4d N = 4 String Islands from Asymmetric Orbifolds

Elisa Iris Marieni

Based on WIP with G. Aldazabal, E. Andrés, A. Font, K. Narain, I. Zadeh

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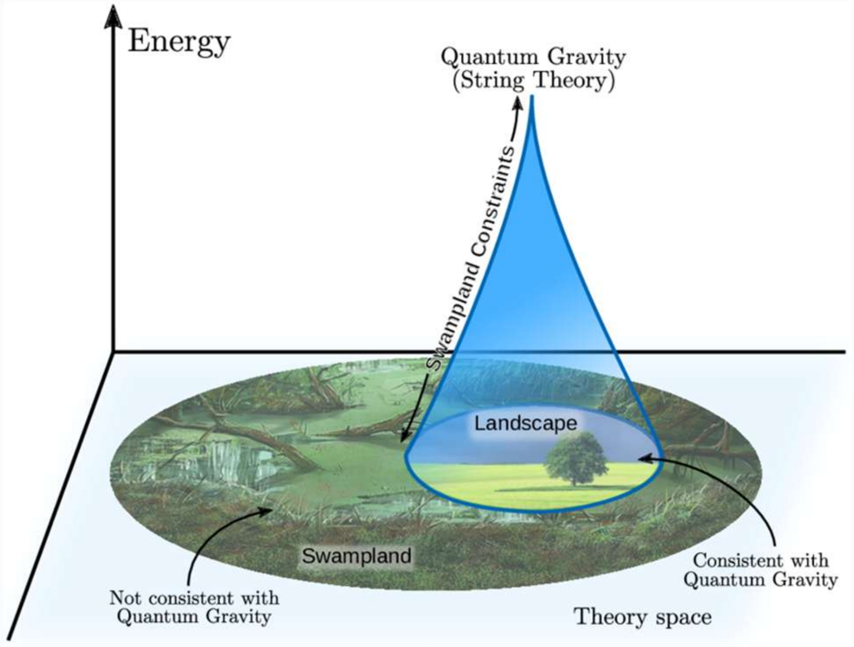
Motivation



Picture: [van Beest, Calderón-Infante, Mirfendereski, Valenzuela; 20]

Since the landscape is finite, it's worth studying:

-understand Quantum Gravity

-moduli space of CFTs

Different approach to study the landscape

Asymmetric Orbifolds

Non geometric construction, give a controlled way of finding isolated points in the landscape: ISLANDS

String Islands: what and why

Low energy theories of compactified string theories contain many massless scalars

e.g. Type II on T^6

$$(\pm 2) + 8(\pm 3/2) + 28(\pm 1) + 56(\pm 1/2) + \boxed{70(0)}$$

$\mathcal{N} = 8$ gravity multiplet

Moduli: continuous parameters of the moduli space of vacua

Theories with reduced number of moduli are interesting phenomenologically (susy, rank of gauge group…)

Islands have no moduli except the dilaton: they are isolated in the landscape and standard compactification techniques cannot reach them

They are truly unexplored regions in the landscape!

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Asymmetric Orbifolds can!

They are truly unexplored regions in the landscape!

(Geometric) Toroidal orbifolds

Toroidal obifolds describe strings propagating on a singular space obtained by gauging a discrete isometry of the torus [Dixon, Harvey, Vafa, Witten; 85, 86]

$$R^{1,9-\mathrm{D}} \times T^{\mathrm{D}}/Z_N$$

Worldsheet dofs are subject to twisted boundary conditions

$$X(\sigma^0, \sigma^1 + 2\pi) = g^\ell X(\sigma^0, \sigma^1) \quad ; \qquad X(\sigma^0 + 2\pi\tau_2, \sigma^1 + 2\pi\tau_1) = g^k X(\sigma^0, \sigma^1)$$

The partition can be organised as a sum over twisted sectors

$$Z = \frac{1}{N} \sum_{\ell,k=0}^{N-1} Z\left(g^\ell, g^k\right) \quad ; \quad Z\left(g^\ell, g^k\right) = Tr_{\mathcal{H}^\ell}\left(e^{-\beta H}\right)$$

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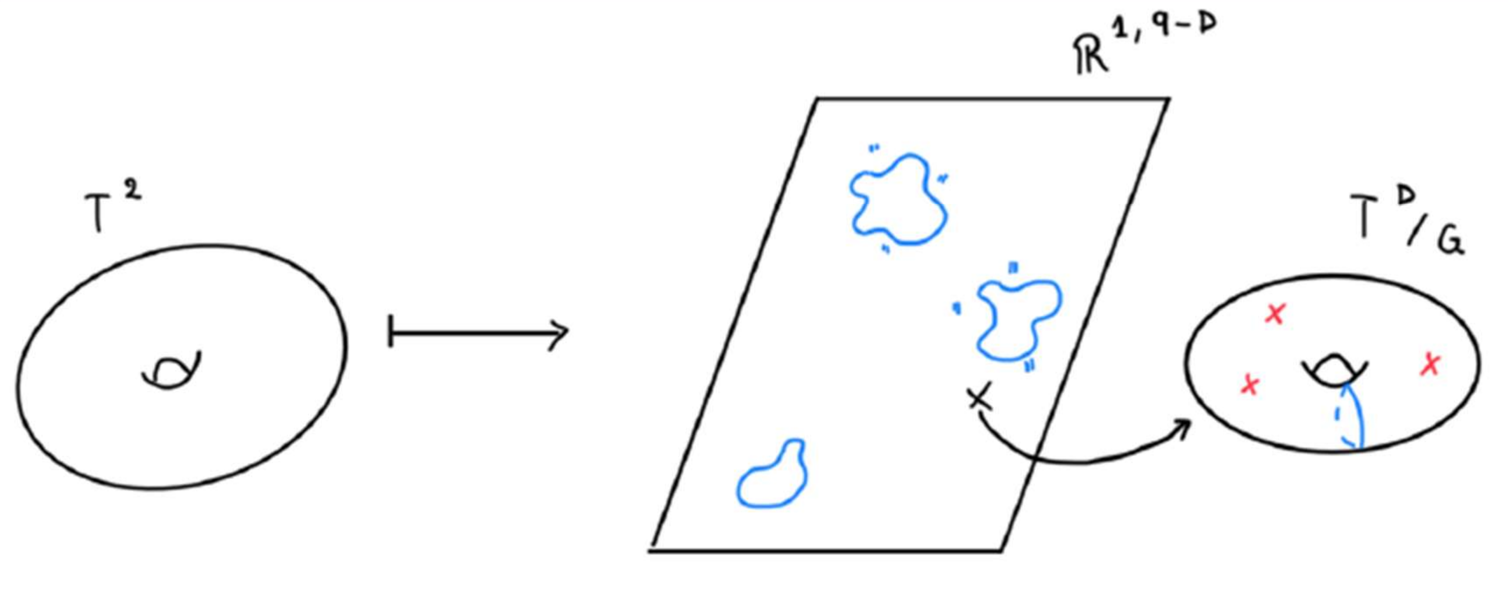
EFT of type IIA on T^6/Z_3 :

$$\mathcal{N} = 2 \qquad (\pm 2) + 2(\pm 3/2) + 37(\pm 1) + 74(\pm 1/2) + 76(0)$$

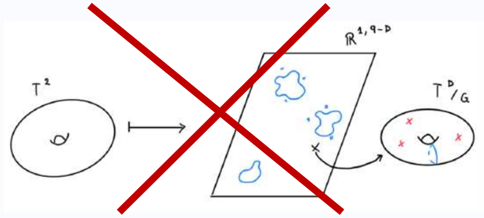
$$G_2(4) + H_2(4) + 36 V_2(4)$$



Despite the space being singular, string dynamics is smooth!

Singularities are resolved thanks to the introduction of localised string states: twisted sectors

Asymmetric Toroidal Orbifolds [Narain, Samradi, Vafa; 87]



No geometric meaning

The partition function of strings compactified on T^D is organised as a sum over quantised states (winding and momenta) living in a lattice $\Gamma_{D,D}$

Instead of modding by an isometry of T^D, we mod out by an isometry of $\Gamma_{D,D}$, that acts asymmetrically on L/R-movers

Inspired by heterotic string theory: $\Gamma_{p,q} = \Gamma_{16+D,D}$
[Narain, Samradi, Vafa; 87], [Aldazabal et al; 25]

In type II we will act only on L-movers: breaks half SUSY
[Dabholkar, Harvey; 98]], [Mizoguchi; 01], [Baykara et al; 25]

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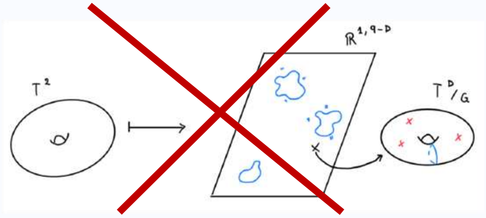

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All nice properties of orbifolds survive: exactly solvable theory

Way of getting 4d thy directly from 10d without any internal space. No internal geometry

In 4d: way to get pure N=4 supergravity with no matter

Type II island construction

DH island construction [Dabholkar, Harvey; 98]:

1. Type II on T^D: go to a point in the moduli space where $\Gamma_{D,D} = (P_L, P_R)$, $P_{L,R} \in \Lambda_w(\mathfrak{g})$ and $P_L - P_R \in \Lambda_r(\mathfrak{g})$.

2. This means that the Weyl group $\mathcal{W}(\mathfrak{g})$ is an isometry of $P_{L,R}$ separately!

3. We choose a generator Θ of Z_n and act asymmetrically: $\Theta = (\Theta_L, 0)$

4. L-movers are completely rotated and the right movers are untouched!

Consistency conditions restrict the allowed actions: modular invariance, supersymmetry, no extra massless states.

4d N=4 Island Candidates

eigenvalue	order	algebra realisation
$(\frac{1}{2},\frac{1}{2},\frac{1}{2})$	4	$\mathbf{D_4 \times A_1{}^2}$ $\mathbf{A_1{}^6}$ $\mathbf{D_6}$
$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$	6	$\mathbf{A_2{}^3}$
$(\frac{1}{2},\frac{1}{2},\frac{1}{3})$	6	$\mathbf{A_1{}^4 \times A_2}$ $\mathbf{D_4 \times A_2}$
$(\frac{1}{6},\frac{1}{6},\frac{2}{3})$	6	$\mathbf{E_6}$
$(\frac{1}{8},\frac{3}{8},\frac{3}{4})$	8	$\mathbf{D_6}$
$(\frac{1}{4},\frac{1}{2},\frac{1}{2})$	8	$\mathbf{A_3 \times A_1{}^3}$
$(\frac{1}{9},\frac{2}{9},\frac{5}{9})$	9	$\mathbf{E_6}$
$(\frac{1}{2},\frac{1}{3},\frac{1}{3})$	12	$\mathbf{A_1{}^2 \times A_2{}^2}$
$(\frac{1}{6},\frac{1}{2},\frac{1}{2})$	12	$\mathbf{D_4 \times A_1{}^2}$
$(\frac{1}{6},\frac{1}{6},\frac{1}{2})$	12	$\mathbf{D_6}$
$(\frac{1}{12},\frac{5}{12},\frac{2}{3})$	12	$\mathbf{E_6}$
$(\frac{1}{3},\frac{1}{4},\frac{1}{4})$	12	$\mathbf{A_2 \times D_4}$
$(\frac{1}{3},\frac{1}{4},\frac{3}{4})$	12	$\mathbf{A_2 \times D_4}$
$(\frac{1}{7},\frac{2}{7},\frac{4}{7})$	14	$\mathbf{A_6}$
$(\frac{1}{5},\frac{2}{5},\frac{1}{3})$	15	$\mathbf{A_4 \times A_2}$
$(\frac{1}{9},\frac{2}{9},\frac{4}{9})$	18	$\mathbf{E_6}$
$(\frac{1}{10},\frac{3}{10},\frac{1}{2})$	20	$\mathbf{D_6}$
$(\frac{1}{2},\frac{1}{5},\frac{2}{5})$	20	$\mathbf{A_1{}^2 \times A_4}$
$(\frac{1}{3},\frac{1}{4},\frac{1}{2})$	24	$\mathbf{A_2 \times A_3 \times A_1}$
$(\frac{1}{6},\frac{1}{4},\frac{1}{2})$	24	$\mathbf{D_5 \times A_1}$
$(\frac{1}{5},\frac{2}{5},\frac{2}{3})$	30	$\mathbf{A_4 \times A_2}$

Actions that break supersymmetry to $\mathcal{N} = 4$
We want to fully rotate the left movers to get islands

The ones in red appeared in [Mizoguchi; 01], who claimed to have the full classification of 4d islands

We have many new island candidates

Explicitly constructed $D_4 \times A_1^2$, D_6 islands
A_1^6 ruled out

Explicitly constructed: we found an archipelago of islands!

The Spectrum

The partition function can be expanded as a series:

$$Z = n_{1B} - n_{1F} + (n_{2B} - n_{2F})(q\bar{q})^{m_1} + (n_{3B} - n_{3F})(q\bar{q})^{m_2} + \dots$$

By choosing the appropriate orbifold action we can remove all extra moduli!

Type II on T^6

$$(\pm 2) + 8(\pm 3/2) + 28(\pm 1) + 56(\pm 1/2) + \boxed{70(0)}$$

$\mathcal{N} = 8$ gravity multiplet

Our asymmetric orbifolds

$$(\pm 2) + 4(\pm 3/2) + 6(\pm 1) + 4(\pm 1/2) + \boxed{2(0)}$$

axidilaton

$\mathcal{N} = 4$ gravity multiplet

And… Heterotic!

Similar classification can be carried out for heterotic strings as well

We found 27 candidates 4d N=4 heterotic islands

Summary and Outlook

- Asymmetric orbifolds are a convenient construction to reach islands (alternative to flux stabilisation)
- They provide a controlled way of breaking supersymmetry and reducing the rank (heterotic)
- We have populated the landscape of 4d N=4 islands with many candidates (and constructed explicitly some of them!)
- General lessons to be learned?

  - archipelagos

  - supersymmetry enhancement

  - S-duals

THANK YOU

Extra Slides

Heterotic on T^D has even self-dual lattice: $\Gamma_{16+D,D}$

Act with asymmetric orbifold on s left-movers:

$$I = (16 + D - s, D) \ ; \ N = (s, 0)$$

Rank reduction: - N has no roots

- $s = 16 + D$ (for maximal reduction)

Unique (24,0) lattice with no roots: Leech.

Automorphisms of Leech and invariant lattices have been classified [Höhn, Mason; 15]:

$$I' \ ; \ N$$

Then gluing construction to build $\Gamma_{16+D,D}$ [Nikulin; 80]

Let us consider toroidal compactification of type II string theory on the torus

$$Z = \frac{1}{\left(\sqrt{\tau_2}\eta\bar{\eta}\right)^{8-\mathrm{D}}} \frac{1}{(\eta\bar{\eta})^4} \left(\sum_{p,r\in v} - \sum_{p,r\in s} \right) q^{\frac{1}{2}p^2} \bar{q}^{\frac{1}{2}r^2} \frac{1}{\eta^D \bar{\eta}^D} \sum_{(P_L,P_R)} q^{\frac{1}{2}\mathrm{P_L}^2} \bar{q}^{\frac{1}{2}\mathrm{P_R}^2}$$

$$q = e^{2\pi i\tau}, \qquad \tau = \tau_1 + i\tau_2, \qquad \eta = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)$$

$$p, r \in SO(8), \qquad (P_L, P_R) \in \Gamma_{D,D}$$

T^6/Z_n orbifold: action

$\Theta = (\Theta_L, \Theta_R)$ automorphism of $\Gamma_{6,6}$, g orbifold generator.

Action on bosonised fermions:

$$g\colon \left|p\right> \longrightarrow e^{2\pi i p\cdot v_f}\left|p\right> \;;\; v_f = (0, t_1, t_2, t_3)$$

Action on complexified compact left-moving directions:

$$gX_a = e^{2\pi i \;\; {}_a} X_a \;\;;\;\; a = 1, .., 3 \;\;;\;\; t = (t_1, t_2, t_3)$$

Action on the lattice vectors defined up to a phase:

$$g : |P_L, P_R > \longrightarrow e^{2\pi i P\cdot v} |\Theta_L P_L, P_R >$$

Shift vector: v

Values of v_f, t_a, v constrained by:

- Modular invariance $Z(g^j, g^n) = Z(g^j, 1)$
- Crystallographic action (Θ automorphism of integral lattice)
- Supersymmetry: how much?
- Z_n action : $g^n = 1$
- Operator interpretation : $Z(g^j, g^k)$ is indeed $Tr_{\mathcal{H}_j}(g^k q^{L_0} \bar{q}^{\overline{L_0}})$
- No extra massless states: island. $\Longrightarrow$ v

- Proper crystallographic eigenvalues t_a of Z_n up to 6d have been classified by [Erler, Klemm; 92]

- Supersymmetry preserving 6d actions have been classified by [Dixon, Harvey, Vafa, Witten; 86] ($SO(6)$ elements)

$$\pm t_1 \pm t_2 \pm t_3 = 0 \ mod \ 2$$

$$n \sum_i t_a = 0 \ mod \ 2$$

- Z_n conjugacy classes of the Weyl groups of ADE algebras are also known