Primordial Power Spectrum from a deformed bouncing background

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Outline

Hamiltonian Cosmology: the Isotropic Universe

The cosmological action and Hamiltonian contain two parts:

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right)$$

$$\mathcal{H} = \mathcal{H}_G(a, p_a) + \mathcal{H}_M(\rho, \phi, \ldots) \approx 0$$

From EoM, find Friedmann equation linking expansion to matter:

$$\boxed{H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{\kappa}{3} \rho}$$

Solve Friedmann eq. for scale factor $a(t)$ (we often use $v = a^3$):

$$a(t) \propto (t - t_0)^{\frac{2}{3(1+w)}} \qquad v(t) \propto (t - t_0)^{\frac{2}{1+w}}$$

Big Bang/Big Crunch singularities at $t = t_0$.

Close to singularities, high E regime $\Rightarrow$ Quantum effects

Outline

Hamiltonian Cosmology

Deformed Commutation Relations

Bouncing Background

Perturbations

Summary

Deformed Commutation Relations

- Modification of standard Heisenberg commutators.
- Inspired by the Generalised Uncertainty Principle (GUP).
- Introduce quantum gravity effects easily on any system.
- Well-parametrized by functions of momentum: $\boxed{[\hat{q}, \hat{p}] = i\, f(\hat{p})}$
- Problems: operator ordering, different representations, ...

$\hat{q}\,\psi(p) = i\, f(p)\, \partial_p \psi$ $\qquad \hat{p}\,\psi(p) = p\,\psi(p)$ $\qquad$ (and measure for EVs)

OR $\qquad \hat{q}\,\psi(p) = i\, \partial_p \psi$ $\qquad \hat{p}\,\psi(p) = g(p)\,\psi(p)$ $\qquad g^{-1} = \int dp/f(p)$

To avoid inconsistencies, we use the semiclassical limit.

Maggiore (PLB 1993). Kempf, Mangano, Mann (PRD 1995).

Semiclassical Limit and the Cut-Off Deformation

The semiclassical limit of DCRs is quick and versatile:

simply downgrade to deformed Poisson brackets: $\{q, p\} = f(p)$

$$\dot{q} = \{q, \mathcal{H}\} = \frac{\partial \mathcal{H}}{\partial p} f(p) \qquad \dot{p} = \{p, \mathcal{H}\} = -\frac{\partial \mathcal{H}}{\partial q} f(p)$$

We use the so-called Cut-Off Deformation:

$$\boxed{f(p) = \sqrt{1 - \frac{\mu^2 p^2}{\hbar^2}}}$$

It clearly implements a cut-off on the momentum: $|p| < 1/\mu$.

The standard case is recovered in the limit $\mu \to 0$.

In FLRW: $q \propto v = a^3$ and $p = p_v \propto \dot{v}/v$ so that $[\mu] = [v] = 1$.

Battisti (PRD 2009). **GB**, Giovannetti, Montani (IJGMMP 2022).

Outline

Deformed FLRW Model

We consider flat isotropic FLRW with a scalar field ϕ:

$$\mathcal{H} = -\frac{3\kappa}{16 V_0} p_v^2 v + \frac{1}{V_0} \frac{p_\phi^2}{2v} + V_0 \, v \, W(\phi) \approx 0$$

$$p_v^2 = \frac{16 V_0^2}{3\kappa} \rho \qquad \rho = \frac{1}{V_0^2} \frac{p_\phi^2}{2v^2} + W \qquad \{v, p_v\} = 2 \, f(p_v)$$

From EoMs, we obtain modified Friedmann equation:

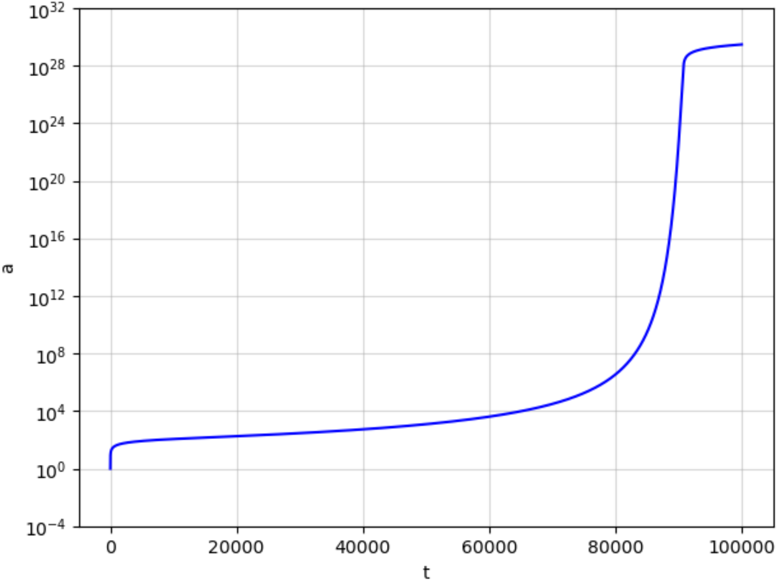
$$H^2 = \left(\frac{\dot{v}}{3v}\right)^2 = \frac{\kappa}{3} \rho \boxed{\left(1 - \frac{\rho}{\rho_\mu}\right)} \qquad \rho_\mu = \frac{3\hbar^2 \kappa}{16 V_0^2 \mu^2}$$

With $V_0 \propto \Delta^{\frac{3}{2}}$ and $\mu = 1$, this is the same as LQC.

Montani, Mantero, Bombacigno, Cianfrani, **GB** (PRD 2019).

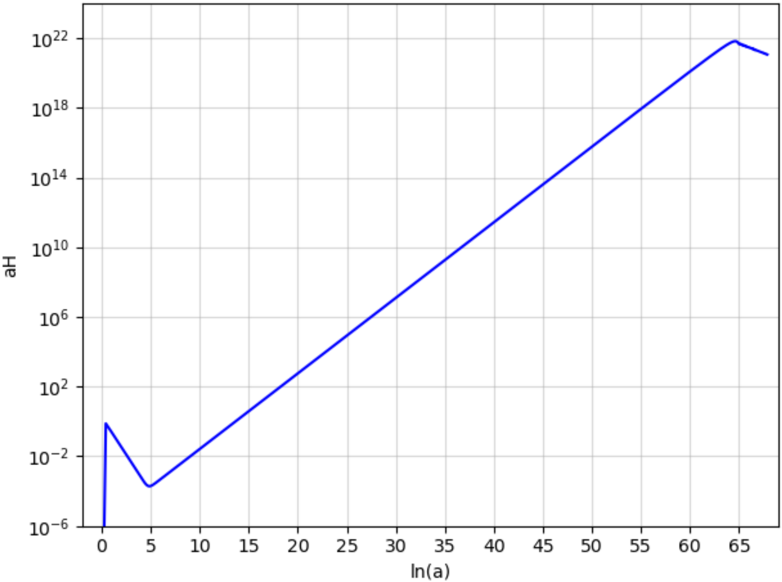
Deformed Background

Same as LQC: big bounce followed by inflation.



Deformed Background

Standard initial conditions, roughly 60 e-folds of inflation.



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Perturbations and Mukhanov-Sasaki Equation

Now we introduce perturbations on the metric and on the field.

Define conformal time η and the Mukhanov-Sasaki variable:

$$\mathrm{d}t = a\,\mathrm{d}\eta \qquad \xi = a\left(\delta\phi_{\mathrm{GI}} + \frac{\phi'\Phi_{\mathrm{B}}}{aH}\right)$$

From second variation of the action, obtain Hamiltonian for ξ_k:

$$\mathcal{H} = \sum_k \mathcal{H}_k = \frac{1}{2}\sum_k \pi_k^2 + \omega_k^2\xi_k^2 \qquad \pi_k = \xi_k'$$

$$\boxed{\omega_k^2 = k^2 + s = k^2 - \frac{z''}{z}} \qquad z = z(\eta) = a\sqrt{\epsilon} = a\sqrt{-\frac{\dot{H}}{H^2}}$$

Mukhanov, Feldman, Brandenberger (Physics Reports 1992).

The Time-Dependent Frequency

The term in the time-dependent frequency amounts to

$$-s = \frac{z''}{z} = 2\left(\frac{a'}{a}\right)^2 + 2\left(\frac{a''}{a'}\right)^2 - 2\frac{a''}{a} + 4\frac{a'}{a}\frac{\phi''}{\phi'} - 2\frac{a''}{a'}\frac{\phi''}{\phi'} - \frac{a'''}{a'} + \frac{\phi'''}{\phi'}$$

The classical case yields:

$$\frac{z''}{z} = -\frac{\kappa}{6}\phi'^2 - \frac{16\kappa}{3}a^2 W - a^2\frac{\partial^2 W}{\partial\phi^2} - 6\frac{a'\phi'}{a\rho}\frac{\partial W}{\partial\phi} + 6\kappa\frac{a^2}{\rho}W^2$$

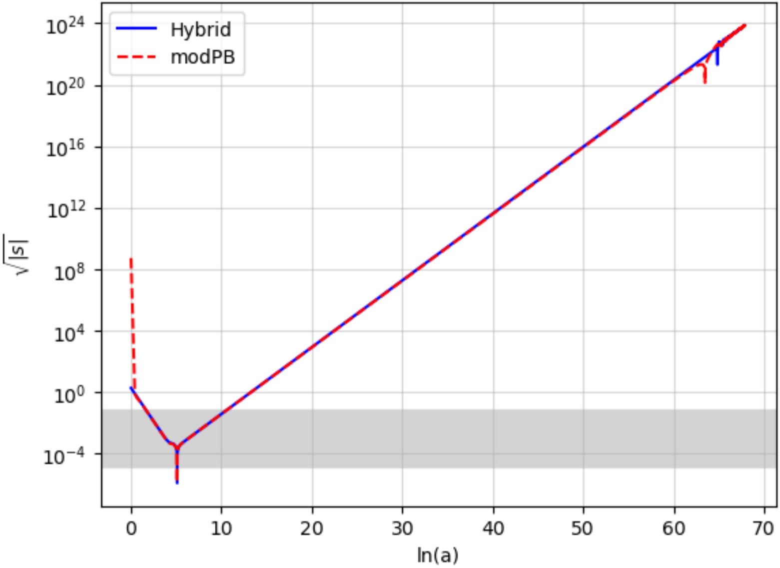
With our deformed bouncing background ($x = \rho/\rho_\mu$):

$$\frac{z''}{z} = -\frac{\kappa}{6}\phi'^2 \boxed{\frac{1-22x+4x^2}{1-x}} - \frac{16\kappa}{3}a^2 W \boxed{\frac{8-16x+17x^2}{8(1-x)}}$$

$$-a^2\frac{\partial^2 W}{\partial\phi^2} - 6\frac{a'\phi'}{a\rho}\frac{\partial W}{\partial\phi} \boxed{\frac{1-2x}{1-x}} + 6\kappa\frac{a^2}{\rho}W^2 \boxed{\frac{1-2x+2x^2}{1-x}}$$

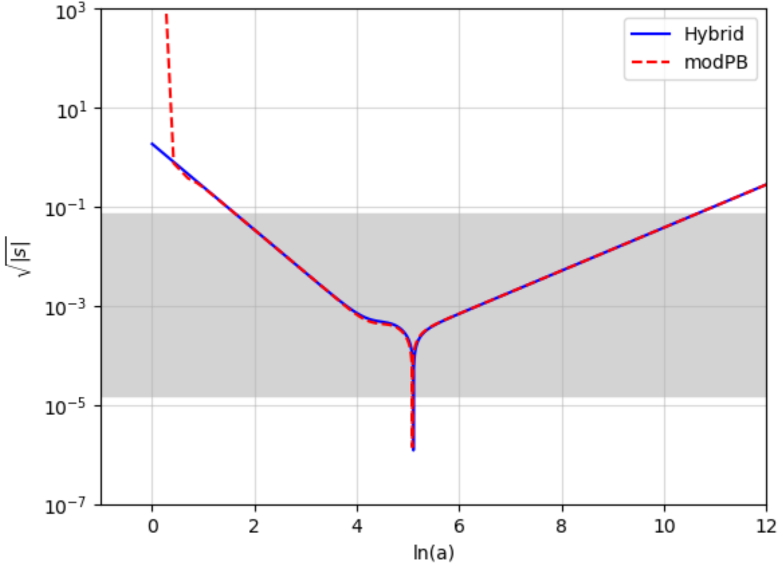
Results and Comparison with LQC

The time-dependent frequency compared with Hybrid LQC:



Results and Comparison with LQC

We don't expect much difference with respect to Hybrid LQC.



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Summary and Outlook

- Deformed Commutation Relations are versatile and easily introduce cut-off effects in any system
- The semiclassical Cut-Off deformation reproduces exactly the same background dynamics as effective LQC
- The deformed background yields a modified time-dependent frequency term in the Mukhanov-Sasaki equation
- The time-dependent frequency is very similar to Hybrid LQC in the observable window
- Next steps: more consistent deformed quantization and full quantum approach to perturbations?

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Soon to appear on arXiv.

Thanks for your attention!