\bar{\Psi}\left(\not{\partial} - i\epsilon' e\ \not{B} + M_{\rm MCP}\right)\Psi$$

Number Theory for Millicharged Particles

Lee, Takahashi, **Tsai**, 2603.12320, 2606.07735

Naturally light mCP? Chiral gauge symmetry!

Chiral symmetry leads to gauge anomaly: anomaly cancellation

Corresponding to the number theory: “PTE” problem

Sharp predictions of the mCP spectra

mCPs with Chiral Symmetry

Lee, Takahashi, **Tsai**, 2603.12320

We introduce an additional chiral U(1)$_X$ gauge symmetry in addition to the hidden U(1)$_H$ gauge symmetry

$$\bar{\Psi}\Psi = \psi_1\psi_2 + \text{h.c.}$$

Dirac spinor Weyl spinor

$$\psi_1\psi_2 = \psi_1^T(i\sigma_2)\psi_2$$

	ψ_1	ψ_2
Vector-like $U(1)_{\mathrm{H}}$	$+1$	-1
Chiral $U(1)_{\mathrm{X}}$	a	$-b$

Credits: Fuminobu Takahashi (Tohoku U)

If $a \neq b$, the mass term is forbidden by the chiral $U(1)_{\mathrm{X}}$

mCPs with Natural Mass Terms

Lee, Takahashi, **Tsai**, 2603.12320

Like the Standard Model, we can have a new Higgs to generate the mass naturally, i.e.

$$\bar{\Psi}\Psi = \psi_1\psi_2 + \text{h.c.}$$

Dirac spinor   Weyl spinor

$$\psi_1\psi_2 = \psi_1^T(i\sigma_2)\psi_2$$

	ψ_1	ψ_2	ϕ
Vector-like $U(1)_\mathrm{H}$	$+1$	-1	0
Chiral $U(1)_\mathrm{X}$	a	$-b$	1

Credits: Fuminobu Takahashi (Tohoku U)

When $U(1)_\mathrm{X}$ is spontaneously broken, a small fermion mass is induced:

$$yM_c\left(\frac{\phi^{(*)}}{M_c}\right)^{|a-b|}\psi_1\psi_2 + \text{h.c.}$$

$y = \mathcal{O}(1)$ complex const.

M_c : UV cutoff scale

$$\epsilon = \frac{\langle\phi\rangle}{M_c},\quad M_{12} \simeq M_c\epsilon^{|a-b|}$$

Mass Hierarchy & Anomaly Cancellation

Lee, Takahashi, **Tsai**, 2603.12320

In general, we may introduce $2n$ left-handed Weyl fermions ψ_i with the following charges

	ψ_i	ψ_j	ϕ
Vector-like $U(1)_{\mathrm{H}}$	$+1$	-1	0
Chiral $U(1)_{\mathrm{X}}$	a_i	$-b_i$	1

$$i = 1, \ldots, n$$
$$j = n+1, \ldots, 2n$$

with $a_i \neq b_j$

mCP masses are induced after SSB of $U(1)_{\mathrm{X}}$:

$$\epsilon = \frac{\langle \phi \rangle}{M_c}$$

$$y_{ij} M_c \left(\frac{\phi^{(*)}}{M_c} \right)^{|a_i - b_j|} \psi_i \psi_j + \text{h.c.}$$

$$y_{ij} = \mathcal{O}(1)$$

$n \times n$ mass matrix:

$$M_{ij} \simeq M_c \epsilon^{|a_i - b_j|}$$

Hierarchical masses can be naturally realized.

Credits: Fuminobu Takahashi (Tohoku U)

Matching to Number Theoretical Studies

Anomaly cancellation conditions

Impose strong constraints on the charge assignments

- $U(1)_X$ - graviton-graviton $\quad \sum^{n} a_i = \sum^{n} b_i$
- $\left(U(1)_X\right)^2 - U(1)_H \quad \sum_{i=1}^{n} a_i^2 = \sum_{i=1}^{n} b_i^2$
- $\left(U(1)_X\right)^3 \quad \sum_{i=1} a_i^3 = \sum_{i=1} b_i^3$

Note that $\left(U(1)_H\right)^2 - U(1)_X$ is the same as $U(1)_X$ - graviton-graviton, and $\left(U(1)_H\right)^3$ is trivial due to the vector-like nature of $U(1)_H$.

Lee, Takahashi, **Tsai**, 2603.12320 (hep-ph).
Other anomaly cancellation considerations: Batra, Dobrescu, Spivak, arXiv:0510181, Nakayama, Takahashi, Yanagida, 1102.4688, **Lohitsiri, Tong, 1907.00514**

Matching to Number Theory

Degree-3 Prouhet–Tarry–Escott (PTE) Problem

Physics	Number Theory	
$U(1)_{\mathrm{X}}$ - gravitational anomaly	first power sum	$\sum^{n} a_i = \sum^{n} b_i$
$\left(U(1)_{\mathrm{X}}\right)^2 - U(1)_{\mathrm{H}}$ anomaly	second power sum	$\sum_{i=1}^{n} a_i^2 = \sum_{i=1}^{n} b_i^2$
$\left(U(1)_{\mathrm{X}}\right)^3$ anomaly	third power sum	$\sum_{i=1} a_i^3 = \sum_{i=1} b_i^3$
$U(1)_{\mathrm{X}}$ charge assignments	two integer multisets A, B	
anomaly-free chiral sector	degree-3 PTE solution	

Credits: Junseok Lee (Tohoku U)

The smallest possible size is **n=k+1** (k=3, n=4 in this specific case)
Such solutions are called **ideal solutions** **(focus of this talk)**.

Symmetric Solutions: Abundance of Symmetric "Locus"

There are solutions with good properties: **symmetric solutions.**

The solution is called symmetric when it remains unchanged under the sign flip, $\{a_i\} = \{-a_i\}$ and $\{b_i\} = \{-b_i\}$.

e.g.) $A = \{-8, -1, 1, 8\}, \quad B = \{-7, -4, 4, 7\}$

Credits: Junseok Lee (Tohoku U)

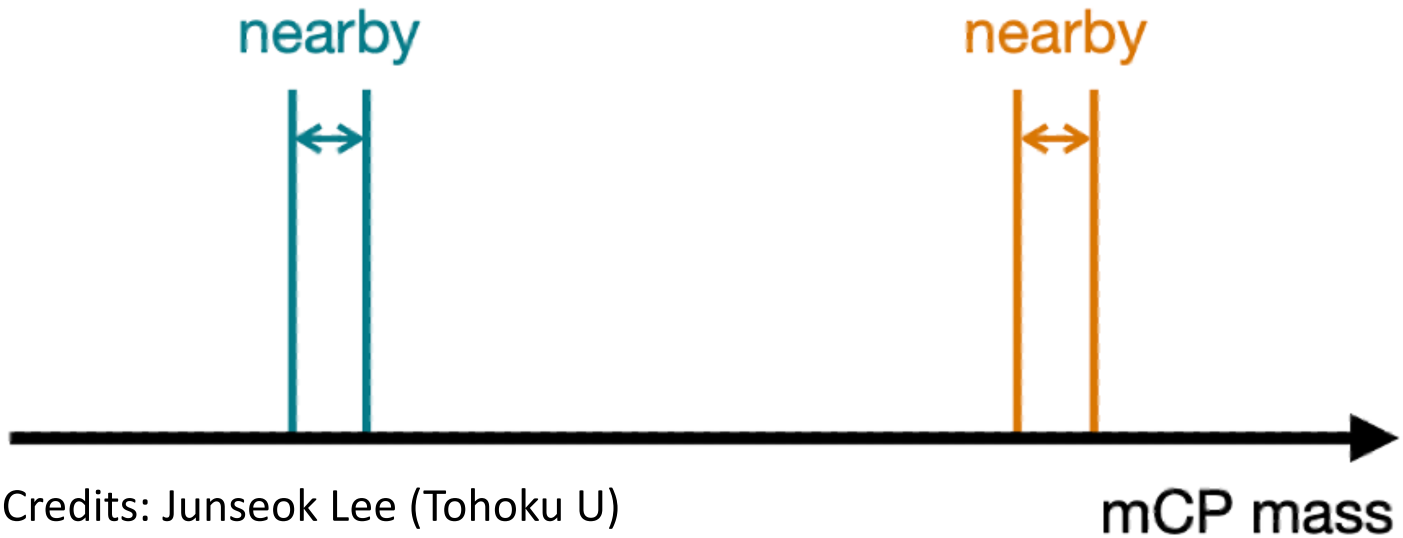
PTE Refs: Goldbach and Euler (1750-51), Prouhet (1851), Escott (1910), Tarry (1895); Borwein (2002) for a review.

Pure math paper: symmetry solutions is abundant among full solutions
Tsai, Lee, Takahashi, 2606.07735 (math.NT)

$$|a_i|,\ |b_i| \leq H, \quad N_{\mathrm{sym}}(H) = \frac{4\log 2}{3\pi^2} H^3 \log H + O(H^3).$$

Ideal Symmetric Solutions Lead to Doublet Structures

Lee, Takahashi, **Tsai**, 2603.12320 (hep-ph)



Credits: Junseok Lee (Tohoku U)

- Degeneracy is of higher order of $\epsilon = \frac{\langle \phi \rangle}{M_c}$
- Impacting experiments and cosmological searches

Summary

Based on our **Lee-Takahashi-Tsai (LTT)** construction:

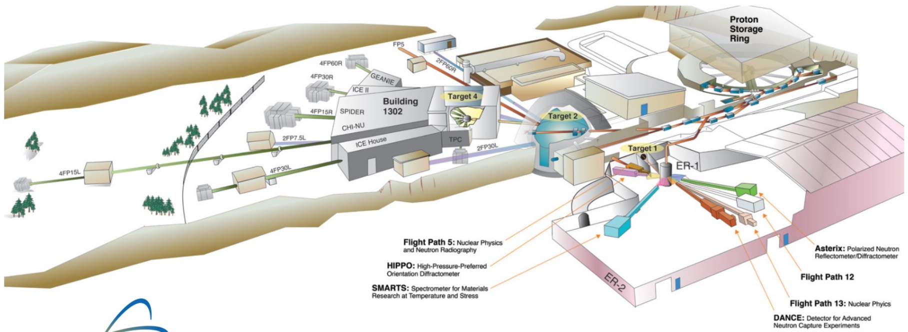
1. **Chiral symmetry provides naturally light mCPs**

2. **Anomaly cancellation conditions: degree-3 PTE problem**

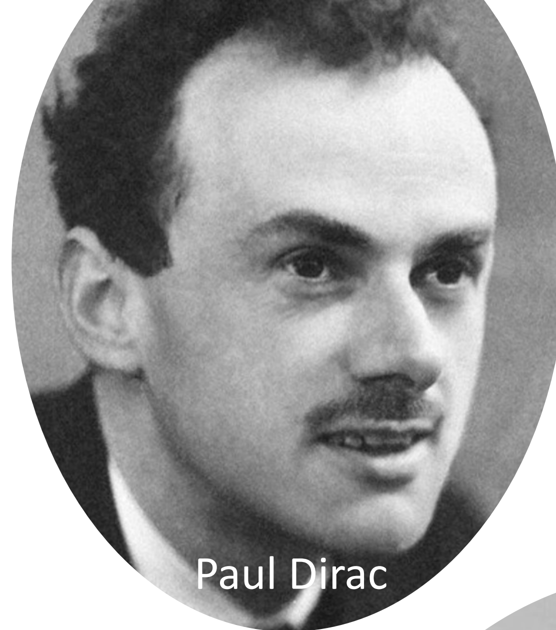
3. **Sharpe prediction of minimal 4 mCP states (ideal sol.)**

4. **Symmetric solutions dominant: leading to doublet LTT spectra**

5. New number theory result:
   Symmetry abundance of the PTE solutions



Los Alamos
NATIONAL LABORATORY
EST. 1943

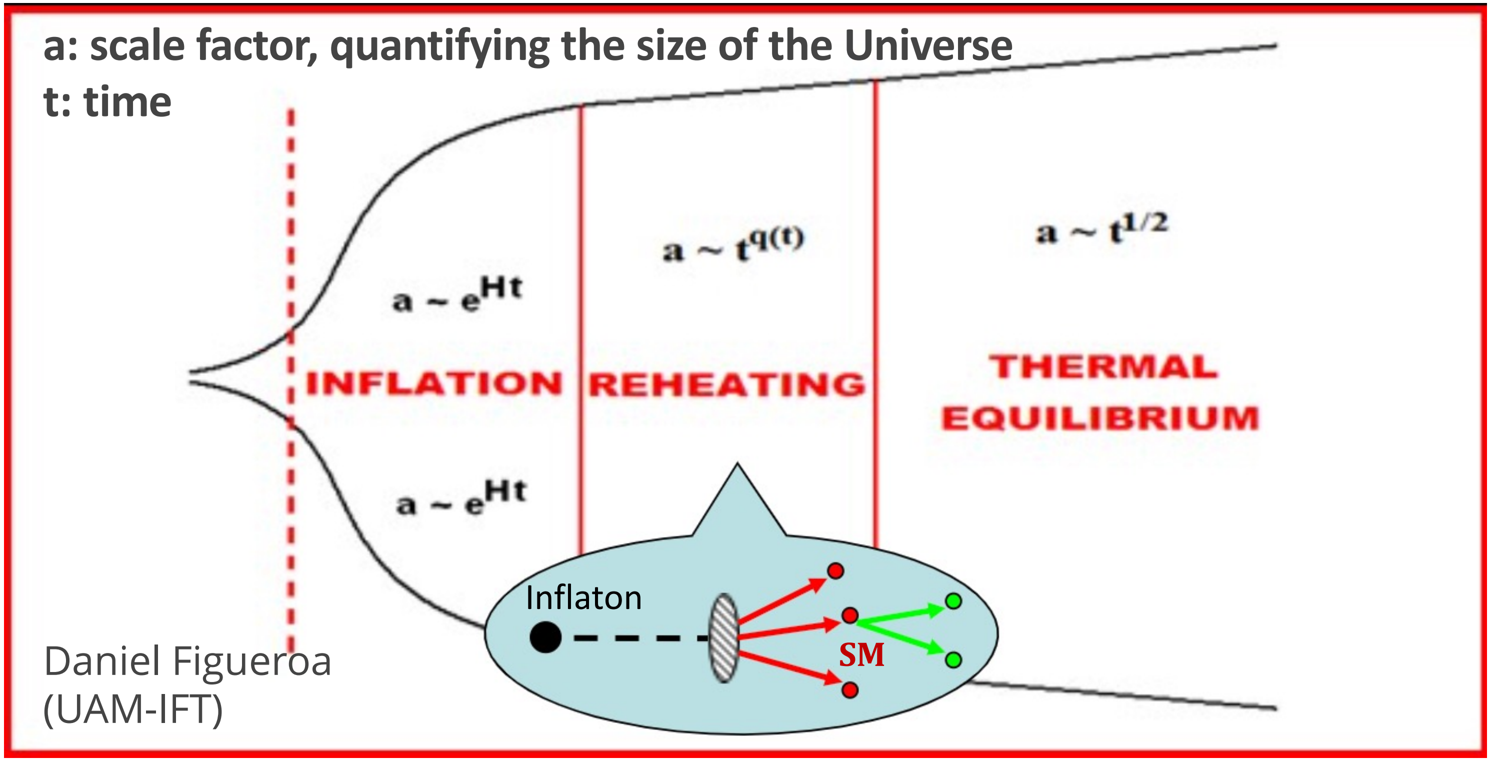

Paul Dirac

Cosmic Millicharge Background & Reheating Probes

Probing Reheating Cosmology with Experimental Searches for mCPs


Frederick Reines

Probing Inflation & Reheating Cosmology

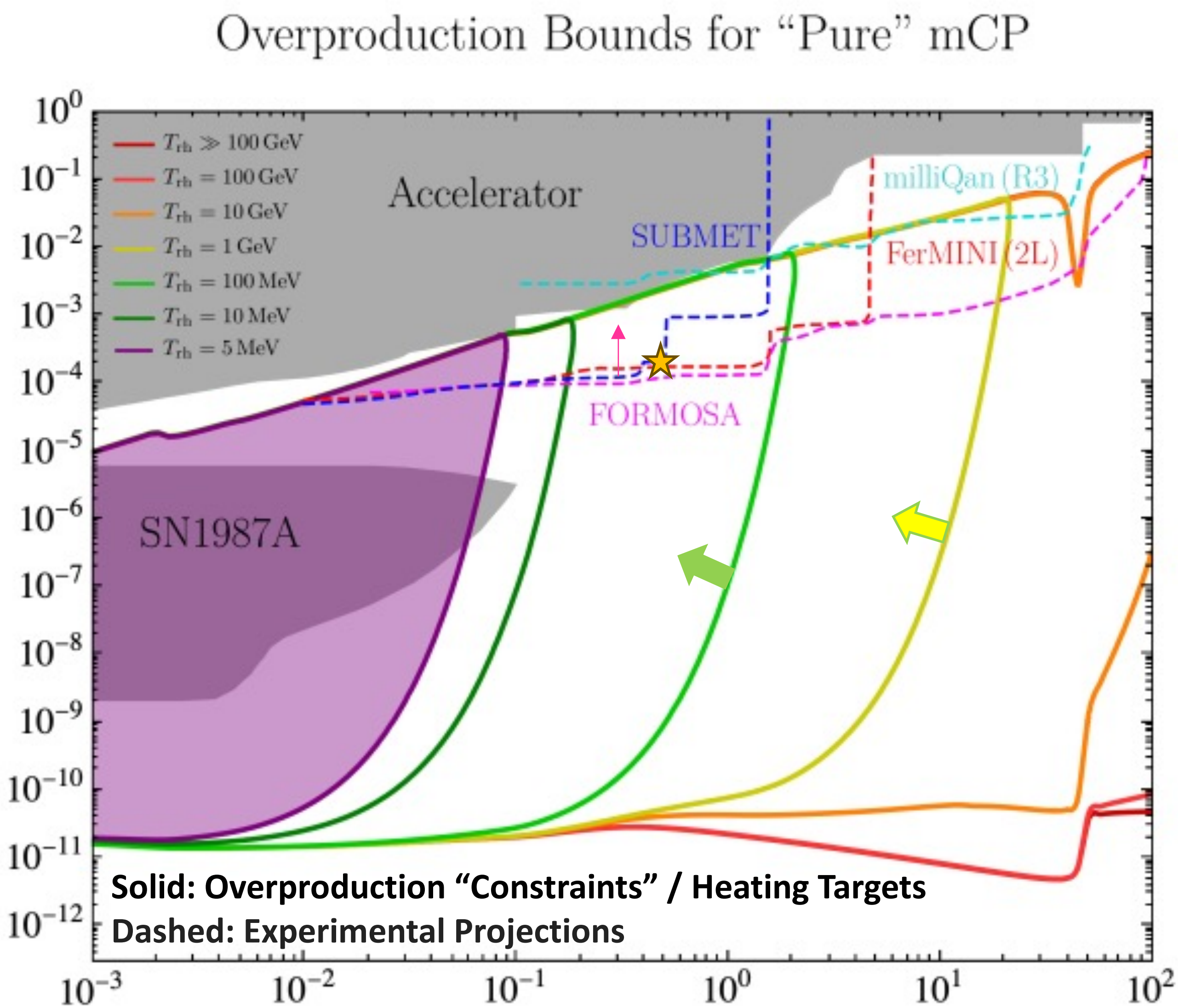


We know very little about the reheating phase; washed out in later stage

$T_{\rm rh}$ is the reheating temperature; BBN is the big bang nucleosynthesis

$$T_{\rm GUT}(\sim 10^{16}\,{\rm GeV}) \gtrsim T_{\rm rh} \gtrsim T_{\rm BBN}(\sim {\rm MeV})$$

Millicharge Search as a Probe for Reheating Temperature

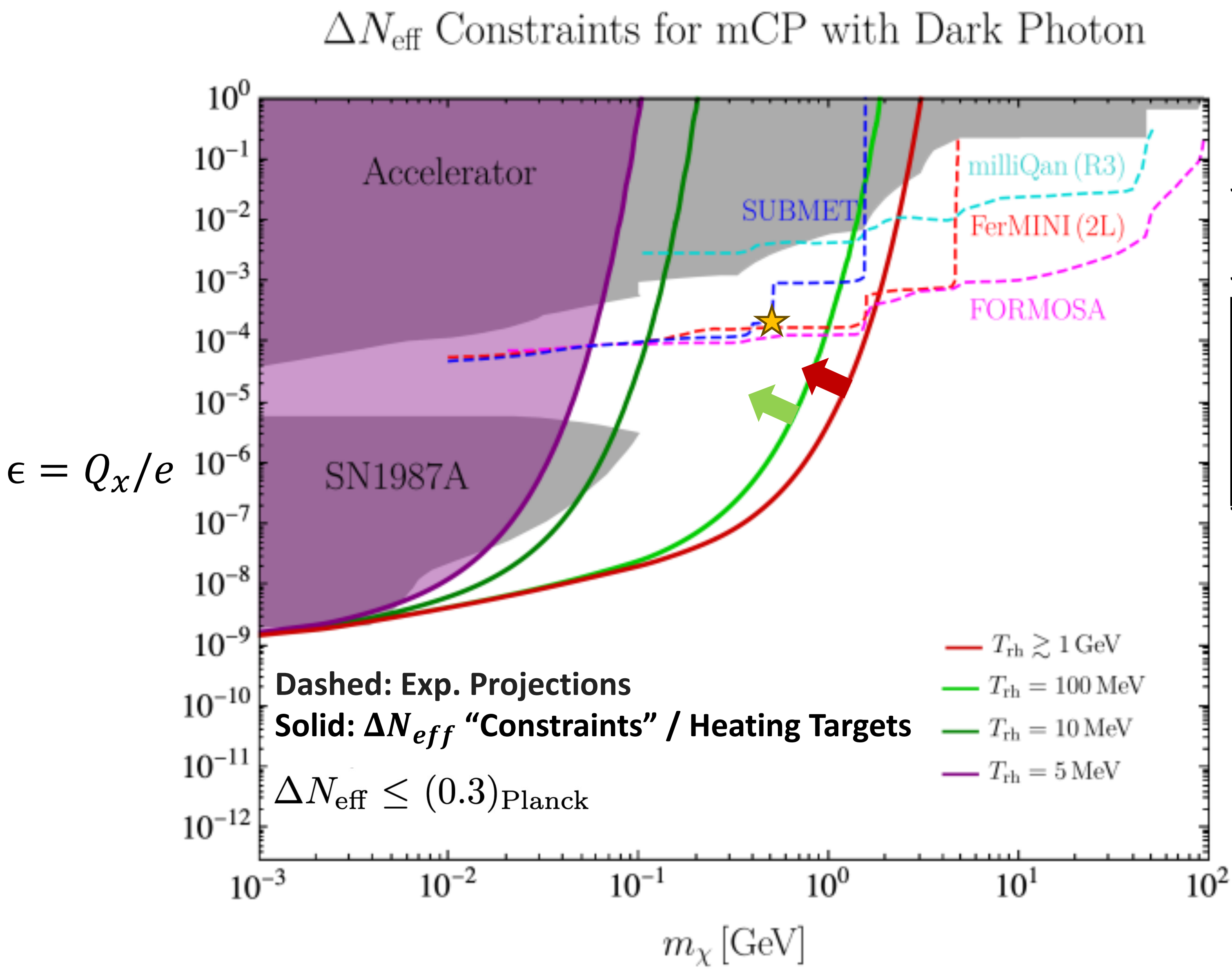


Charge in unit of electron charge

Mass of millicharge particle (GeV)

Gan, **Tsai**, 2308.07951, *JHEP* 2025

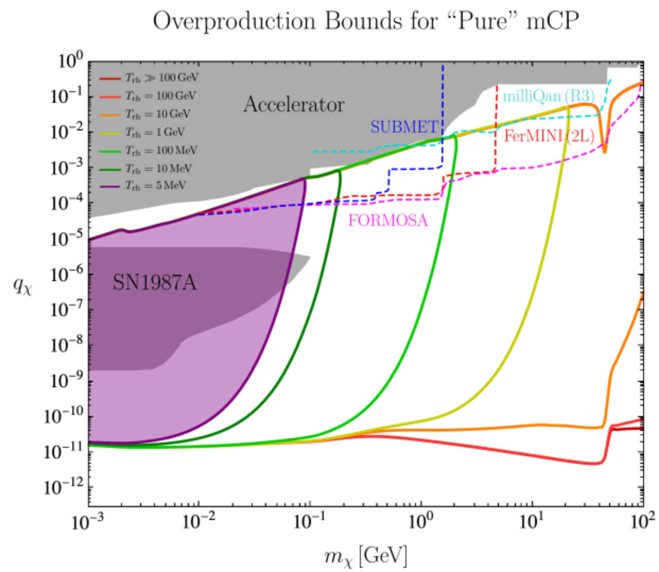
Millicharge Search as a Probe for Reheating Temperature

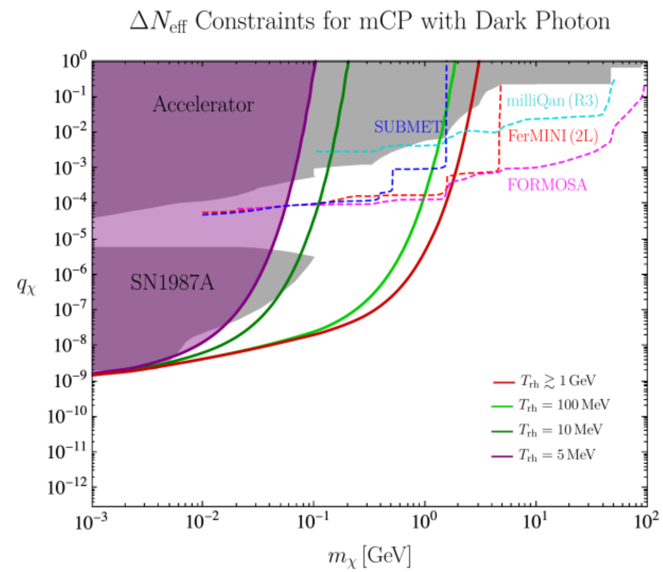


Gan, **Tsai**, 2308.07951, *JHEP* 2025

Testing Reheat Temperatures in Both Cases

Gan, **Tsai**, *JHEP* 2025, 2308.07951





Note: Our study naturally extend to higher temperatures.

We only just focus on the MeV to GeV region as a demonstration, also for the interest of the accelerator program

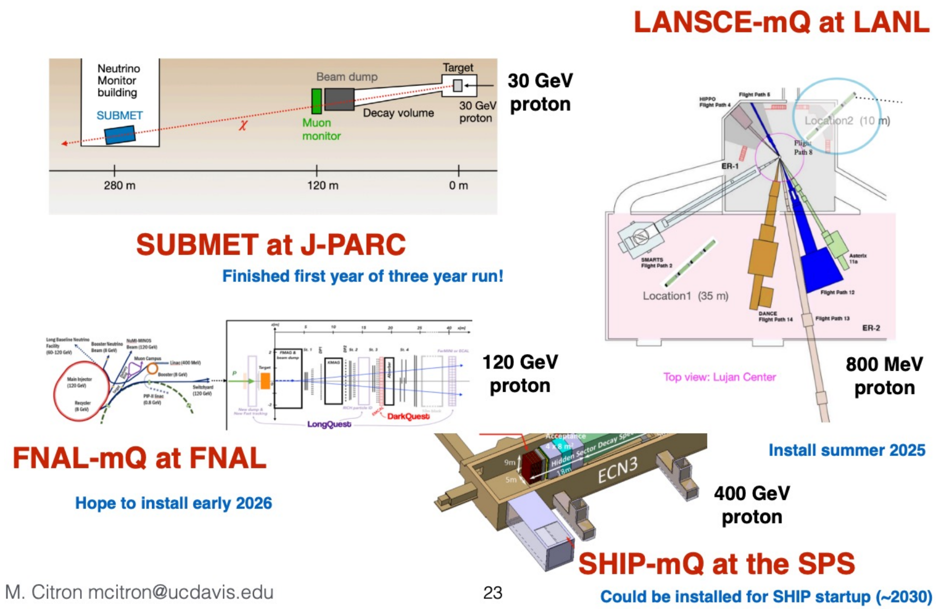
Dedicated Scintillation-Based Millicharged Experiments

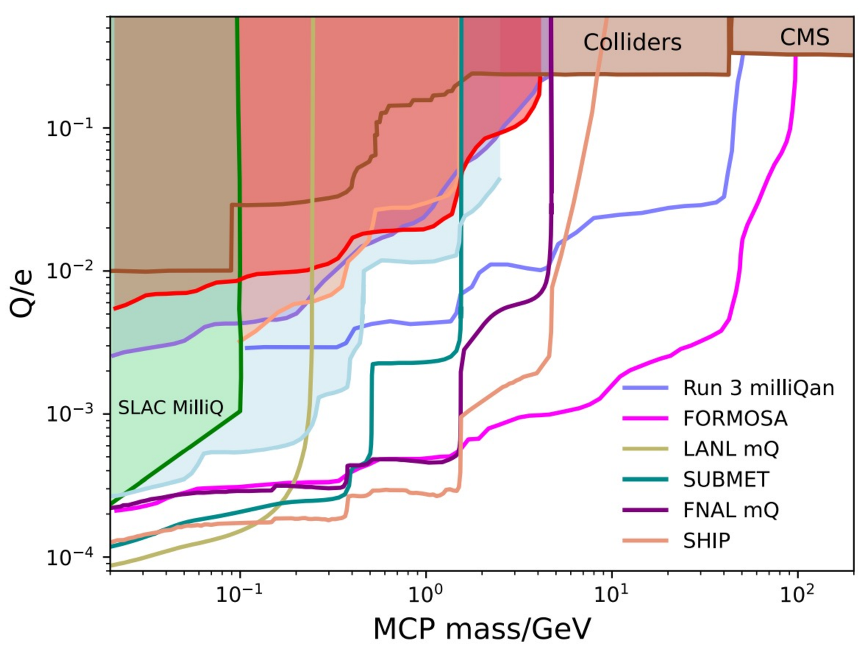
1. **FORMOSA @CERN**
- **Proposed it & conducted flux calculations with PhD Student**
  Abari, Kling, **Tsai**, *PRD* (2021)
- Demonstrator installed & taking data

2. **LANSCE-mQ @LANL**
- **A tiny "pathfinder" has taken data**
- **Tsai** et al., *PRD* (2025), 2407.07142

Samantha Kelly (PhD Student) at LANSCE@LANL

3. **LongQuest-mQ @Fermilab**
- **Study mCP & Long-Lived Particles**
- **Tsai** et al., *PRL* (2021)





Updates:

- **For dedicated searches:**

1. **FORMOSA**: <u>specifically featured on US P5 report</u>

2. **LANL LDRD $165k funding (2025)**

3. Demonstrators are built for **FORMOSA** & **LANSCE-mQ**

Students:
Undergrads: Bailloeul (UCD)
PhDs: Hwang (Boston U) , Kelly (UCD), Lee (Tohoku, theory); many **posdocs**…

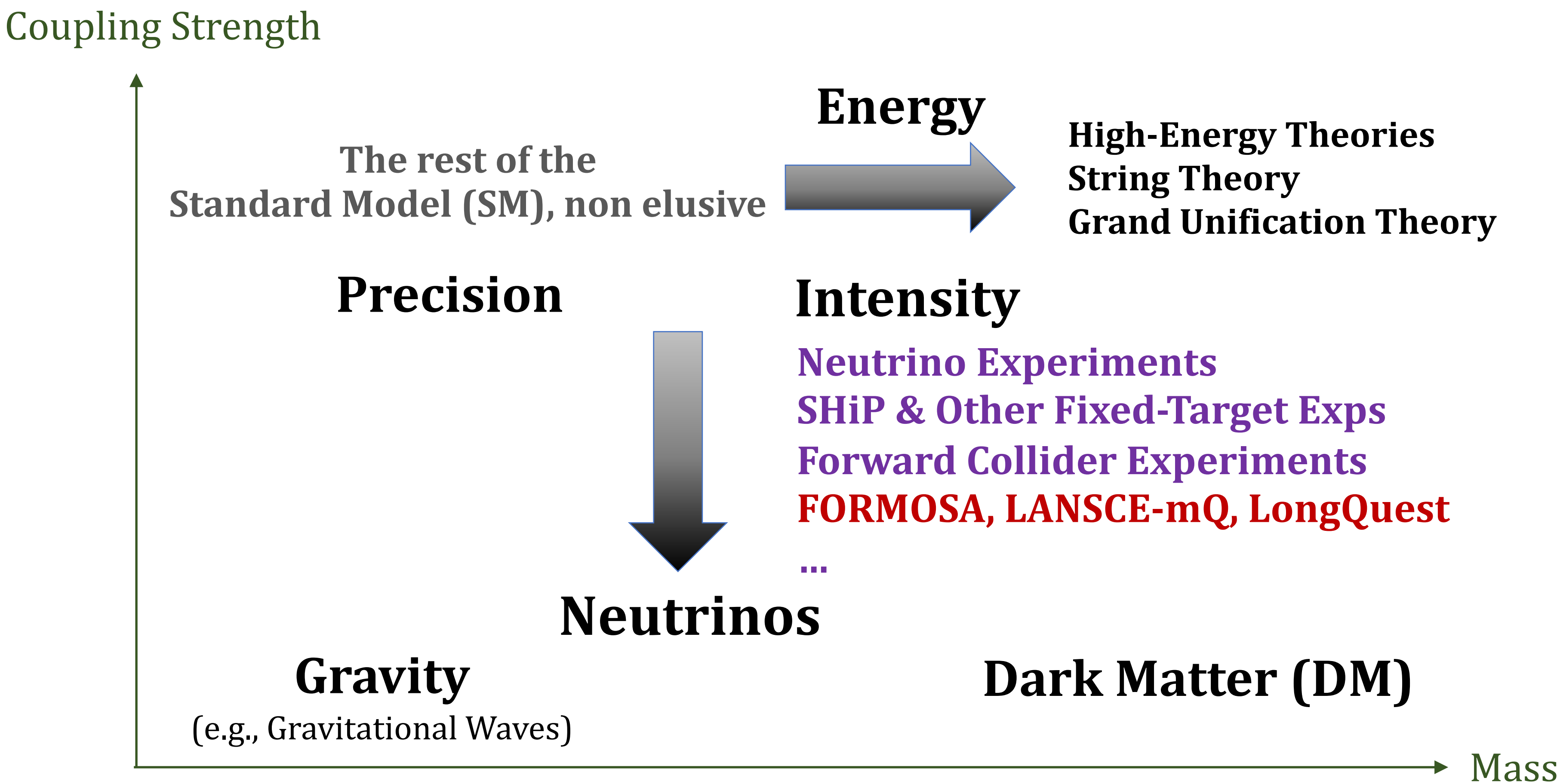
Summary

1. Bridging: new results in QFT (anomaly cancellation), Math (number theory), and Phenomenology (LTT mCP Spectral Predictions for experiments and cosmology)

2. Probing mCPs to reveal cosmology (reheating cosmology)

3. Many dedicated small-scale experiments are being built to study mCPs

Summary: Important Explorations in Particle Physics



- **Quantum Number Theory:** 2603.12320
- **Symmetry Abundance in PTE Solutions (Pure Math):** 2606.07735
- **Cosmic Millicharged Particles:** 2308.07951