CSIC
Consejo Superior de Investigaciones Científicas
Institute of Space Sciences
Excelencia María de Maeztu
MaX
Marco de Autoevaluación de la Excelencia

Tidal Love Numbers of Black Holes Dressed by Dark Matter

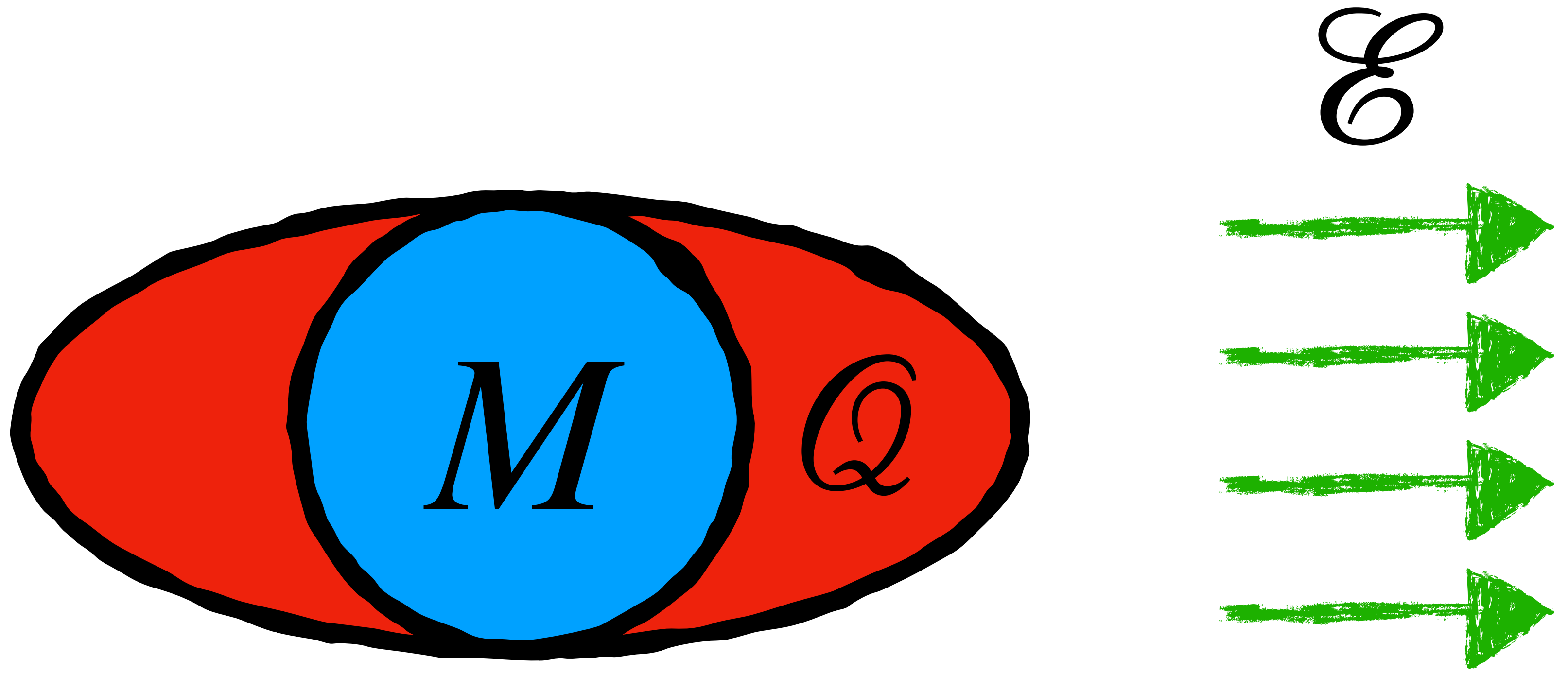
Simone D'Onofrio Institute of Space Sciences (ICE-CSIC), Barcelona

D'Onofrio, Datta & Maselli, arXiv:2605.02633

Newtonian Love number

Newtonian potential

$$\Phi(r) = \underbrace{\frac{M}{r}}_{} \overbrace{- \frac{1}{2}\mathcal{E} r^2}^{\text{Tidal Field}} \underbrace{+ \frac{\mathcal{Q}}{r^3}}_{\text{Induced Response}}$$



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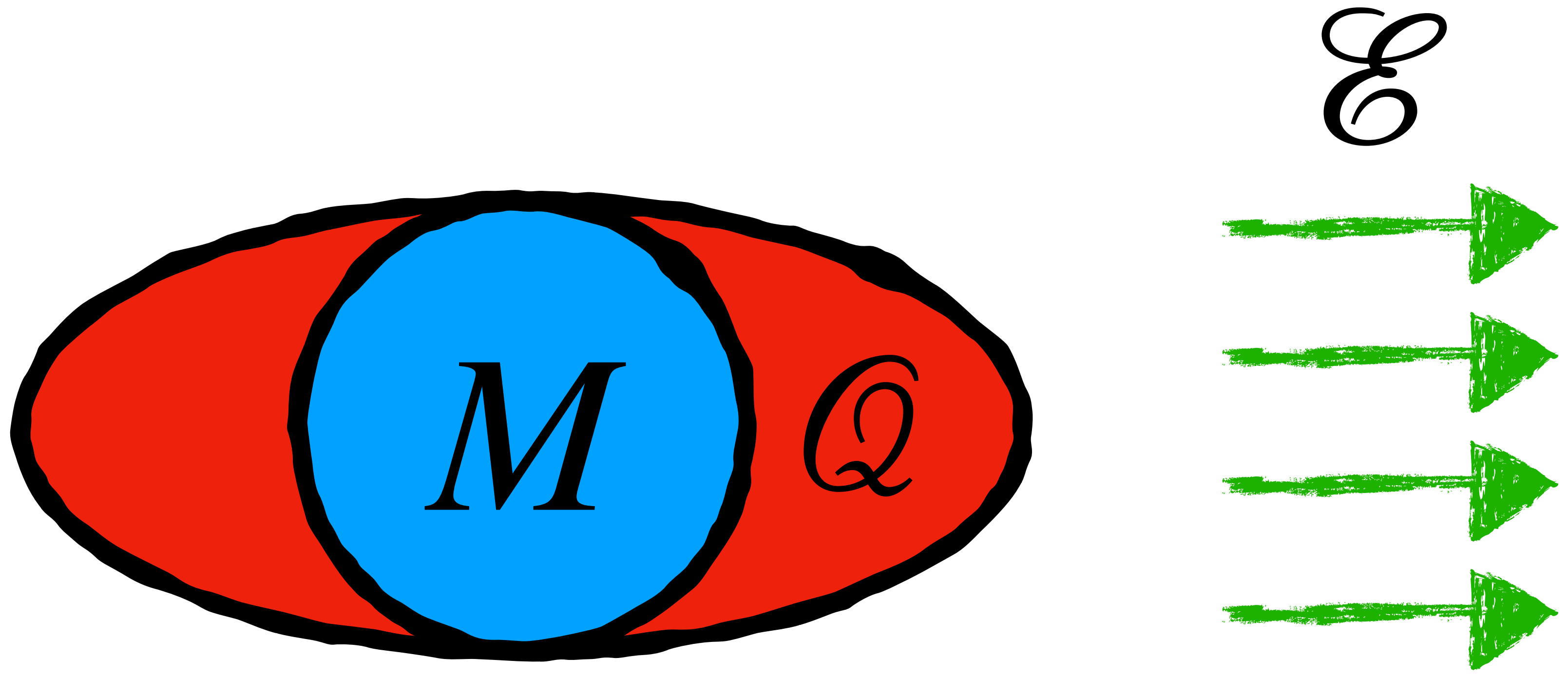

Augustus Love (1863, 1940)

Tidal deformability λ
(dimensionful)

$$\mathcal{Q} = \lambda \mathcal{E}$$

Tidal Love number k_2
(dimensionless)

$$k_2 \propto \frac{\lambda}{L^5}$$

Love, Proc. A 82 (551): 73–88 (1909)

Relativistic Love number

Now the perturbation is **metric**, not a scalar, so it splits by **parity**

Polar/Electric
even

Axial/Magnetic
odd

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Far from the source, the field **must** settles into the **ACMC structure**, where **internal** ($\mathcal{Q}_{\ell m}$) and **tidal** ($\mathcal{E}_{\ell m}$) multipoles are read off without coordinate ambiguity.

Asymptotically-mass-centered (ACMC)

$$g_{tt} = -1 + \frac{2M}{r} + \sum_{\ell \geq 2, m} \left(\frac{1}{r^{\ell+1}} \mathcal{Q}_{\ell m} + r^{\ell} \mathcal{E}_{\ell m} \right) Y_{\ell m}$$

Thorne, Rev. Mod. Phys. 52, 299 (1980)

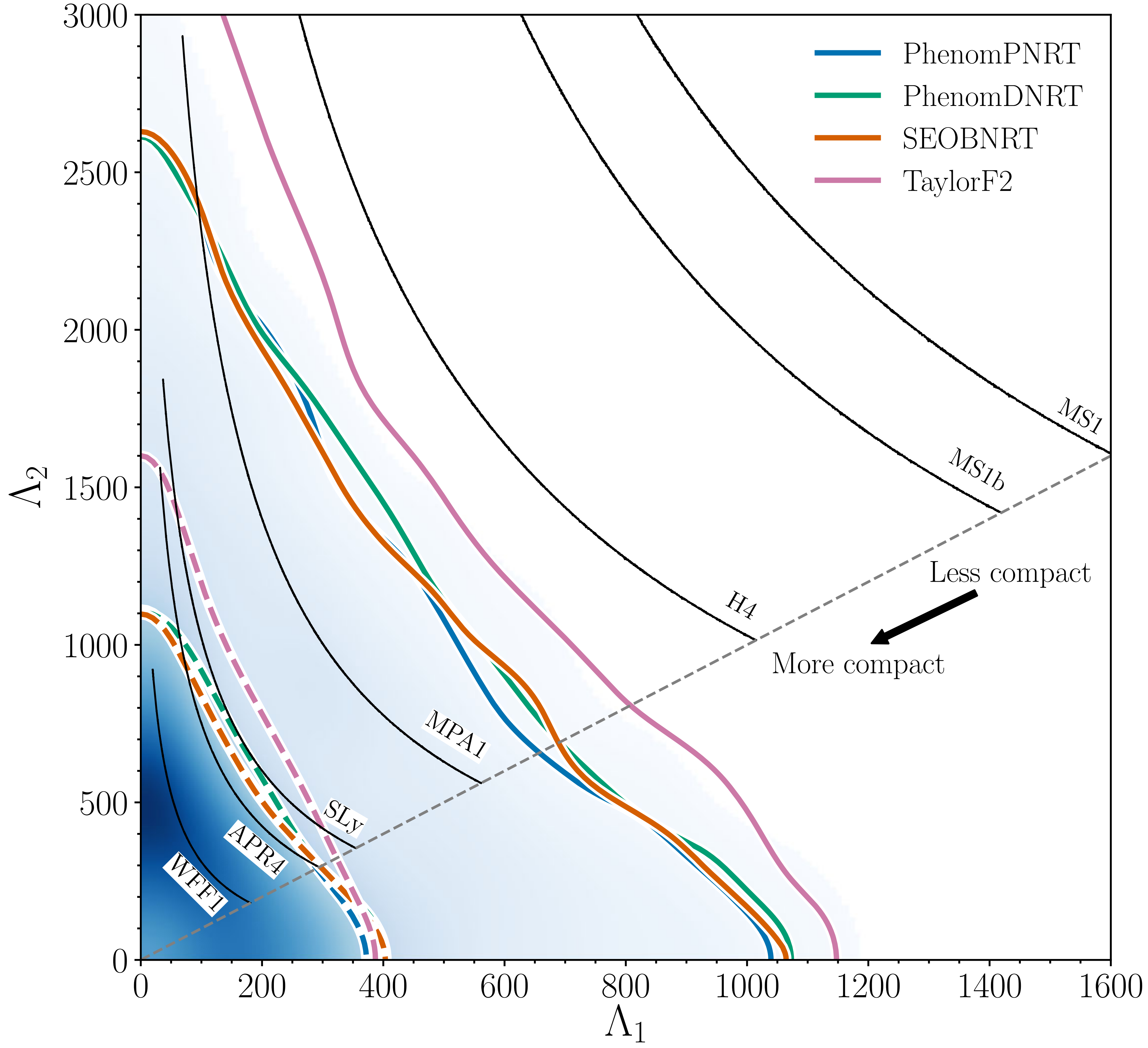
Tides are measurable

$$\psi_{\text{tidal}} \propto \overbrace{\underbrace{\lambda_2 v^5}_{} }^{\text{5PN}} + \overbrace{\lambda_2 v^6}^{\text{6PN}} + \overbrace{\sigma_2 v^6}^{} + \dots$$

5PN, 6PN; Polar ($\lambda_2 v^5 + \lambda_2 v^6$), Axial ($\sigma_2 v^6$)

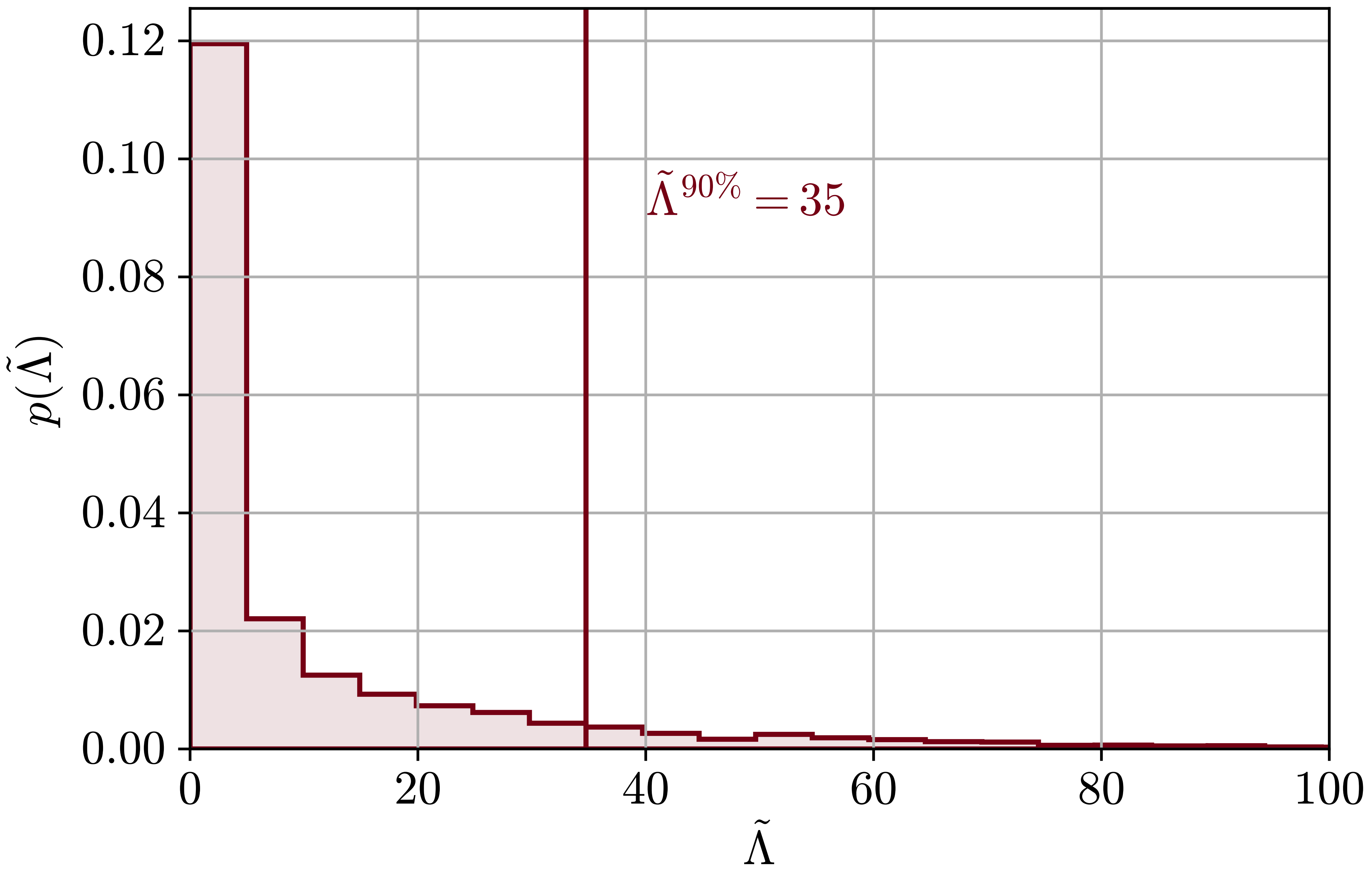
BNS *GW170817*

LVK, arXiv:1805.11579



BBH *GW250114*

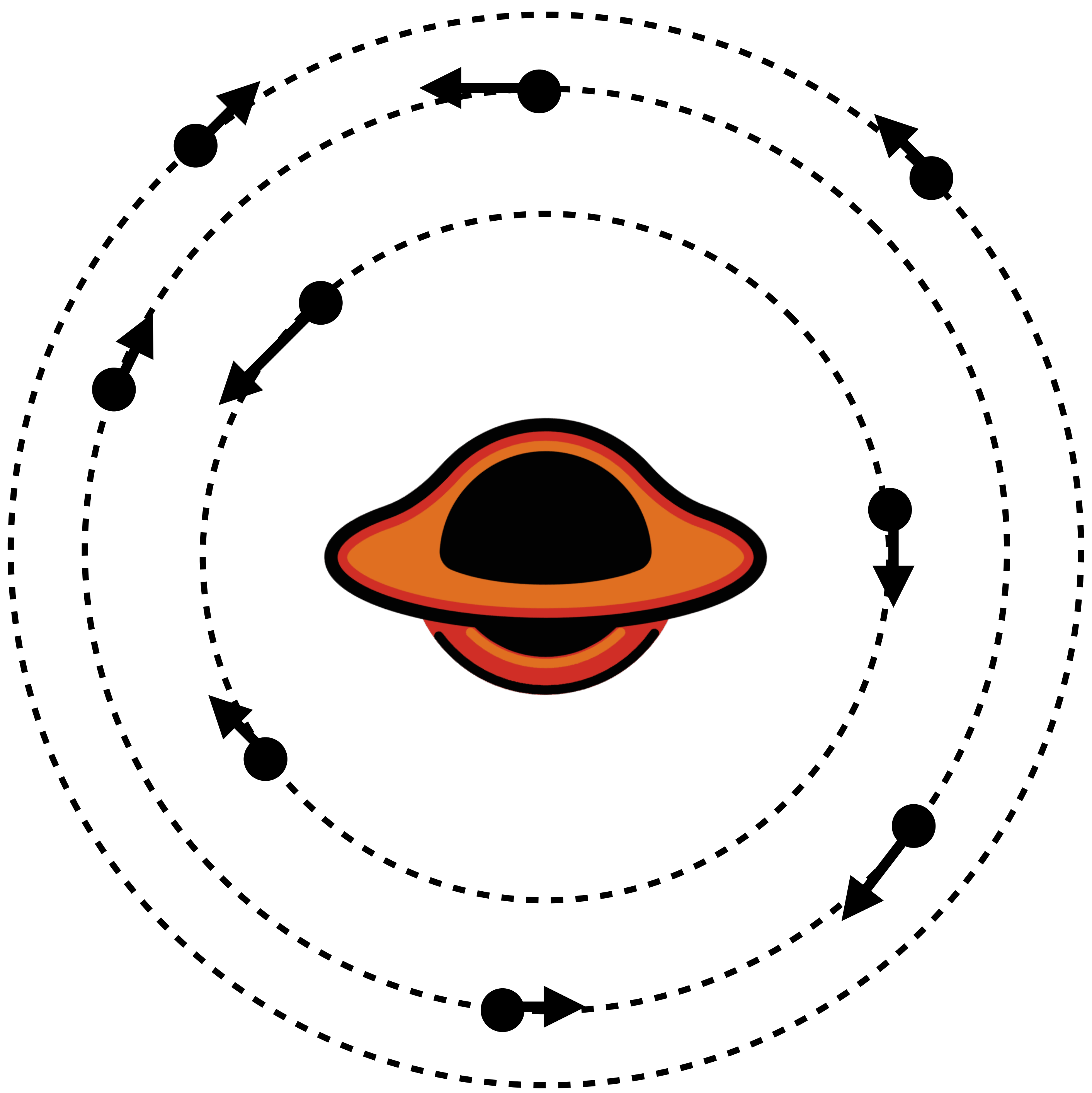
Andrés-Carcasona & Caneva Santoro, arXiv:2512.01918



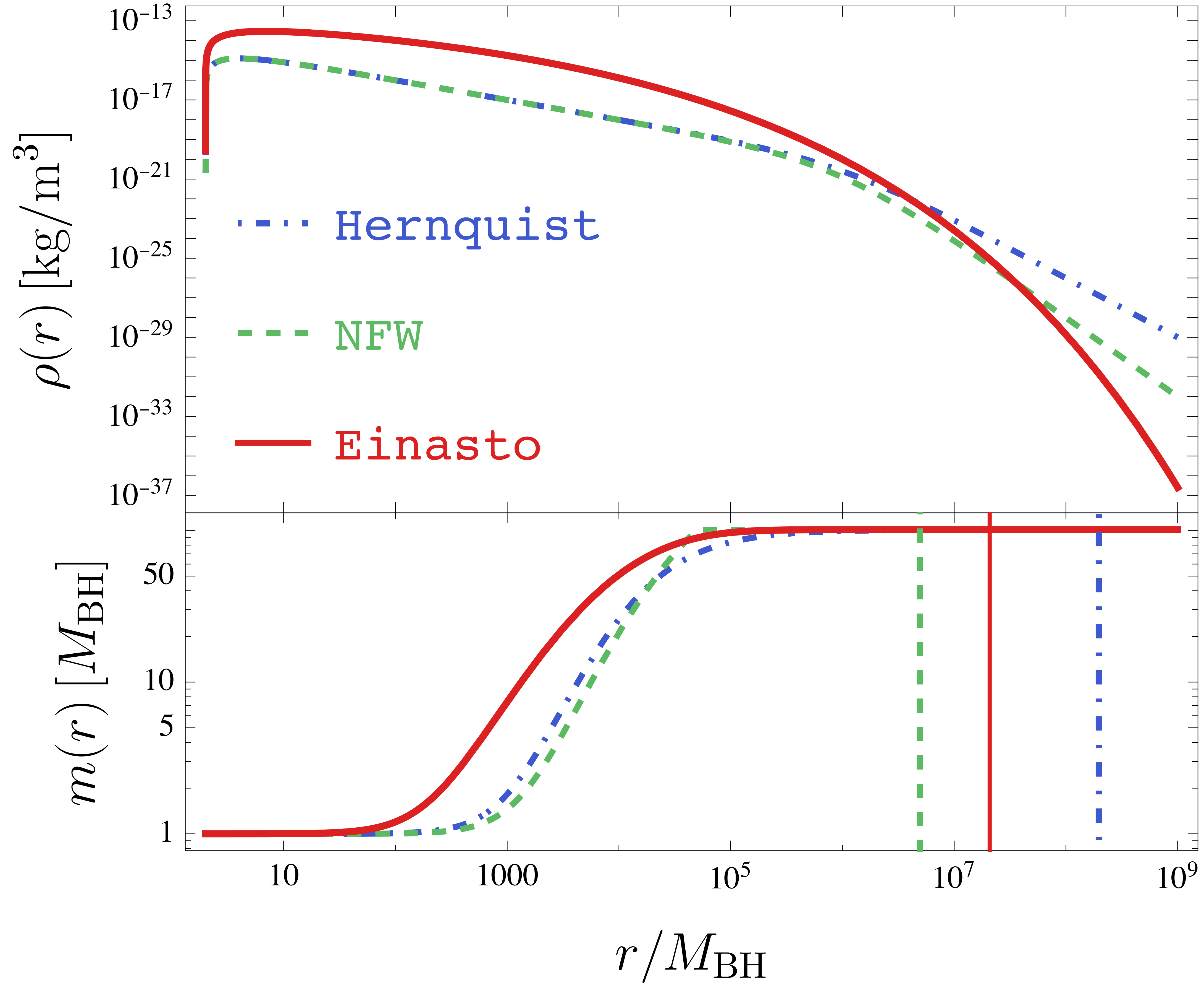
Black Holes in dark matter halos

Einstein Cluster

anisotropic fluid with no radial pressure



$$\rho(r) = \left(1 - \frac{2M_{BH}}{r}\right) \rho_{DM}(r)$$



The framework

Valid for axial perturbations of any anisotropic fluids in a spherically symmetric background.

$$T_{\mu\nu} = (\rho + p_t)u_\mu u_\nu + p_t g_{\mu\nu} + (p_r - p_t)k_\mu k_\nu$$

Axial/Magnetic

$$\delta g_{\mu\nu} = \sum_{\ell,m} \begin{pmatrix} 0 & 0 & h_0^{\ell m}(r,t)S_\theta^{\ell m} & h_0^{\ell m}(r,t)S_\phi^{\ell m} \\ * & 0 & h_1^{\ell m}(r,t)S_\theta^{\ell m} & h_1^{\ell m}(r,t)S_\phi^{\ell m} \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix}$$

Coupled time-dependent axial equations

$$e^{-\nu}\dot{h}_0 - e^{-\lambda}h_1' - \frac{1}{r^2}\left(2m - 4\pi r^3(\bar{\rho} - \bar{p}_r)\right)h_1 = 0$$

$$e^{-\nu}\left(\dot{h}_0' - \ddot{h}_1\right) - \frac{2e^{-\nu}}{r}\dot{h}_0 - \left(\frac{(\ell-1)(\ell+2)}{r^2} + 16\pi(\bar{p}_t - \bar{p}_r)\right)h_1 = -\frac{4(\bar{p}_t - \bar{p}_r)}{\bar{\rho} + \bar{p}_t}U_k$$

$$e^{-\lambda}\left(h_0'' - \dot{h}_1'\right) - 4\pi r(\bar{\rho} + \bar{p}_r)\left(h_0' - \dot{h}_1\right) - \frac{2}{r}e^{-\lambda}\dot{h}_1 - \frac{1}{r^2}\left(\ell(\ell+1) - \frac{4m}{r} + 8\pi r^2(\bar{\rho} + 2\bar{p}_t - \bar{p}_r)\right)h_0 = -4e^{\nu}U$$

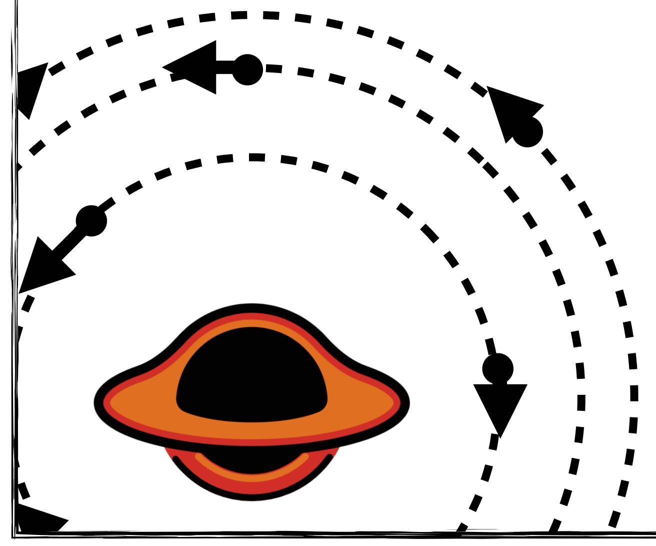
D'Onofrio, Datta & Maselli, arXiv:2605.02633

The framework

Irrotational master equation

$$e^{-\lambda}h_0'' - 4\pi r(\bar{\rho} + \bar{p}_r)h_0' - \frac{1}{r^2}\left(\ell(\ell+1) - \frac{4m}{r} - 8\pi r^2(\bar{\rho} + \bar{p}_r)\right)h_0 = 0$$

In the case of an ***Einstein cluster***, $p_r = 0$:



$$\left(1 - \frac{2m}{r}\right)h_0'' - \frac{m'}{r}h_0' - \left(\frac{\ell(\ell+1)}{r^2} - \frac{4m}{r^3} - \frac{2m'}{r^2}\right)h_0 = 0$$

Case study: Dark matter environment

Analytical

1. *Small compactness expansion* $C = M/a_0$
2. *Solving analytically* h_0 *up to first order in* C
3. *Compute the asymptotic behavior of the solution and* ***extract Love numbers***

Numerical

1. *Integrate the master equation up to* R_{99}
2. *Link it to the vacuum solution up to* R_{ext}
3. *Use the Hinderer formula for compact objects*

Hinderer, arXiv:0711.2420

Must agree!

Hernquist profile

Asymptotic solution different from ACMC :

$$h_0(r) \sim \overbrace{\sum_{i=-\infty}^{3} a_i r^i}^{\text{ACMC}} + \underbrace{\log\left(\frac{r}{R_s}\right)\sum_{i=2}^{\infty} b_i r^{-i}}_{\text{Extra terms}}$$

Cardoso & Duque, arXiv:1912.07616

R_s *is an arbitrary length scale introduced to render the argument of the logarithm dimensionless*

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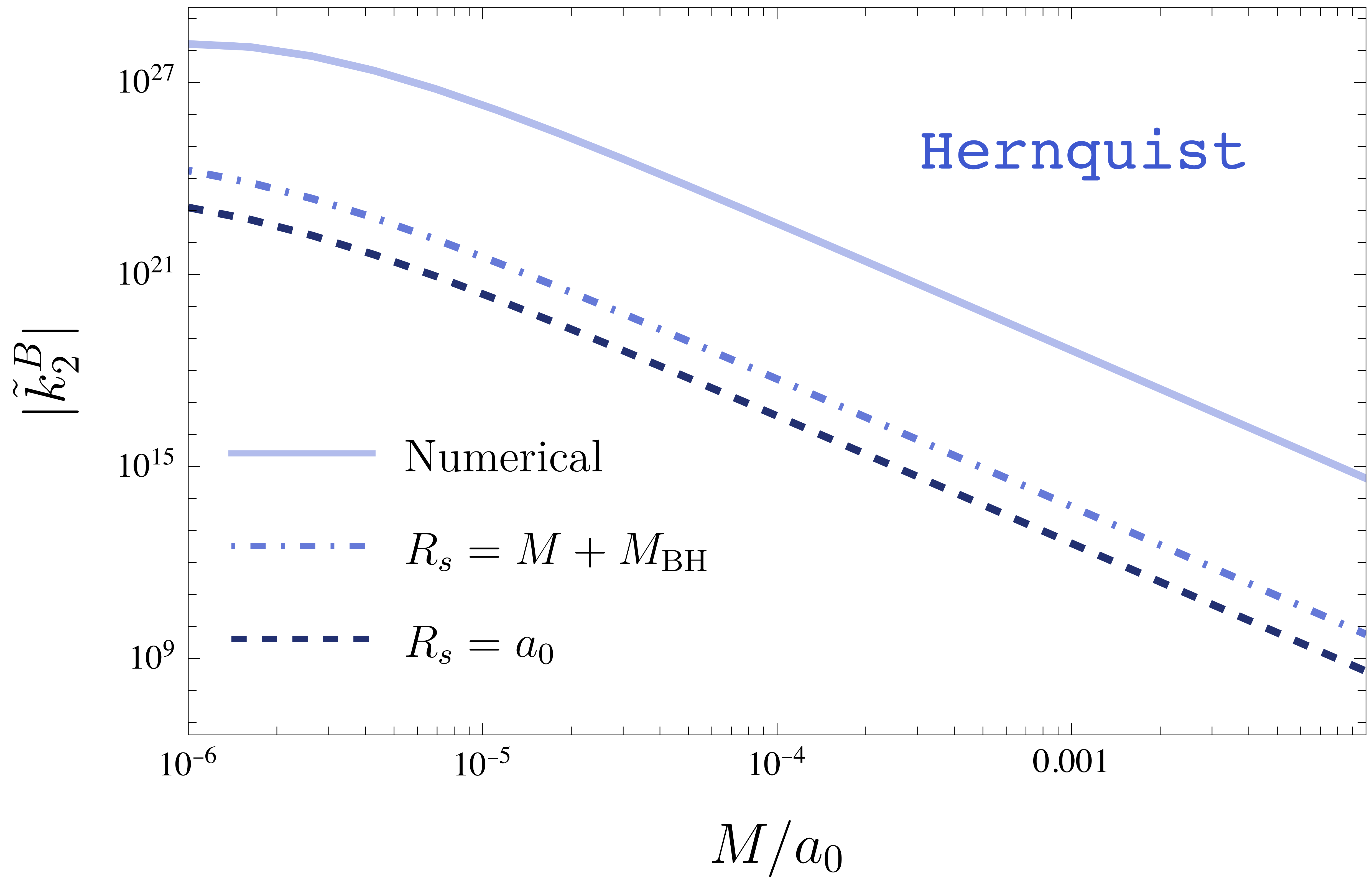
R_s is an arbitrary length scale introduced to render the argument of the logarithm dimensionless

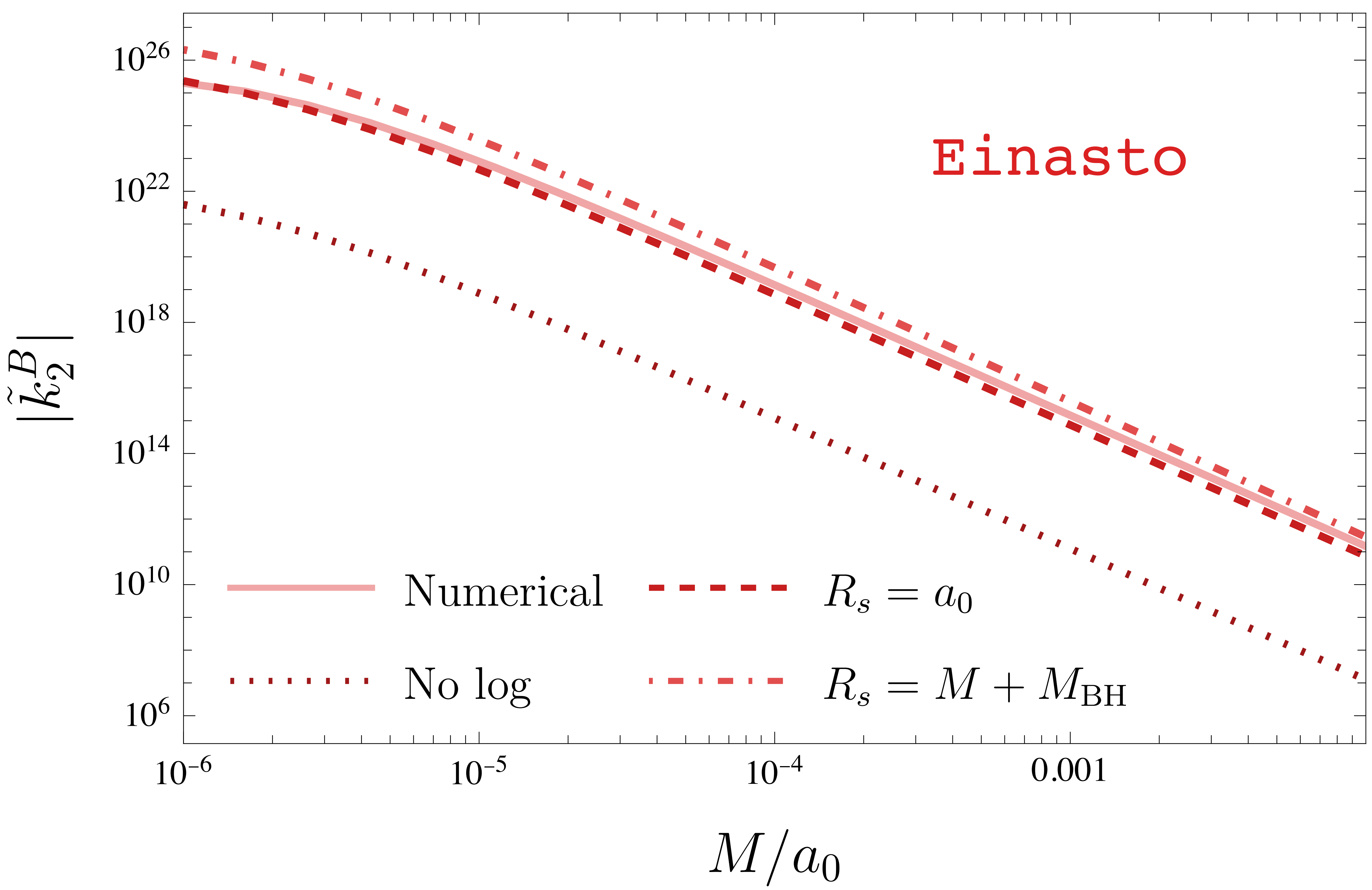
Ignoring these terms :

$$k_2^B \sim \frac{4\, a_0^4 M \left(1 - 4\log(a_0/R_s)\right)}{(M + M_{BH})^5}$$

Cardoso et al. , arXiv:2109.00005
Chakraborty et al. , arXiv:2412.14831

The analytic Love number fails





D’Onofrio, Datta & Maselli, arXiv:2605.02633

NFW profile

*For NFW we **must** include a cut-off for the profile because its total mass diverges*

NFW profile

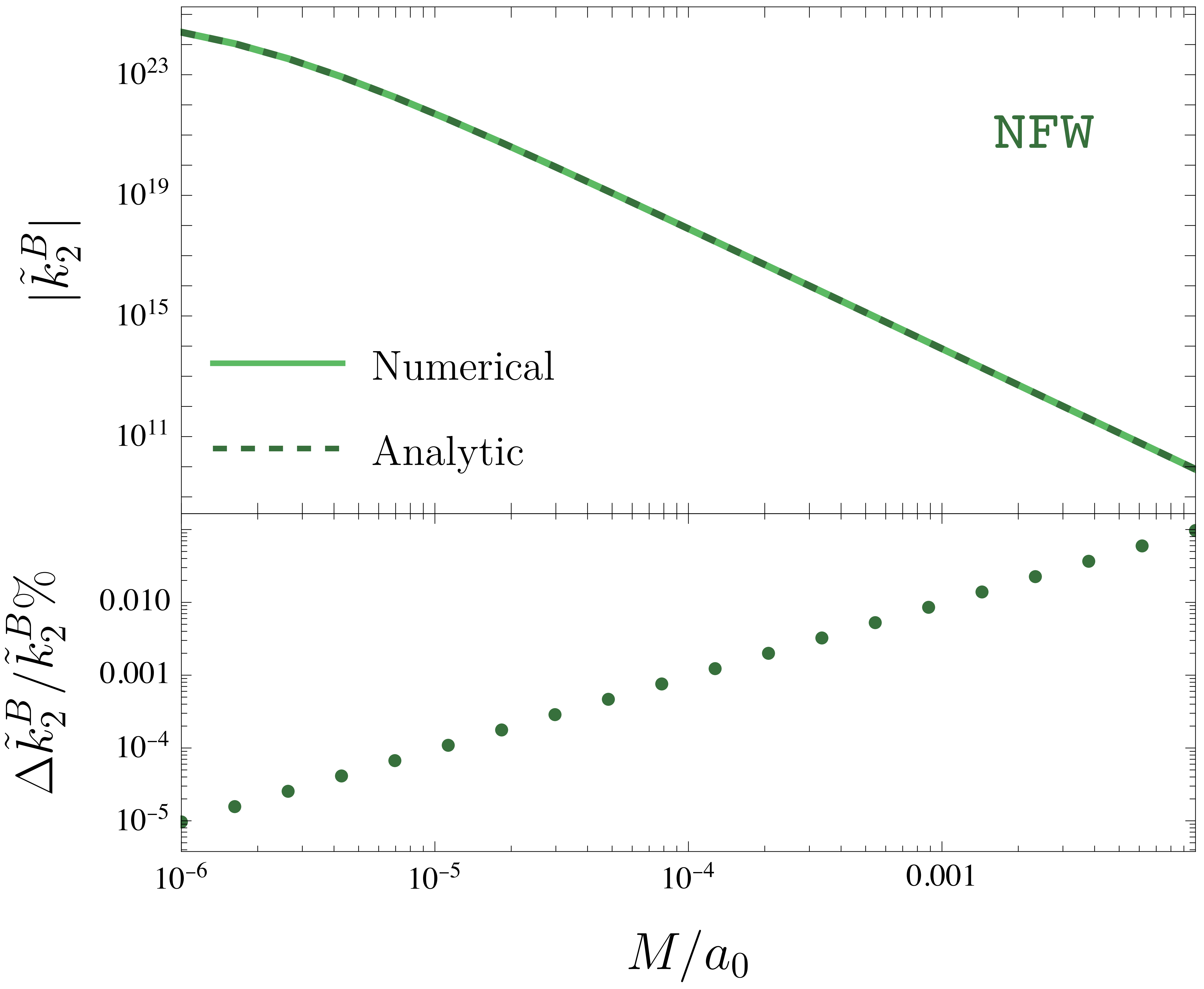
For NFW we ***must*** *include a cut-off for the profile because its total mass diverges*

Asymptotic solution is consistent with the ACMC : $h_0(r) \sim \sum_{i=-\infty}^{3} a_i r^i$ ✔

$$k_2^B \sim -\frac{4a_0^4 M}{(M+M_{BH})^5} + \frac{M r_c^2(3r_c^3 - 5a_0 r_c^2 + 10a_0^2 r_c - 30a_0^3)}{15(M+M_{BH})^5\left[r_c + (a_0 + r_c)\log\frac{a_0}{a_0+r_c}\right]}$$

***D'Onofrio**, Datta & Maselli, arXiv:2605.02633*

NFW: analytic and numerical agree



D'Onofrio, Datta & Maselli, arXiv:2605.02633

The fix: give the halo an edge

Analytically we ***never recover the vacuum*** *for Hernquist and Einasto*

We ***impose a cut-off*** *for these profiles at* $r_o = R_{99}$ *and compute the analytical solution*

The fix: give the halo an edge

*Analytically we **never recover the vacuum** for Hernquist and Einasto*

*We **impose a cut-off** for these profiles at $r_o = R_{99}$ and compute the analytical solution*

~~$\log\left(\frac{r}{R_s}\right)\sum_{i=2}^{\infty} b_i r^{-i}$~~ ***No more logs in the asymptotic expansion!***

ACMC

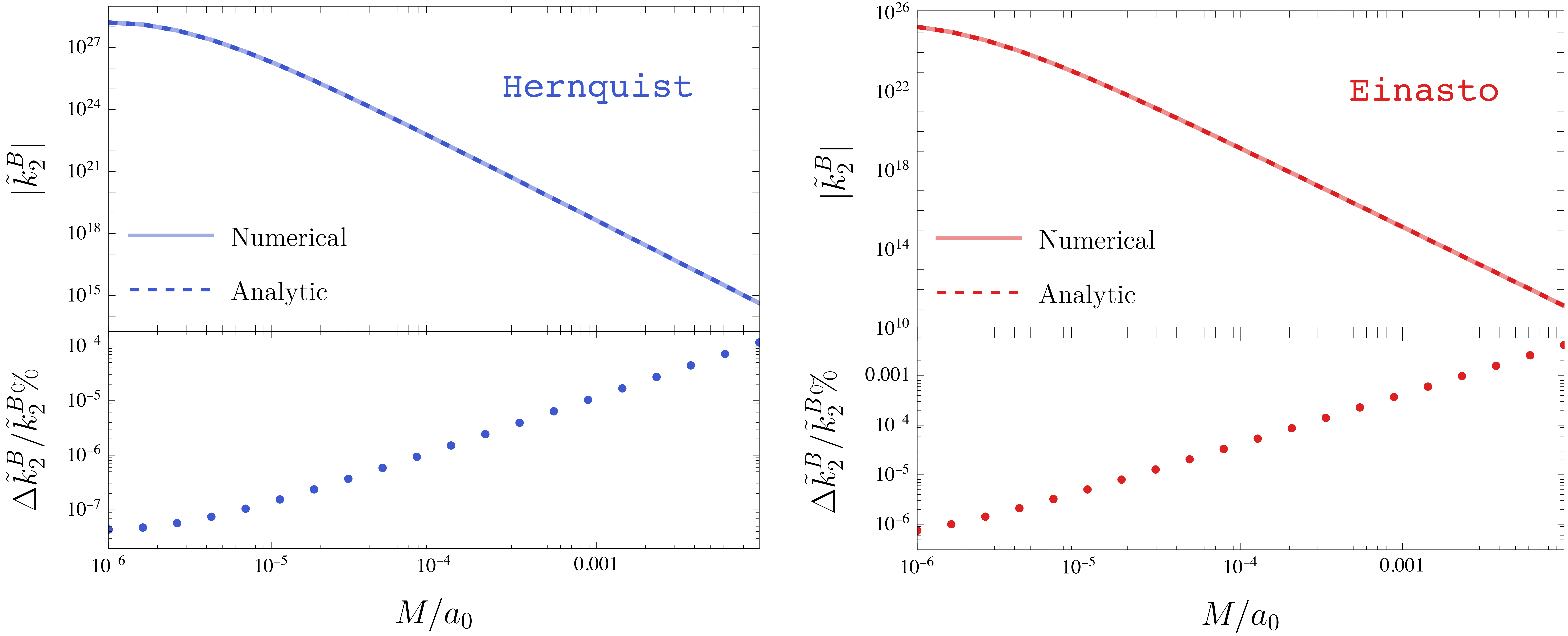
$$h_0(r) \sim \sum_{i=-\infty}^{3} a_i r^i$$

✔

*For **Hernquist**:*

$$k_2^B \sim -\frac{4Ma_0(60a_0^4 + 90a_0^3 r_o + 20a_0^2 r_o^2 - 5a_0 r_o^3 + 2r_o^4)}{15(M+M_{BH})^5 r_o} + \frac{16Ma_0^4(a_0+r_o)^2 \log\left(\frac{a_0+r_o}{a_0}\right)}{15(M+M_{BH})^5 r_o^2}$$

With a cutoff: agreement restored



D'Onofrio, Datta & Maselli, arXiv:2605.02633

Conclusions

- *We developed the framework of axial perturbations of anisotropic fluids in a spherically symmetric background.*
- *We computed the tidal Love number of a BH surrounded by different DM profiles.*
- *We gave a physical interpretation and a solution to the logarithm problem in the asymptotic solution.*
- *We proved that DM halos give BHs large axial tidal deformability, a potential environmental signature for future detectors.*

Thanks for the attention! :)

donofrio@ice.csic.es

BACKUP

Relativistic Love number

Now the perturbation is **metric**, not a scalar, so it splits by **parity** into the asymptotically-mass-centered (ACMC) form:

Polar/Electric

even

$$g_{tt} = -1 + \frac{2M}{r} + \sum_{\ell \geq 2, m} \left(\frac{1}{r^{\ell+1}} \mathcal{Q}_{\ell m} + r^{\ell} \mathcal{E}_{\ell m} \right) Y_{\ell m}$$

$$\mathcal{Q}_{\ell m} = \lambda_\ell \mathcal{E}_{\ell m} \qquad k_\ell^E \propto \frac{\lambda_\ell}{R^{2\ell+1}}$$

Axial/Magnetic

odd

$$g_{0\phi} = \sum_{\ell \geq 2} \left(\frac{1}{r^{\ell}} S_{\ell m} + r^{\ell+1} H_{\ell m} \right) \sin\theta \partial_\theta Y_{\ell m}$$

$$S_{\ell m} = \sigma_\ell H_{\ell m} \qquad k_\ell^B \propto \frac{\sigma_\ell}{R^{2\ell+1}}$$

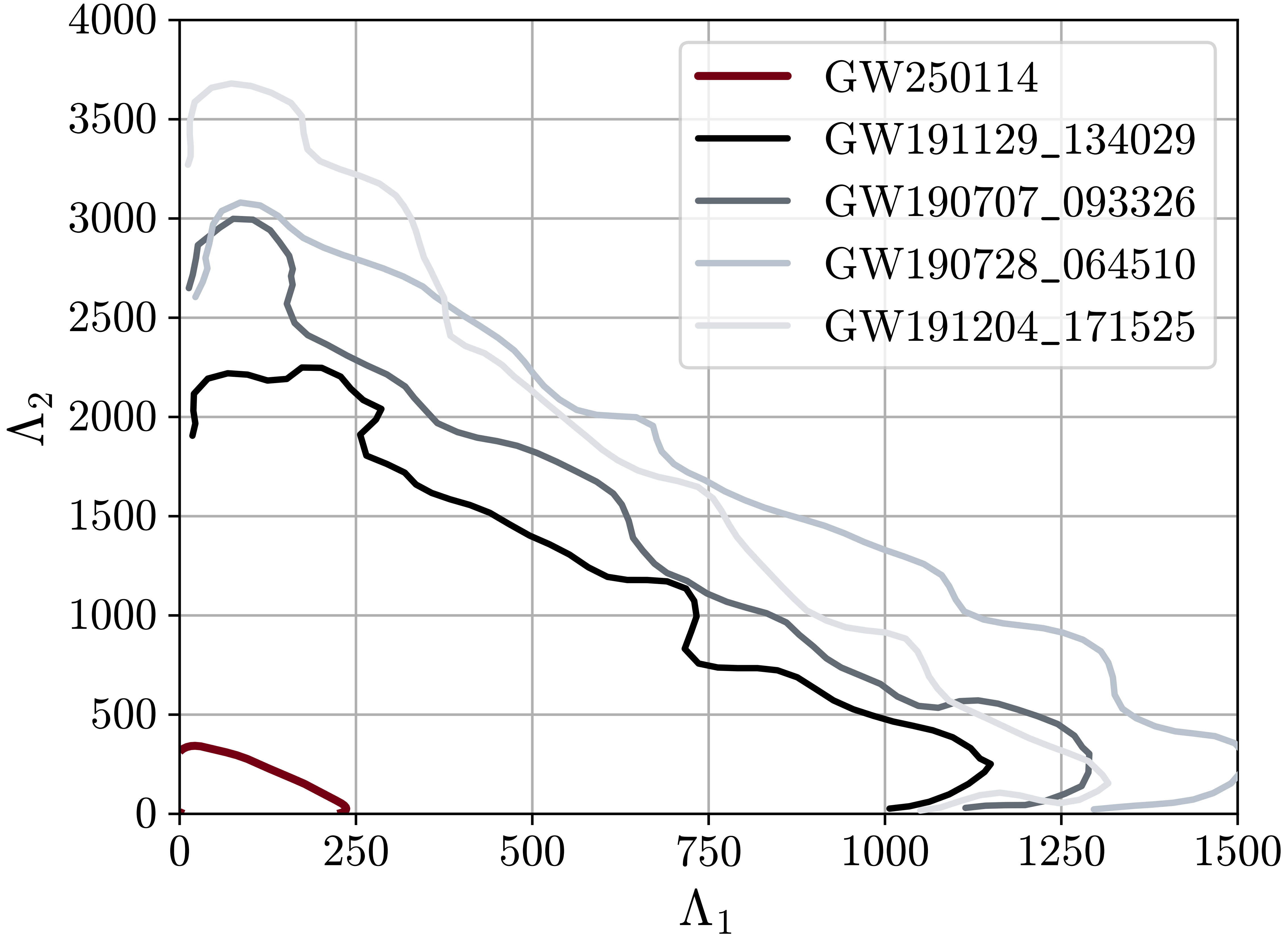
Thorne, Rev. Mod. Phys. 52, 299 (1980)

Polar/Electric
even

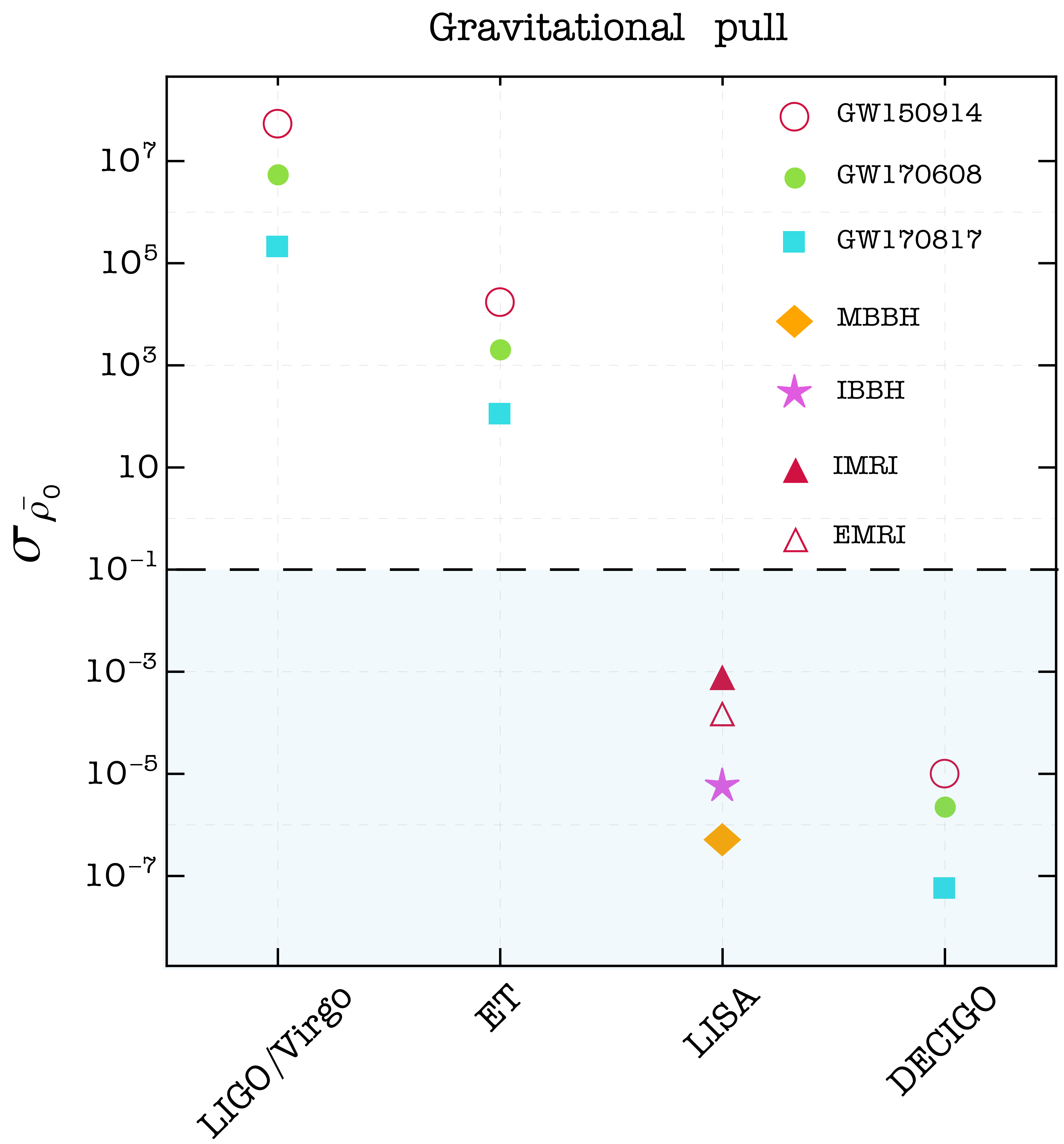
$$g_{tt} = -1 + \frac{2M}{r} + \sum_{\ell \geq 2, m} \left(\frac{2(2\ell - 1)!!}{\ell\,!} \frac{1}{r^{\ell+1}} \mathcal{Q}_{\ell m} + \frac{2}{\ell\,!} r^{\ell} \mathcal{E}_{\ell m} \right) Y_{\ell m}$$

Axial/Magnetic
odd

$$g_{0\phi} = \sum_{\ell \geq 2} \left(\frac{4(2\ell - 1)!!}{(\ell + 1)!} \frac{1}{r^{\ell}} S_{\ell m} - \frac{1}{(\ell + 1)!} r^{\ell+1} H_{\ell m} \right) \sin\theta \partial_{\theta} Y_{\ell m}$$



Andrés-Carcasona & Caneva Santoro, arXiv:2512.01918



Cardoso & Maselli, arXiv:1909.05870

Anisotropic fluid dynamic

General

$$e^{-\nu}\dot{h}_0 - e^{-\lambda}h_1' - \frac{1}{r^2}\Big(2m - 4\pi r^3(\bar{\rho} - \bar{p}_r)\Big)h_1 = 0$$

$$e^{-\nu}\Big(\dot{h}_0' - \ddot{h}_1\Big) - \frac{2e^{-\nu}}{r}\dot{h}_0 - \left(\frac{(\ell-1)(\ell+2)}{r^2} + 16\pi(\bar{p}_t - \bar{p}_r)\right)h_1 = -\frac{4(\bar{p}_t - \bar{p}_r)}{\bar{\rho} + \bar{p}_t}U_k$$

$$e^{-\lambda}\Big(h_0'' - \dot{h}_1'\Big) - 4\pi r(\bar{\rho} + \bar{p}_r)\Big(h_0' - \dot{h}_1\Big) - \frac{2}{r}e^{-\lambda}\dot{h}_1 - \frac{1}{r^2}\left(\ell(\ell+1) - \frac{4m}{r} + 8\pi r^2(\bar{\rho} + 2\bar{p}_t - \bar{p}_r)\right)h_0 = -4e^{\nu}U$$

$$\partial_t\Big(U - 4\pi e^{-\nu}(\bar{\rho} + \bar{p}_t)h_0\Big) + \frac{\bar{p}_r - \bar{p}_t}{e^{\lambda}(\bar{\rho} + \bar{p}_t)}\Big[\partial_r\Big(U_k - 4\pi(\bar{p}_t + \bar{\rho})h_1\Big) + \mathscr{F}(r)\Big(U_k - 4\pi(\bar{p}_t + \bar{\rho})h_1\Big)\Big] = 0$$

Irrotational

$$e^{-\nu}\dot{h}_0 - e^{-\lambda}h_1' - \frac{1}{r^2}\Big(2m - 4\pi r^3(\bar{\rho} - \bar{p}_r)\Big)h_1 = 0$$

$$e^{-\nu}\Big(\dot{h}_0' - \ddot{h}_1\Big) - \frac{2e^{-\nu}}{r}\dot{h}_0 - \frac{(\ell-1)(\ell+2)}{r^2}h_1 = 0$$

$$e^{-\lambda}\Big(h_0'' - \dot{h}_1'\Big) - 4\pi r(\bar{\rho} + \bar{p}_r)\Big(h_0' - \dot{h}_1\Big) - \frac{2}{r}e^{-\lambda}\dot{h}_1 - \frac{1}{r^2}\left(\ell(\ell+1) - \frac{4m}{r} - 8\pi r^2(\bar{\rho} + \bar{p}_r)\right)h_0 = 0$$

$$\frac{d^2\psi}{dr_\star^2} + \left(\omega^2 - e^\nu\left(\frac{\ell(\ell+1)}{r^2} - \frac{6m}{r^3} + 4\pi(\bar{\rho} - \bar{p}_r)\right)\right)\psi = 0$$

Equivalent Ragge-Wheeler equation for the irrotational fluid

Numerical formula

$$k_2^B = \frac{96}{5}\left(\frac{M_{tot}}{M + M_{BH}}\right)^5 \frac{2\xi(y-2) - y + 3}{\mathscr{D}(\xi, y)}$$

$$y = y(R_{ext}) = R_{ext}h_0'^{(ext)}(R_{ext})/h_0^{(ext)}(R_{ext})$$

$$\xi = M_{tot}/R_{ext}$$

$$\mathscr{D}(\xi, y) = 2\xi\left[2\xi^3(y+1) + 2\xi^2 y + 3\xi(y-1) - 3y + 9\right] + 3\left[2\xi(y-2) - y + 3\right]\log(1-2\xi)$$

Analytical method

$$h_0^{(1)}(r) = \Psi_+(r)\int_{2M_{BH}}^{r} dr' \frac{\mathcal{S}(r')\Psi_-(r')}{W(r')} + \Psi_-(r)\int_{r}^{\infty} dr' \frac{\mathcal{S}(r')\Psi_+(r')}{W(r')}$$

$$\Psi_- = A_1 r^3 \left(1 - \frac{2M_{BH}}{r}\right)$$

$$\Psi_+ = -\frac{A_2}{24M_{BH}^5 r}\left[2M_{BH}\left(3r^3 - 3M_{BH}r^2 - 2M_{BH}^2 r - 2M_{BH}^3\right) + 3r^4\left(1 - \frac{2M_{BH}}{r}\right)\log\left(\frac{r}{r - 2M_{BH}}\right)\right]$$