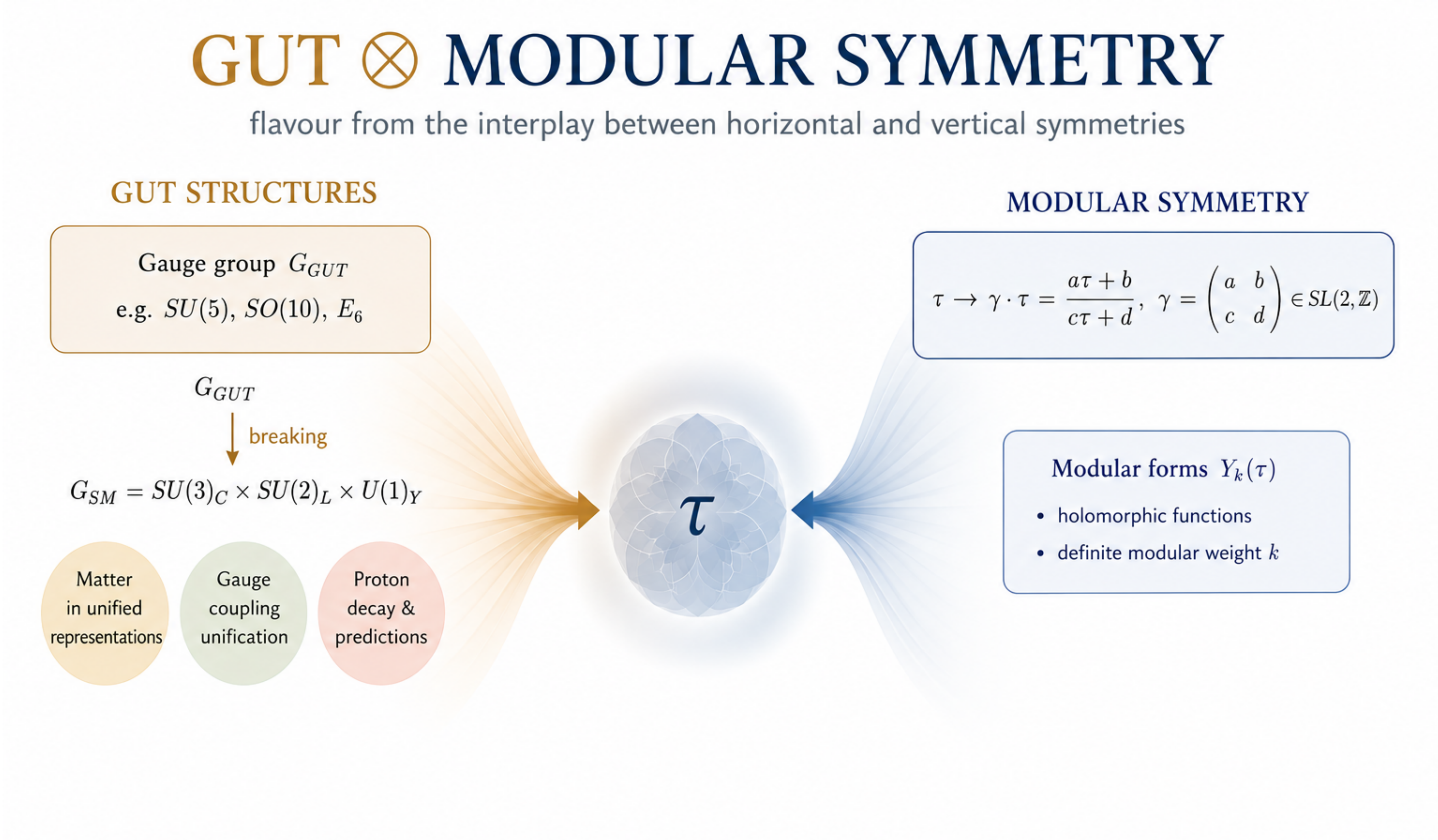
Towards a minimal gCP modular GUT

Marco Carducci

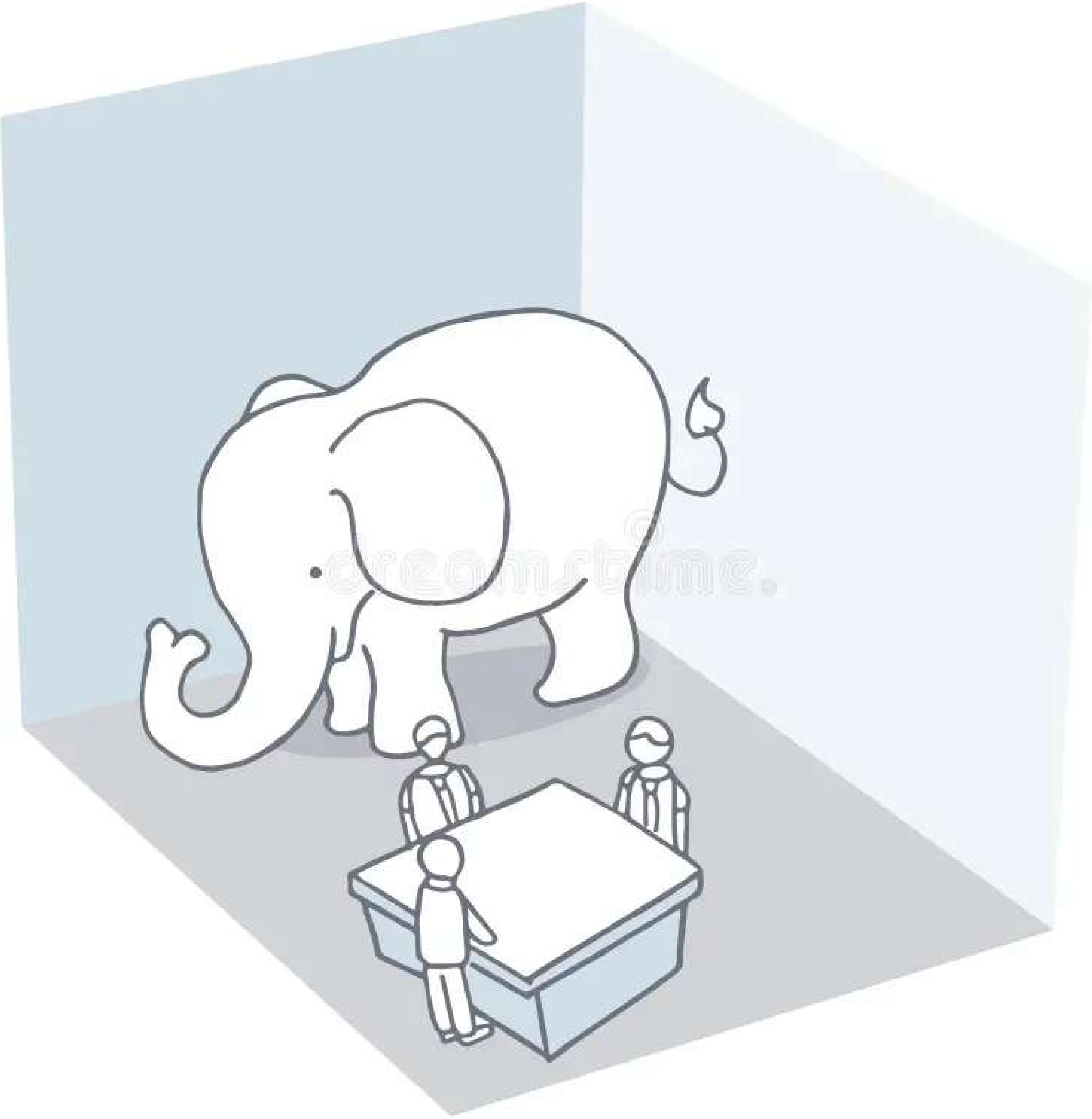
Roma Tre

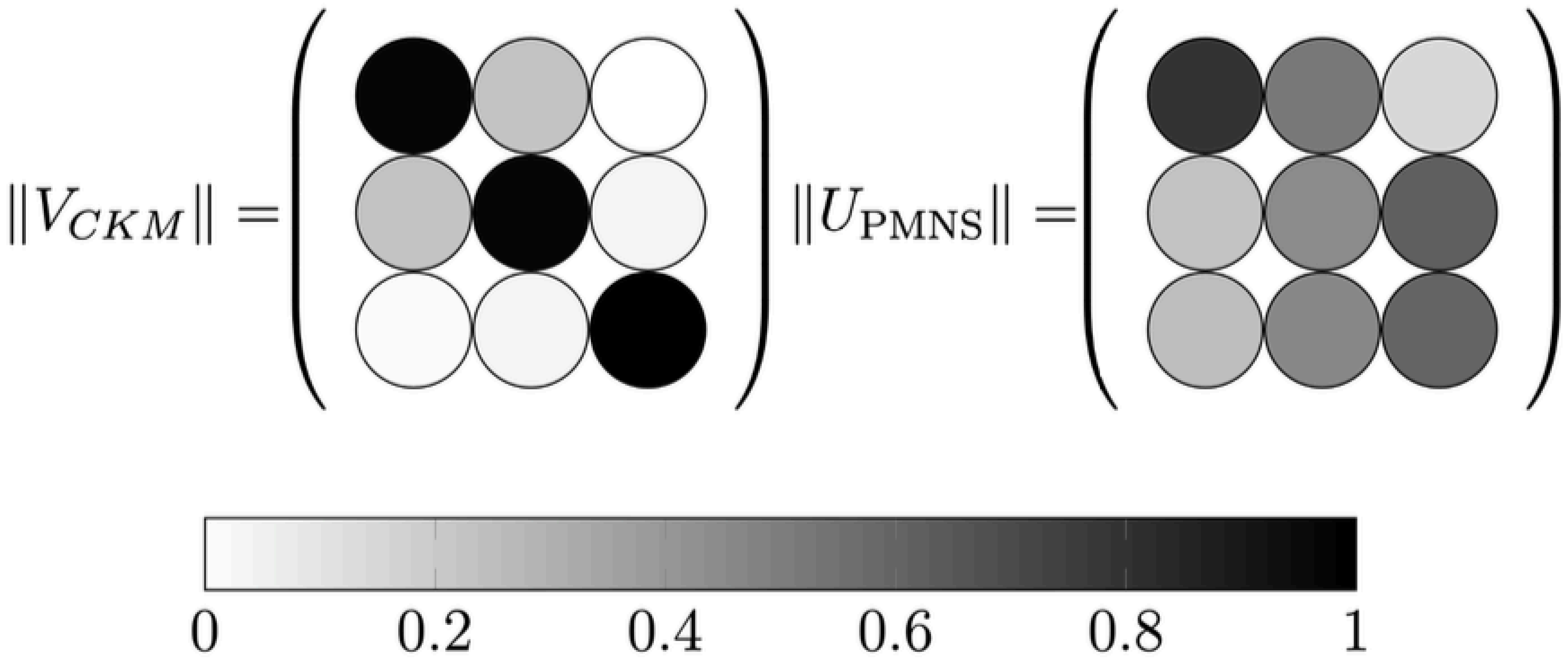
In collaboration with D. Meloni and J. T. Penedo

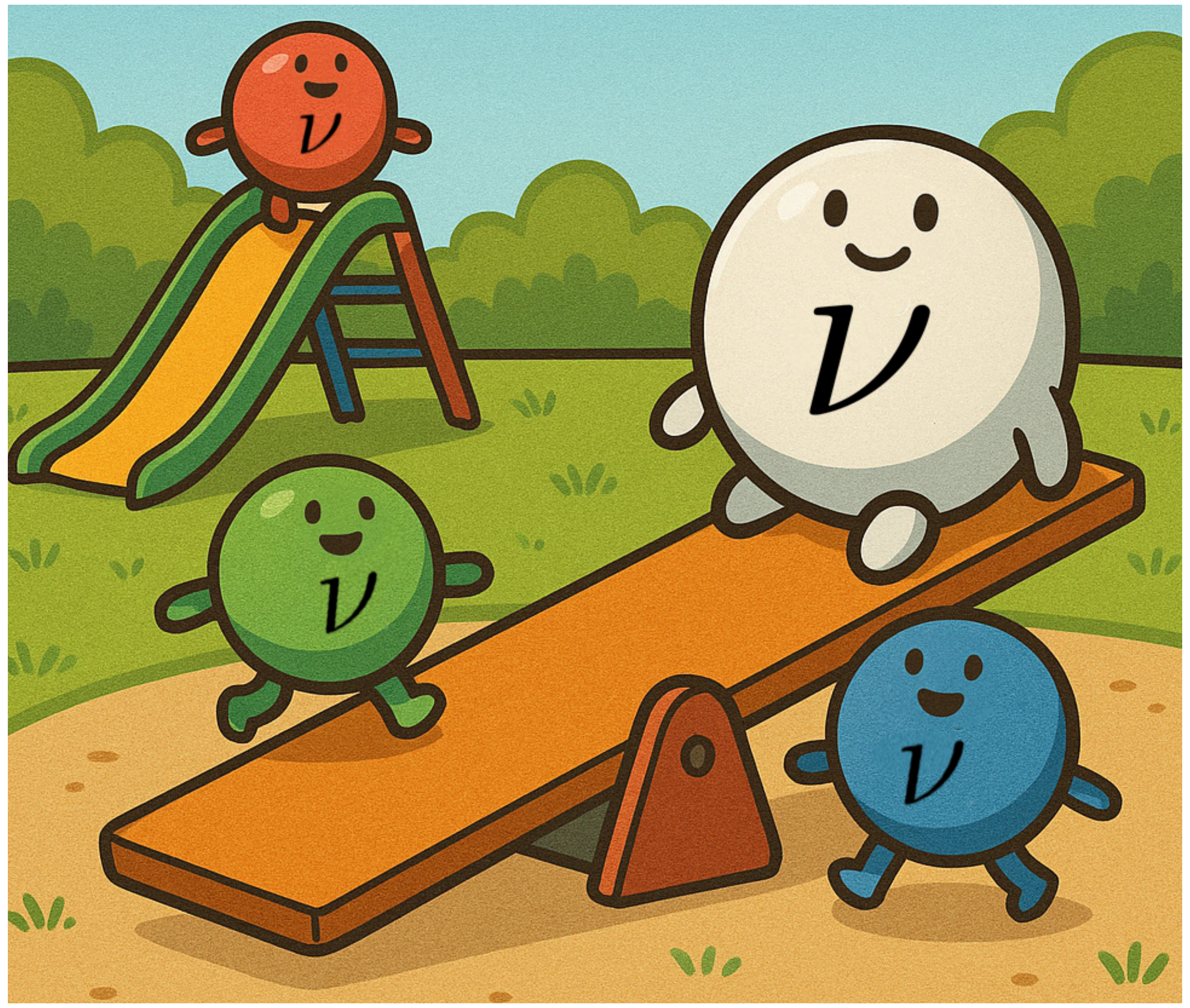


PASCOS 2026 (23 June)

Flavour puzzle



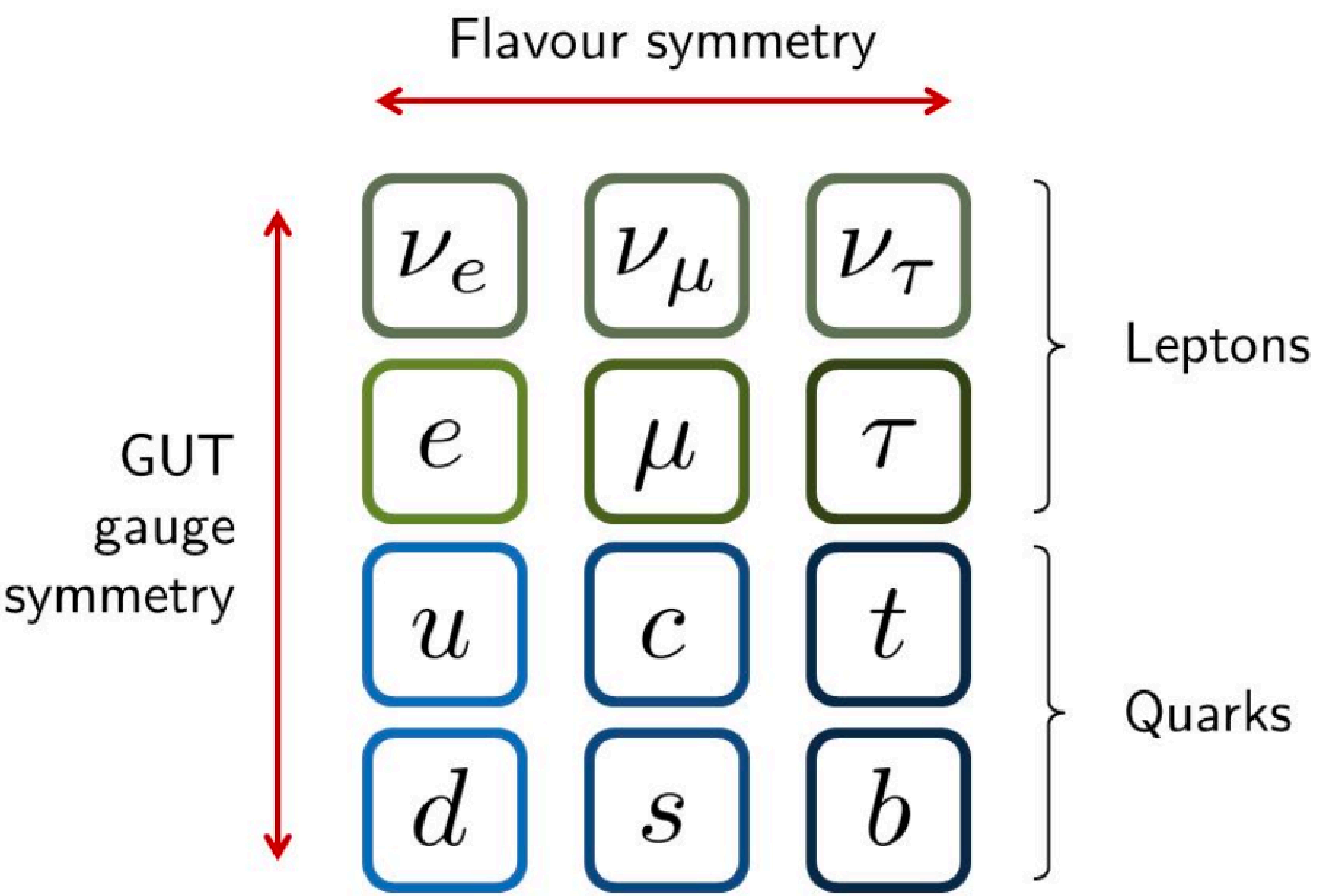




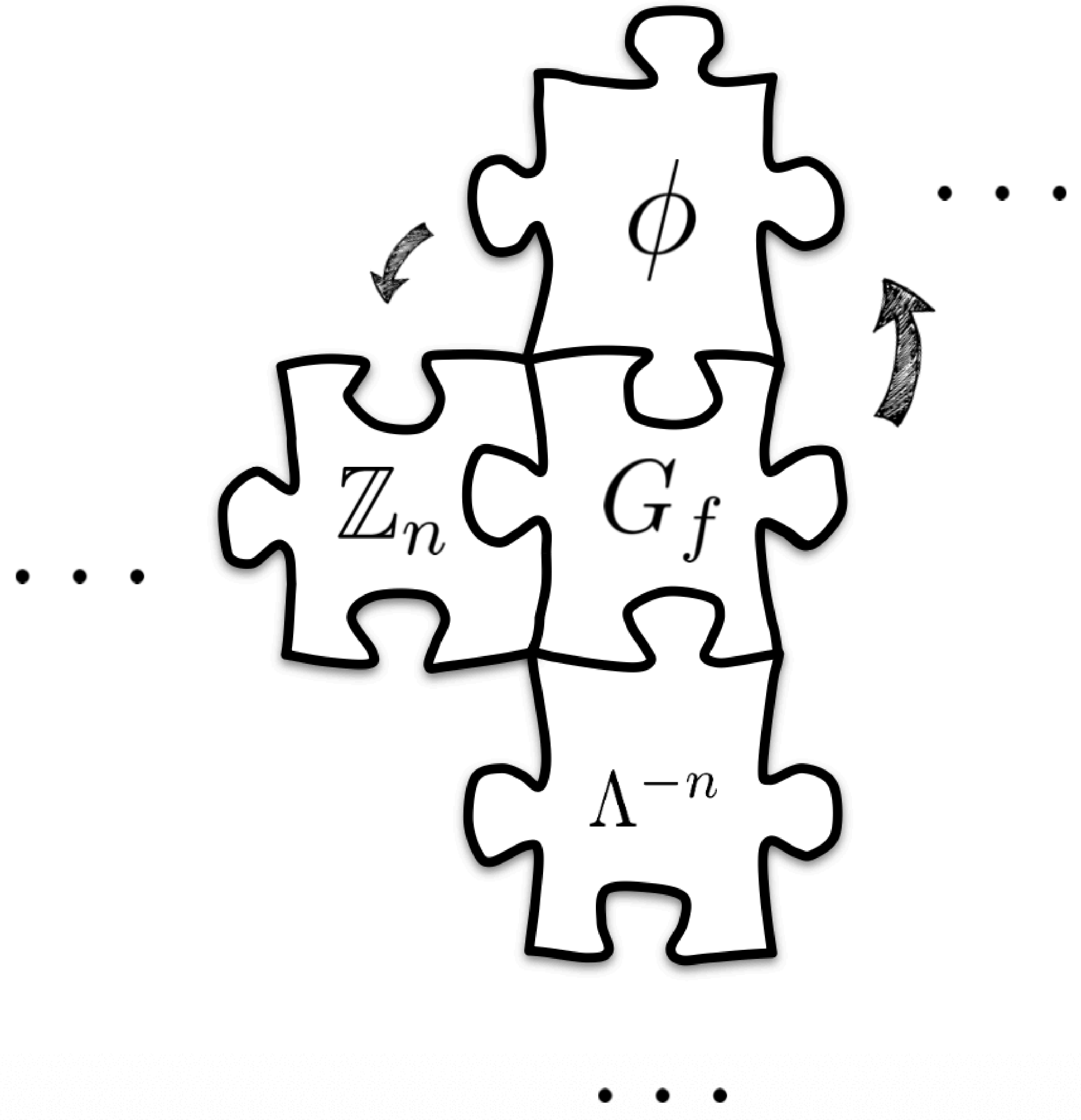
(Adapted from slides of M. Parriciatu)

Flavour puzzle

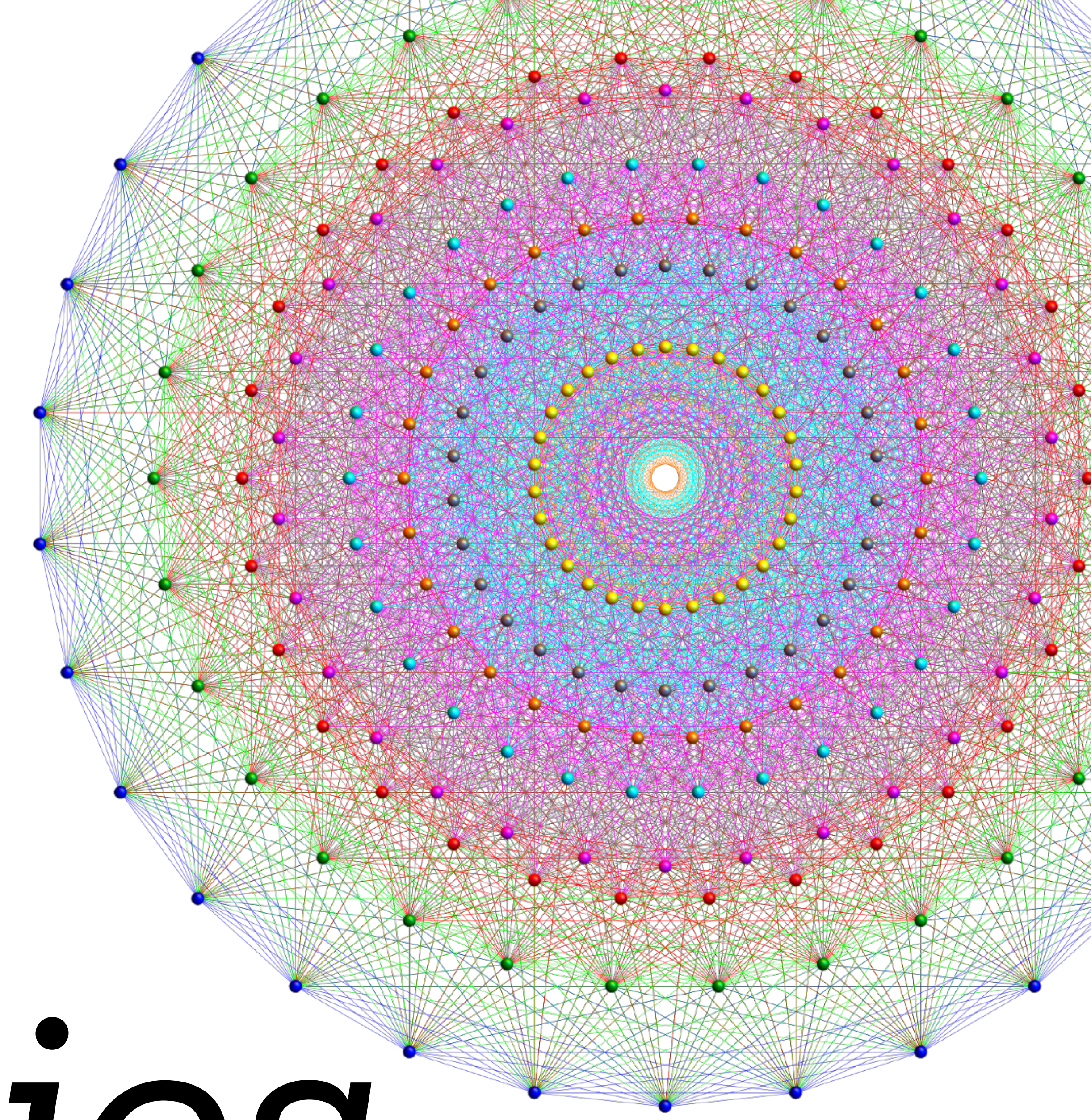
C. D. Froggatt and H. B. Nielsen
Nucl. Phys. B



Attemps...



(Adapted from slides of J. T. Penedo)

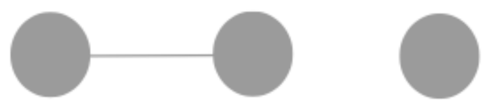


Grand Unified Theories


STANDARD MODEL
STANDARD MODEL interaction
NAME
GRAND UNIFIED THEORY
ALIAS
GUTs
ELECTROMAGNETIC FORCE
dictates the structure of atoms and the behavior of light
electron
DOCUMENT FILE
NAME
Bottom Quark
ALIAS
beauty
CHARGE
-1/3
DOB
1977
BIRTHPLACE
Fermilab
SPIN
1/2
b
Predicted Before Discovered
GUT SCALE
WEAK NUCLEAR FORCE
particles decay
short ranged
electroweak interaction
HYPER K
3 Neutrino Types
electron
TAU
MUON
STRONG NUCLEAR FORCE
u,c,t,d,s,b
Binds quarks together into things like the protons and neutrons in our atoms
Additional Higgses?
N/A ON whereabouts
NAME
SUPERSYMMETRY
NICKNAME
SUSY
SUSPECT ACCOMPLICE
SPARTICLE
CONFIDENTIAL
SUSY
Complicated interactions and more Particles
PROTON
PROTON DECAY ?
DUNE
800 miles
Messy

Supersymmetric SU(5): Particle Content

H. Georgi and S. L. Glashow
Phys. Rev. Lett.

 U(1)

SM group is of **rank 4:** Embed into simple groups of rank 4 or more.

SU(5) Each generation a 10 and $\overline{5}$ (without RH neutrino)

Minimal non-SUSY model ruled out by $\sin^2\theta_W$

Supersymmetric SU(5): Particle Content

B. Bajc and G. Senjanovic
[hep-ph/0612029]-JHEP

SM group is of rank 4: Embed into simple groups of rank 4 or more.

SU(5) Each generation a 10 and $\overline{5}$ (without RH neutrino)

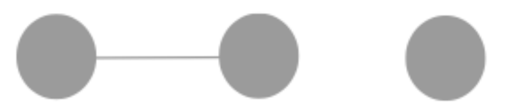
Minimal non-SUSY model ruled out by $\sin^2\theta_W$

$$F \equiv \overline{5}_F = \begin{pmatrix} d^c \\ \epsilon_2\,\ell \end{pmatrix} \qquad 10_F = \begin{pmatrix} \epsilon_3\, u^c & q \\ -q^T & \epsilon_2\, e^c \end{pmatrix} \equiv T$$

$$\Sigma(24) = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(\bar{3},2)} & \Sigma_3 \end{pmatrix} + \frac{1}{2\sqrt{15}} \begin{pmatrix} 2\,\mathbf{1}_3 & 0 \\ 0 & -3\,\mathbf{1}_2 \end{pmatrix} \Sigma_{24}$$

Supersymmetric SU(5): Particle Content

B. Bajc and G. Senjanovic
[hep-ph/0612029]-JHEP



U(1)

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Minimal non-SUSY model ruled out by $\sin^2\theta_W$

Irrep

Label

$$F \equiv \overline{5}_F = \begin{pmatrix} d^c \\ \epsilon_2\,\ell \end{pmatrix} \qquad 10_F = \begin{pmatrix} \epsilon_3\,u^c & q \\ -q^T & \epsilon_2\,e^c \end{pmatrix} \equiv T$$

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Adjoint

Supersymmetric SU(5): Superpotential

D. Emmanuel-Costa and S. Wiesenfeldt

[hep-ph/0302272]-Nucl. Phys. B

$$W_Y = \sqrt{2}\left(Y_5^{ij}\,\overline{H}_a\,T_i^{ab}\,F_{jb} + h_1^{ij}\,\overline{H}_a\,\frac{\Sigma_b^a}{\Lambda}\,T_i^{bc}\,F_{jc} + h_2^{ij}\,\overline{H}_a\,T_i^{ab}\,\frac{\Sigma_b^c}{\Lambda}\,F_{jc}\right) +$$

$$+\frac{1}{4}\epsilon_{abcde}\left(Y_{10}^{ij}\,T_i^{ab}\,T_j^{cd}\,H^e + f_1^{ij}\,T_i^{ab}\,T_j^{cd}\,\frac{\Sigma_f^e}{\Lambda}\,H^f + f_2^{ij}\,T_i^{ab}\,T_j^{cf}\,H^d\,\frac{\Sigma_f^e}{\Lambda}\right)$$

Supersymmetric SU(5): Superpotential

D. Emmanuel-Costa and S. Wiesenfeldt

[hep-ph/0302272]-Nucl. Phys. B

$$W_Y = \sqrt{2}\left(Y_5^{ij}\,\overline{H}_a\,T_i^{ab}\,F_{jb} + h_1^{ij}\,\overline{H}_a\,\frac{\Sigma_b^a}{\Lambda}\,T_i^{bc}\,F_{jc} + h_2^{ij}\,\overline{H}_a\,T_i^{ab}\,\frac{\Sigma_b^c}{\Lambda}\,F_{jc}\right) +$$

$$+\,\frac{1}{4}\epsilon_{abcde}\left(Y_{10}^{ij}\,T_i^{ab}\,T_j^{cd}\,H^e + f_1^{ij}\,T_i^{ab}\,T_j^{cd}\,\frac{\Sigma_f^e}{\Lambda}\,H^f + f_2^{ij}\,T_i^{ab}\,T_j^{cf}\,H^d\,\frac{\Sigma_f^e}{\Lambda}\right)$$

NR operators

Supersymmetric SU(5): Superpotential

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$$W_Y = \sqrt{2}\left(Y_5^{ij}\,\overline{H}_a\,T_i^{ab}\,F_{jb} + h_1^{ij}\,\overline{H}_a\,\frac{\Sigma_b^a}{\Lambda}\,T_i^{bc}\,F_{jc} + h_2^{ij}\,\overline{H}_a\,T_i^{ab}\,\frac{\Sigma_b^c}{\Lambda}\,F_{jc}\right) + $$

$$+\frac{1}{4}\epsilon_{abcde}\left(Y_{10}^{ij}\,T_i^{ab}\,T_j^{cd}\,H^e + f_1^{ij}\,T_i^{ab}\,T_j^{cd}\,\frac{\Sigma_f^e}{\Lambda}\,H^f + f_2^{ij}\,T_i^{ab}\,T_j^{cf}\,H^d\,\frac{\Sigma_f^e}{\Lambda}\right)$$

$$\downarrow \quad \langle\Sigma\rangle = v_\Sigma\,diag\,(2,2,2,-3,-3)$$

Supersymmetric SU(5): Superpotential

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$$W_Y = \sqrt{2}\left(Y_5^{ij}\,\overline{H}_a\,T_i^{ab}\,F_{jb} + h_1^{ij}\,\overline{H}_a\,\frac{\Sigma_b^a}{\Lambda}\,T_i^{bc}\,F_{jc} + h_2^{ij}\,\overline{H}_a\,T_i^{ab}\,\frac{\Sigma_b^c}{\Lambda}\,F_{jc}\right) +$$

$$+\frac{1}{4}\epsilon_{abcde}\left(Y_{10}^{ij}\,T_i^{ab}\,T_j^{cd}\,H^e + f_1^{ij}\,T_i^{ab}\,T_j^{cd}\,\frac{\Sigma_f^e}{\Lambda}\,H^f + f_2^{ij}\,T_i^{ab}\,T_j^{cf}\,H^d\,\frac{\Sigma_f^e}{\Lambda}\right)$$

$$\langle\Sigma\rangle = v_\Sigma\; diag\,(2,2,2,-3,-3)$$

$$Y_u = Y_{10} + 3\frac{v_\Sigma}{\Lambda}f_1^S + \frac{1}{4}\frac{v_\Sigma}{\Lambda}\left(3f_2^S + 5f_2^A\right)$$

$$Y_d = Y_5 - 3\frac{v_\Sigma}{\Lambda}h_1 + 2\frac{v_\Sigma}{\Lambda}h_2$$

$$Y_e = Y_5 - 3\frac{v_\Sigma}{\Lambda}h_1 - 3\frac{v_\Sigma}{\Lambda}h_2$$

Supersymmetric SU(5): Superpotential

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$$+\frac{1}{4}\epsilon_{abcde}\left(Y_{10}^{ij}\,T_i^{ab}\,T_j^{cd}\,H^e + f_1^{ij}\,T_i^{ab}\,T_j^{cd}\,\frac{\Sigma_f^e}{\Lambda}\,H^f + f_2^{ij}\,T_i^{ab}\,T_j^{cf}\,H^d\,\frac{\Sigma_f^e}{\Lambda}\right)$$

$$\langle\Sigma\rangle = v_\Sigma\; diag\,(2,2,2,-3,-3)$$

$$Y_u = Y_{10} + 3\frac{v_\Sigma}{\Lambda}f_1^S + \frac{1}{4}\frac{v_\Sigma}{\Lambda}\left(3f_2^S + 5f_2^A\right)$$

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NR corrections

Supersymmetric SU(5): Superpotential

$$\frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2}$$

As a further natural step , one can also consider additional NR operators

$$\frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2}$$

Supersymmetric SU(5): Superpotential

$$W^{(2)}_{Y_u} = \frac{1}{4}\epsilon_{abcde}\left(k_1\, T^{ab}\, T^{cd}\, H^e\, \frac{Tr\Sigma^2}{\Lambda^2} + k_2\, T^{ap}\, T^{bf}\, H^c\, \frac{\Sigma^d_p\, \Sigma^e_f}{\Lambda^2} + \right.$$

$$\left. + \; k_3\, T^{ab}\, T^{cd}\, H^p\, \frac{\Sigma^d_f\, \Sigma^e_p}{\Lambda^2} + k_4\, T^{ab}\, T^{nm}\, H^c\, \frac{\Sigma^d_n\, \Sigma^e_m}{\Lambda^2} + k_5\, T^{ab}\, T^{cg}\, H^d\, \frac{\Sigma^f_g\, \Sigma^e_f}{\Lambda^2}\right)$$

$$W^{(2)}_{Y_{de}} = \sqrt{2}\left(g_1 \overline{H}_a\, T^{ab}\, F_b\, \frac{Tr\Sigma^2}{\Lambda^2} + g_2\, \overline{H}_a\, \frac{\Sigma^a_b\, \Sigma^b_c}{\Lambda^2} T^{cd}\, F_d + \right.$$

$$\left. + \; g_3\, \overline{H}_a\, T^{ba}\, \frac{\Sigma^c_b\, \Sigma^d_c}{\Lambda^2} F_d + g_4\, \overline{H}_a \frac{\Sigma^a_b\, \Sigma^d_c}{\Lambda^2}\, T^{bc}\, F_d\right)$$

LiE code

Supersymmetric SU(5): Neutrinos

C. Biggio and L. Calibbi

[1007.3750]-JHEP

$$W_{Y_\nu} \propto F_a \Sigma^a_{F\,b} H^b \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_1 \end{pmatrix}$$

Supersymmetric SU(5): Neutrinos

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[1007.3750]-JHEP

$$W_{Y_\nu} \propto F_a \Sigma^a_{Fb} H^b \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_1 \end{pmatrix}$$

Second viable mass

$$W^{NR}_{Y_\nu} = n_1 F_a \, \Sigma^a_{Fb} \, \frac{\Sigma^b_c}{\Lambda} \, H^c + n_2 F_a \, \frac{\Sigma^a_b}{\Lambda} \, \Sigma^b_{Fc} \, H^c + n_3 F_a \, H^a \, \Sigma^c_{Fd} \, \frac{\Sigma^d_c}{\Lambda} + $$
$$+ n_4 F_a \, \Sigma^a_{Fb} \, H^b \, \frac{Tr \Sigma^2}{\Lambda^2} + n_5 F_a \, \Sigma^a_{Fb} \, \frac{\Sigma^b_c \, \Sigma^c_d}{\Lambda^2} \, H^d + n_6 F_a \, \frac{\Sigma^a_b \, \Sigma^c_d}{\Lambda^2} \, \Sigma^b_{Fc} \, H^d + $$
$$+ n_7 F_a \, \frac{\Sigma^a_b \, \Sigma^b_c}{\Lambda^2} \, \Sigma^c_{Fd} \, H^d + n_8 F_a \, \frac{\Sigma^a_b \, \Sigma^d_c}{\Lambda^2} \, H^b \, \Sigma^c_{Fd} + n_9 F_a \, H^a \, \frac{\Sigma^b_c \, \Sigma^c_d}{\Lambda^2} \, \Sigma^d_{Fb}$$

Supersymmetric SU(5): Neutrinos

C. Biggio and L. Calibbi

[1007.3750]-JHEP

$$W_{Y_\nu} \propto F_a \Sigma^a_{Fb} H^b \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_1 \end{pmatrix}$$

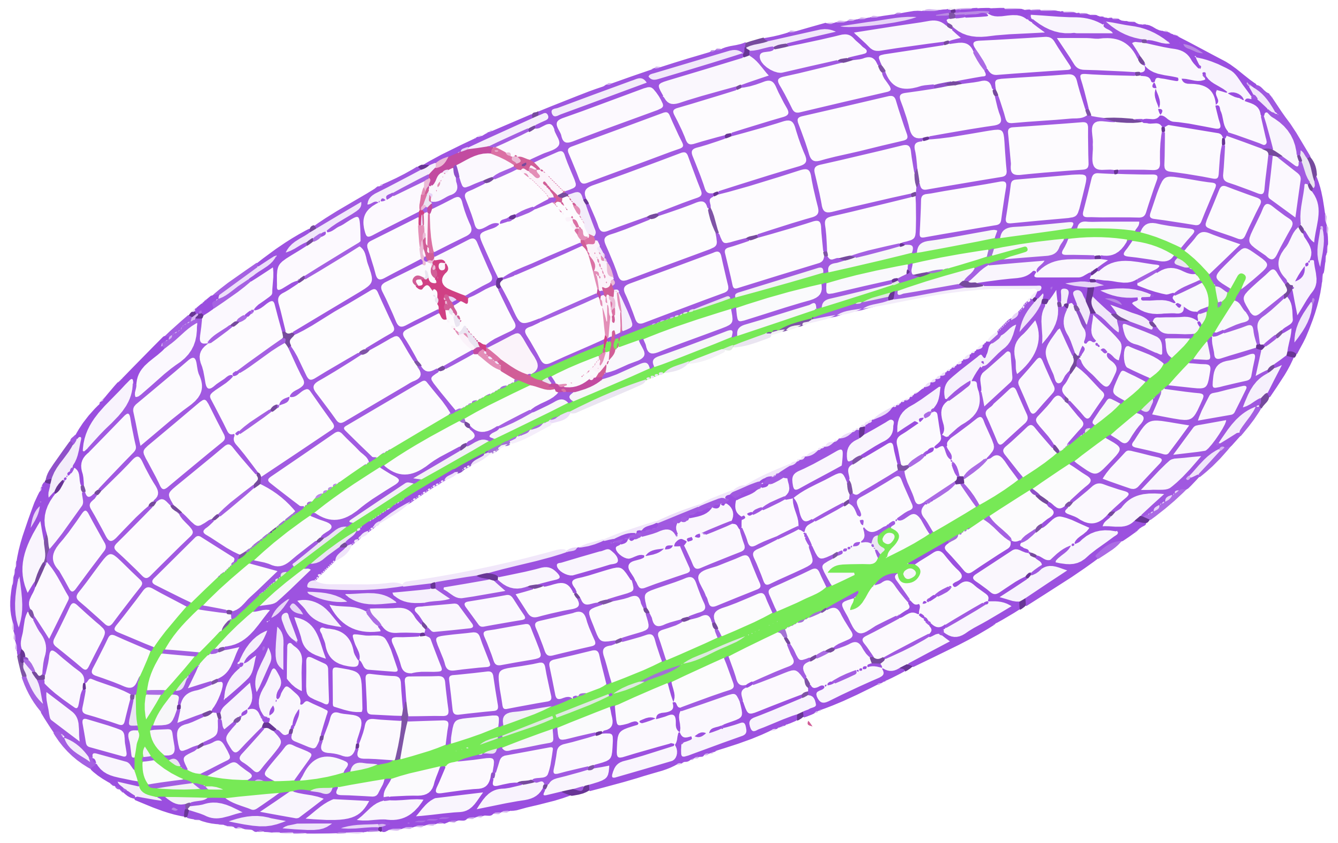
Second viable mass

$$W^{NR}_{Y_\nu} = n_1 F_a \, \Sigma^a_{Fb} \, \frac{\Sigma^b_c}{\Lambda} \, H^c + n_2 F_a \, \frac{\Sigma^a_b}{\Lambda} \, \Sigma^b_{Fc} \, H^c + n_3 F_a \, H^a \, \Sigma^c_{Fd} \, \frac{\Sigma^d_c}{\Lambda} +$$

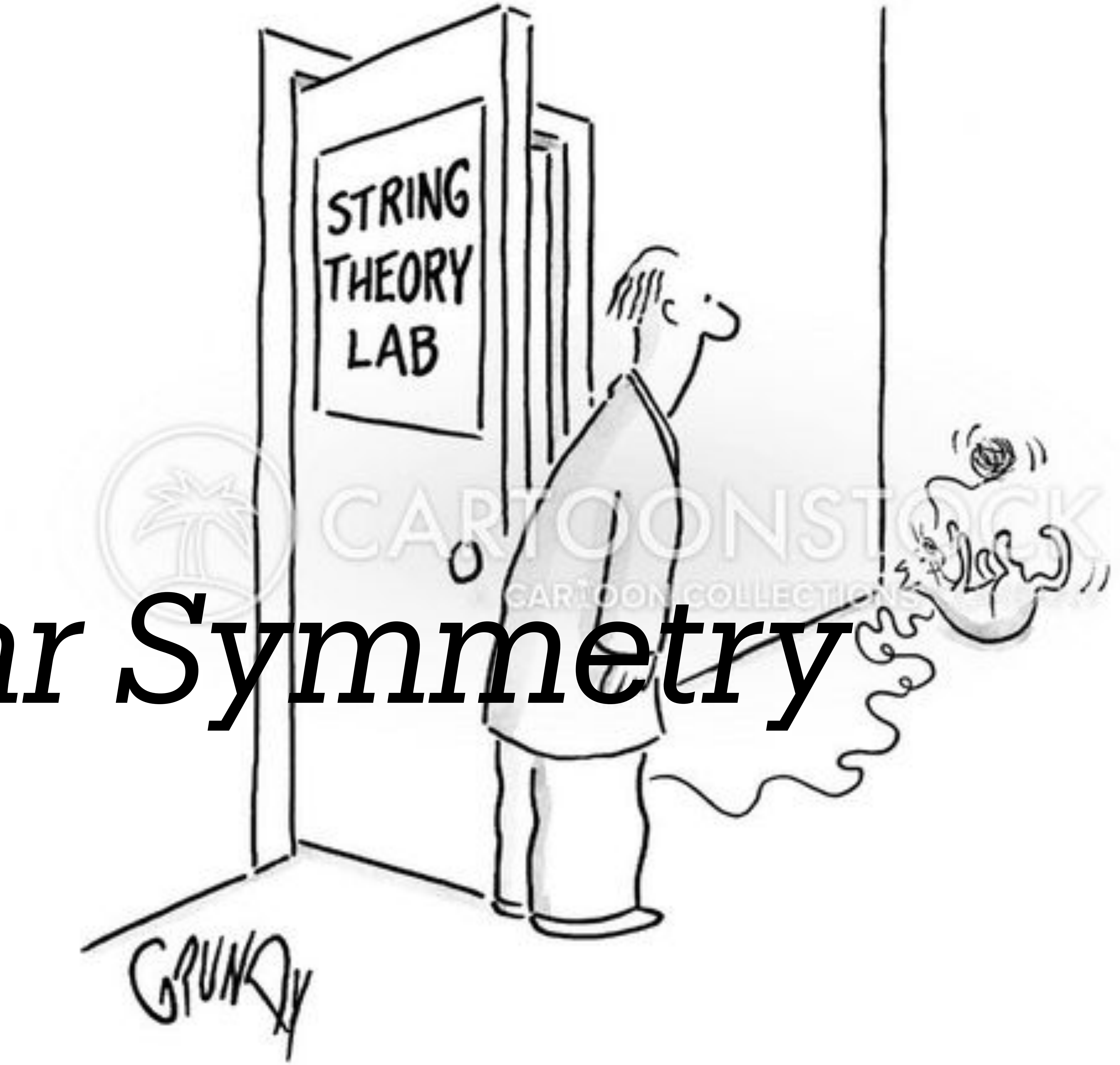
NR

$$+ n_4 F_a \, \Sigma^a_{Fb} \, H^b \, \frac{Tr \Sigma^2}{\Lambda^2} + n_5 F_a \, \Sigma^a_{Fb} \, \frac{\Sigma^b_c \, \Sigma^c_d}{\Lambda^2} \, H^d + n_6 F_a \, \frac{\Sigma^a_b \, \Sigma^c_d}{\Lambda^2} \, \Sigma^b_{Fc} \, H^d +$$

$$+ n_7 F_a \, \frac{\Sigma^a_b \, \Sigma^b_c}{\Lambda^2} \, \Sigma^c_{Fd} \, H^d + n_8 F_a \, \frac{\Sigma^a_b \, \Sigma^d_c}{\Lambda^2} \, H^b \, \Sigma^c_{Fd} + n_9 F_a \, H^a \, \frac{\Sigma^b_c \, \Sigma^c_d}{\Lambda^2} \, \Sigma^d_{Fb}$$



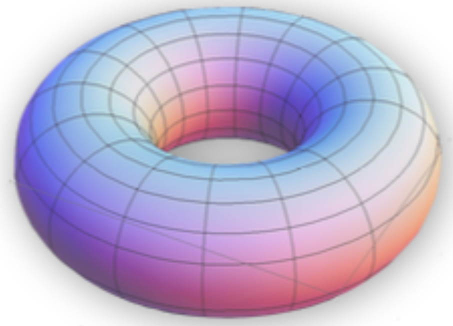
Modular Symmetry



Modular symmetry: Lattice

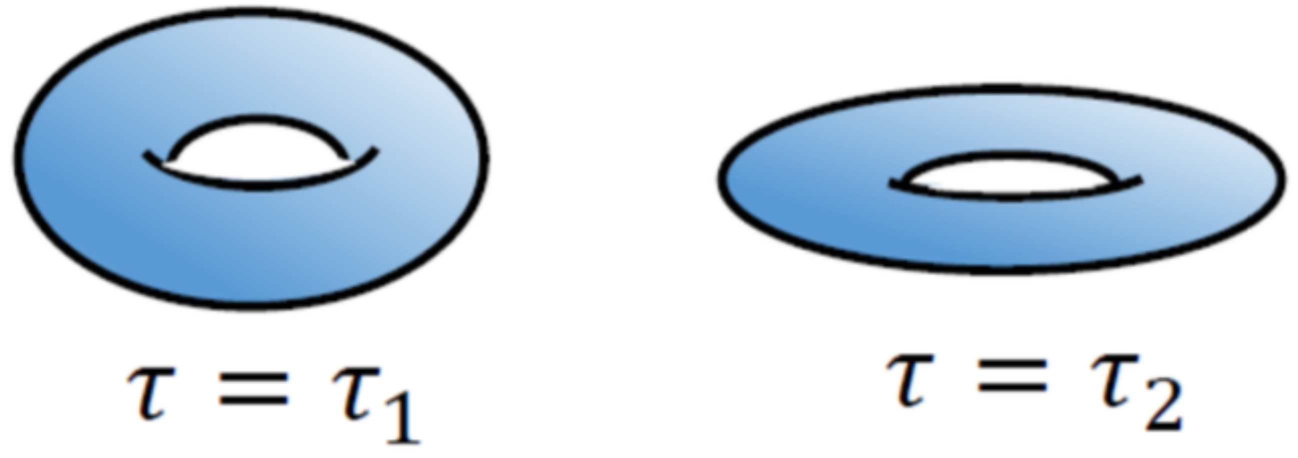
F. Feruglio
[1706.08749]

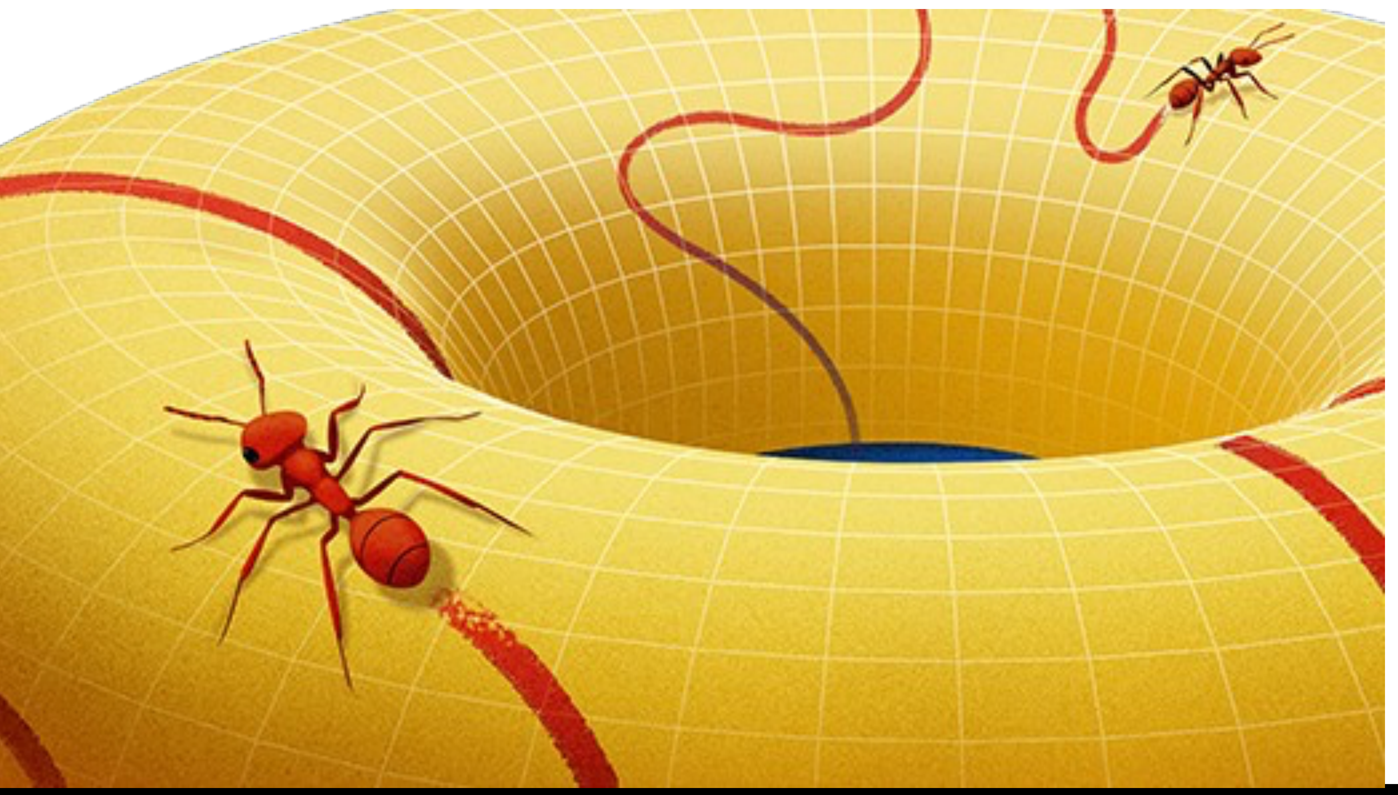
Discrete lattice Λ
$\omega_1, \omega_2 \in \mathbb{C}$

$\mathbb{C}/\Lambda \equiv$ 

$$\tau \equiv \frac{\omega_2}{\omega_1}$$

Modulus





(Adapted from slides of M. Parriciatu)

Modular symmetry: Modular Transf.

F. Feruglio
[1706.08749]

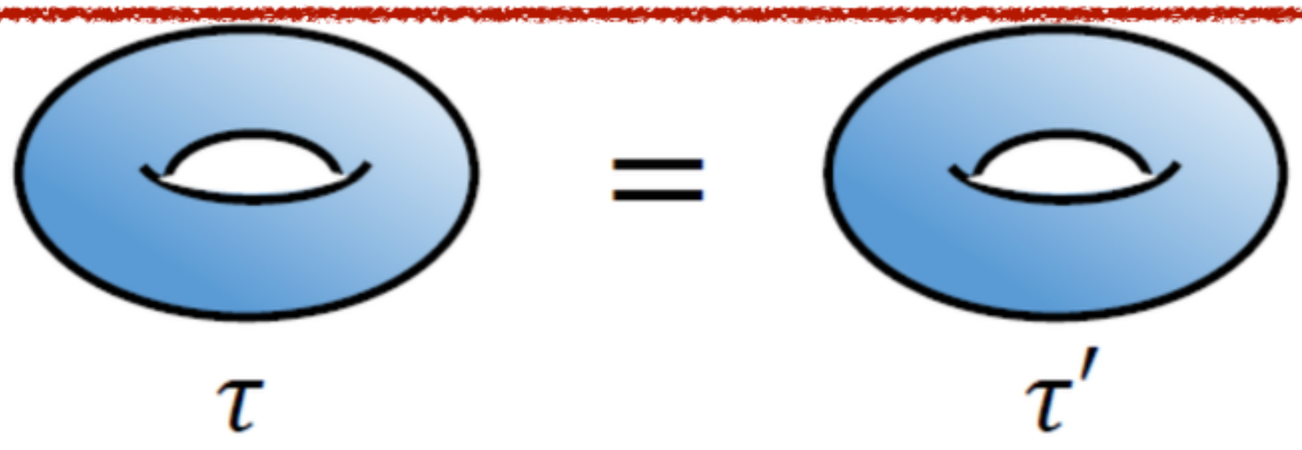
Change basis?

$$\begin{pmatrix}\omega_2' \\ \omega_1'\end{pmatrix} = \gamma \begin{pmatrix}\omega_2 \\ \omega_1\end{pmatrix} \equiv \begin{pmatrix}a & b \\ c & d\end{pmatrix}\begin{pmatrix}\omega_2 \\ \omega_1\end{pmatrix} \qquad a, b, c, d \in \mathbb{Z}$$

$\gamma \in \mathrm{SL}(2,\mathbb{Z}) \equiv$ Modular group $\equiv \Gamma$

$$\tau \equiv \frac{\omega_2}{\omega_1} \xrightarrow{\mathrm{SL}(2,\mathbb{Z})} \tau' = \frac{a\,\omega_2 + b\,\omega_1}{c\,\omega_2 + d\,\omega_1} = \frac{a\tau + b}{c\tau + d}$$

$\tau \to \tau'$

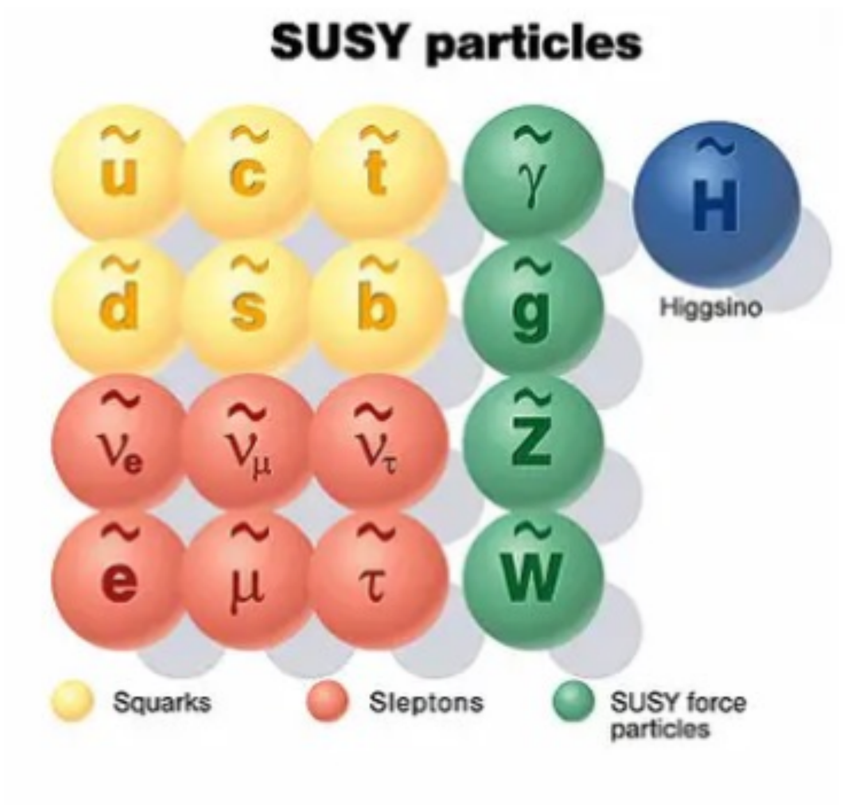


Modular symmetry: Superfields

F. Feruglio

[1706.08749]

Superfields transformations



$$\begin{cases} \tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} & \gamma \in \Gamma \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases}$$

Usual matter fields ($\varphi^{(I)}$)

Unitary irrep. of $\Gamma_N \subset \Gamma$ ($\rho^{(I)}(\gamma)$)

$N = 1, 2, 3, \dots$ "Level"

Modular symmetry: Discrete Groups

F. Feruglio
[1706.08749]

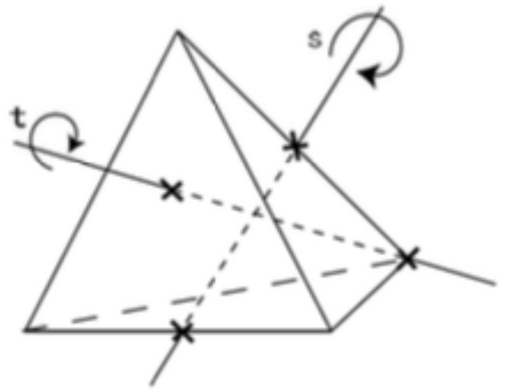
Finite modular group

$$\Gamma_N$$

for $N \leq 5$ isomorphic to

$N = 2$	$N = 3$	$N = 4$	$N = 5$
S_3	A_4	S_4	A_5

non-abelian discrete groups



Modular symmetry: Modular Forms

F. Feruglio
[1706.08749]

$$Y(\tau) \to \left(c\tau + d\right)^{k_Y} \rho_Y(\gamma)\, Y(\tau)$$

Yukawa couplings are now promoted to modular forms

Modular symmetry: gCP

Impose CP symmetry on the model

- gCP $\Rightarrow$ $\alpha_i \in \mathbb{R}$

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

Only source of CPV in the model is the VEV of τ

$$<\tau> = \mathrm{Re}\,\tau + i\,\mathrm{Im}\,\tau$$

(Adapted from slides of M. Parriciatu)

The Model

	T	T_3	F_1	F_2	F_3	H_5	$\overline{H}_5$	Σ
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{24}$
$\Gamma_2 \simeq S_3$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$
Weight	-1	6	-1	-3	0	0	0	3

The model: Superpotential

$$W_{TT} = a_1 (TT)_2 \frac{\Sigma^2}{\Lambda^2} H_5\, Y_2^{(4)} + a_2 (TT) \frac{\Sigma^2}{\Lambda^2} H_5\, Y_1^{(4)} + a_3 (TT_3) \frac{\Sigma}{\Lambda} H_5\, Y_{2,1}^{(8)} + a_4 (TT_3) \frac{\Sigma}{\Lambda} H_5\, Y_{2,2}^{(8)}$$
$$+ a_5 (T_3 T) \frac{\Sigma}{\Lambda} H_5\, Y_{2,1}^{(8)} + a_6 (T_3 T) \frac{\Sigma}{\Lambda} H_5\, Y_{2,2}^{(8)} + a_7 (T_3 T_3) H_5\, Y_1^{(12)}$$

The model: Superpotential

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$$+ a_5(T_3T) \frac{\Sigma}{\Lambda} H_5\, Y_{2,1}^{(8)} + a_6(T_3T) \frac{\Sigma}{\Lambda} H_5\, Y_{2,2}^{(8)} + a_7(T_3T_3) H_5\, Y_1^{(12)}$$

$$\downarrow \quad \begin{pmatrix} \epsilon^2_{2\times 2} & \epsilon \\ \epsilon & 1 \end{pmatrix}, \quad \epsilon \equiv \frac{v_\Sigma}{\Lambda}$$

$$Y_u \propto \begin{pmatrix} -\frac{v_\Sigma^2}{\Lambda^2} a_1 (Y_{\mathbf{2}}^{(4)})_1 + \frac{v_\Sigma^2}{\Lambda^2} a_2 Y_{\mathbf{1}}^{(4)} & \frac{v_\Sigma^2}{\Lambda^2} a_1 (Y_{\mathbf{2}}^{(4)})_2 & \frac{v_\Sigma}{\Lambda} a_3 (Y_{\mathbf{2},1}^{(8)})_1 + \frac{v_\Sigma}{\Lambda} a_4 (Y_{\mathbf{2},2}^{(8)})_1 \\ \frac{v_\Sigma^2}{\Lambda^2} a_1 (Y_{\mathbf{2}}^{(4)})_2 & \frac{v_\Sigma^2}{\Lambda^2} a_1 (Y_{\mathbf{2}}^{(4)})_1 + \frac{v_\Sigma^2}{\Lambda^2} a_2 Y_{\mathbf{1}}^{(4)} & \frac{v_\Sigma}{\Lambda} a_3 (Y_{\mathbf{2},1}^{(8)})_2 + \frac{v_\Sigma}{\Lambda} a_4 (Y_{\mathbf{2},2}^{(8)})_2 \\ \frac{v_\Sigma}{\Lambda} a_5 (Y_{\mathbf{2},1}^{(8)})_1 + \frac{v_\Sigma}{\Lambda} a_6 (Y_{\mathbf{2},2}^{(8)})_1 & \frac{v_\Sigma}{\Lambda} a_5 (Y_{\mathbf{2},1}^{(8)})_2 + \frac{v_\Sigma}{\Lambda} a_6 (Y_{\mathbf{2},2}^{(8)})_2 & a_7 Y_{\mathbf{1}}^{(12)} \end{pmatrix}^T$$

The model: Superpotential

$$W_{TT} = a_1 (TT)_2 \frac{\Sigma^2}{\Lambda^2} H_5\, Y_2^{(4)} + a_2 (TT) \frac{\Sigma^2}{\Lambda^2} H_5\, Y_1^{(4)} + a_3 (TT_3) \frac{\Sigma}{\Lambda} H_5\, Y_{2,1}^{(8)} + a_4 (TT_3) \frac{\Sigma}{\Lambda} H_5\, Y_{2,2}^{(8)}$$
$$+ a_5 (T_3 T) \frac{\Sigma}{\Lambda} H_5\, Y_{2,1}^{(8)} + a_6 (T_3 T) \frac{\Sigma}{\Lambda} H_5\, Y_{2,2}^{(8)} + a_7 (T_3 T_3) H_5\, Y_1^{(12)}$$

$$\begin{pmatrix} \epsilon^2_{2\times 2} & \epsilon \\ \epsilon & 1 \end{pmatrix}, \quad \epsilon \equiv \frac{v_\Sigma}{\Lambda}$$

$$Y_u \propto \begin{pmatrix} -\frac{v_\Sigma^2}{\Lambda^2} a_1 (Y_{\mathbf{2}}^{(4)})_1 + \frac{v_\Sigma^2}{\Lambda^2} a_2 Y_{\mathbf{1}}^{(4)} & \frac{v_\Sigma^2}{\Lambda^2} a_1 (Y_{\mathbf{2}}^{(4)})_2 & \frac{v_\Sigma}{\Lambda} a_3 (Y_{\mathbf{2},1}^{(8)})_1 + \frac{v_\Sigma}{\Lambda} a_4 (Y_{\mathbf{2},2}^{(8)})_1 \\ \frac{v_\Sigma^2}{\Lambda^2} a_1 (Y_{\mathbf{2}}^{(4)})_2 & \frac{v_\Sigma^2}{\Lambda^2} a_1 (Y_{\mathbf{2}}^{(4)})_1 + \frac{v_\Sigma^2}{\Lambda^2} a_2 Y_{\mathbf{1}}^{(4)} & \frac{v_\Sigma}{\Lambda} a_3 (Y_{\mathbf{2},1}^{(8)})_2 + \frac{v_\Sigma}{\Lambda} a_4 (Y_{\mathbf{2},2}^{(8)})_2 \\ \frac{v_\Sigma}{\Lambda} a_5 (Y_{\mathbf{2},1}^{(8)})_1 + \frac{v_\Sigma}{\Lambda} a_6 (Y_{\mathbf{2},2}^{(8)})_1 & \frac{v_\Sigma}{\Lambda} a_5 (Y_{\mathbf{2},1}^{(8)})_2 + \frac{v_\Sigma}{\Lambda} a_6 (Y_{\mathbf{2},2}^{(8)})_2 & a_7 Y_{\mathbf{1}}^{(12)} \end{pmatrix}^T$$

O(1)

The model: Superpotential

$$W_{TT} = a_1(TT)_2\frac{\Sigma^2}{\Lambda^2}H_5\,Y_2^{(4)} + a_2(TT)\frac{\Sigma^2}{\Lambda^2}H_5\,Y_1^{(4)} + a_3(TT_3)\frac{\Sigma}{\Lambda}H_5\,Y_{2,1}^{(8)} + a_4(TT_3)\frac{\Sigma}{\Lambda}H_5\,Y_{2,2}^{(8)}$$
$$+ a_5(T_3T)\frac{\Sigma}{\Lambda}H_5\,Y_{2,1}^{(8)} + a_6(T_3T)\frac{\Sigma}{\Lambda}H_5\,Y_{2,2}^{(8)} + a_7(T_3T_3)H_5\,Y_1^{(12)}$$

Rank one

$$\begin{pmatrix} \epsilon^2_{2\times 2} & \epsilon \\ \epsilon & 1 \end{pmatrix}, \quad \epsilon \equiv \frac{v_\Sigma}{\Lambda}$$

$$Y_u \propto \begin{pmatrix} -\frac{v_\Sigma^2}{\Lambda^2}a_1(Y_{\mathbf{2}}^{(4)})_1 + \frac{v_\Sigma^2}{\Lambda^2}a_2Y_{\mathbf{1}}^{(4)} & \frac{v_\Sigma^2}{\Lambda^2}a_1(Y_{\mathbf{2}}^{(4)})_2 & \frac{v_\Sigma}{\Lambda}a_3(Y_{\mathbf{2},1}^{(8)})_1 + \frac{v_\Sigma}{\Lambda}a_4(Y_{\mathbf{2},2}^{(8)})_1 \\ \frac{v_\Sigma^2}{\Lambda^2}a_1(Y_{\mathbf{2}}^{(4)})_2 & \frac{v_\Sigma^2}{\Lambda^2}a_1(Y_{\mathbf{2}}^{(4)})_1 + \frac{v_\Sigma^2}{\Lambda^2}a_2Y_{\mathbf{1}}^{(4)} & \frac{v_\Sigma}{\Lambda}a_3(Y_{\mathbf{2},1}^{(8)})_2 + \frac{v_\Sigma}{\Lambda}a_4(Y_{\mathbf{2},2}^{(8)})_2 \\ \frac{v_\Sigma}{\Lambda}a_5(Y_{\mathbf{2},1}^{(8)})_1 + \frac{v_\Sigma}{\Lambda}a_6(Y_{\mathbf{2},2}^{(8)})_1 & \frac{v_\Sigma}{\Lambda}a_5(Y_{\mathbf{2},1}^{(8)})_2 + \frac{v_\Sigma}{\Lambda}a_6(Y_{\mathbf{2},2}^{(8)})_2 & a_7Y_{\mathbf{1}}^{(12)} \end{pmatrix}^T$$

The model: Superpotential

$$W_{TF} = B_1(TF_1)\frac{\Sigma^2}{\Lambda^2}\overline{H}_5\, Y_2^{(4)} + B_2(TF_2)\frac{\Sigma^2}{\Lambda^2}\overline{H}_5\, Y_2^{(2)} + B_3(TF_3)\frac{\Sigma}{\Lambda}\overline{H}_5\, Y_2^{(2)} + \\ + B_4(T_3F_2)\frac{\Sigma}{\Lambda}\overline{H}_5\, Y_1^{(6)} + B_5(T_3F_3)\overline{H}_5\, Y_{1'}^{(6)}$$

$$W_\nu = D_1(F_1\Sigma)\frac{\Sigma}{\Lambda}H_5\, Y_1^{(8)} + D_2(F_2\Sigma)\frac{\Sigma^2}{\Lambda^2}H_5\, Y_{1'}^{(6)} + D_3(F_3\Sigma)\frac{\Sigma}{\Lambda}H_5\, Y_1^{(6)}$$

The model: Superpotential

$$W_{TF} = B_1(TF_1)\frac{\Sigma^2}{\Lambda^2}\overline{H}_5\, Y_2^{(4)} + B_2(TF_2)\frac{\Sigma^2}{\Lambda^2}\overline{H}_5\, Y_2^{(2)} + B_3(TF_3)\frac{\Sigma}{\Lambda}\overline{H}_5\, Y_2^{(2)} + B_4(T_3F_2)\frac{\Sigma}{\Lambda}\overline{H}_5\, Y_1^{(6)} + B_5(T_3F_3)\overline{H}_5\, Y_{1'}^{(6)}$$

$$\longrightarrow \quad Y_d \propto \begin{pmatrix} \frac{v_\Sigma^2}{\Lambda^2} b_1 (Y_{\mathbf{2}}^{(4)})_2 & \frac{v_\Sigma^2}{\Lambda^2} b_2 (Y_{\mathbf{2}}^{(2)})_1 & \frac{v_\Sigma}{\Lambda} b_3 (Y_{\mathbf{2}}^{(2)})_2 \\ -\frac{v_\Sigma^2}{\Lambda^2} b_1 (Y_{\mathbf{2}}^{(4)})_1 & \frac{v_\Sigma^2}{\Lambda^2} b_2 (Y_{\mathbf{2}}^{(2)})_2 & -\frac{v_\Sigma}{\Lambda} b_3 (Y_{\mathbf{2}}^{(2)})_1 \\ 0 & \frac{v_\Sigma}{\Lambda} b_4 Y_{\mathbf{1}}^{(6)} & b_5 Y_{\mathbf{1'}}^{(6)} \end{pmatrix}^T$$

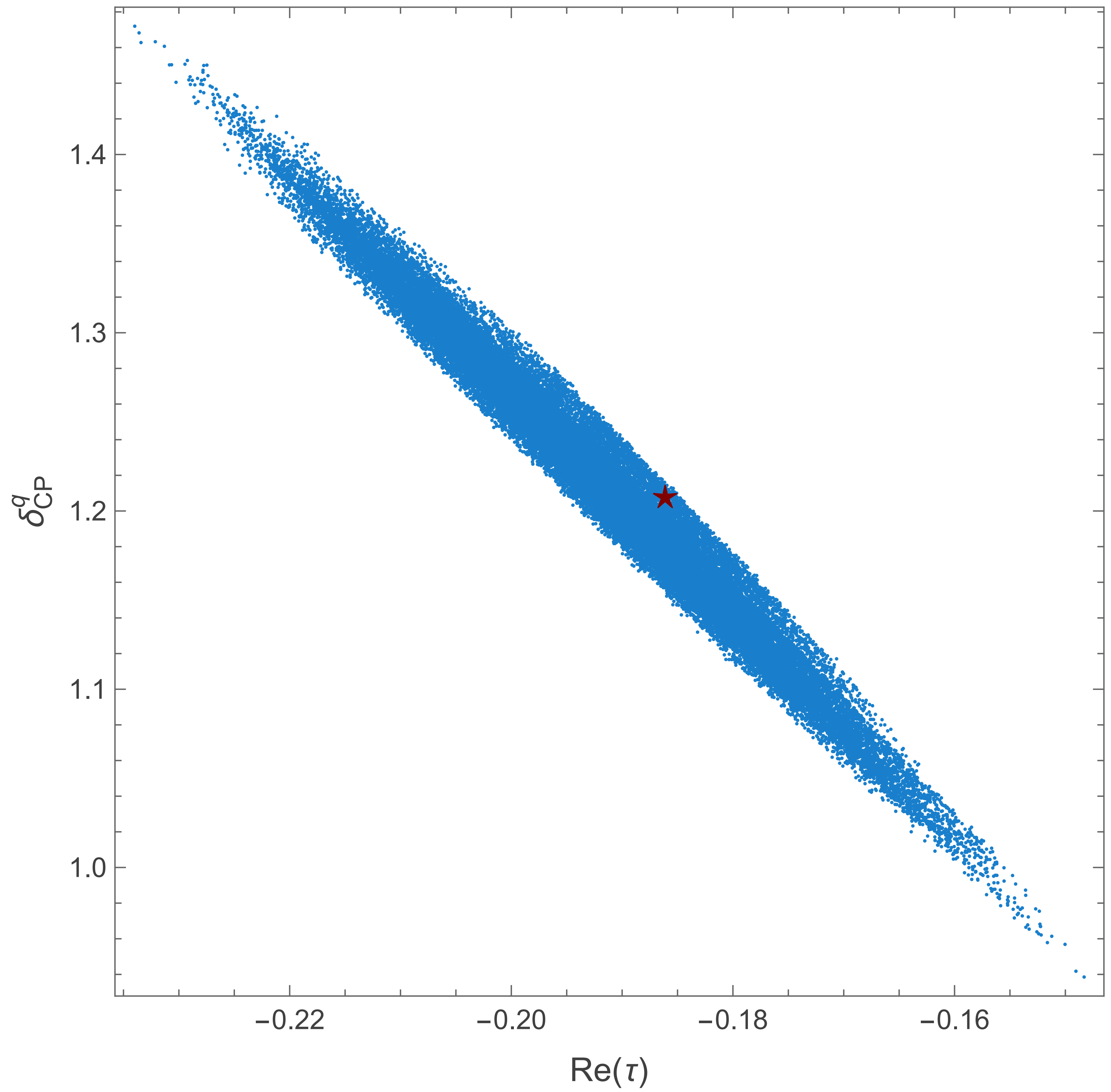
Modular Zero

$$W_\nu = D_1(F_1\Sigma)\frac{\Sigma}{\Lambda}H_5\, Y_1^{(8)} + D_2(F_2\Sigma)\frac{\Sigma^2}{\Lambda^2}H_5\, Y_{1'}^{(6)} + D_3(F_3\Sigma)\frac{\Sigma}{\Lambda}H_5\, Y_1^{(6)} \quad \longrightarrow \quad m_\nu \propto Y_\nu^* M^{-1} Y_\nu^\dagger$$

$$M = diag(G_{N^c}, G_{\Sigma_F^c})$$

Type I+III seesaw

The model: Numerical Procedure



$$\chi^2 = 0.363$$

Deformed Bottom-Tau unification

$$y_b/y_\tau \sim 0.789$$

Correlation region in the quark sector between the CP-violating phase δ^q_{CP} and the real part of τ, which constitutes the sole source of CP violation

The model: Outlook

First successful gCP implementation in a Modular-GUT scenario

The model: Outlook

First successful gCP implementation in a Modular-GUT scenario

MORE TO BE DONE...

- Top-Down $\leftrightarrow$ Bottom-Up Unification
- Moduli Stabilization
- Choice of Modular Weights

Thank You

Roma tre Neutrino Theory Group