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(2) ROMA TRE UNIVERSITÀ DEGLI STUDI

Quark hierarchies and spontaneous CP violation from Siegel modular forms

Matteo Parriciatu(1)(2)

from a work in collaboration with M. Carducci, D. Meloni, J.T. Penedo

[arXiv:2604.21979]

June 23 2026 Sheffield

Particle Physics is "finished", but not understood

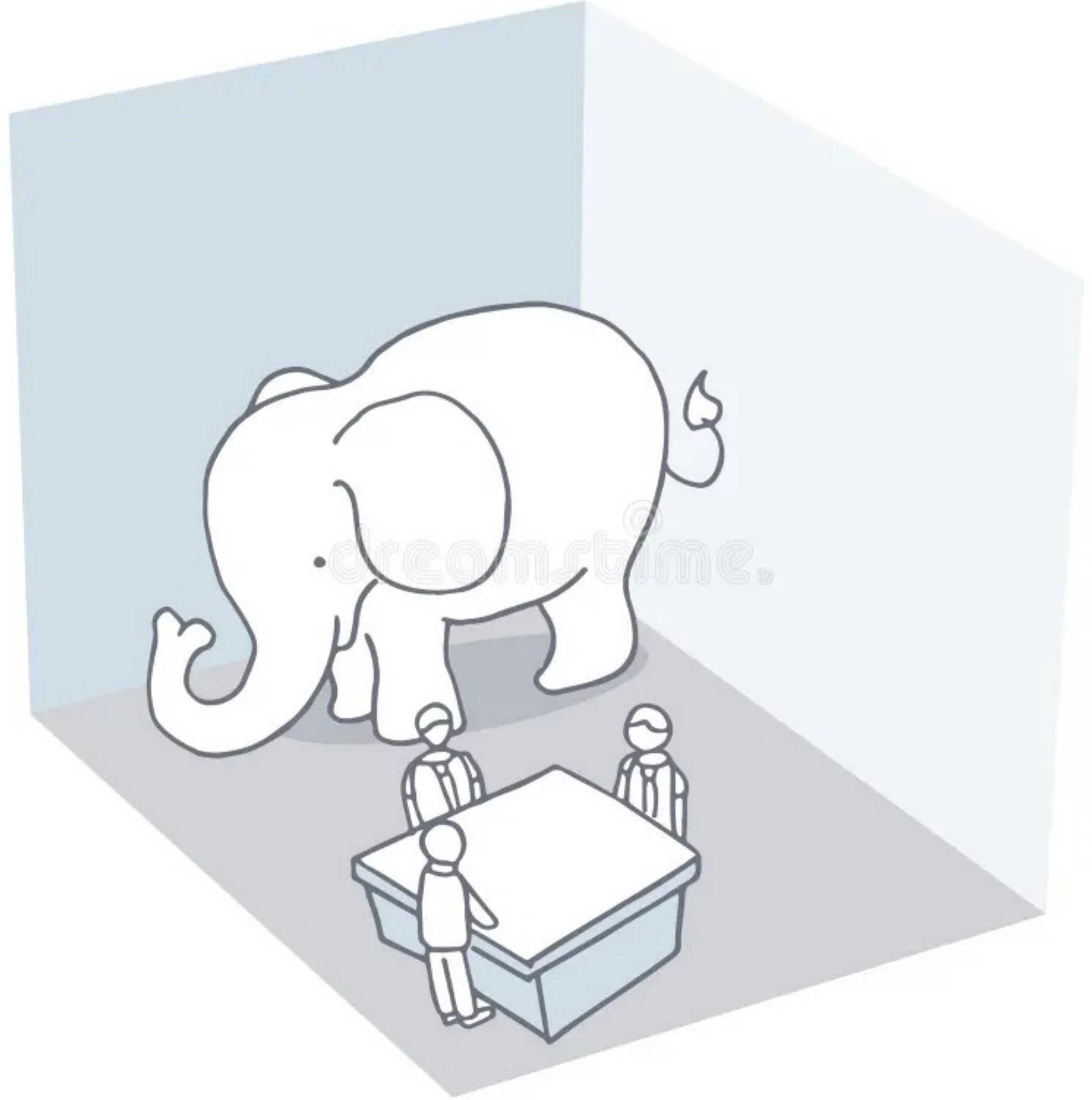
$$\mathscr{L}^{SM} = \mathscr{L}_{gauge} + \mathscr{L}_{higgs} + \sum_{d,i} c_i^{[d]} \frac{\mathcal{O}_i^{d\geq 5}}{\Lambda^{d-4}} \; ?$$

Why the fermion replicas?

$U(3)^5$ symmetry

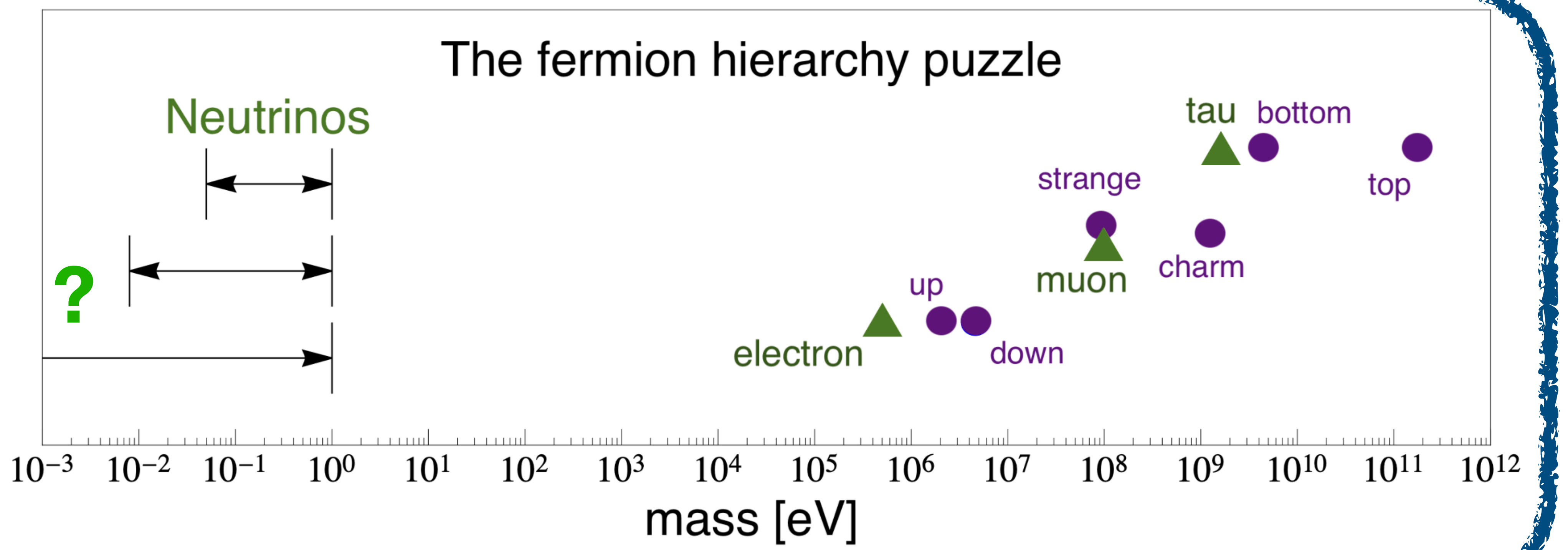
precision $\gtrsim 10\,\%$

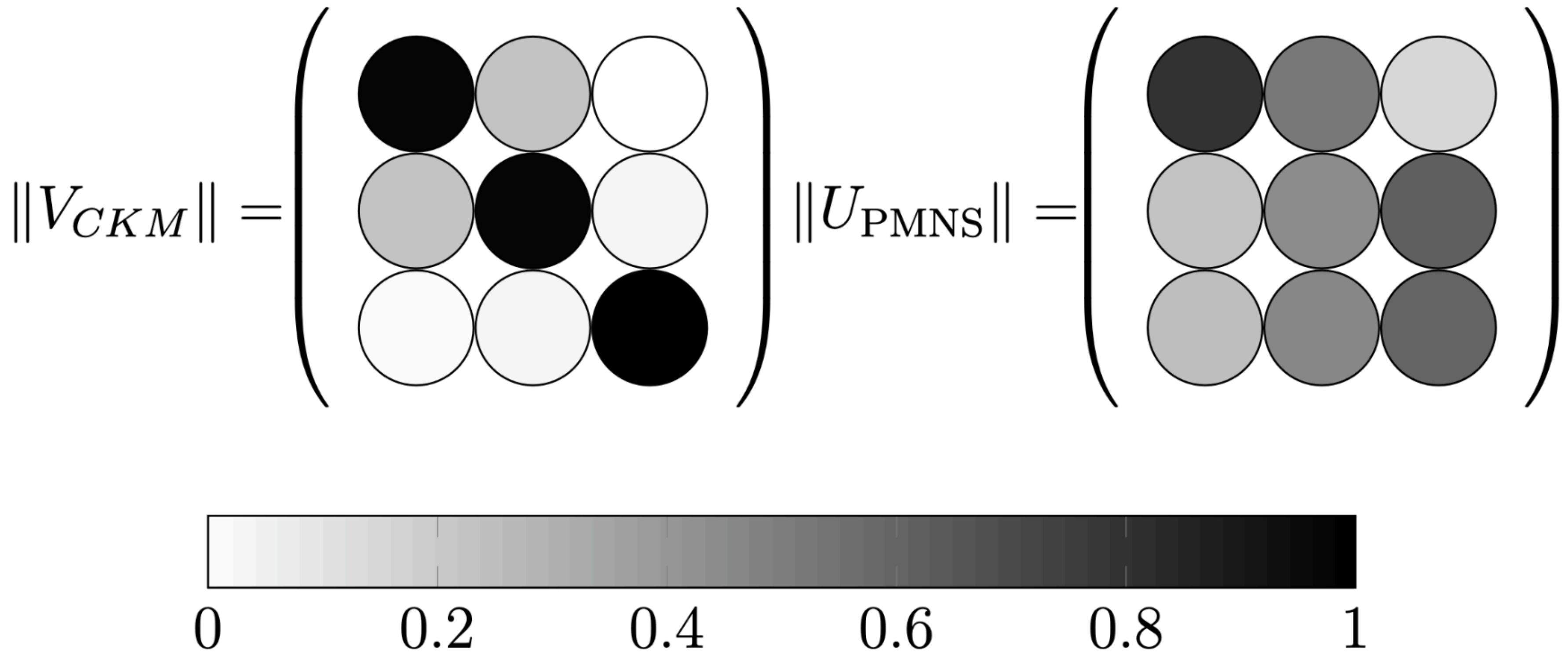
Many free parameters;
lack of calculability;
hierarchy problem of the Higgs;
and so on...



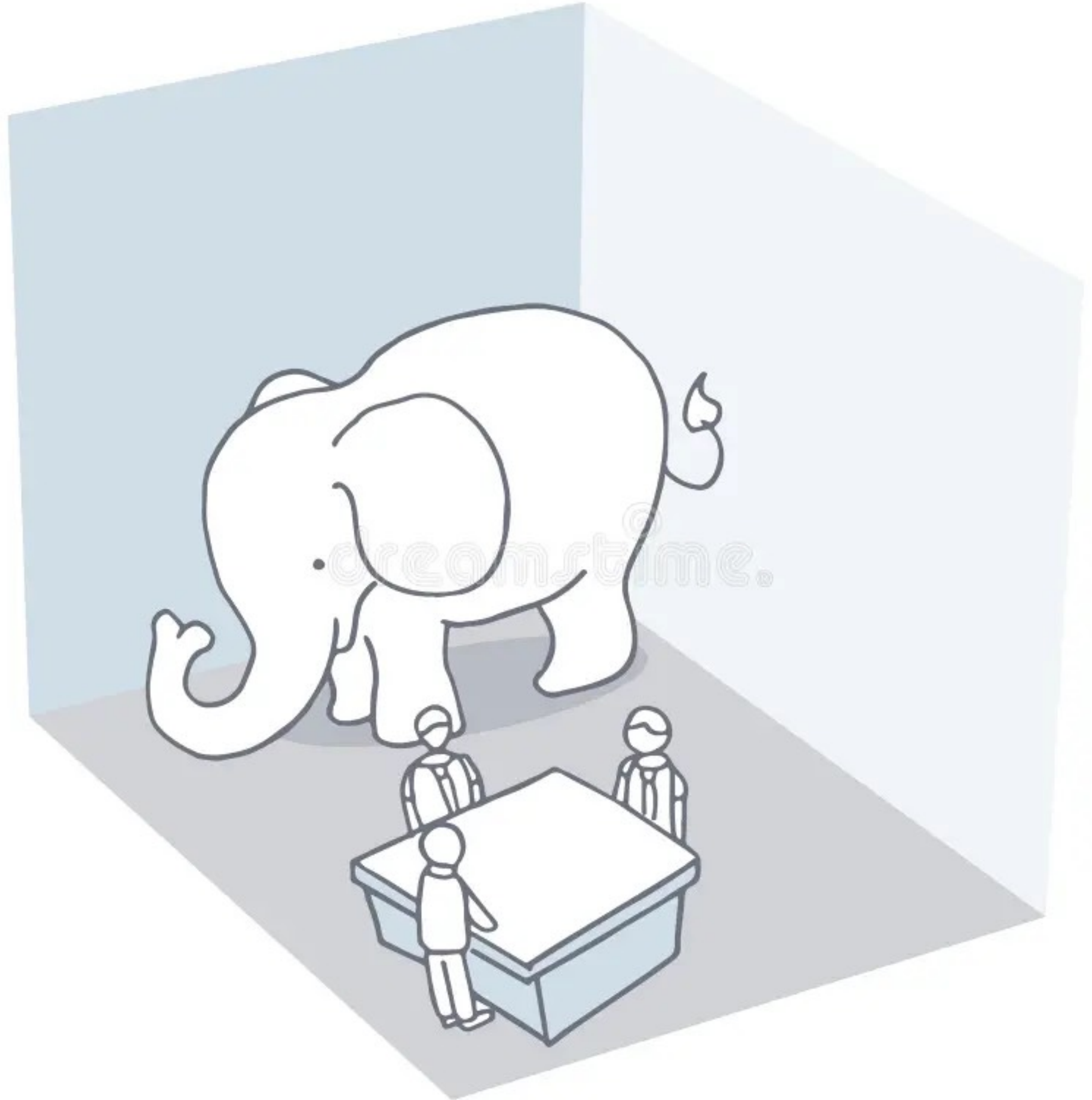
▲ leptons

● quarkss

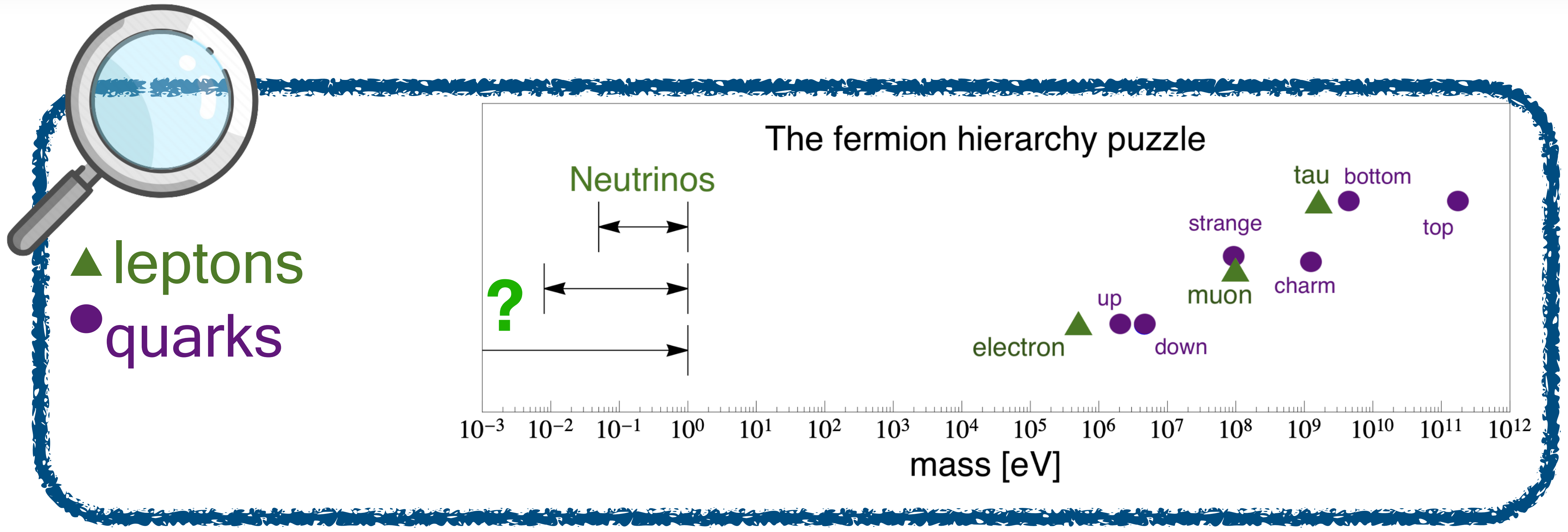


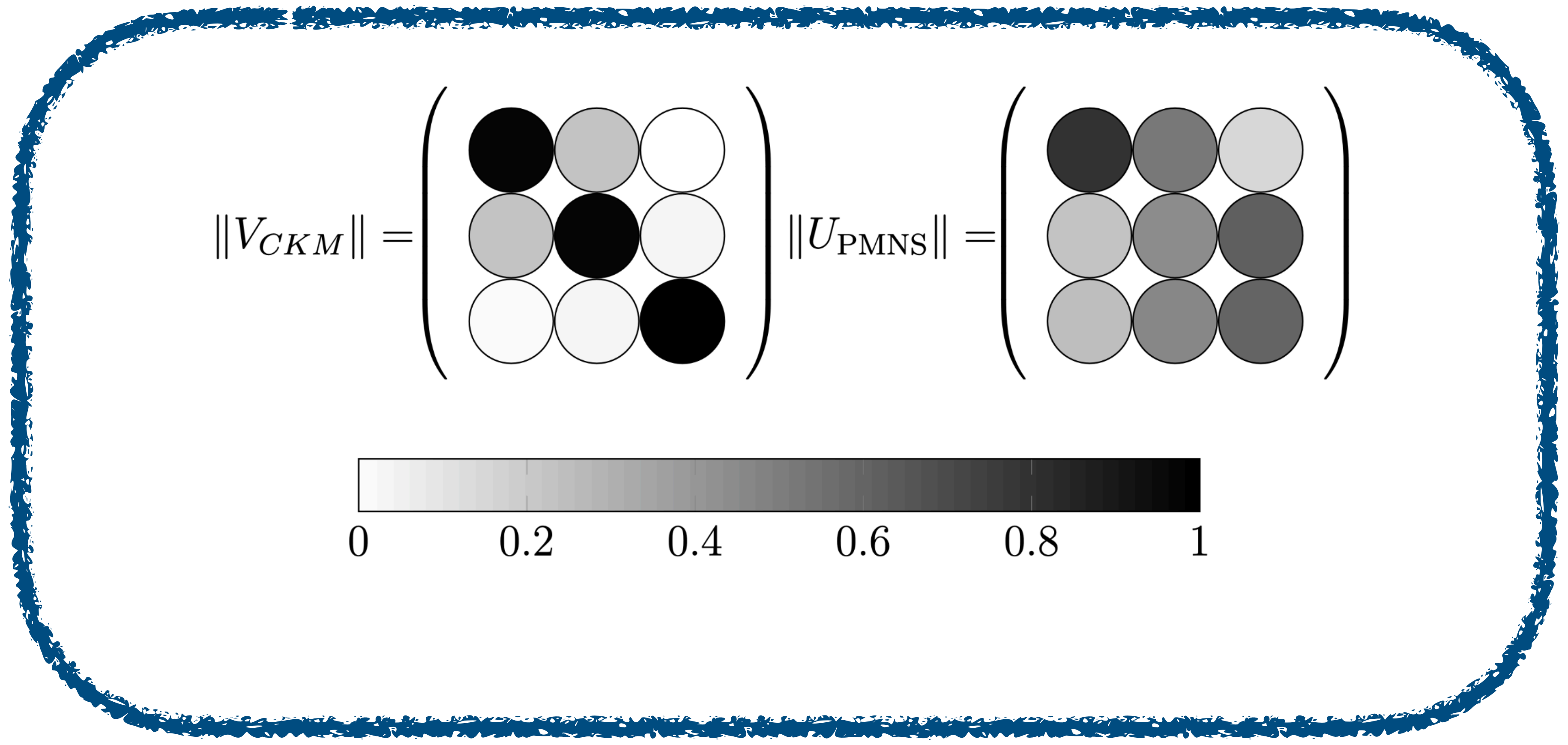


The peculiar breaking of $U(3)^5$ tells us Nature did not “just throw” dice



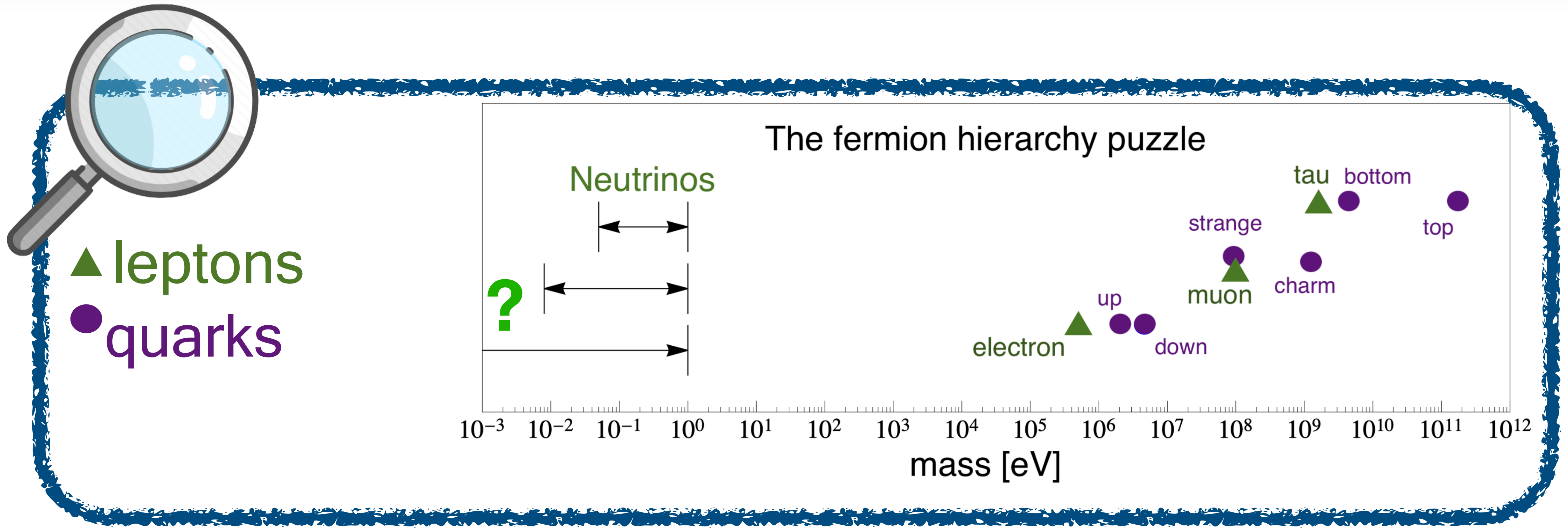
Solution to Flavour is a matter of taste

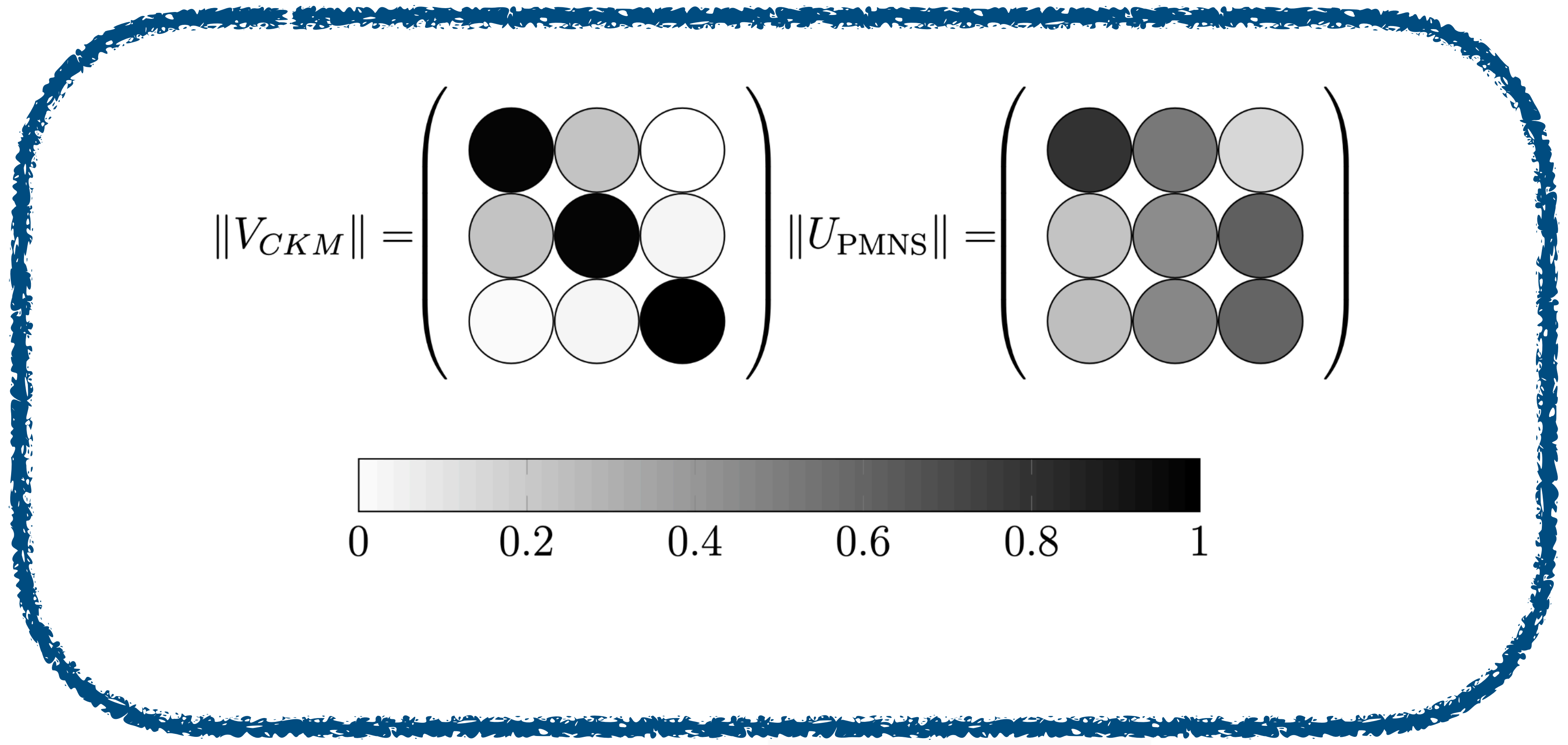


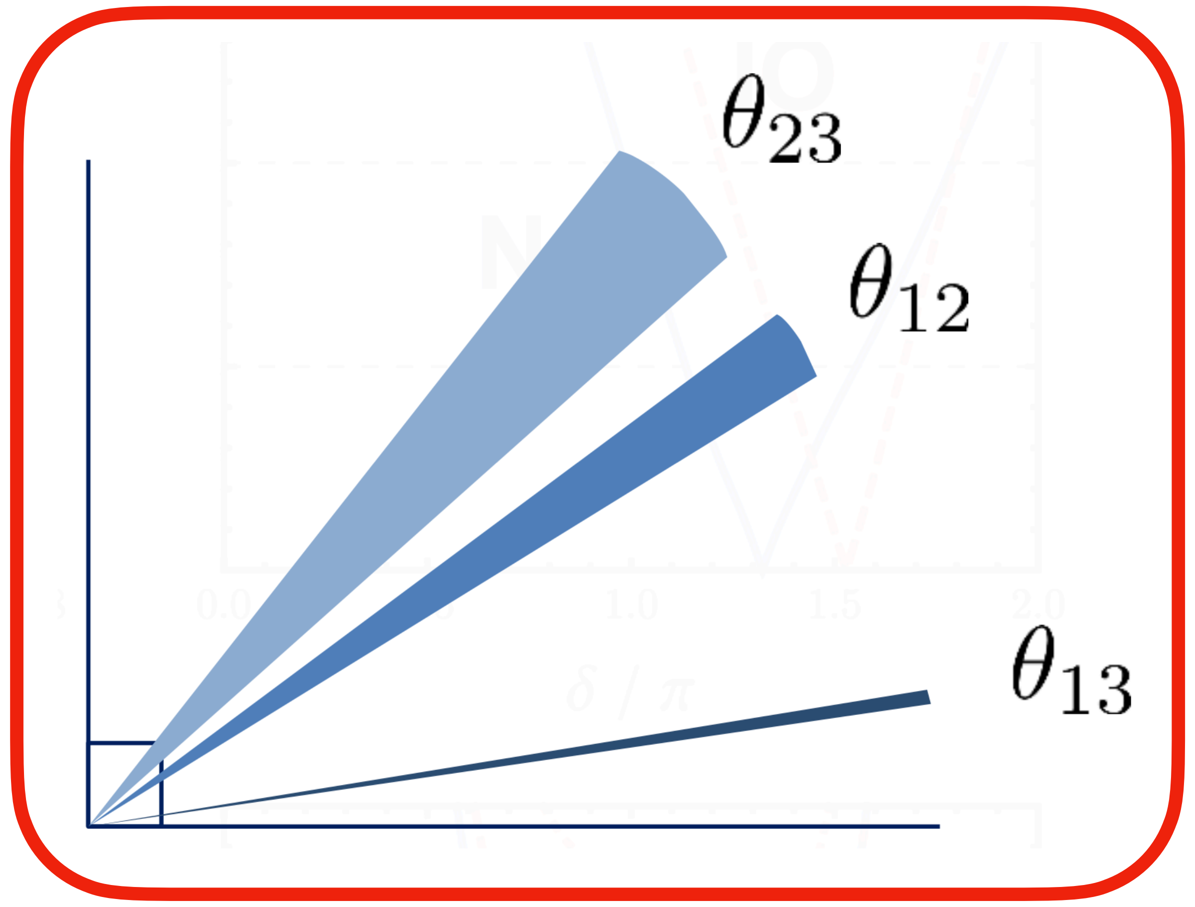


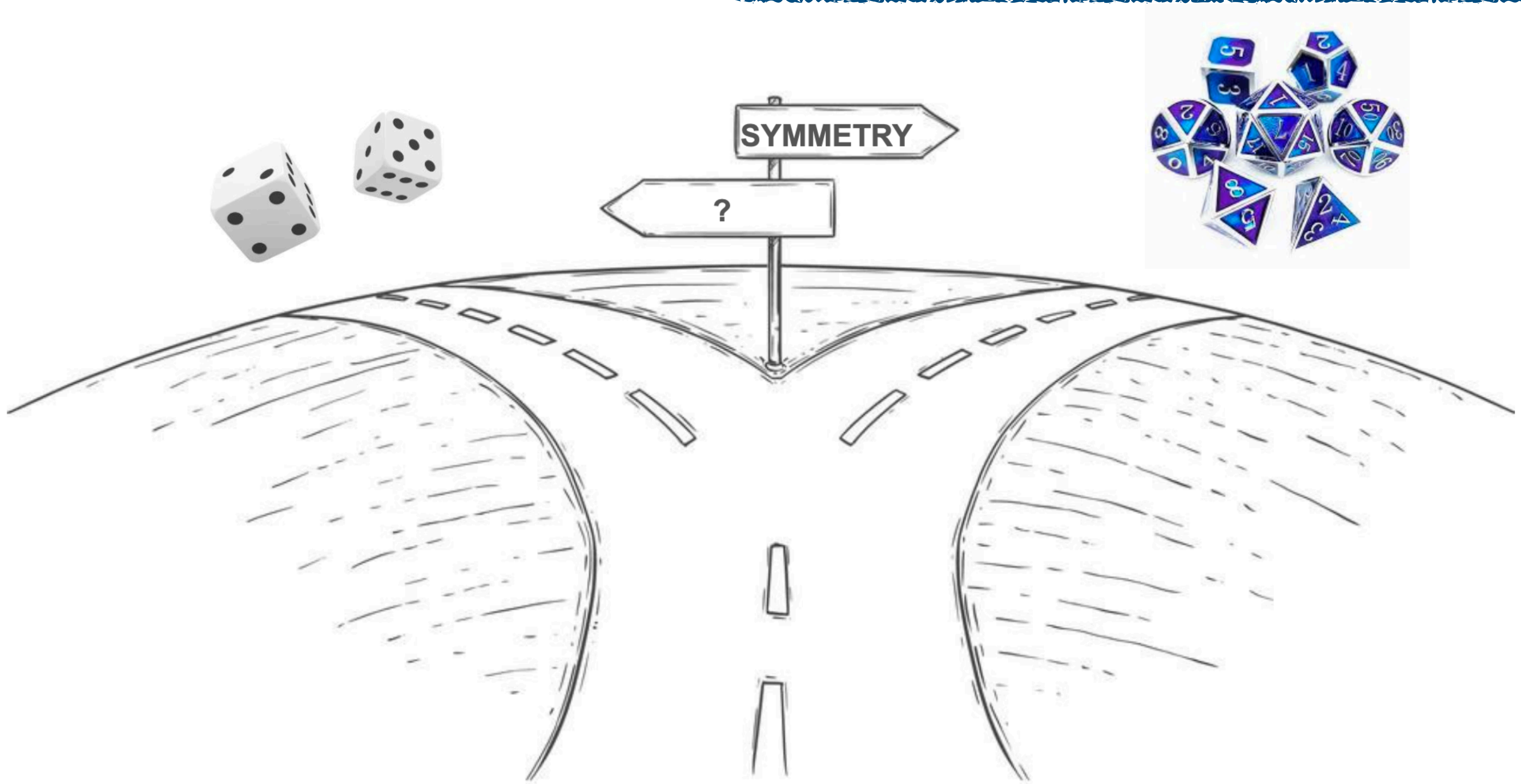
[1] “Asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg doesn’t have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses.” From *Model Physicist*, CERN Courier, 13 October 2017.

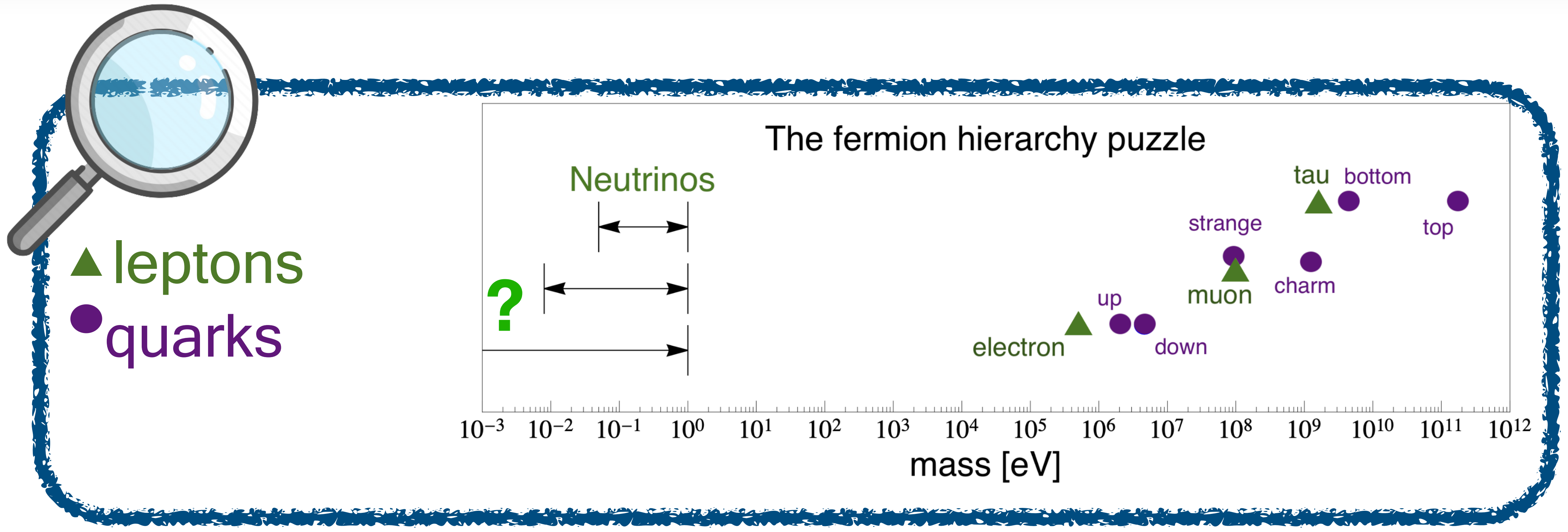
[Steven Weinberg, 2013]

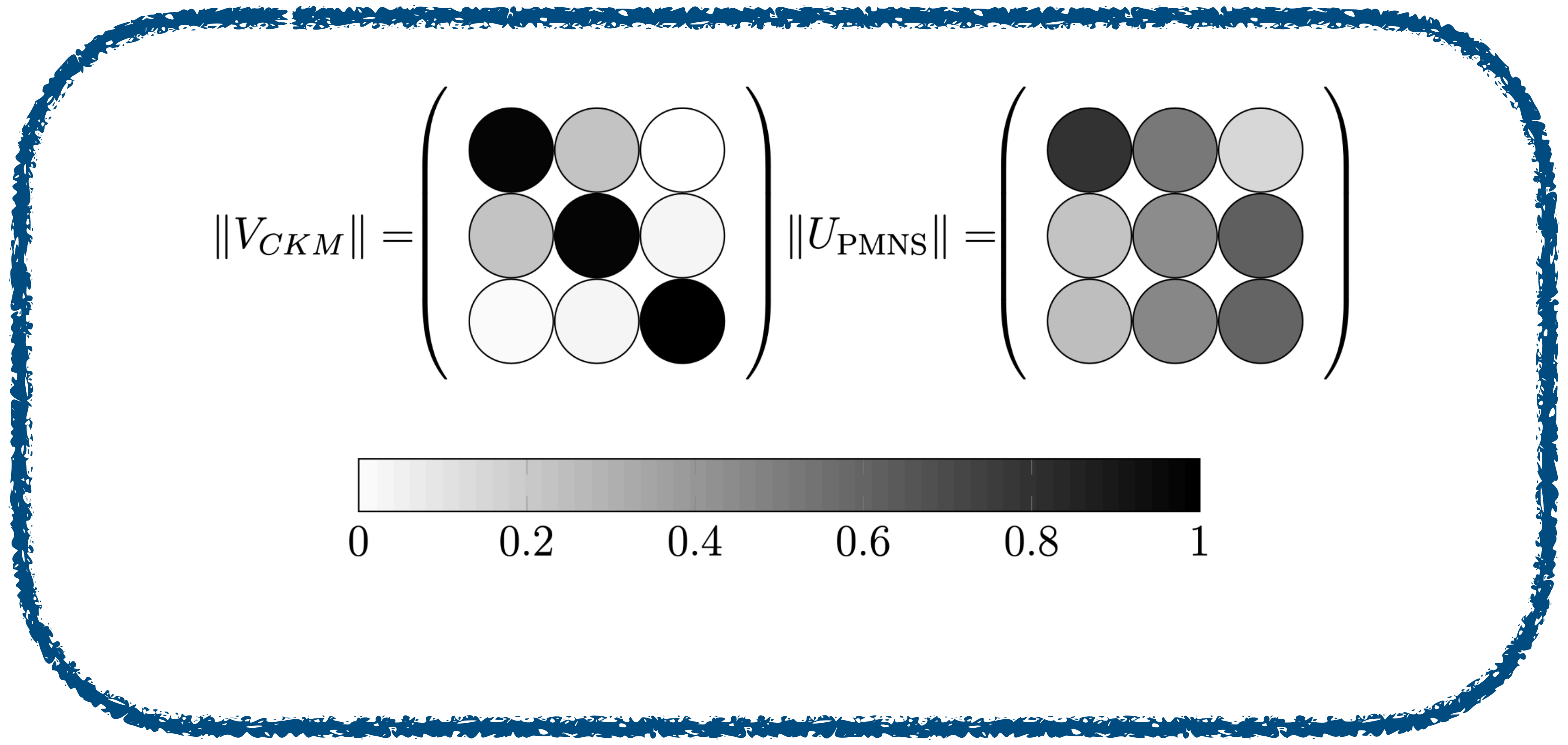

leptons
quarks
The fermion hierarchy puzzle
Neutrinos
?
electron
up
down
muon
strange
charm
tau
bottom
top
mass [eV]


$\|V_{CKM}\| =$
$\|U_{\text{PMNS}}\| =$
0
0.2
0.4
0.6
0.8
1


θ_{23}
θ_{12}
θ_{13}


SYMMETRY
?





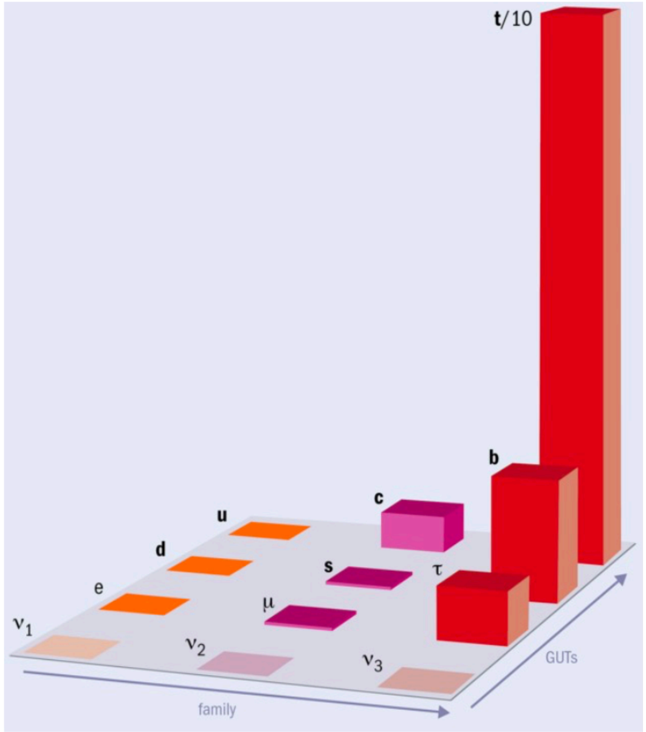
New physics is not immediately testable

But these puzzles could teach us new ways of understanding what we have

- Many examples in the history of Physics…

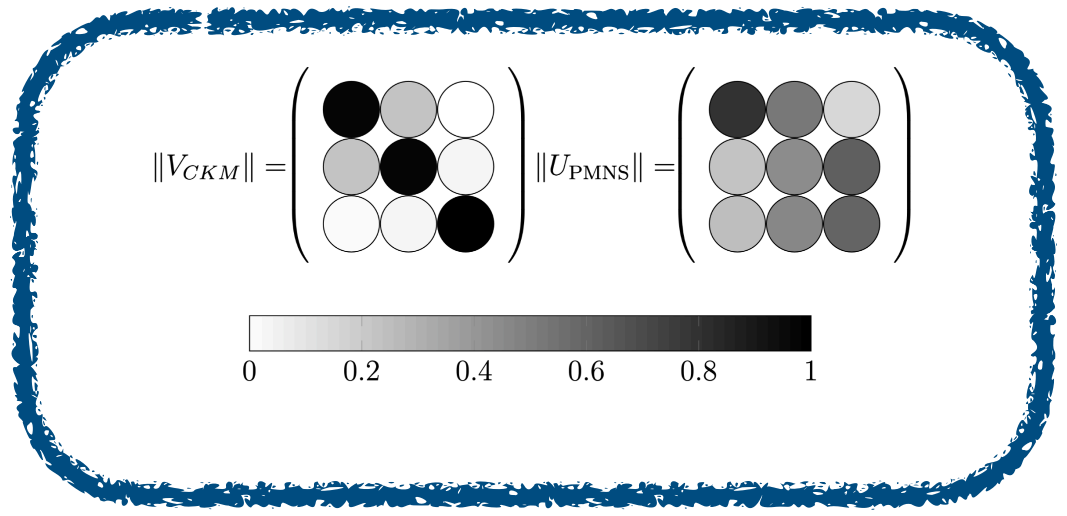
Can multiple aspects of flavour be explained by a single mechanism?

For many years we did:



e.g. Froggatt-Nielsen mechanism

X



e.g. discrete or continuous flavour symmetries

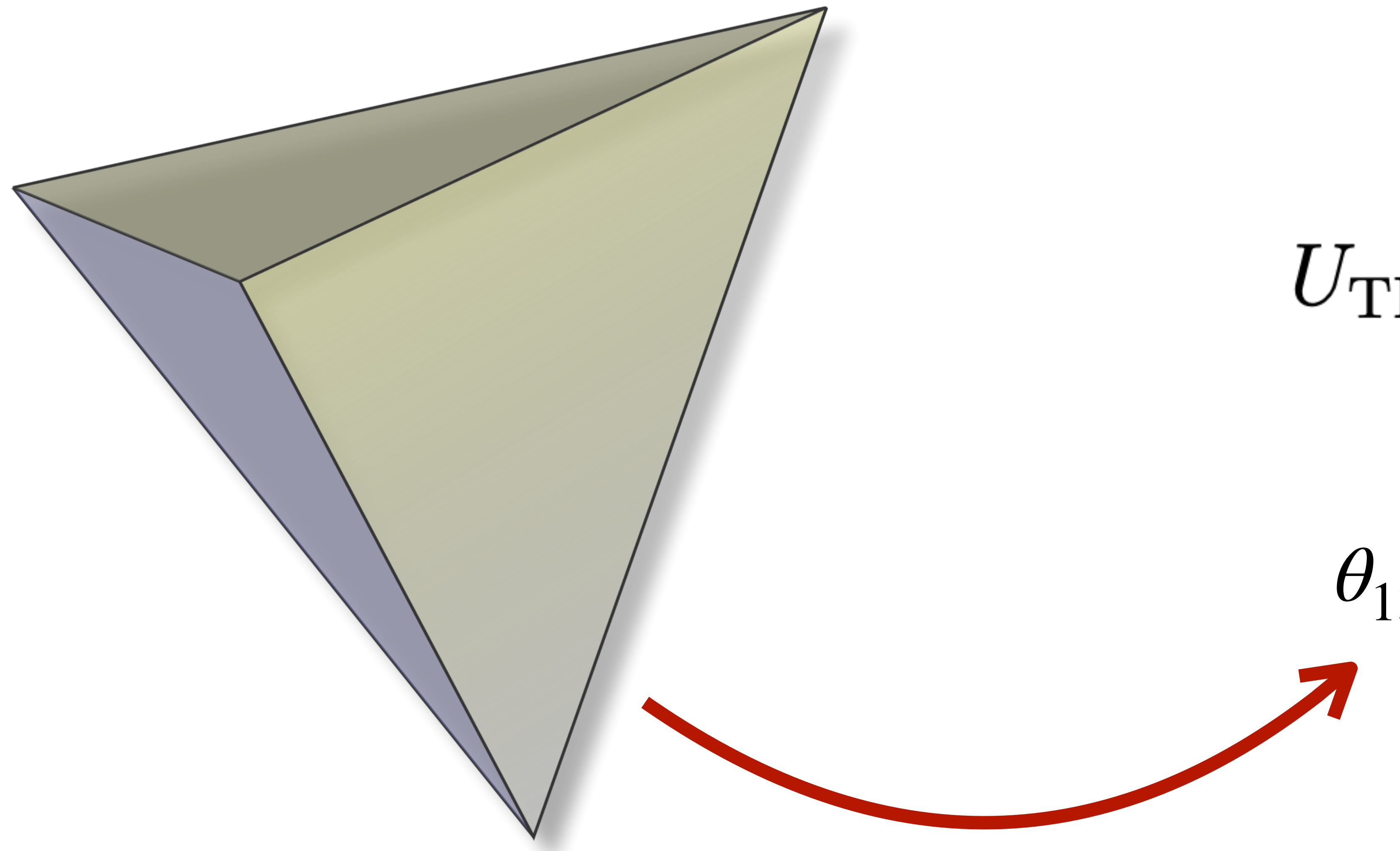
$$\left(\frac{<\varphi>}{\Lambda}\right)^{n_{ij}} Q_i H_u u_j^c + \ldots$$

Flavour symmetry breaking $<\varphi>$ is engineered

Neutrino data was compatible with discrete rotations!

Harrison, Perkins, Scott

Phys.Lett.B 530 (2002) 167



EFT with scalar "flavons" ϕ_i

$$\mathscr{W}_{Yukawa} \supset \frac{\alpha}{\Lambda} E^c (L\phi_i)_{\mathbf{1}} H_d$$

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{13} = 0 \quad \theta_{12} \simeq 35^\circ \quad \theta_{23} = 45^\circ$$

Flavour rotation under $G_{sym} \sim \{S_3, A_4, S_4, \ldots\}$

$$\begin{pmatrix} L_e \\ L_\mu \end{pmatrix} \to \begin{pmatrix} L_e + \sqrt{2} L_\mu \\ \sqrt{2} L_e - L_\mu \end{pmatrix}$$

Altarelli, Feruglio

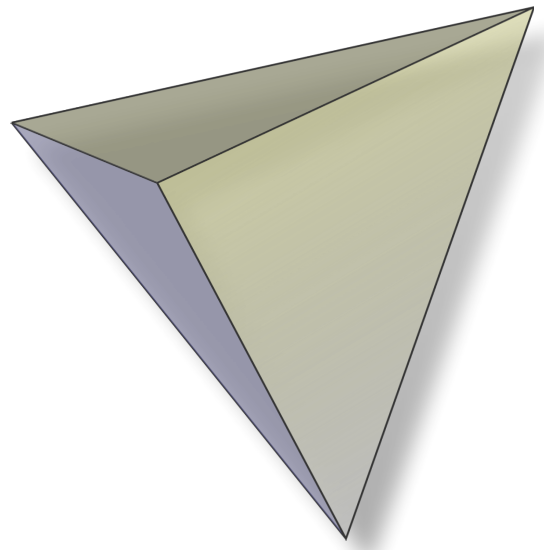
New J.Phys. 6 (2004) 106

Nucl.Phys.B 720 (2005) 64-88

Nucl.Phys.B 741 (2006) 215-235

and more…

Shortcomings of the traditional approach



$\theta_{13} = 0 \quad \theta_{12} \simeq 35^\circ \quad \theta_{23} = 45^\circ$

Tri-bimaximal compatible with data until 2012

✗ $U_{\text{PMNS}} = U^0_{\text{TBM}} + \ldots \text{corrections}$

$\downarrow$ $\theta_{13} = 0$ — Automatic

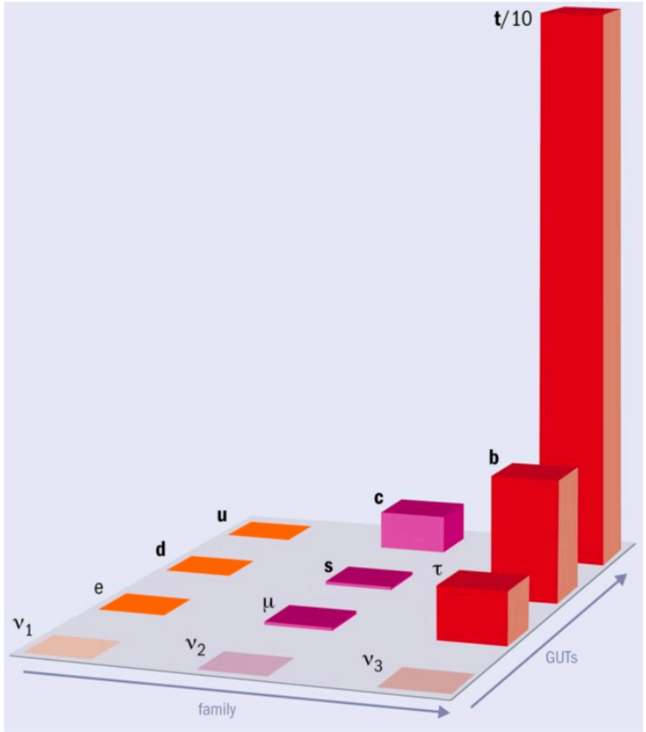
$\downarrow$ $\theta_{13} \approx 8.5^\circ$? → $\delta\theta_{23}\ ,\ \delta\theta_{12} \approx 8.5^\circ$

✗ $V(\phi_i) \rightarrow$ Mess!

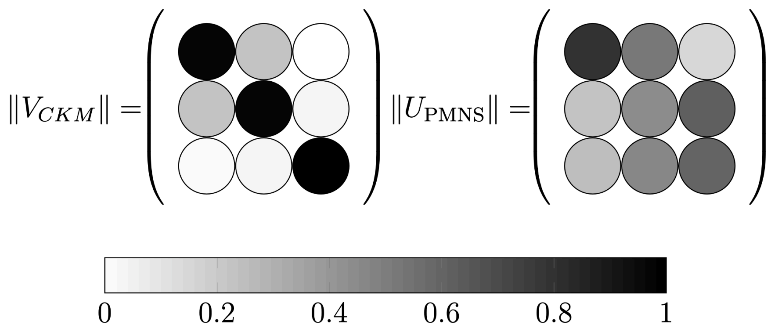
✗ $\alpha_i \ \gamma_i \ \beta_i \ \varrho_i \ \varsigma_i \ \eta_i$

Can multiple aspects of flavour be explained by a single mechanism?

For many years we did:



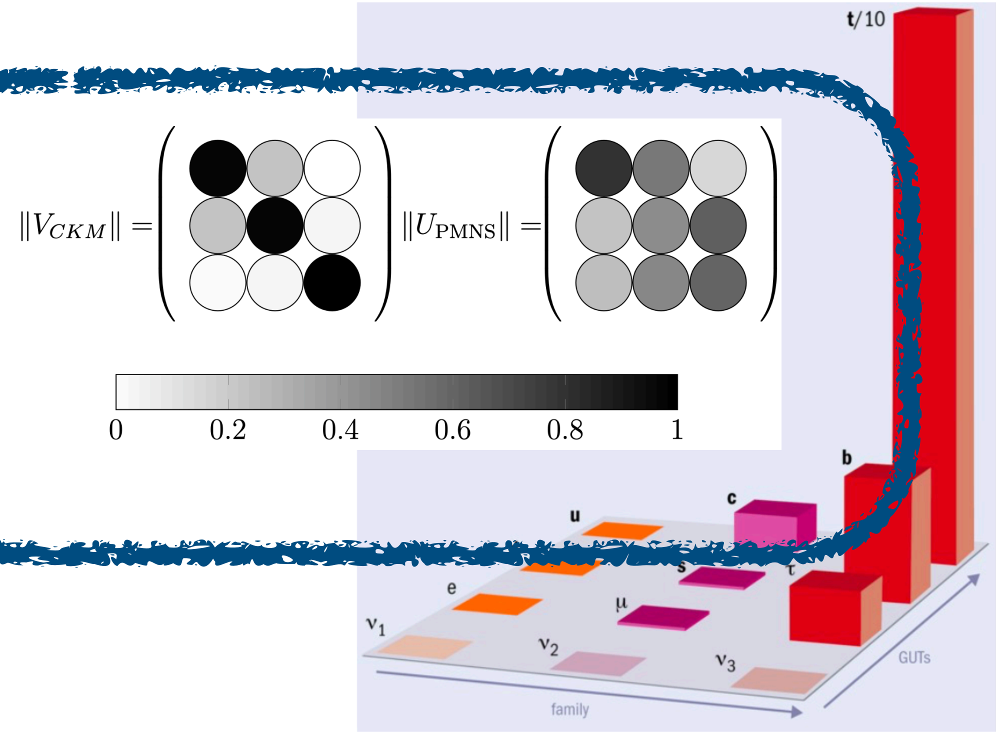
X



e.g. Froggatt-Nielsen mechanism

e.g. discrete or continuous flavour symmetries

An alternative:



with the SAME mechanism

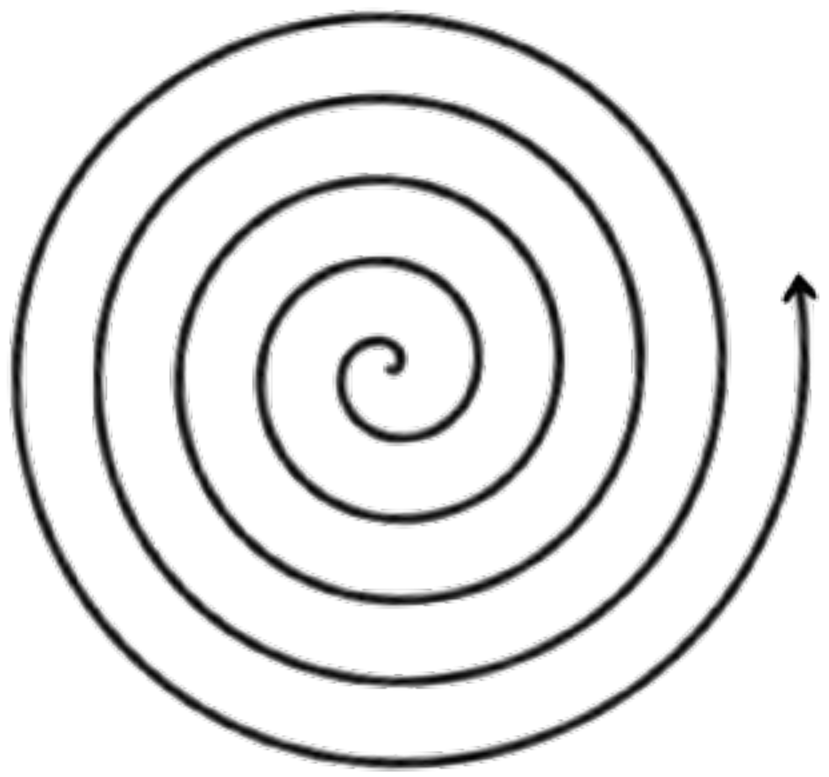
Can multiple aspects of flavour be explained by a single mechanism?

Some biases:

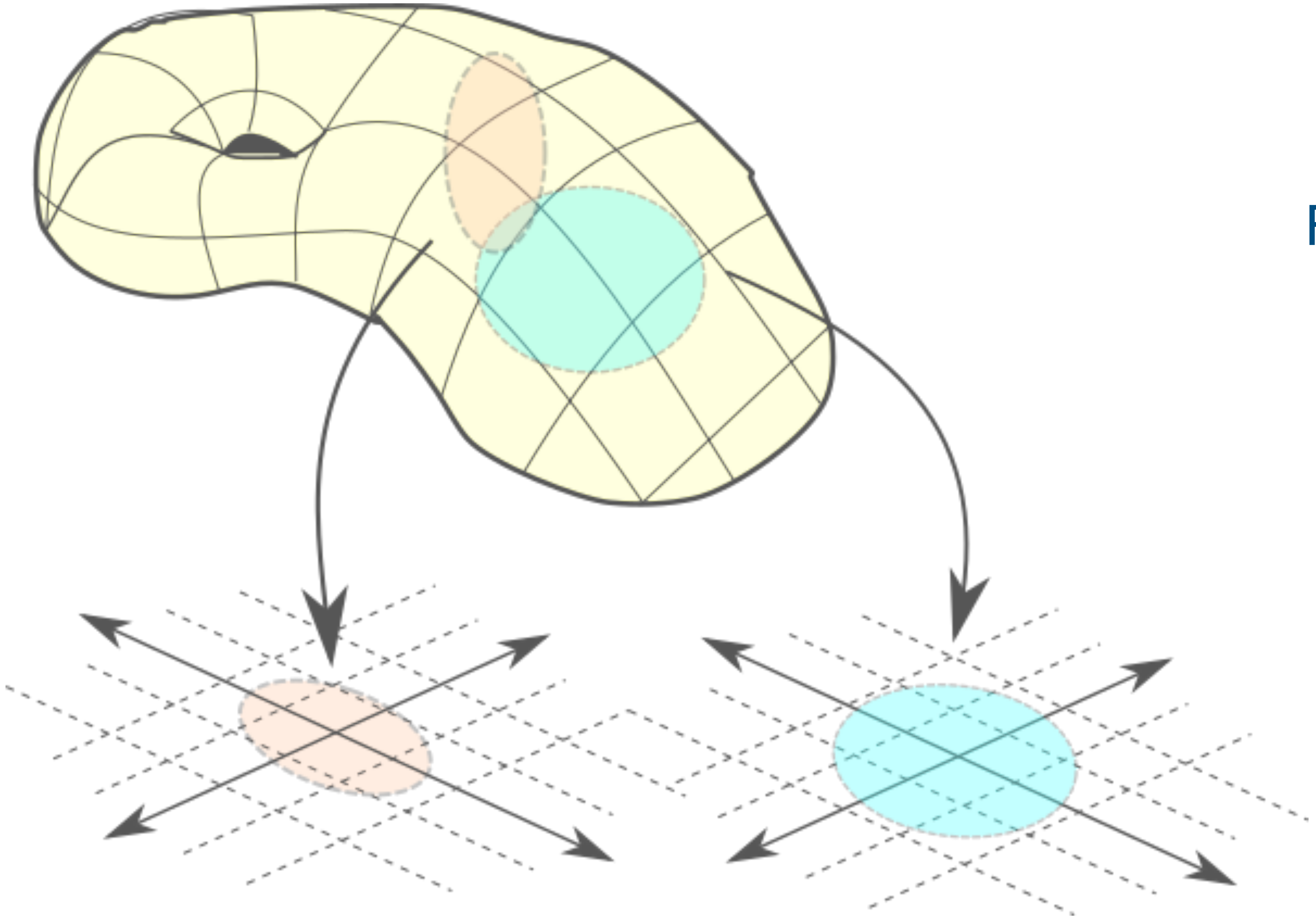
- Mass hierarchies MUST be explained simultaneously with the mixings
- SUSY as the most immediate extension of SM
- String-inspired
- CP should be spontaneously violated

Formidable task!

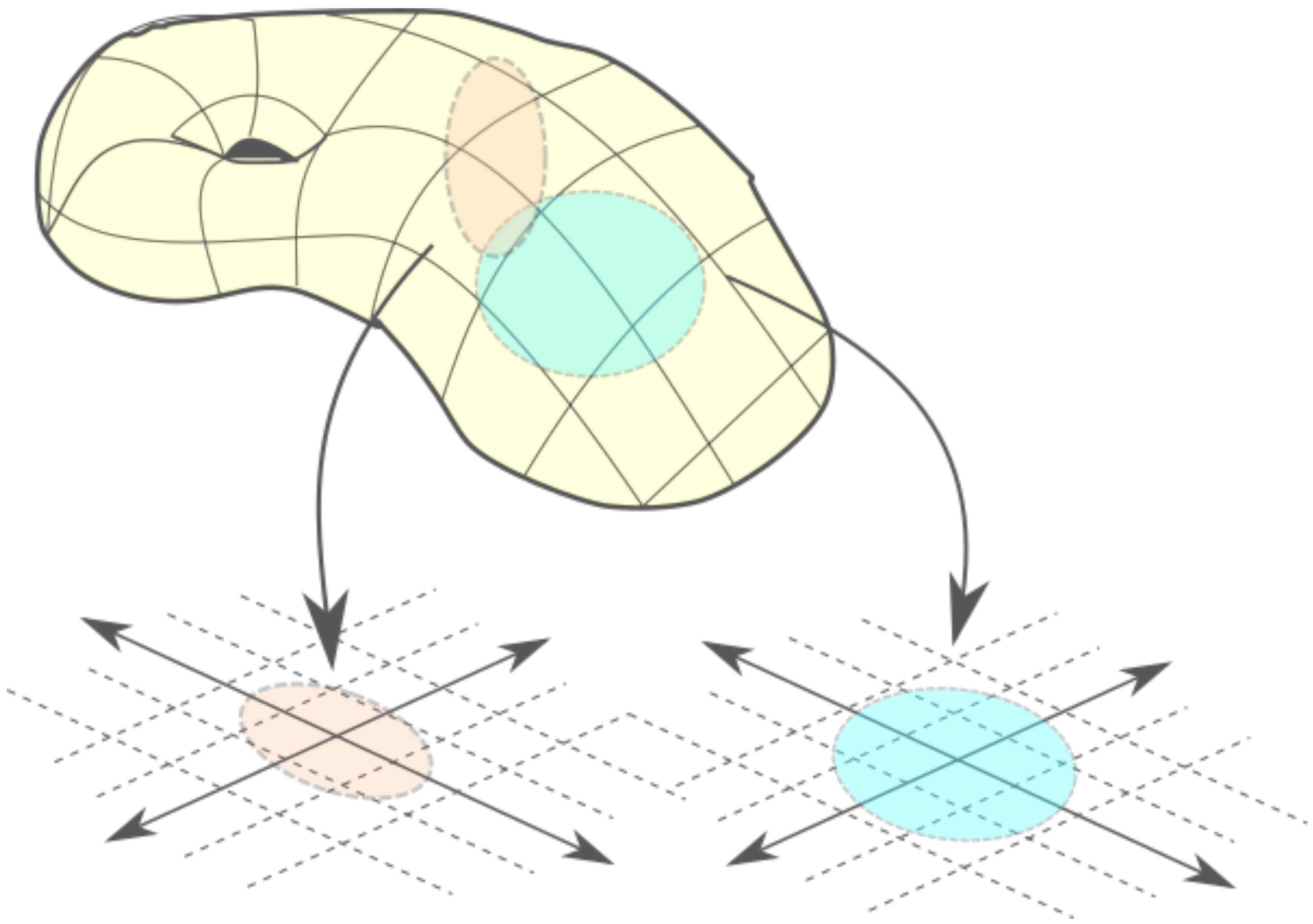
If you think you're going in circles…



Change your perspective



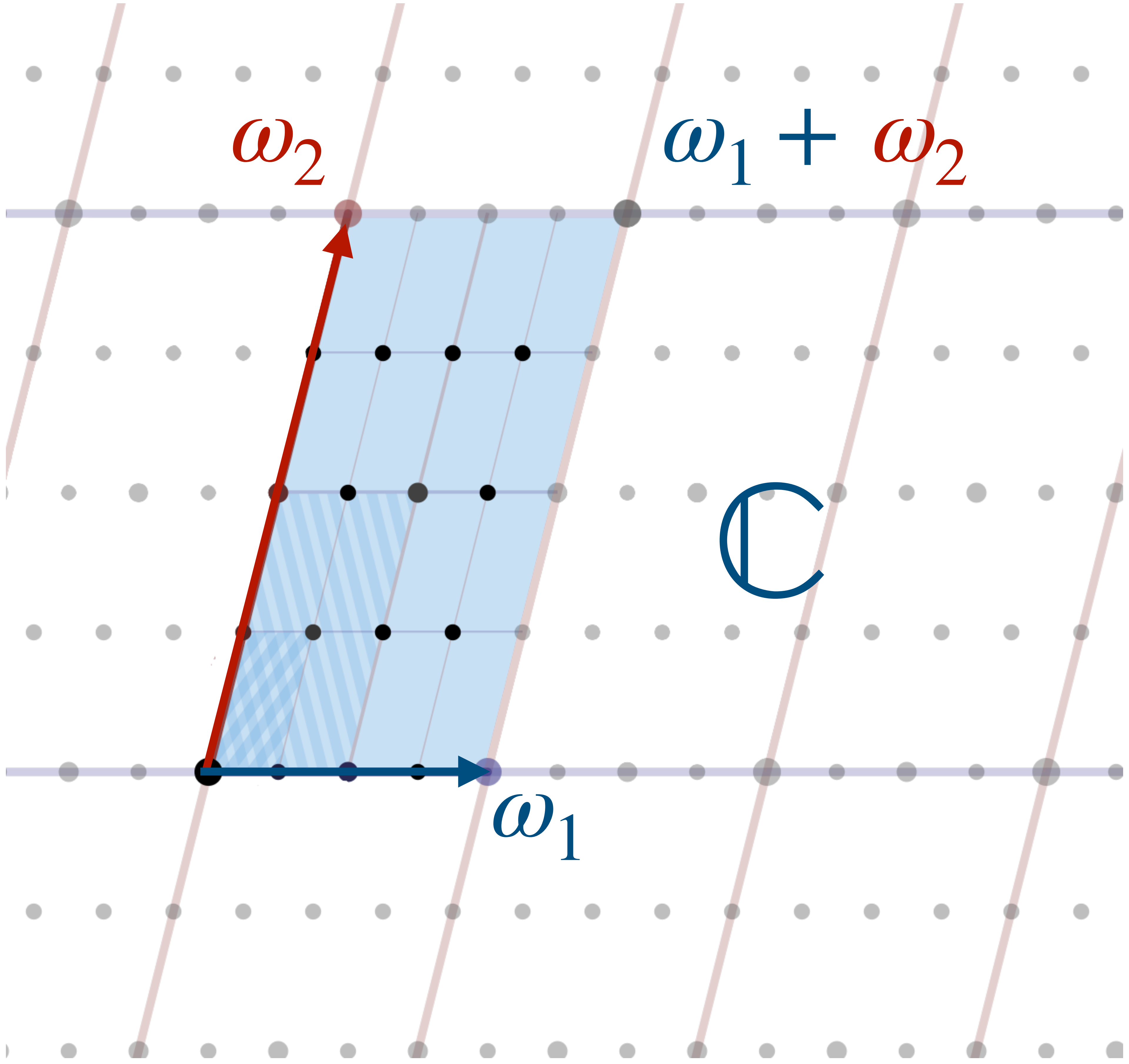
Fix the structure of the symmetry-breaking vacuum FIRST



φ breaks flavour symmetry

$$\text{VEV} = \langle \varphi \rangle$$

takes geometrically constrained values

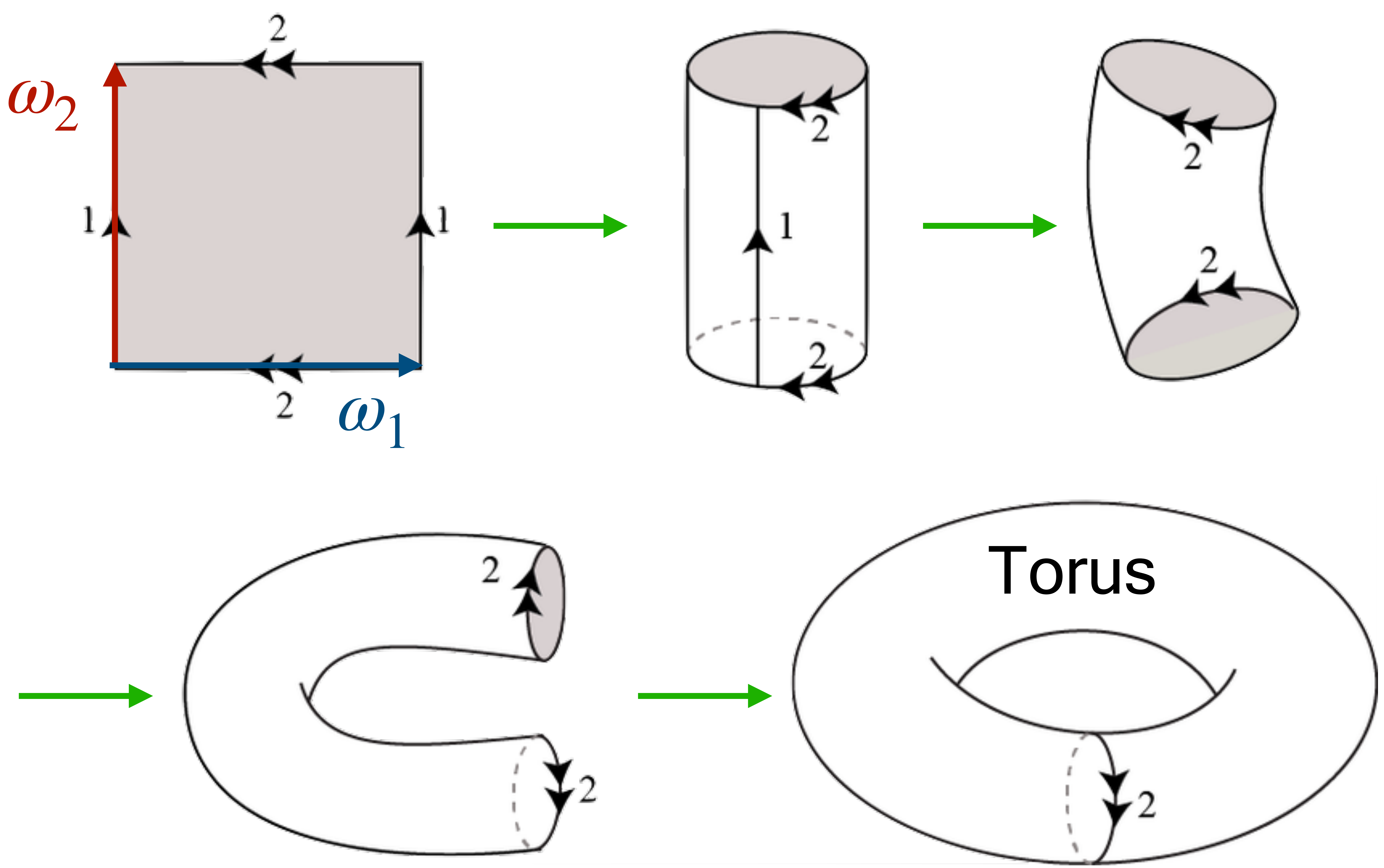


Change lattice basis

$$\begin{pmatrix} \omega_1' \\ \omega_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}$$

$a, b, c, d \in \mathbb{Z}$ $\qquad ad - bc = 1$

Area unchanged


2
ω_2
1
1
2
ω_1
2
1
2
2
2
2
2
Torus
2

"Shape" of the Torus

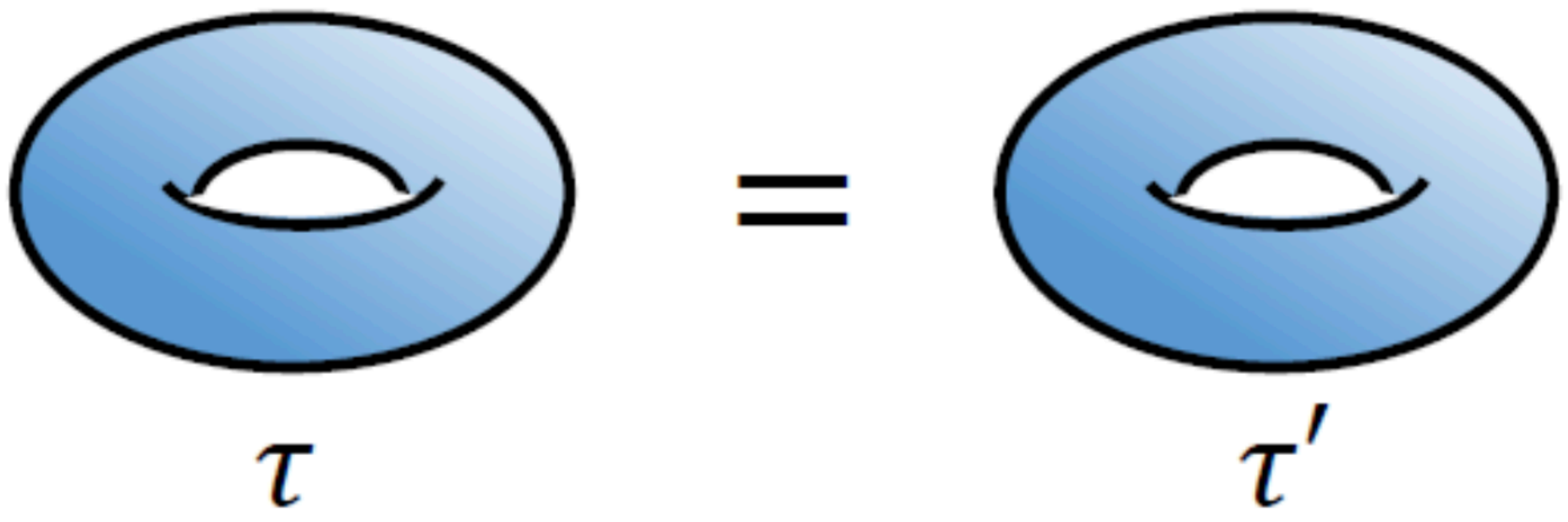
$$\tau \equiv \frac{\omega_2}{\omega_1} \in \mathbb{C}$$

$\tau = \tau_1$ $\tau = \tau_2$

$$\begin{pmatrix} \omega_1' \\ \omega_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}$$

$$\tau \quad \longrightarrow \quad \tau' = \frac{a\,\omega_2 + b\,\omega_1}{c\,\omega_2 + d\,\omega_1} \equiv \frac{a\tau + b}{c\tau + d}$$

$\tau \to \tau'$



Shape UNCHANGED

$$\gamma \in \mathrm{SL}(2,\mathbb{Z}) \equiv \text{Modular group} \equiv \Gamma$$

[1706.08749]
Ferruccio Feruglio

$$\tau \xrightarrow{\gamma} \tau' = \frac{a\tau + b}{c\tau + d}$$

$a, b, c, d \in \mathbb{Z}$

$$(y_{jk}\bar{L}_j H \ell_{Rk} + \text{h.c.})$$

[1706.08749]
Ferruccio
Feruglio

Γ

MAIN IDEA:
Yukawa couplings
are modular forms
in the UV theory

$$y_{jk} \rightarrow Y(\tau)$$

non-linearly realized symmetry!

$< \tau > \equiv$ VEV

$$M_e \sim \sum_i \alpha_i \begin{pmatrix} f_{11}(\tau) & f_{12}(\tau) & \dots \\ \dots & \dots & \dots \\ \dots & \dots & f_{33}(\tau) \end{pmatrix}$$

$f_{ij} \equiv$ pre-determined functions of τ

$$(y_{jk}\bar{L}_j H \ell_{Rk} + \text{h.c.})$$

[1706.08749]
Ferruccio Feruglio

Γ

$<\tau>\equiv$ VE

MAIN IDEA:
Yukawa couplings are modular forms in the UV theory

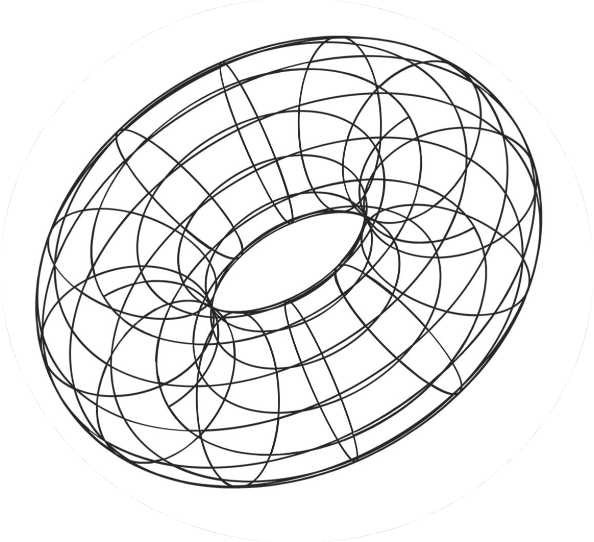
WARNING
BOTTOM UP APPROACH

$$y_{jk} \to Y(\tau)$$

non-linearly realized symmetry!

$$M_e \sim \sum_i \alpha_i \begin{pmatrix} f_{11}(\tau) & f_{12}(\tau) & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & f_{33}(\tau) \end{pmatrix}$$

$f_{ij} \equiv$ pre-determined functions of τ



$$\int d^4x\, d^6y\, \mathscr{L}_{10D} \Longrightarrow \int d^4x\, \mathscr{L}_{EFT}$$

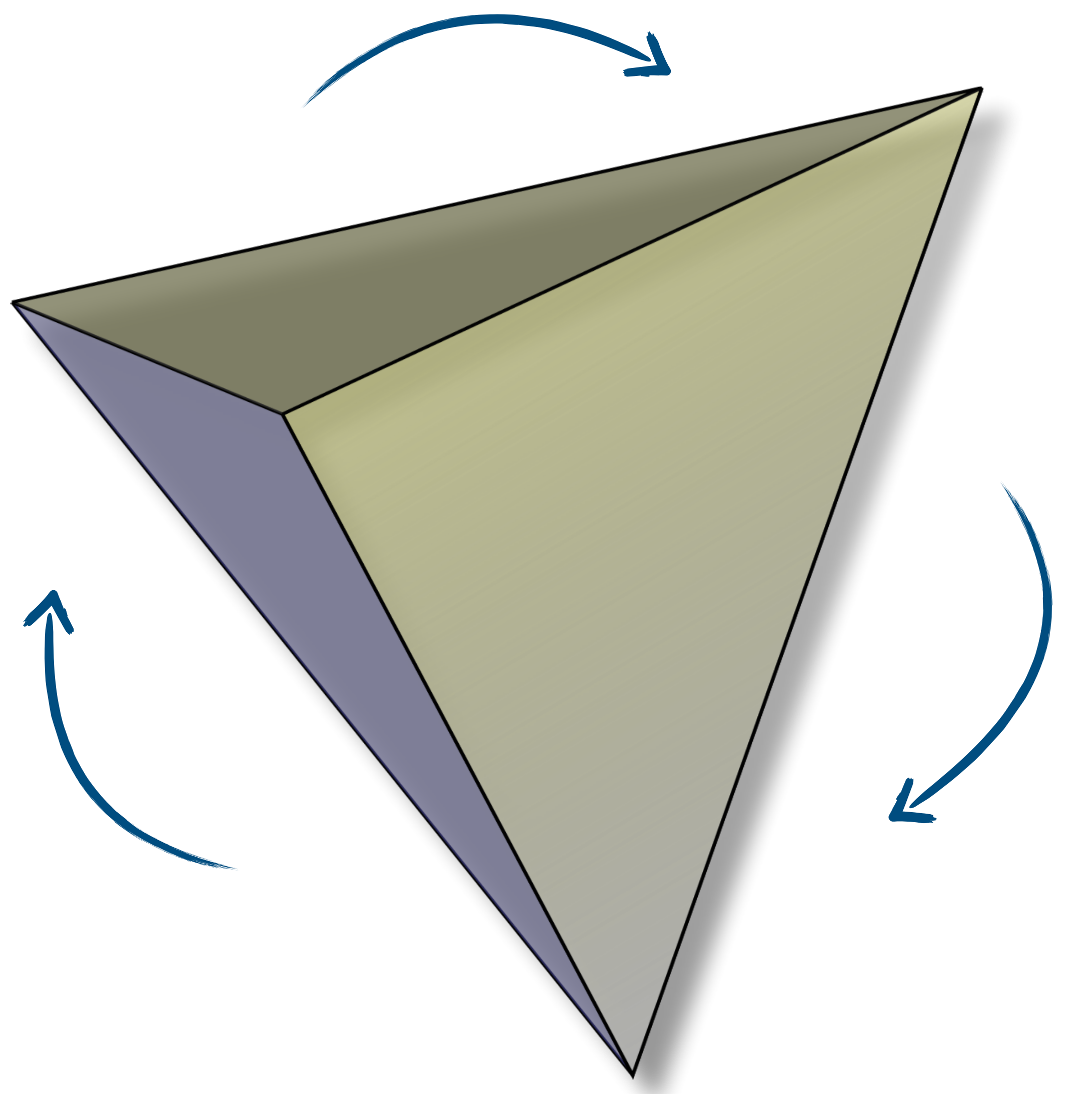
What is a modular form? $Y(\tau)$

- $Y(\gamma(\tau)) = (c\tau + d)^k Y(\tau)$ Holomorphic in: $\{\tau \in \mathbb{C} \,|\, \mathrm{Im}(\tau) > 0\}$
- "Weight" $k > 0$

Very constraining!

The modular group exists
in a FINITE and
DISCRETE version

$$\Gamma_N \cong$$

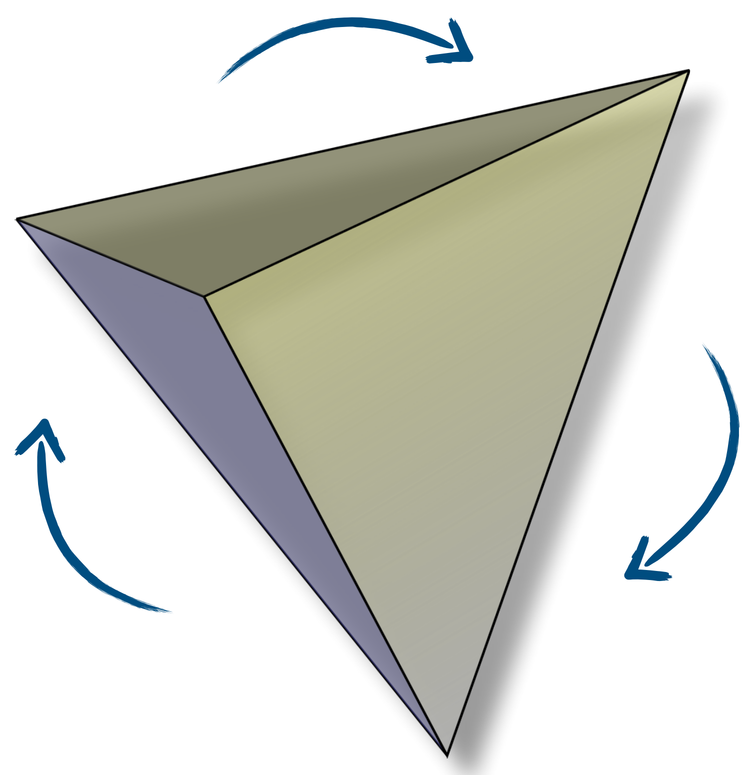


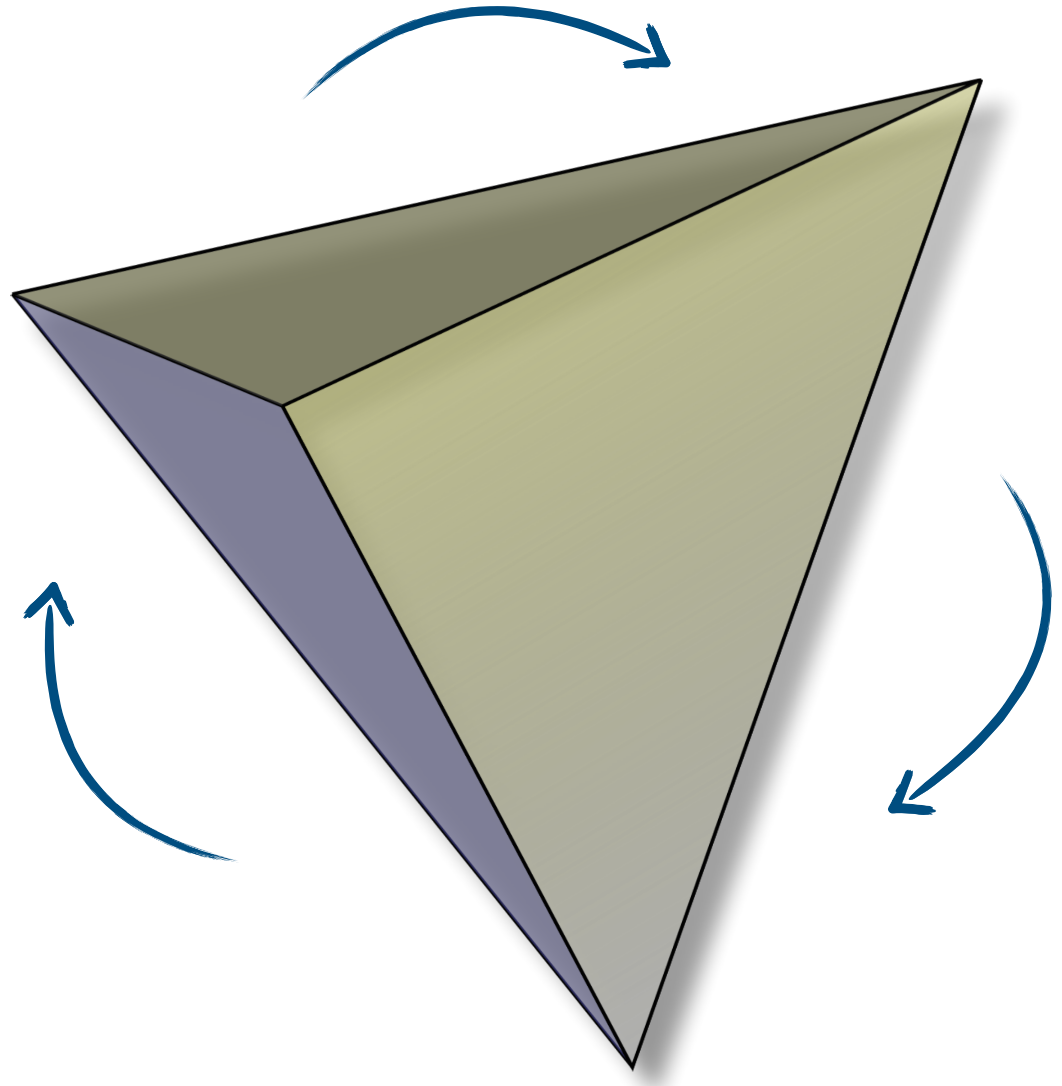
A finite version of Γ describes flavour

Feruglio's Ansatz

$$\Gamma(N) = \{\gamma \in SL(2,\mathbb{Z}) \mid \gamma \equiv 1_2 \,\mathrm{mod}(N)\}$$

$$\Gamma_N = \Gamma/\Gamma(N) \cong$$





Flavour rotation under $\Gamma_N \cong \{S_3, A_4, S_4, \ldots\}$

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} \to (c\tau + d)^3 \begin{pmatrix} \psi_e + \sqrt{2}\psi_\mu \\ \sqrt{2}\psi_e - \psi_\mu \end{pmatrix}$$

Example

Modular factor

Weight $k = 3$

$$\mathcal{W}(\Phi) = \sum (Y_{I_1 \ldots I_n}(\tau)\, \varphi^{(I_1)} \ldots \varphi^{(I_n)})_{\mathbf{1}}$$

Modular invariance if;

$$\begin{cases} \rho \otimes \rho_{I_1} \otimes \rho_{I_2} \ldots \otimes \rho_{I_n} \supset \mathbf{1} \\ k_Y = k_{I_1} + k_{I_2} + \ldots + k_{I_n} \end{cases}$$

$\longrightarrow$ Usual

$\longrightarrow$ Novelty

$$Y_{I_1 \ldots I_n}(\tau) \to (c\tau + d)^{k_Y} \rho(\gamma) Y_{I_1 \ldots I_n}(\tau)$$

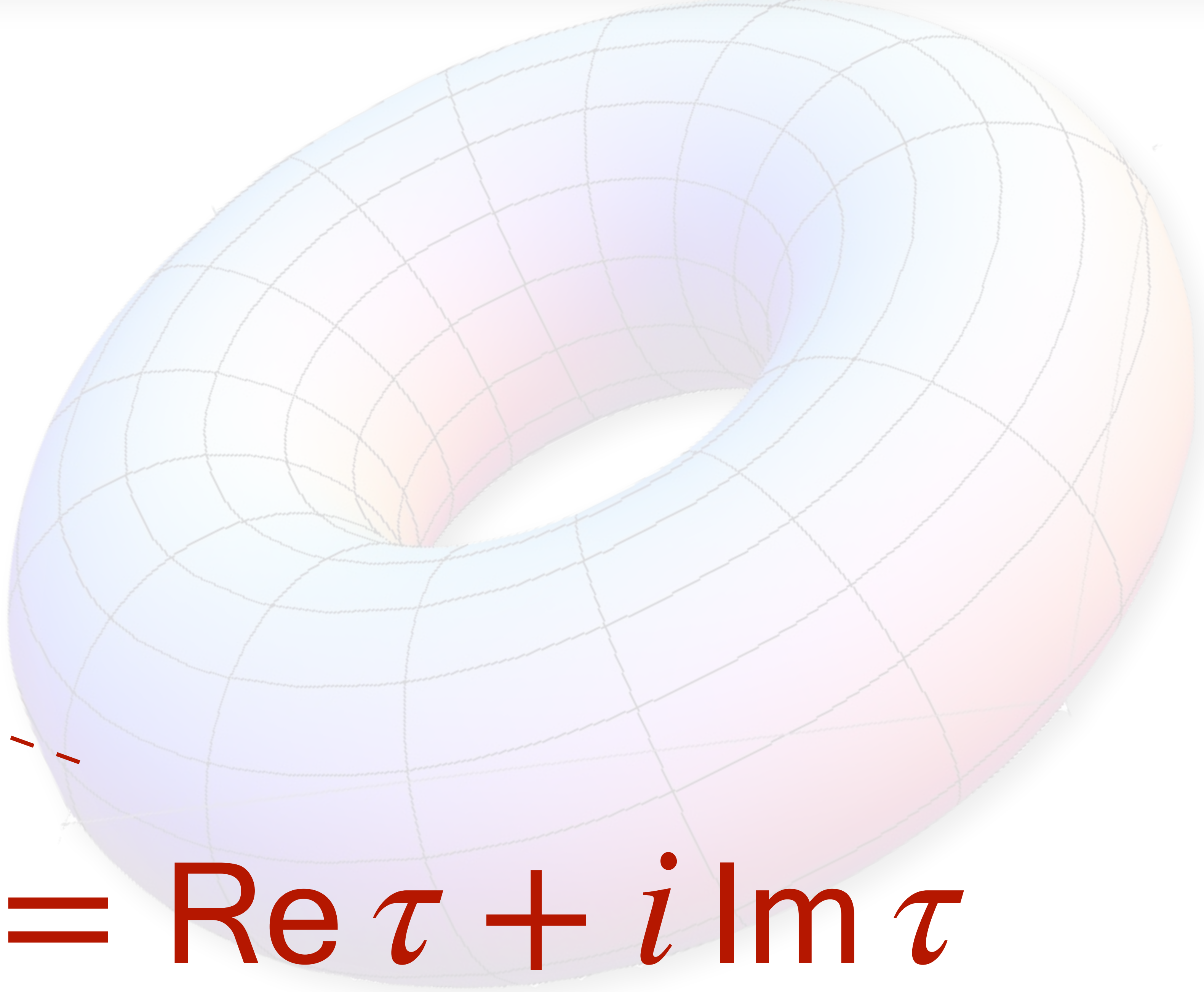
Yukawa: modular forms of weight k_Y

$$\varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)}$$

Superfields with modular charges $-k_I$

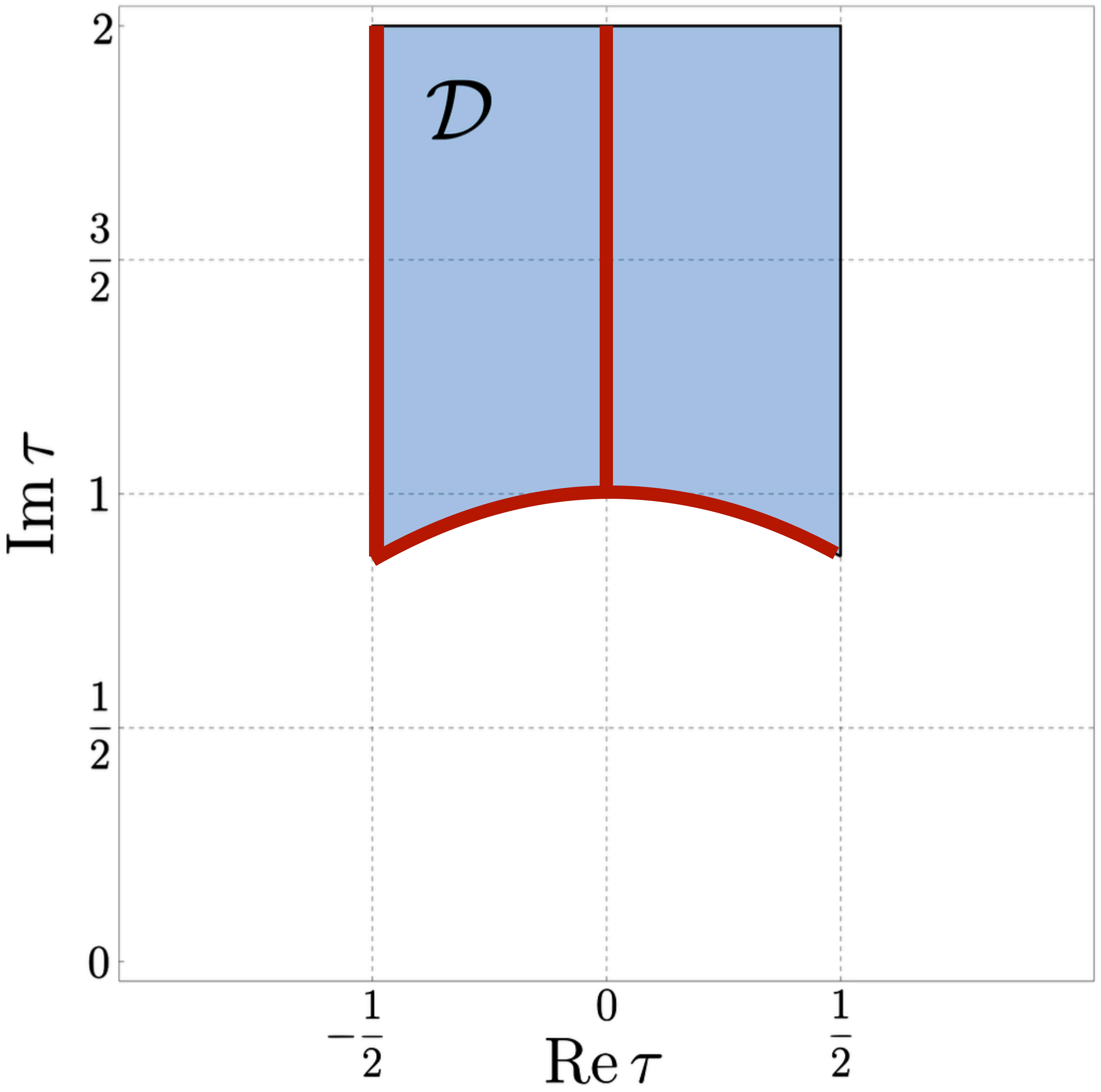
$$(c\tau + d)^{k_Y} (c\tau + d)^{-\sum k_{I_n}} = 1$$

$$Y(\tau) \sim \sum_{n}^{\infty} e^{2\pi i n \tau}$$



$$\tau = \mathrm{Re}\,\tau + i\,\mathrm{Im}\,\tau$$

- ✓ τ controls fermion masses
- ✓ τ controls fermion mixing
- ✓ τ controls δ_{CP}



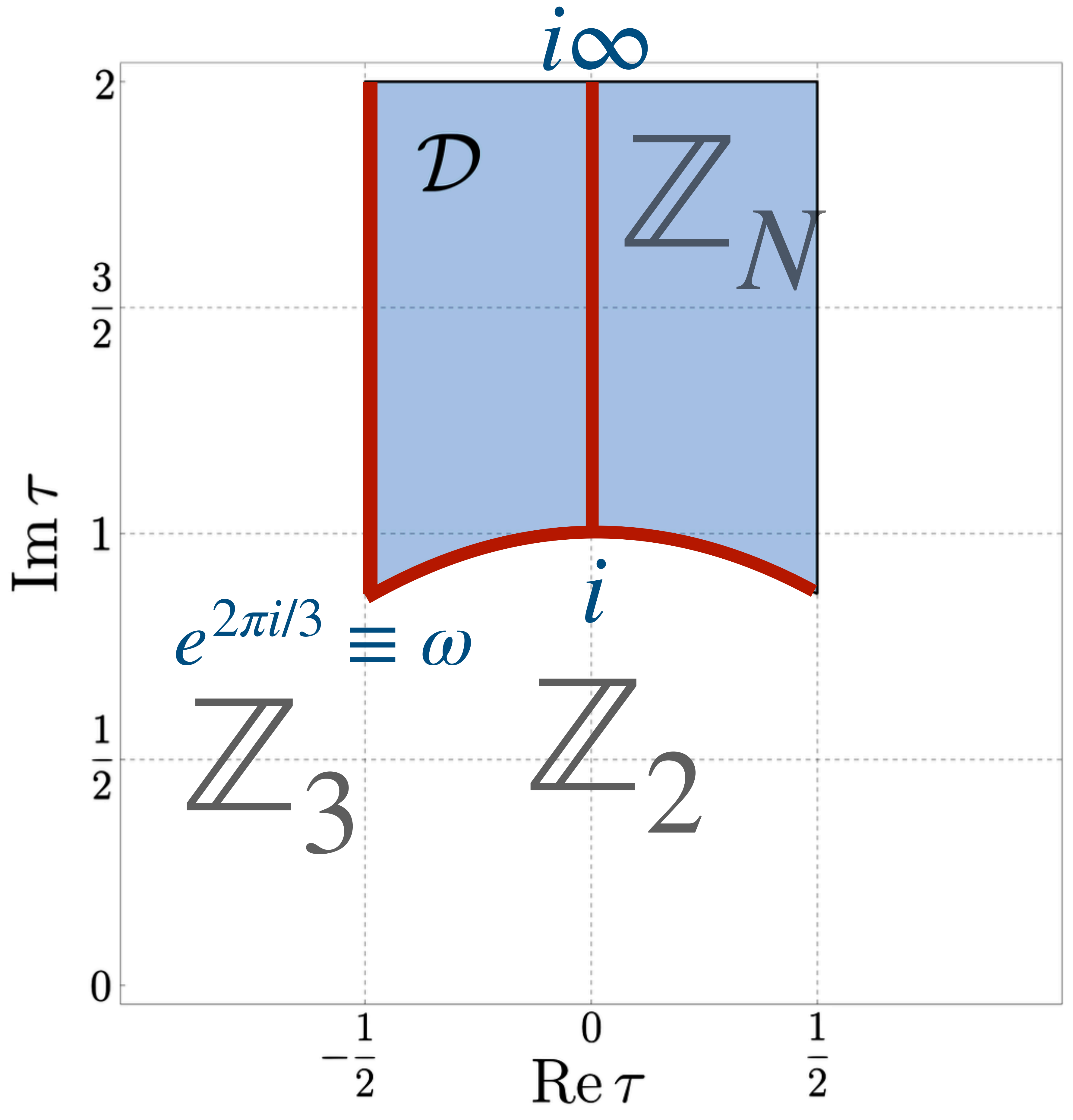
$$\mathcal{D} = \left\{ \tau \in \mathbb{C} \,:\, \mathrm{Im}\,\tau > 0\,,\;\; |\mathrm{Re}\,\tau| \leq \frac{1}{2}\,,\;\; |\tau| \geq 1 \right\}$$

- Every $\tau \notin \mathscr{D}$ can be mapped in $\tau' \in \mathscr{D}$ through Γ transformation
- CP conserving values

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

Only source of CPV in the model is the VEV of τ



JHEP 04 (2021) 206 Novichkov, Penedo, Petcov

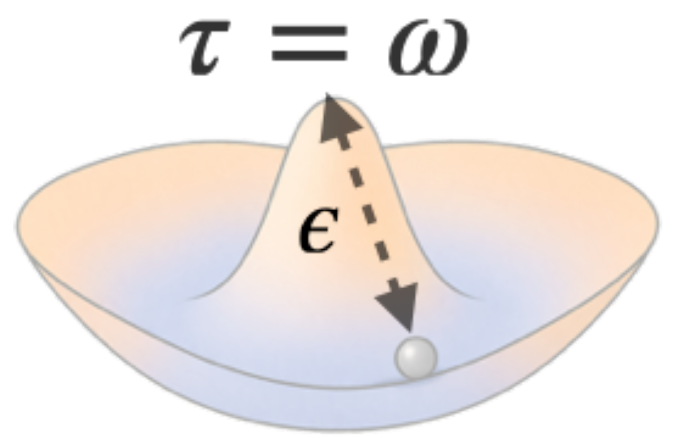
Modular-Proximity-Induced-Hierarchies (MPIH)

$$\epsilon \sim |\tau - \tau_{sym}|$$

- $\tau = \tau_{\text{sym}}$ $\qquad \epsilon = |\tau - \tau_{\text{sym}}| > 0$

$$M \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow M \sim \begin{pmatrix} 1 & \epsilon^{\cdots} & \epsilon^{\cdots} \\ \epsilon^{\cdots} & \epsilon^{\cdots} & \epsilon^{\cdots} \\ \epsilon^{\cdots} & \epsilon^{\cdots} & \epsilon^{\cdots} \end{pmatrix}$$

- $m_\ell = m_\tau(1,\, \epsilon\,,\, \epsilon^3)$



- The pattern depends on both Γ'_N and the weights

Everything from ϵ?

[2505.21405]

Granelli, Meloni, Parriciatu, Penedo, Petcov

$m_\nu \to 0$ as $\epsilon \to 0$

Smallness of light neutrino masses

$$m_e/m_\mu \simeq \frac{1}{200}$$

$$m_\mu/m_\tau \simeq \frac{1}{17}$$

$$\epsilon \equiv |\tau - \tau_{sym}|$$

$$\Delta M \sim |\epsilon|^r \times M \,.$$

L-conserving for $\epsilon \to 0$

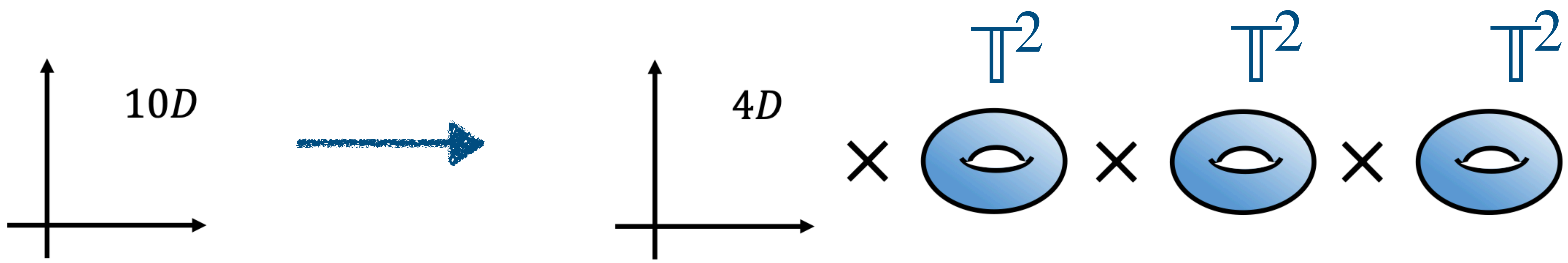
CP conserving (in gCP mode)

$\delta_{CP}, \to 0$ as $\epsilon \to 0$

$$\delta_{CP} \to 0 \quad \text{as} \quad \epsilon \to 0$$

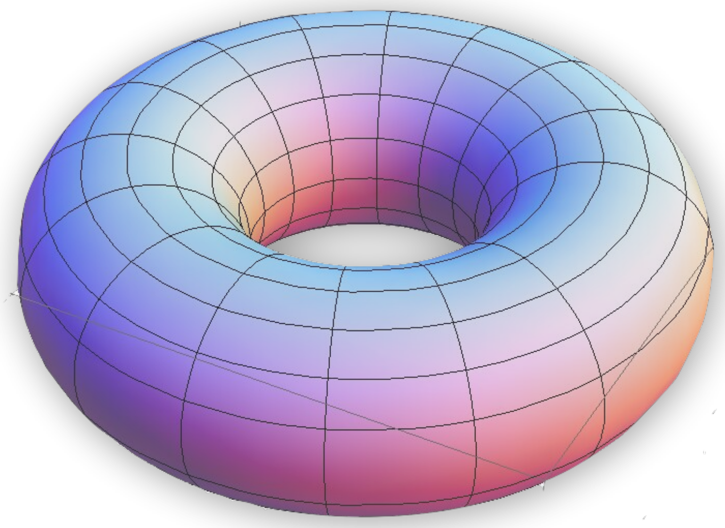
As the quarks teach us,
this is NOT desirable [2307.14410]

$$\delta_{CP}^{CKM} \sim 69^\circ$$

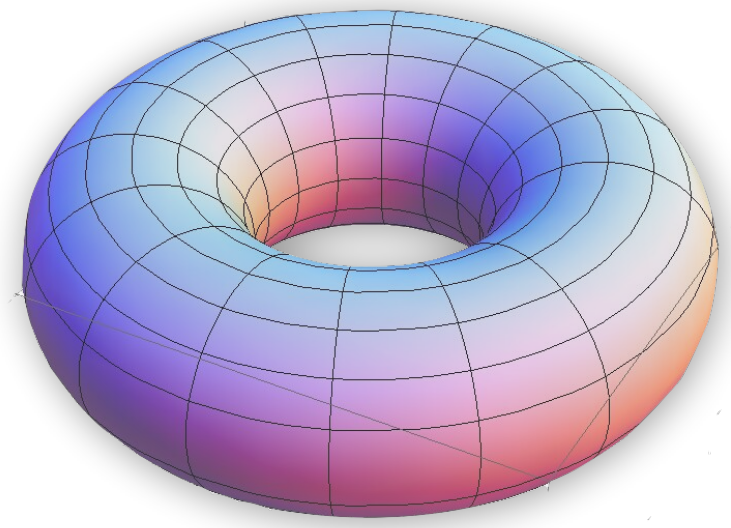


One modulus is just a toy model?

Multiple moduli?



X



Mass hierarchies

$\tau_1 \sim \{\omega, i, i\infty\}$

CP violation

$\tau_2 \in \mathscr{D}$

OR

genus 2

[2010.07952]

Ding, Feruglio, Liu

$$\tau = \begin{pmatrix} \tau_1 & \tau_3 \\ \tau_3 & \tau_2 \end{pmatrix}$$

Two rings to rule them all...

Carducci, Meloni, Parriciatu, Penedo

[arXiv:2604.21979]

Symplectic symmetry 

$$\tau = \begin{pmatrix} \tau_1 & \tau_3 \\ \tau_3 & \tau_2 \end{pmatrix} \xrightarrow{Sp(4,\mathbb{Z})} \tau' = (A\tau + B)(C\tau + D)^{-1}$$

$$\det \text{Im}\,\tau > 0 \quad , \quad \text{Tr}\ \text{Im}\,\tau > 0$$

$$Y(\gamma(\tau)) = [\det(C\tau + D)]^k\, Y(\tau)$$

Siegel modular forms

A second ansatz needed 

$$\Gamma(n) = \{\gamma \in SL(4,\mathbb{Z}) \,|\, \gamma \equiv 1_4 \bmod (n)\}$$

$$|\Gamma_n| = n^{10} \prod_{p|n} \left(1 - \frac{1}{p^2}\right)\left(1 - \frac{1}{p^4}\right)$$

$$|\Gamma_2| = |\Gamma_4| = 720 \quad , \quad |\Gamma_3| = 51840 \quad , \quad |\Gamma_5| = 9.36 \times 10^6$$

$\Gamma_2 \cong S_6$ lowest irrep is **5**…

Need additional constraints!

Consider $H \subset Sp(4,\mathbb{Z})$

$$H\tau = \tau \ , \qquad \tau \in \Sigma$$

Invariant region of genus 2

Choose $N(H)$ as the flavour modular group

Genus 2 ansatz

Choose $N(H)$ as the flavour modular group

Genus 2 ansatz

$$N_n(H) = N(H)/N(H,n)$$

Now suited for flavour model building!

everything from genus 1 is now generalizable: gCP, MPIH…

Quarks at genus $g = 2$

		Fixed region Σ	Stabilizer $\bar{H} \equiv H/\{\pm\mathbb{1}\}$	$N_2(H)$
dimension two	$\mathcal{T}_1$:	$\begin{pmatrix}\tau_1 & 0\\ 0 & \tau_2\end{pmatrix}$	$\mathbb{Z}_2$	$(S_3 \times S_3) \rtimes \mathbb{Z}_2$
	$\mathcal{T}_2$:	$\begin{pmatrix}\tau_1 & \tau_3\\ \tau_3 & \tau_1\end{pmatrix}$	$\mathbb{Z}_2$	$S_4 \times \mathbb{Z}_2$
dimension one	$\mathcal{O}_1$:	$\begin{pmatrix}i & 0\\ 0 & \tau_2\end{pmatrix}$	$\mathbb{Z}_4$	D_6
	$\mathcal{O}_2$:	$\begin{pmatrix}\omega & 0\\ 0 & \tau_2\end{pmatrix}$	$\mathbb{Z}_6$	$S_3 \times \mathbb{Z}_3$
	$\mathcal{O}_3$:	$\begin{pmatrix}\tau_1 & 0\\ 0 & \tau_1\end{pmatrix}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	D_6
	$\mathcal{O}_4$:	$\begin{pmatrix}\tau_1 & 1/2\\ 1/2 & \tau_1\end{pmatrix}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$D_4 \times \mathbb{Z}_2$
	$\mathcal{O}_5$:	$\begin{pmatrix}\tau_1 & \tau_1/2\\ \tau_1/2 & \tau_1\end{pmatrix}$	S_3	$S_3 \times S_3$
dimension zero	$\mathcal{Z}_1$:	$\begin{pmatrix}\zeta & \zeta+\zeta^{-2}\\ \zeta+\zeta^{-2} & -\zeta^{-1}\end{pmatrix}$	$\mathbb{Z}_5$	$\mathbb{Z}_5$
	$\mathcal{Z}_2$:	$\begin{pmatrix}\xi & \frac{1}{2}(\xi-1)\\ \frac{1}{2}(\xi-1) & \xi\end{pmatrix}$	S_4	S_4
	$\mathcal{Z}_3$:	$\begin{pmatrix}i & 0\\ 0 & i\end{pmatrix}$	$(\mathbb{Z}_4 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$	D_4
	$\mathcal{Z}_4$:	$\begin{pmatrix}\omega & 0\\ 0 & \omega\end{pmatrix}$	$S_3 \times \mathbb{Z}_6$	$\mathbb{Z}_3 \times S_3$
	$\mathcal{Z}_5$:	$\frac{i\sqrt{3}}{3}\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$	D_6	D_6
	$\mathcal{Z}_6$:	$\begin{pmatrix}\omega & 0\\ 0 & i\end{pmatrix}$	$\mathbb{Z}_{12}$	$\mathbb{Z}_6$

available flavour groups

$$\omega = e^{2\pi i/3}$$

$$\xi = \frac{1}{3}(1 + i2\sqrt{2})$$

$$\zeta = e^{2\pi i/5}$$

Generalizing MPIH mechanism:

$$d(\tau_{(g)}, \tau'_{(g)}) = \left(\sum_{k=1}^{g} \ln^2 \frac{1+\sqrt{\lambda_k}}{1-\sqrt{\lambda_k}} \right)^{1/2}$$

This distance

leaves invariant

$$ds^2 = \mathrm{Tr}\left(Y^{-1} d\tau_{(g)}\, Y^{-1} d\bar{\tau}_{(g)}\right)$$

this genus 2 metric

and λ_k are eigenvalues of

$$\mathcal{D}_g(\tau_{(g)}, \tau'_{(g)}) = (\tau_{(g)} - \tau'_{(g)})(\tau_{(g)} - \bar{\tau}'_{(g)})^{-1}(\bar{\tau}_{(g)} - \bar{\tau}'_{(g)})(\bar{\tau}_{(g)} - \tau'_{(g)})^{-1}$$

this cross-ratio

"Trajectories" at genus 2

$\Sigma \rightarrowtail \Sigma^*$		$H_0^*/\langle H \rangle$	$\Delta\,\mathrm{dim}$
$\mathcal{T}_2$	$\mathcal{Z}_2$	$\mathbb{Z}_2$	2
$\mathcal{T}_1$	$\mathcal{Z}_3$	D_4	2
$\mathcal{T}_2$		$\mathbb{Z}_2 \times \mathbb{Z}_2$	2
$\mathcal{O}_1$		$\mathbb{Z}_2$	1
$\mathcal{O}_3$		$\mathbb{Z}_2$	1
$\mathcal{T}_1$	$\mathcal{Z}_4$	$S_3 \times \mathbb{Z}_3$	2
$\mathcal{T}_2$		$\mathbb{Z}_6$	2
$\mathcal{O}_2$		$\mathbb{Z}_3$	1
$\mathcal{O}_3$		$\mathbb{Z}_3$	1
$\mathcal{T}_2$	$\mathcal{Z}_5$	$\mathbb{Z}_2$	2
$\mathcal{O}_5$		$\mathbb{Z}_2$	1
$\mathcal{T}_1$	$\mathcal{Z}_6$	$\mathbb{Z}_6$	2
$\mathcal{O}_1$		$\mathbb{Z}_3$	1
$\mathcal{O}_2$		$\mathbb{Z}_2$	1
$\mathcal{T}_1$	$\mathcal{O}_1$	$\mathbb{Z}_2$	1
$\mathcal{T}_1$	$\mathcal{O}_2$	$\mathbb{Z}_3$	1
$\mathcal{T}_1$	$\mathcal{O}_3$	$\mathbb{Z}_2$	1
$\mathcal{T}_2$		$\mathbb{Z}_2$	1
$\mathcal{T}_2$	$\mathcal{O}_4$	$\mathbb{Z}_2$	1
$\mathcal{T}_2$	$\mathcal{O}_5$	1	1

check the
residual symmetries!

We restrict to subspace

$$\begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{pmatrix}$$

→

Here $\Gamma \cong (S_3 \times S_3) \rtimes \mathbb{Z}_2$

Left quarks

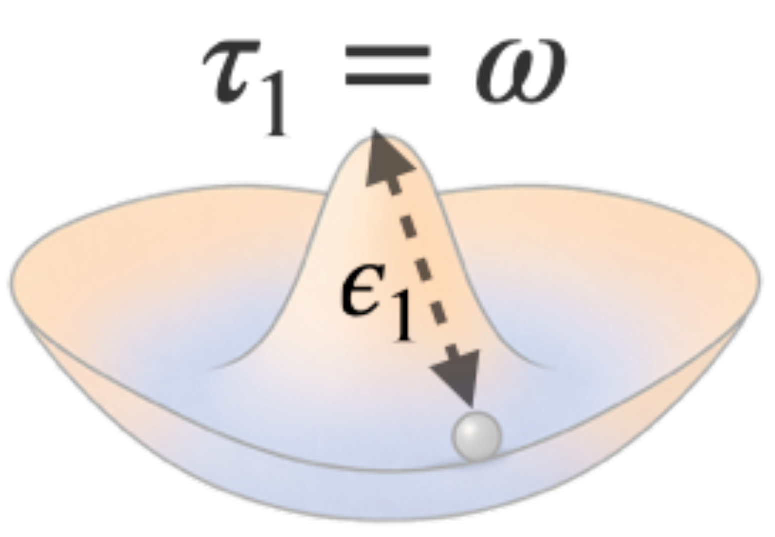
$$Q \sim \mathbf{2} \oplus \mathbf{1}_a$$

Right quarks

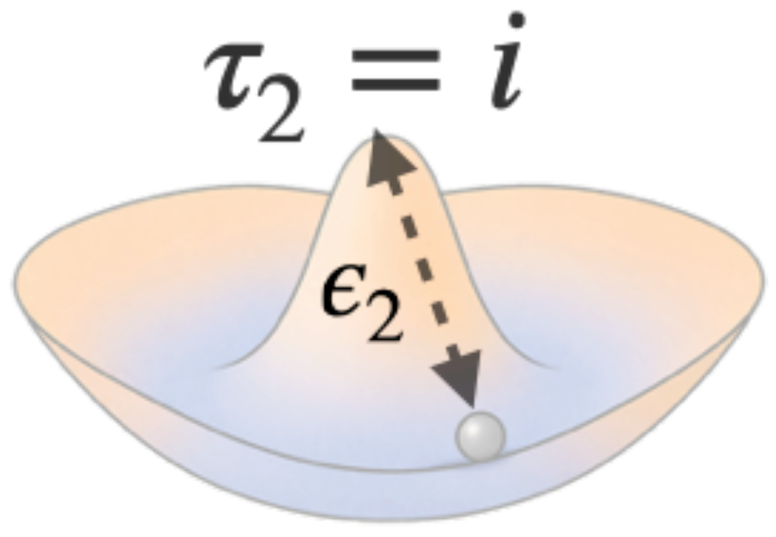
$$u^c, d^c \sim \mathbf{1}_a \oplus \mathbf{1}_b \oplus \mathbf{1}_c$$

$$\begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{pmatrix} \rightarrowtail \begin{pmatrix} \omega & 0 \\ 0 & \tau_2 \end{pmatrix}, \begin{pmatrix} \omega & 0 \\ 0 & i \end{pmatrix}, \begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix}, \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$$

MPIH at genus $g = 2$



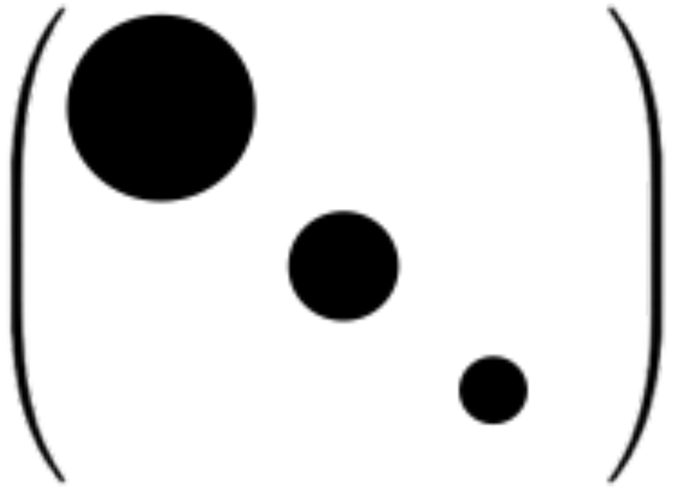
$$\epsilon^3 \equiv \underbrace{\left|\frac{\tau_1 - \omega}{\tau_1 - \omega^2}\right|^2}_{\epsilon_1} \cdot \underbrace{\left|\frac{\tau_2 - i}{\tau_2 + i}\right|}_{\epsilon_2}$$



$\arg(\epsilon_1) \neq \arg(\epsilon_2)$

More complex phases available for CP violation!

$M_{quarks} \sim$



$$= \begin{pmatrix} 1 & & \\ & \epsilon & \\ & & \epsilon^3 \end{pmatrix}$$

$$V_{CKM} = \mathbb{1} + \epsilon\, V_1 + \epsilon^2 V_2 + \ldots.$$

$$Y_u = \begin{pmatrix} \alpha_1 (Y_{\mathbf{2}}^{(10)})_2 & \alpha_2 (Y_{\mathbf{2},1}^{(12)})_1 + \alpha_3 (Y_{\mathbf{2},2}^{(12)})_1 & \alpha_4 (Y_{\mathbf{2}}^{(10)})_1 \\ -\alpha_1 (Y_{\mathbf{2}}^{(10)})_1 & \alpha_2 (Y_{\mathbf{2},1}^{(12)})_2 + \alpha_3 (Y_{\mathbf{2},2}^{(12)})_2 & \alpha_4 (Y_{\mathbf{2}}^{(10)})_2 \\ 0 & \alpha_5 Y_{\mathbf{1}}^{(10)} & \alpha_6 Y_{\mathbf{1}}^{(8)} \end{pmatrix},$$

$$Y_d = \begin{pmatrix} \beta_1 (Y_{\mathbf{2}}^{(10)})_1 & \beta_2 (Y_{\mathbf{2},1}^{(12)})_2 + \beta_3 (Y_{\mathbf{2},2}^{(12)})_2 & \beta_4 (Y_{\mathbf{2}}^{(10)})_1 \\ -\beta_1 (Y_{\mathbf{2}}^{(10)})_2 & \beta_2 (Y_{\mathbf{2},1}^{(12)})_1 + \beta_3 (Y_{\mathbf{2},2}^{(12)})_1 & \beta_4 (Y_{\mathbf{2}}^{(10)})_2 \\ 0 & \beta_5 Y_{\mathbf{1}''}^{(10)} & \beta_6 Y_{\mathbf{1}}^{(8)} \end{pmatrix},$$

Mostly $\mathcal{O}(1)$ parameters from the fit

Existence of a viable model is
non-trivial, very constrained!

$$\begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{pmatrix} \xrightarrow{VEV} \begin{pmatrix} \omega & 0 \\ 0 & i \end{pmatrix}$$

$$Y_u \sim \begin{pmatrix} \epsilon & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \\ 0 & \epsilon^2 & \epsilon^2 \end{pmatrix}, \qquad Y_d \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ 0 & \epsilon & \epsilon^2 \end{pmatrix}$$

$$\epsilon_1 \sim \epsilon_2 \equiv \epsilon$$

$$\begin{pmatrix} \omega & 0 \\ 0 & i \end{pmatrix}$$

$m_{up} \sim (1, \epsilon, \epsilon^3)$ $m_{down} \sim (1, \epsilon^2, \epsilon^2)$

$$\epsilon \sim 10^{-2} \sim \lambda^2$$

$$|V| \sim \begin{pmatrix} 1 - \mathcal{O}(10^2)|\epsilon|^2 & \mathcal{O}(10)|\epsilon| & \mathcal{O}(10)|\epsilon|^2 \\ \mathcal{O}(10)|\epsilon| & 1 - \mathcal{O}(10^2)|\epsilon|^2 & |\epsilon| \\ \mathcal{O}(10^2)|\epsilon|^2 & |\epsilon| & 1 - \mathcal{O}(10)|\epsilon|^2 \end{pmatrix} + \mathcal{O}(|\epsilon|^3)\,,$$

$$J_{\mathrm{CP}} = |\mathrm{Im}(V_{11}^* V_{12} V_{21} V_{22}^*)| \sim \mathcal{O}(10^3)\,|\epsilon|^4\,.$$

First time!

Just a proof of concept…

The road is still long…

Thank you for the attention

Roma Tre Neutrino Group

SCAN ME

BACKUP SLIDES

Fit results of 2604.21979

Region	$\mathcal{O}_2$	$\mathcal{Z}_4$	$\mathcal{Z}_6$
τ_1	$\omega + 0.047$	$\omega + 0.072$	$\omega + 0.019$
τ_2	$-0.196 + 2.220\,i$	$-0.537 + 0.927\,i$	$0.029 + 0.996\,i$
α_1/α_5	-0.751	-0.388	-0.286
α_2/α_5	56.00	1.205	-12.08
α_3/α_5	1.303	-7.124	-3.685
α_4/α_5	1.594	-1.178	-0.464
α_6/α_5	0.035	0.188	0.067
β_1/β_6	-1.209	0.076	-0.076
β_2/β_6	0.034	-55.42	0.206
β_3/β_6	0.046	0.977	14.05
β_4/β_6	30.22	7.492	0.291
β_5/β_6	2.444	0.223	-0.084
$v_u\,\alpha_5$, GeV	15.30	16.04	23.23
$v_d\,\beta_6$, GeV	0.370	0.250	0.462

Region	$\mathcal{O}_2$	$\mathcal{Z}_4$	$\mathcal{Z}_6$
y_u/y_t	$5.383 \cdot 10^{-6}$	$5.396 \cdot 10^{-6}$	$2.304 \cdot 10^{-6}$
y_c/y_t	$2.674 \cdot 10^{-3}$	$2.650 \cdot 10^{-3}$	$2.687 \cdot 10^{-3}$
y_d/y_b	$6.711 \cdot 10^{-4}$	$7.185 \cdot 10^{-4}$	$7.370 \cdot 10^{-4}$
y_s/y_b	$1.367 \cdot 10^{-2}$	$1.406 \cdot 10^{-2}$	$1.359 \cdot 10^{-2}$
θ_{12}	$2.274 \cdot 10^{-1}$	$2.275 \cdot 10^{-1}$	$2.273 \cdot 10^{-1}$
θ_{13}	$3.167 \cdot 10^{-3}$	$2.770 \cdot 10^{-3}$	$3.201 \cdot 10^{-3}$
θ_{23}	$3.784 \cdot 10^{-2}$	$4.150 \cdot 10^{-2}$	$3.562 \cdot 10^{-2}$
δ_{CP}	1.211	1.204	1.217
$\lvert\epsilon_1\rvert$	0.027	0.042	0.011
$\lvert\epsilon_2\rvert$	–	0.040	0.015
$\chi^2_{\min}$	0.11	1.41	0.97

Mostly $\mathcal{O}(1)$ parameters from the fit

Modular symmetry protected seesaw

- Charged-leptons mass hierarchies controlled by the same ϵ which breaks L-symmetry, and gives $m_\nu \neq 0$, $\Delta M \neq 0$!

Model	Group	$\tau_{\rm sym}$	L	E^c	N^c	ΔM
A	A_4	ω	$(\mathbf{1},+2)\oplus(\mathbf{1}',+4)\oplus(\mathbf{1}'',+6)$	$(\mathbf{3},0)$	$(\mathbf{3},0)$	$\sim m_\nu$
B			$(\mathbf{1}',+1)\oplus(\mathbf{1}'',+3)\oplus(\mathbf{1},+5)$	$(\mathbf{3},+1)$	$(\mathbf{3},+1)$	$\sim \epsilon\, M$
C	S_4'	$i\infty$	$(\hat{\mathbf{1}},+2)\oplus(\mathbf{1},+3)\oplus(\hat{\mathbf{1}},+4)$	$(\mathbf{3},+1)$	$(\mathbf{3},+1)$	$\sim \epsilon^2\, M$
D			$(\hat{\mathbf{1}}',+2)\oplus(\mathbf{1},+3)\oplus(\hat{\mathbf{1}}',+4)$			

Let's focus on this

Notation: $\psi \sim (\rho, k_\psi)$

An example: model C

Group
$\Gamma'_4 \simeq S'_4$

L	E^c	N^c	ΔM
$(\hat{\mathbf{1}},+2)\oplus(\mathbf{1},+3)\oplus(\hat{\mathbf{1}},+4)$	$(\mathbf{3},+1)$	$(\mathbf{3},+1)$	$\sim \epsilon^2 M$

$\tau_{sym} \simeq i\infty$

$$M_R = \Lambda \begin{pmatrix} \frac{2}{\sqrt{3}}\left(Y_{\mathbf{2}}^{(2)}\right)_1 & 0 & 0 \\ 0 & \left(Y_{\mathbf{2}}^{(2)}\right)_2 & -\frac{1}{\sqrt{3}}\left(Y_{\mathbf{2}}^{(2)}\right)_1 \\ 0 & -\frac{1}{\sqrt{3}}\left(Y_{\mathbf{2}}^{(2)}\right)_1 & \left(Y_{\mathbf{2}}^{(2)}\right)_2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & \epsilon^2 & 1 \\ 0 & 1 & \epsilon^2 \end{pmatrix}$$

L-conserving for $\epsilon \to 0$

$$\epsilon \sim q^{1/4} = e^{i\pi\tau/2}$$

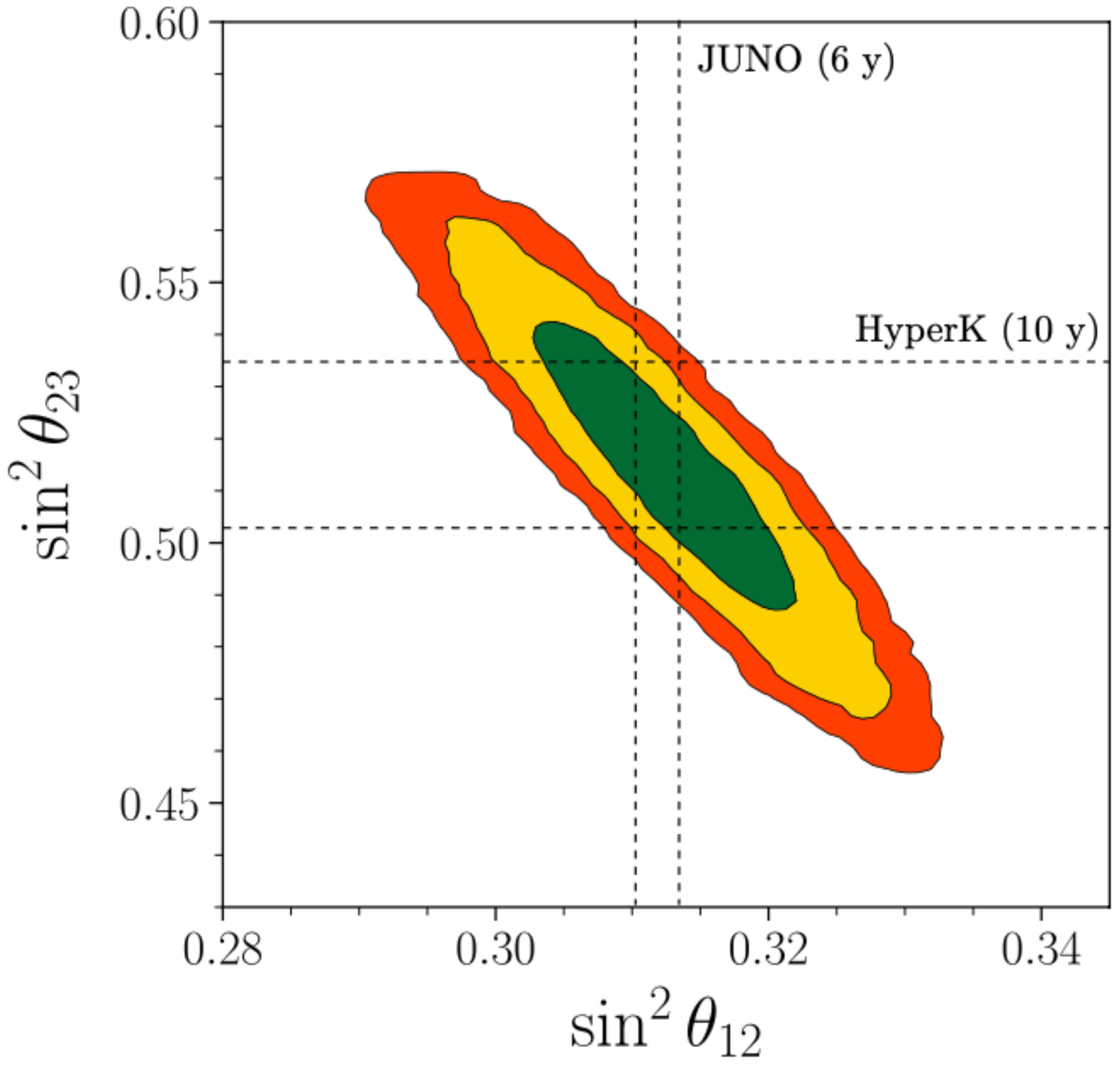
$$Y_D = \begin{pmatrix} g_1\left(Y_{\hat{\mathbf{3}}'}^{(3)}\right)_1 & g_1\left(Y_{\hat{\mathbf{3}}'}^{(3)}\right)_3 & g_1\left(Y_{\hat{\mathbf{3}}'}^{(3)}\right)_2 \\ g_2\left(Y_{\mathbf{3}}^{(4)}\right)_1 & g_2\left(Y_{\mathbf{3}}^{(4)}\right)_3 & g_2\left(Y_{\mathbf{3}}^{(4)}\right)_2 \\ g_3\left(Y_{\hat{\mathbf{3}}'}^{(5)}\right)_1 & g_3\left(Y_{\hat{\mathbf{3}}'}^{(5)}\right)_3 & g_3\left(Y_{\hat{\mathbf{3}}'}^{(5)}\right)_2 \end{pmatrix}$$

$$Y_e = \begin{pmatrix} \alpha_1\left(Y_{\hat{\mathbf{3}}'}^{(3)}\right)_1 & \alpha_1\left(Y_{\hat{\mathbf{3}}'}^{(3)}\right)_3 & \alpha_1\left(Y_{\hat{\mathbf{3}}'}^{(3)}\right)_2 \\ \alpha_2\left(Y_{\mathbf{3}}^{(4)}\right)_1 & \alpha_2\left(Y_{\mathbf{3}}^{(4)}\right)_3 & \alpha_2\left(Y_{\mathbf{3}}^{(4)}\right)_2 \\ \alpha_3\left(Y_{\hat{\mathbf{3}}'}^{(5)}\right)_1 & \alpha_3\left(Y_{\hat{\mathbf{3}}'}^{(5)}\right)_3 & \alpha_3\left(Y_{\hat{\mathbf{3}}'}^{(5)}\right)_2 \end{pmatrix}$$

$$\sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

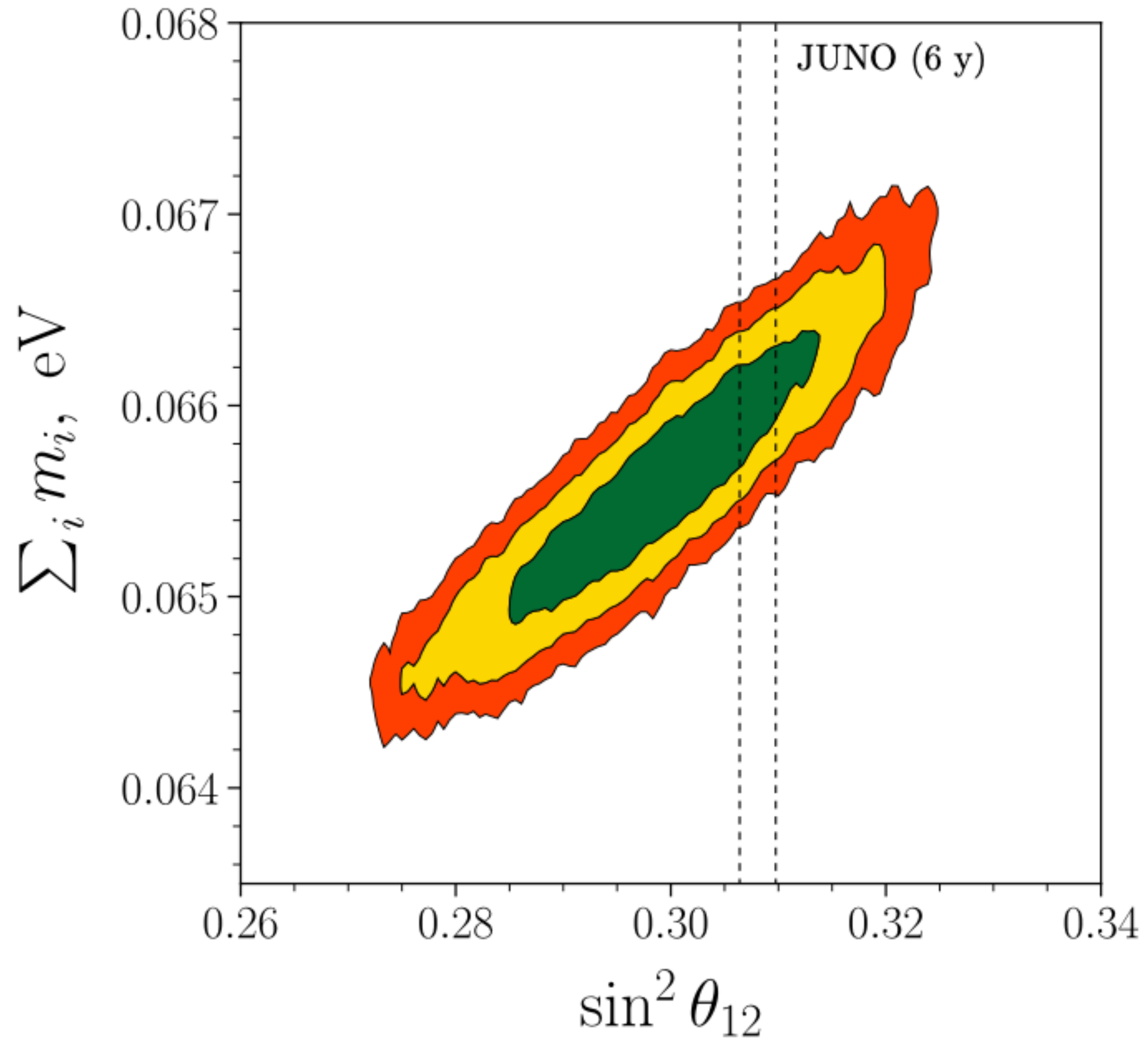
Correlations and pheno

MODEL C



9 free params (includes Λ and τ)

MODEL D



11 free params (includes Λ and τ)

1σ

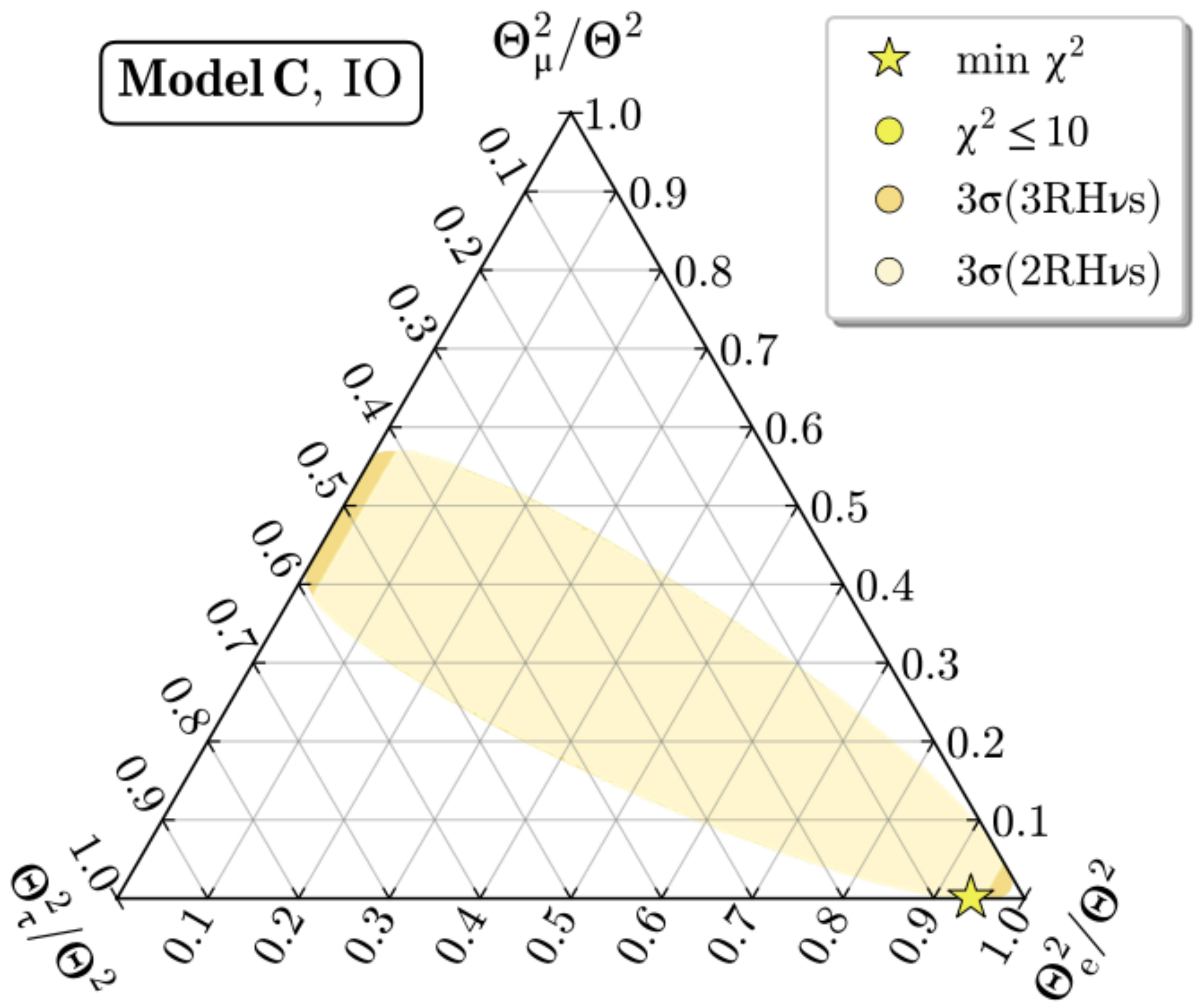
2σ

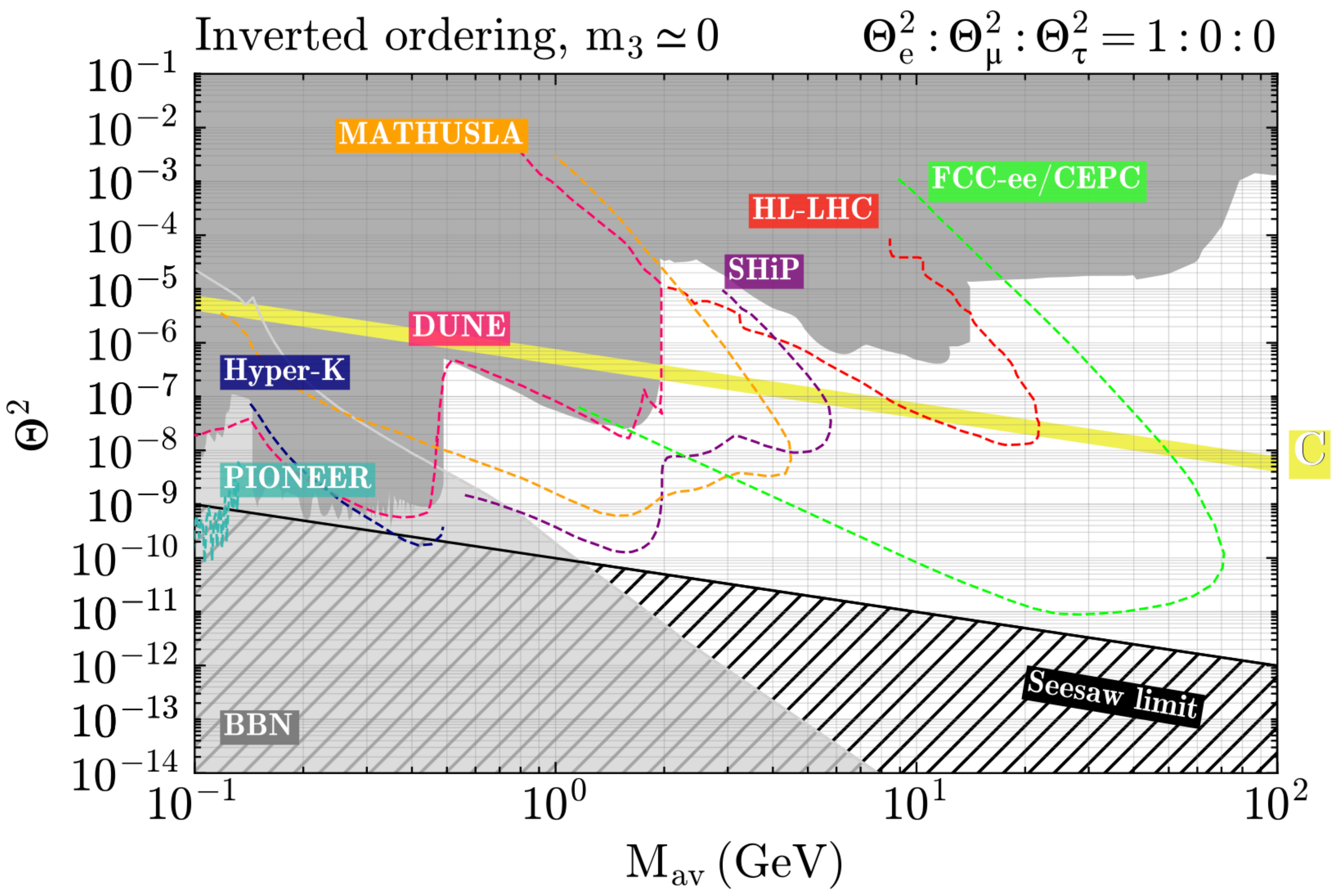
3σ

Pheno: model C (Inverted Ordering)

$$\Theta_{\alpha j} \equiv v_u \hat{Y}_{\alpha j}/M_j .$$

$$\Theta_\alpha^2 \equiv \sum_{j=1}^{3} |\Theta_{\alpha j}|^2 \quad \text{and} \quad \Theta^2 \equiv \sum_{\alpha=e,\,\mu,\,\tau} \Theta_\alpha^2 .$$





$$M_{av} = \frac{\sum_i M_i}{3}$$

The numerics of the benchmarks

Model (ordering)	A (NO)	B (NO)	C (IO)	D (NO)
$\mathrm{Re}\,\tau$	$-0.472^{+0.017}_{-0.028}$	$-0.475^{+0.036}_{-0.024}$	$[-1/2, +1/2]$	$[-1/2, +1/2]$
$\mathrm{Im}\,\tau$	$0.892^{+0.019}_{-0.042}$	$0.912^{+0.015}_{-0.062}$	$2.55^{+0.07}_{-0.04}$	$2.54^{+0.13}_{-0.12}$
$\hat{\alpha}_2/\hat{\alpha}_1$	$-0.286^{+0.674}_{-0.106}$	$0.590^{+0.125}_{-0.083}$	$-0.593^{+0.102}_{-0.078}$	$1.25^{+0.34}_{-0.44}$
$\hat{\alpha}_{3(,1)}/\hat{\alpha}_1$	$-0.160^{+0.145}_{-0.481}$	$-0.0502^{+0.0501}_{-0.1540}$	$0.0339^{+0.0026}_{-0.0024}$	$0.0531^{+0.0062}_{-0.0081}$
$\hat{\alpha}_{3,2}/\hat{\alpha}_1$	$0.0202^{+0.0094}_{-0.0514}$	$-0.0185^{+0.0037}_{-0.0062}$	—	$0.158^{+0.025}_{-0.032}$
$\hat{g}_2/\hat{g}_1$	$0.712^{+0.052}_{-0.076}$	$0.219^{+0.011}_{-0.008}$	$0.0799^{+0.0137}_{-0.0125}$	$7.1^{+15.0}_{-2.2}$
$\hat{g}_{3(,1)}/\hat{g}_1$	$0.491^{+0.196}_{-0.461}$	$0.336^{+0.062}_{-0.161}$	$13.2^{+1.1}_{-1.2}$	$0.0985^{+0.0197}_{-0.0984}$
$\hat{g}_{3,2}/\hat{g}_1$	$0.214^{+0.119}_{-0.551}$	$0.289^{+0.018}_{-0.020}$	—	$-0.21^{+0.17}_{-5.60}$
$v_d\,\hat{\alpha}_1$, GeV	$0.594^{+0.070}_{-0.121}$	$0.363^{+0.047}_{-0.055}$	$0.176^{+0.005}_{-0.007}$	$0.161^{+0.015}_{-0.013}$
$v_u^2\,\hat{g}_1^2/\Lambda$, eV	$0.0328^{+0.0145}_{-0.0070}$	$0.0568^{+0.0192}_{-0.0120}$	$0.00032^{+0.00005}_{-0.00003}$	$0.0104^{+0.0053}_{-0.0094}$
m_e/m_μ	$0.0048^{+0.0006}_{-0.0005}$	$0.0047^{+0.0006}_{-0.0005}$	$0.0048^{+0.0005}_{-0.0006}$	$0.0049^{+0.0004}_{-0.0006}$
m_μ/m_τ	$0.0560^{+0.0115}_{-0.0114}$	$0.0571^{+0.0114}_{-0.0121}$	$0.0577^{+0.0109}_{-0.0134}$	$0.0580^{+0.0108}_{-0.0113}$
r	$0.0297^{+0.0022}_{-0.0021}$	$0.0297^{+0.0023}_{-0.0021}$	$0.0300^{+0.0023}_{-0.0021}$	$0.0299^{+0.0019}_{-0.0021}$
$\sin^2\theta_{12}$	$0.307^{+0.034}_{-0.030}$	$0.308^{+0.032}_{-0.030}$	$0.312^{+0.019}_{-0.018}$	$0.308^{+0.024}_{-0.033}$
$\sin^2\theta_{13}$	$0.0220^{+0.0017}_{-0.0014}$	$0.0221^{+0.0014}_{-0.0014}$	$0.0222^{+0.0016}_{-0.0014}$	$0.0220^{+0.0017}_{-0.0013}$
$\sin^2\theta_{23}$	$0.506^{+0.067}_{-0.065}$	$0.507^{+0.065}_{-0.062}$	$0.519^{+0.049}_{-0.058}$	$0.507^{+0.070}_{-0.051}$

The numerics of the benchmarks

Model (ordering)	A (NO)	B (NO)	C (IO)	D (NO)
m_1, eV	$< 10^{-4}$	$< 10^{-4}$	$0.0491^{+0.0002}_{-0.0002}$	$0.0054^{+0.0009}_{-0.0010}$
m_2, eV	$0.00864^{+0.00028}_{-0.00029}$	$0.00865^{+0.00029}_{-0.00028}$	$0.0499^{+0.0001}_{-0.0002}$	$0.0102^{+0.0005}_{-0.0006}$
m_3, eV	$0.0501^{+0.0001}_{-0.0002}$	$0.0501^{+0.0002}_{-0.0002}$	$< 10^{-4}$	$0.0504^{+0.0002}_{-0.0002}$
$\Sigma_i m_i$, eV	$0.0588^{+0.0001}_{-0.0001}$	$0.0588^{+0.0001}_{-0.0001}$	$0.0990^{+0.0003}_{-0.0004}$	$0.0660^{+0.0015}_{-0.0015}$
$m_{\beta\beta}$, meV	$1.49^{+0.31}_{-0.28}$	$1.49^{+0.27}_{-0.28}$	$17.8^{+1.7}_{-1.8}$	$1.69^{+0.13}_{-0.11}$
δ	$\simeq 0,\ \pi$	$\simeq 0$	$\simeq 0$	$\simeq \pi$
$\alpha_{21(23)}$	$\simeq \pi$	$\simeq \pi$	$\simeq 0$	$\simeq \pi$
α_{31}	—	—	—	$\simeq 0$
ΔM	$0.0416^{+0.0019}_{-0.0018}$ eV	$(0.149^{+0.051}_{-0.034})\ M$	$(0.003^{+0.015}_{-0.003})\ M$	$(0.010^{+0.014}_{-0.010})\ M$
$\lvert\epsilon(\tau)\rvert$	$0.0218^{+0.0047}_{-0.0046}$	$0.0292^{+0.0076}_{-0.0067}$	$0.0182^{+0.0013}_{-0.0018}$	$0.0184^{+0.0039}_{-0.0034}$
$(m_e/m_\tau)/\lvert\epsilon\rvert^{2,3}$	$0.563^{+0.167}_{-0.111}$	$0.317^{+0.123}_{-0.088}$	$45.8^{+3.1}_{-3.1}$	$44.9^{+19.4}_{-16.0}$
$(m_\mu/m_\tau)/\lvert\epsilon\rvert$	$2.57^{+0.05}_{-0.08}$	$1.96^{+0.20}_{-0.24}$	$3.16^{+0.39}_{-0.52}$	$3.14^{+0.20}_{-0.37}$
min $N\sigma$	0.412	0.411	0.548	0.552
max par. ratio	49.6	54.1	165	71.8
min(max par. ratio)	32.1	40.5	133	55.2

Potential shortcomings

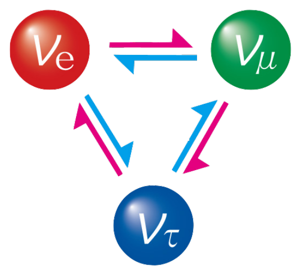
Bottom-up problems

Kähler potential

Normalisation of the modular forms

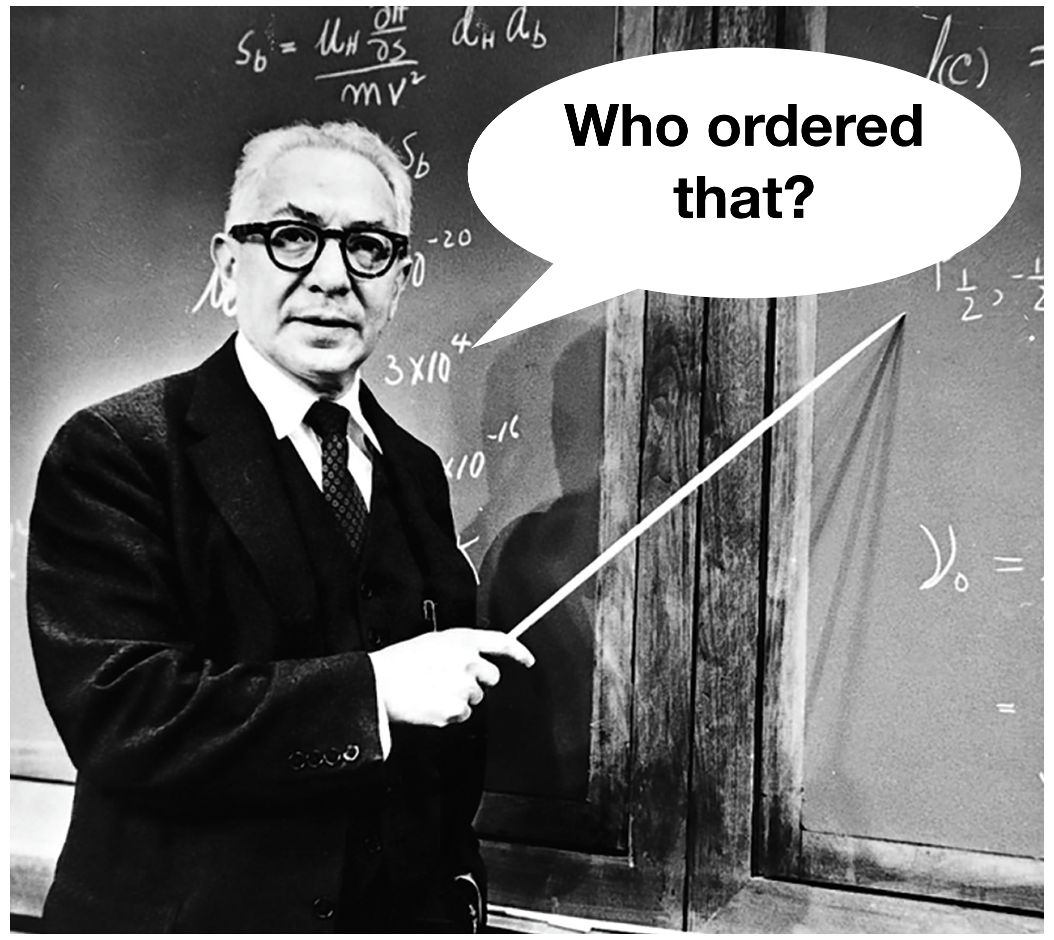
Modulus stabilization

SUSY-dependent theory



Three “copies”, different masses

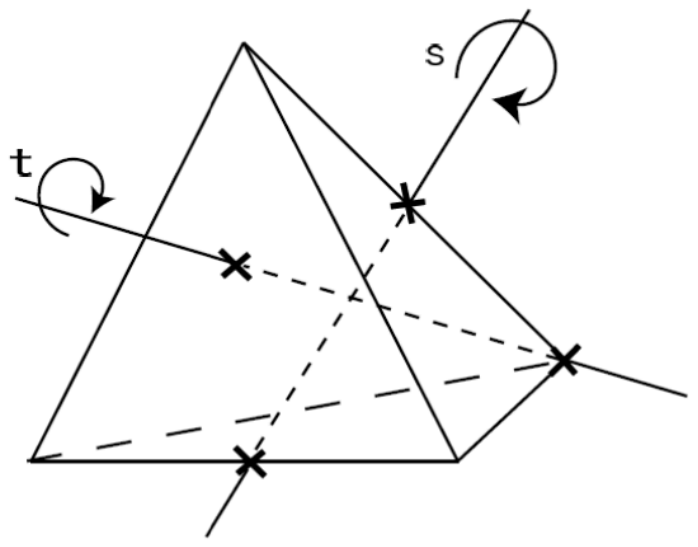
ν_e	ν_μ	ν_τ
e	μ	τ



Isidor I. Rabi

non-abelian discrete symmetries

$$S_3 \quad A_4 \quad S_4 \quad A_5 \ldots\ldots$$



$$\mathscr{W}_{Yukawa} \supset \frac{\alpha}{\Lambda} E^c (L\phi_i)_{\mathbf{1}} H_d$$

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-bimaximal compatible with data until 2012

$$\theta_{13} = 0 \quad \theta_{12} \simeq 35^\circ \quad \theta_{23} = 45^\circ$$

Not zero

F. P. An *et al.*, “Observation of electron-antineutrino disappearance at daya bay,” *Phys. Rev. Lett.* **108** (Apr, 2012)

The Modular symmetry approach

Modular-invariant SUSY action

$$\mathcal{S} = \int d^4x \int d^2\theta d^2\bar{\theta}\, K(\Phi, \bar{\Phi}) + \left[\int d^4x \int d^2\theta\, \mathcal{W}(\Phi) + \text{h.c.}\right]$$

Kähler potential

- Gives the kinetic terms after the modulus acquires a VEV
- A minimalistic form is chosen

Superpotential

- Holomorphic function of superfields
- Encodes the Higgs Yukawa interactions

$$\sigma \equiv \Lambda_\tau \tau$$

The superfields transform as:

$$\begin{cases} \tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases}, \quad \text{with} \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_N$$

Action is invariant if, under Γ_N:

$$\begin{cases} \mathcal{W}(\Phi) \to \mathcal{W}(\Phi) \\ K(\Phi, \bar{\Phi}) \to \underbrace{K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi})}_{\text{Kähler transformation}} \end{cases}$$

- $\theta, \bar{\theta}$ Grassmann spinor coordinates
- $\Phi = (\tau, \varphi)$ Chiral superfields
- φ Usual matter supermultiplets
- $\rho(\gamma)$ Unitary representation of Γ_N

The Kähler potential...

- Minimalist choice for the Kähler

$$K(\Phi, \bar{\Phi}) = -h\Lambda_\tau^2 \log(-i\tau + i\bar{\tau}) + \sum_I (-i\tau + i\bar{\tau})^{-k_I} |\varphi^{(I)}|^2$$

$\Lambda_\tau \equiv$ dimensions of mass

Satisfies $K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi})$ under Γ_N

$h \equiv$ positive constant

- In a bottom-up approach, this is just a choice

- Corrections of the Kähler potential can spoil the predictivity of the model

M.-C. Chen, S. Ramos-Sánchez, and M. Ratz, "A note on the predictions of models with modular flavor symmetries," *Physics Letters B* **801** (Feb, 2020) 135153.

- This question is an open one

The Modular symmetry approach

The group generators

▶ Finite modular group can be defined: $\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}) \middle| \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

subgroups of Γ N=1,2,3...called "level"

$$\overline{\Gamma} \equiv \Gamma/\{\pm\mathbb{I}\}$$
$$\overline{\Gamma}(N) \equiv \Gamma(N)/\{\pm\mathbb{I}\}$$

▶ Generators S and T of the modular group Γ_N

$$\tau \xrightarrow{S} -\frac{1}{\tau} \qquad \tau \xrightarrow{T} \tau + 1 \qquad S^2 = T^N = (ST)^3 = \mathbb{I}$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

▶ S_3 Generators S and T satisfy:

$$S^2 = T^2 = (ST)^3 = \mathbb{I}$$

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I},$$

Backup slides

Numerical procedure

- Define a "figure of merit", i.e. chi-square for every set of parameters $l(p_i) \equiv \sqrt{\chi^2(p_i)}$

$$\chi^2(p_i) = \sum_{j=1}^{6} \left(\frac{q_j(p_i) - q_j^{\text{b-f}}}{\sigma_j} \right)^2$$

$$p_i = \{\tau, \beta/\alpha, \gamma/\alpha, ..., g'/g, g_p/g, ...\}$$

$$q_j = \{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, m_e/m_\mu, m_\mu/m_\tau, r\}$$

- Define a "potential" with a given temperature T and a threshold

$$V(p_i) = \begin{cases} l(p_i) & , \quad l(p_i) \le l_{\text{max}} \\ +\infty & , \quad \text{otherwise} \end{cases}$$

P. P. Novichkov, J. T. Penedo, S. T. Petcov, and A. V. Titov, "Modular S_4 models of lepton masses and mixing," (2019)

- At iteration "t", generate a new point from a Gaussian centred on the previous one →
- Accept the new point with a probability given by:

$$P_\alpha = \min[1, \exp(V(p_i^{(t)}) - V(p_i'))/T]$$

Non-standard interactions

tests of modulus couplings

G-J. Ding, FF, 2003.13448

non standard neutrino interactions

$$\mathcal{L} = i \sum_{f=e,e^c,\nu} \bar{f}\,\bar{\sigma}^\mu \partial_\mu f + \frac{1}{2}\partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha - \frac{1}{2} M_\alpha^2 \varphi_\alpha^2 - (m_e + \mathcal{Z}_\alpha^e \varphi_\alpha) e^c e - \frac{1}{2}\nu(m_\nu + \mathcal{Z}_\alpha^\nu \varphi_\alpha)\nu + h.c. + \dots$$

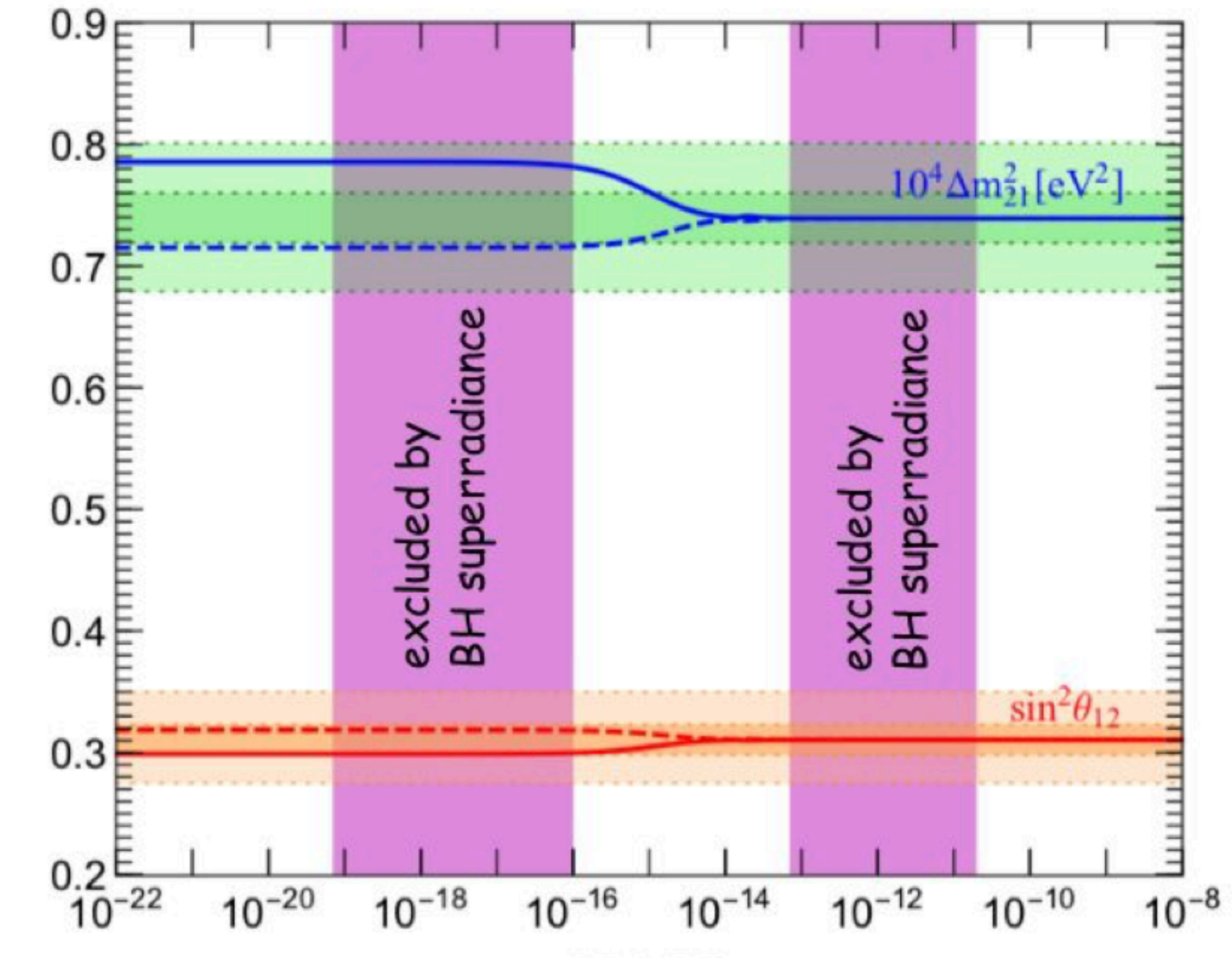
$$\tau = \langle \tau \rangle + \frac{\varphi_u + i\,\varphi_v}{\sqrt{2}}$$

in medium with non-zero electron number density

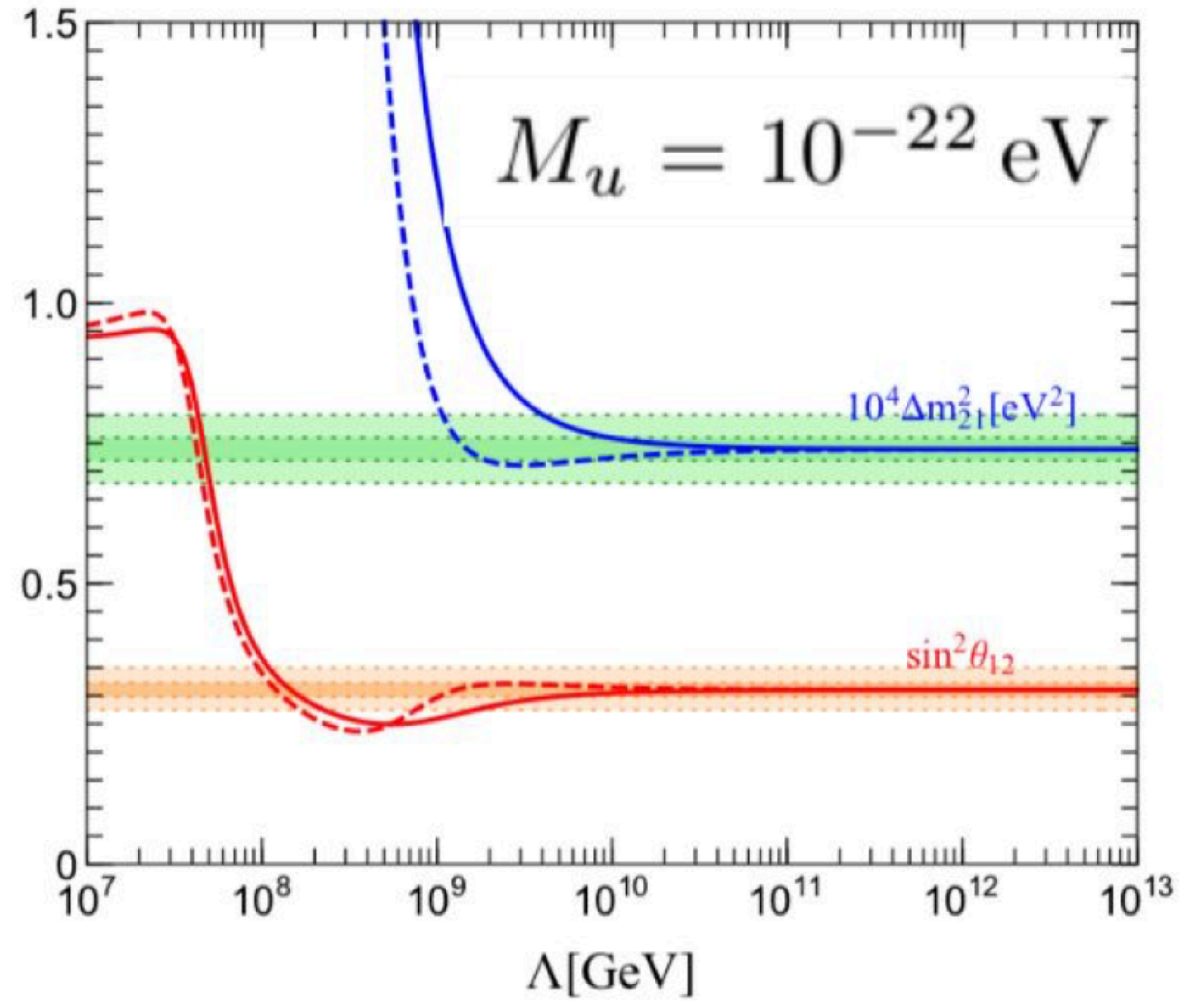
small, unless the modulus is very light

$$\delta m_\nu(0) = -n_e^0 \frac{\mathrm{Re}(\mathcal{Z}^e)\mathcal{Z}^\nu}{M^2(R)},$$

in the sun:



$\Lambda = 5 \times 10^9$ GeV



from Feruglio's slides at Mod. Symmetry Bethe Workshop