Università degli Studi di Padova
INFN PADOVA
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Precision gravitational wave theory at seventh post-Newtonian order

PASCOS 2026, Sheffield

Matteo Pegorin

Based on work with Giacomo Brunello, Manoj K. Mandal, Pierpaolo Mastrolia, Raj Patil, Jonathan Ronca, Sid Smith, Jan Steinhoff, William J. Torres Bobadilla

Università degli studi di Padova, INFN Padova, Max Planck Institute for Gravitational Physics (AEI)

June 23, 2026

Image credit: Argonne Leadership Computing Facility

Overview

EFT approach to compact binary systems

Gravitational waves from compact binary systems

- Coalescing Compact Binary Systems: comprising *black holes* and *neutron stars*

$\implies$ primary source of gravitational waves observed by *gravitational waves observatories*.

- To detect them: *matched filtering* technique.
  To perform parameter estimation: *Bayesian analysis*.

$\implies$ Need for *accurate* theoretical prediction for the *gravitational waveform*

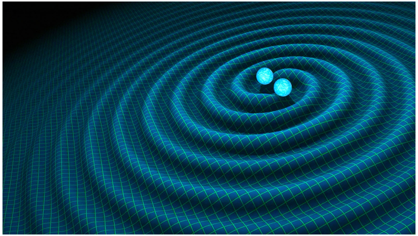

Image credit: R. Hurt, Caltech / JPL

Gravitational waves from compact binary systems

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- To detect them: *matched filtering* technique.
  To perform parameter estimation: *Bayesian analysis*.

  $\Longrightarrow$ Need for *accurate* theoretical prediction for the *gravitational waveform*

- Current LVK catalogs contain nearly 400 compact binary candidates. [LVK GWTC-5.0 (2026)]

- Next-gen GW observatories in 2030s: Einstein Telescope, Cosmic Explorer, LISA [Maggiore et al. (2020); Evans et al. (2023); Colpi et al. (2024); Abac et al. (2025)]

  $\Longrightarrow$ 10x sensitivity improvement and new frequency ranges

  $\Longrightarrow$ More accurate tests of GR, population studies and cosmological measurements.

  $\Longrightarrow$ Theoretical predictions need a commensurate improvement

[Pürrer, Haster (2020); Hu, Veitch (2022); Owen, Haster, Perkins, Cornish, Yunes (2023)]

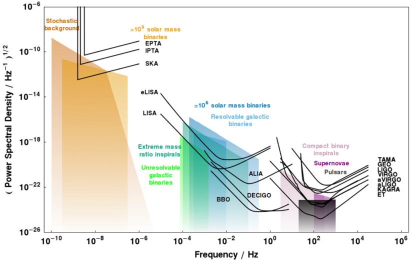

Image credit: Moore, Cole, Berry (arXiv:1408.0740)

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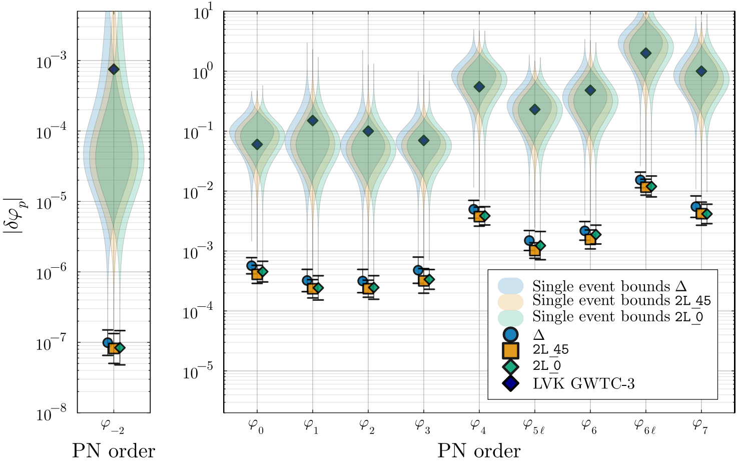


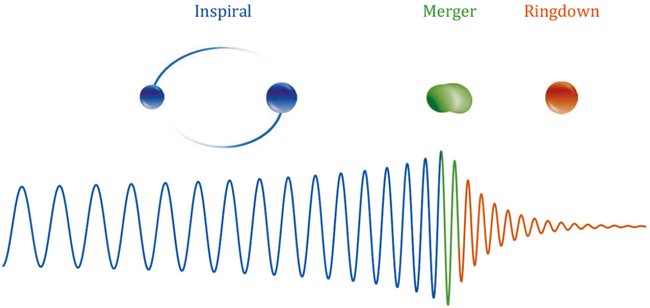
Einstein Telescope: $\mathcal{O}(10^2 \sim 10^4)$ improvement over LVK Tests of GR!

[Begnoni, Del Pozzo, M.P., Pomper, Ricciardone (2025)]

408.0740)

Gravitational waveform modeling

Image credit: Antelis, Moreno (arXiv:1610.03567)



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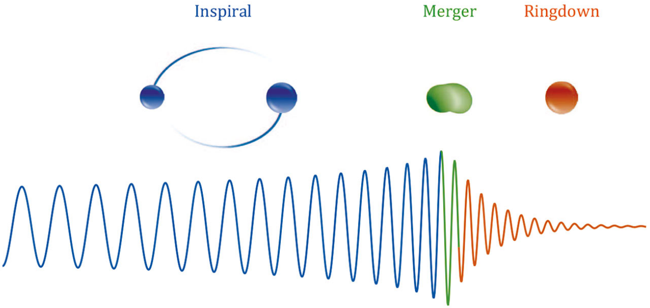


Inspiral

- Post-Newtonian
- Post-Minkowskian
- Self-force

Merger

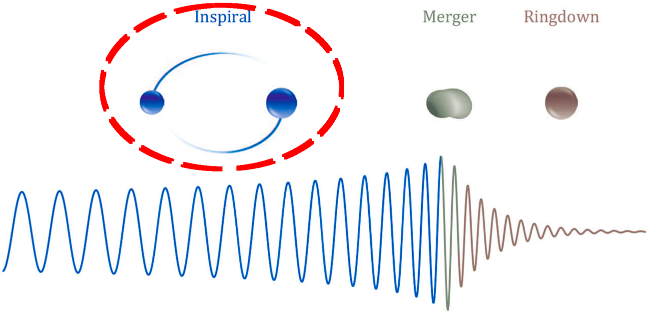
- Numerical Relativity

Ringdown

- QNM - Black hole perturbation theory

Gravitational waveform modeling

Image credit: Antelis, Moreno (arXiv:1610.03567)



Inspiral

- **Post-Newtonian (PN)**
- Post-Minkowskian
- Self-force

Merger

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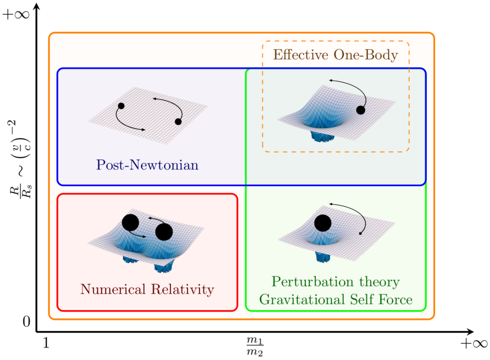
Complete waveforms join this information (Effective One Body, IMRPhenom, surrogate NR models).

Post-Newtonian formalism

- Evaluates dynamics and gravitational wave emission of bound *comparable-mass inspiralling* compact binaries.
- Assumptions:
  - *slowly moving objects* $\frac{v^2}{c^2} \ll 1$
  - *weak gravitational field* $\frac{Gm}{r} \ll 1$

$\Longrightarrow$ *Perturbative expansion* of GR dynamics in the PN parameter $\frac{v^2}{c^2} \sim \frac{Gm}{r} \ll 1$.



[Reviews: Blanchet (2024), Porto (2016), Levi (2020)]

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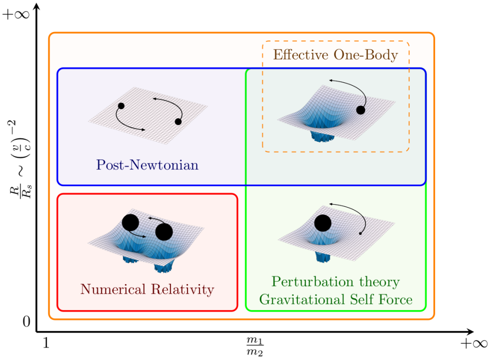
- The *Effective Field Theory (EFT) approach* to compact binary systems connects post-Newtonian to *QFT and multi-loop Feynman integrals* computations. [Goldberger, Rothstein (2006)]



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Post-Newtonian: precision computations

	0PN		1PN		2PN		3PN		4PN		5PN		6PN		7PN		…
1PM	G	$+$	$G\,v^2$	$+$	$G\,v^4$	$+$	$G\,v^6$	$+$	$G\,v^8$	$+$	$G\,v^{10}$	$+$	$G\,v^{12}$	$+$	$G\,v^{14}$	$+$	…
2PM			G^2	$+$	$G^2\,v^2$	$+$	$G^2\,v^4$	$+$	$G^2\,v^6$	$+$	$G^2\,v^8$	$+$	$G^2\,v^{10}$	$+$	$G^2\,v^{12}$	$+$	…
3PM					G^3	$+$	$G^3\,v^2$	$+$	$G^3\,v^4$	$+$	$G^3\,v^6$	$+$	$G^3\,v^8$	$+$	$G^3\,v^{10}$	$+$	…
4PM							G^4	$+$	$G^4\,v^2$	$+$	$G^4\,v^4$	$+$	$G^4\,v^6$	$+$	$G^4\,v^8$	$+$	…
5PM									G^5	$+$	$G^5\,v^2$	$+$	$G^5\,v^4$	$+$	$G^5\,v^6$	$+$	…
6PM											G^6	$+$	$G^6\,v^2$	$+$	$G^6\,v^4$	$+$	…
7PM													G^7	$+$	$G^7\,v^2$	$+$	…
8PM															G^8	$+$	…
⋮																	⋱

[..., Jaranowski, Schäfer, Damour, Gilmore, Ross, Buonanno, Vines, Antonelli, Kavanagh, Khalil, Galley, Edison, Morales, Teng, Almeida, Müller, Leibovich, Bernard, Blanchet, Bohé, Faye, Marsat, Marchand, Will, Wiseman, Iyer, Kidder, Pati, Deruelle, Asada, Futamase, Itoh, Goldberger, Rothstein, Porto, Kol, Smolkin, Shir, Foffa, Sturani, Mastrolia, Sturm, Torres Bobadilla, Blümlein, Maier, Marquard, Bini, Geralico, Henry, Larrouturou, Trestini, Cho, Pardo, Yang, Levi, Steinhoff, McLeod, von Hippel, Kim, Yin, Mandal, Patil, Amalberti, Riva, Brunello, Ronca, Smith, M.P., ...]

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1PM	G	$+\ G\,v^2$	$+\ G\,v^4$	$+\ G\,v^6$	$+\ G\,v^8$	$+\ G\,v^{10}$	$+\ G\,v^{12}$	$+\ G\,v^{14}$	$+\ \dots$
2PM		G^2	$+\ G^2\,v^2$	$+\ G^2\,v^4$	$+\ G^2\,v^6$	$+\ G^2\,v^8$	$+\ G^2\,v^{10}$	$+\ G^2\,v^{12}$	$+\ \dots$
3PM			G^3	$+\ G^3\,v^2$	$+\ G^3\,v^4$	$+\ G^3\,v^6$	$+\ G^3\,v^8$	$+\ G^3\,v^{10}$	$+\ \dots$
4PM				G^4	$+\ G^4\,v^2$	$+\ G^4\,v^4$	$+\ G^4\,v^6$	$+\ G^4\,v^8$	$+\ \dots$
5PM					G^5	$+\ G^5\,v^2$	$+\ G^5\,v^4$	$+\ G^5\,v^6$	$+\ \dots$
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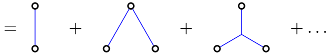
EFT approach to Compact Binary Systems

Full theory (General Relativity + compact objects + ...)

$$S_{\text{tot}}[\{x_a^\mu\}, g_{\mu\nu}] = -2\Lambda^2 \int d^4x \sqrt{-g} \left(R - \frac{1}{2}\Gamma^\mu \Gamma_\mu\right) - \sum_{a=1}^{2} m_a \int d\sigma_a \sqrt{g_{\mu\nu}(x_a) \frac{dx_a^\mu}{d\sigma} \frac{dx_a^\nu}{d\sigma}} + \dots$$

Integrate out gravitational d.o.f.

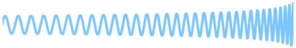
$$e^{iS_{\text{eff}}[\{x_a^\mu\}]} = \int \mathcal{D}g_{\mu\nu}\, e^{iS_{\text{tot}}[\{x_a^\mu\}, g_{\mu\nu}]}$$



Effective action (post-Newtonian corrections)

$$S_{\text{eff}}[\{x_a^\mu\}] = \int dt\, L = \int dt \left(\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + G\frac{m_1 m_2}{r} + \dots\right)$$

Physical observables (gravitational waveform)



[Goldberger, Rothstein (2006)]

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Possible to extend to non-vacuum/beyond GR theories too!

[Placidi, Grilli, Orselli, M.P., Bartolo, Mastrolia (2025)]

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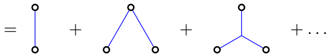

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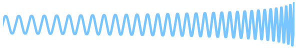
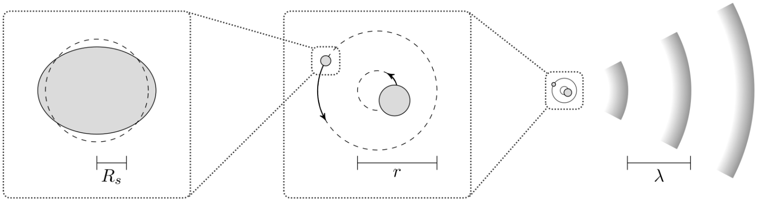


[Goldberger, Rothstein (2006)]

EFT: separation of scales

- v $\ll 1 \implies$ Hierarchy of scales $\implies$ 3 different EFTs:



$R_s \ll r \ll \lambda$

Internal zone

- Compact body d.o.f.: spin and finite size effects

Near zone

- Off-shell istantaneous gravitons
- Binding energy from potential modes, source multipoles

Far zone

- On-shell radiation gravitons
- Gravitational wave flux, hereditary effects

[Goldberger, Rothstein (2006)]

Near zone EFT

- Fundamental action: GR + two compact objects

$$S_{\text{near,UV}}\left[\{x_A^\mu\}, g_{\mu\nu}\right] = -\frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left(R - \frac{1}{2}g_{\mu\nu}\Gamma^\mu\Gamma^\nu\right) - \sum_{A=1,2} m_A \int d\sigma_A \sqrt{g_{\mu\nu}\frac{dx_A^\mu}{d\sigma_A}\frac{dx_A^\nu}{d\sigma_A}} + \dots$$

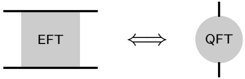
- Method of regions:

$$g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu} + h_{\mu\nu}$$

- Consider only *classical* contributions $\longleftrightarrow$ tree-level EFT diagrams.
- Employ *dimensional regularization*, $d = 3 + \epsilon$.
- Two-body potential: Fourier transform of *multi-loop* massless *two-point Feynman integrals*

[Foffa, Mastrolia, Sturani, Sturm (2016)]

$$\mathcal{V}_{\text{eff}} = i \lim_{d\to 3}\int \frac{d^d\mathbf{p}}{(2\pi)^d}\, e^{i\mathbf{p}\cdot(\mathbf{x}_{(1)}-\mathbf{x}_{(2)})}\left(\;\text{EFT}\;\right),$$



Six-loop diagrams: 6PN static corrections

[Brunello, Mandal, Mastrolia, Patil, M.P., Ronca, Smith, Steinhoff, Torres Bobadilla (2025)]

Post-Newtonian static sector

- Kaluza-Klein decomposition of the metric (scalar ϕ + vector A_i + symmetric tensor σ_{ij}):

$$g_{\mu\nu} = e^{2\phi} \begin{pmatrix} 1 & -A_j \\ -A_i & -e^{-c_d\phi}\gamma_{ij} + A_i A_j \end{pmatrix}, \qquad \gamma_{ij} = \delta_{ij} + \sigma_{ij} , \quad c_d = \frac{2(d-1)}{d-2}.$$

- Full action in the static point-particle limit: [Kol, Smolkin (2008a,b),Kol, Smolkin (2012)]

$$S_{\text{full}}^{(v^0)} = \underbrace{-\frac{1}{16\pi G_d}\int dt\, d^d\mathbf{x}\,\sqrt{\gamma}\left[\frac{c_d}{2}\gamma^{ij}\partial_i\phi\partial_j\phi - R[\gamma] + \frac{1}{2}|\Gamma_i[\gamma]|^2\right]}_{S_{\text{bulk}}=S_{\text{EH}}+S_{\text{GF}}} \underbrace{-\sum_{a=1,2} m_a \int dt\, e^{\phi(\mathbf{x}_a)}}_{S_{\text{pp}}} .$$

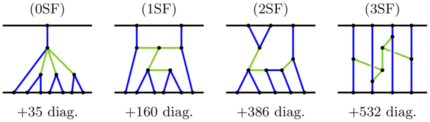
- Static conservative dynamics $\longleftrightarrow$ *Euclidean QFT* in d spatial dimensions.
- Static sector comprises the *highest loop computations* at any fixed PN order.

Six-loop diagram generation and reduction

- *Integrand generation*: `PNTHR` code.

  [PNTHR: Mandal, Mastrolia, Patil, Steinhoff]

  - 1117 PN diagrams contributing to the 6PN static sector.



Six-loop diagram generation and reduction

- *Integrand generation*: PNTHR code.

  [PNTHR: Mandal, Mastrolia, Patil, Steinhoff]

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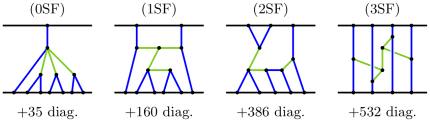
- *Integration-by-part (IBP) reduction*: PRISM code.

  [Chetyrkin, Tkachov (1981); Laporta (2000); Peraro (2019); Smith, Zeng (2025); PRISM: Brunello, Mastrolia, Ronca, Smith, Zeng (in progress)]

$$0 = \int_{\ell_1,\cdots,\ell_6} \frac{\partial}{\partial k_i^\mu} \left(\frac{w^\mu}{D_1^{\alpha_1} \cdots D_{27}^{\alpha_{27}}} \right) , \quad w_\mu \in \{\ell_j, p\} .$$

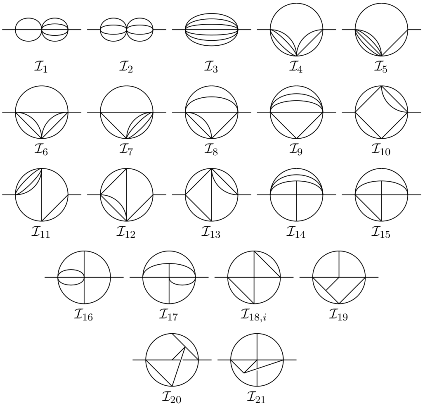
  - Six-loop Feynman integrals, rank ≤ 6.
  - Spanning cuts, syzygies and improved seeding strategy; IBP systems solved over finite fields.



SF order	Diagrams	MIs
0	36	1
1	161	3
2	387	13
3	533	16
Total	1117	21

Six loop Feynman integral evaluation

- *Master integrals (MIs)*:
  - 6-loop massless two-point functions
  - 21 MIs contributing to 6PN.
- Complementary techniques for evaluation:
  - Analytic (1-loop bubble recursion, Gegenbauer Polynomials). [Chetyrkin, Kataev, Tkachov (1980); Kotikov (1996); Bierenbaum, Weinzierl (2003)]
  - High-precision numerical evaluation (`AMFlow`) with `PSLQ` analytical reconstruction via transcendental ansätze. [Liu, Ma (2023)]
  - Checks via `FeynTrop`: numerical evaluation for quasi-finite basis in $d = 5$, Dimensional Recurrence Relations. [Lee (2010a,b); Lee, Mingulov (2016, 2017); Borinsky, Munch, Tellander (2023)]



Six loop F

- *Master*
  - 6-l
  - 21
- Comple
  - Ar Pc
  - Hi PS an
  - Ch qu Re

$$\vdots \qquad\qquad \vdots$$

$$\mathcal{I}_{17} = \frac{c_\epsilon}{(p^2)^3}\left[\frac{1}{24\epsilon^3} + \frac{7}{24\epsilon^2} + \frac{1}{12\epsilon}\left(19 - \frac{47\pi^2}{16}\right) + \mathcal{O}(\epsilon^0)\right],$$

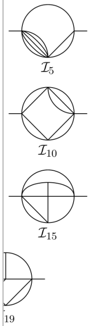
$$\mathcal{I}_{18,2} = \frac{c_\epsilon}{(p^2)^2}\left[\frac{\pi^2}{24\epsilon^2} - \frac{\pi^2}{3\epsilon}\left(1 + \frac{\log(2)}{4}\right) + \mathcal{O}(\epsilon^0)\right],$$

$$\mathcal{I}_{19} = \frac{c_\epsilon}{(p^2)^4}\left[\frac{5}{48\epsilon^3} - \frac{7}{24\epsilon^2} - \frac{1}{48\epsilon}\left(139 + \frac{95\pi^2}{8}\right) + \mathcal{O}(\epsilon^0)\right],$$

$$\mathcal{I}_{20} = \frac{c_\epsilon}{(p^2)^4}\left[\frac{1}{24\epsilon^3} - \frac{1}{24\epsilon}\left(41 + \frac{59}{8}\pi^2\right) + \mathcal{O}(\epsilon^0)\right],$$

$$\mathcal{I}_{21} = \frac{c_\epsilon}{(p^2)^4}\left[\frac{1}{4\epsilon^3} - \frac{1}{4\epsilon^2} - \frac{1}{\epsilon}\left(\frac{7}{2} + \frac{3\pi^2}{32}\right) + \mathcal{O}(\epsilon^0)\right].$$

with $c_\epsilon = \frac{(p^2)^{2+3\epsilon} e^{3\gamma_E \epsilon}}{(4\pi)^{3(\epsilon+2)}}$.



6PN static potential

[Brunello, Mandal, Mastrolia, Patil, M.P., Ronca, Smith, Steinhoff, Torres Bobadilla (2025)]

$$\mathcal{V}_{6\mathrm{PN}}^{G_N^7} = -\frac{G_N^7}{r^7}\left[\frac{5}{16}m_1^7 m_2 + \frac{190}{9}m_1^6 m_2^2 + \frac{37651}{144}m_1^5 m_2^3 + \frac{5852}{9}\frac{m_1^4 m_2^4}{2}\right] + (1 \leftrightarrow 2).$$

- Complete *static* contribution $(G_N^7 v^0)$ at *sixth post-Newtonian order*.
- No ϵ poles; all transcendental constants cancel.
- Fully compatible with known test-particle limit.

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- Complete *static* contribution $(G_N^7 v^0)$ at *sixth post-Newtonian order*.
- No ϵ poles; all transcendental constants cancel.
- Fully compatible with known test-particle limit.
- First contribution computed at *third self-force order*.

Correlation functions framework: 7PN static corrections and beyond

Correlation functions framework

[Brunello, Mandal, Mastrolia, Patil, M.P., Smith, Steinhoff (2026)]

[Mastrolia, M.P. (in prep.)]

- Effective action $S_{\text{eff}}[\{x_a^\mu\}] \longleftrightarrow$ logarithm of path integral

$$e^{\frac{i}{\hbar}S_{\text{eff}}[\{x_a^\mu\}]} = \frac{1}{Z_0}\int \mathcal{D}\phi\,\mathcal{D}\sigma_{ij}\;\exp\left[\frac{i}{\hbar}S_{\text{bulk}}[\phi,\sigma_{ij}]\right]\;\exp\left[\frac{i}{\hbar}S_{\text{pp}}[\phi;\{x_a^\mu\}]\right].$$

$$Z_0 = \int \mathcal{D}\phi\,\mathcal{D}\sigma_{ij}\;e^{\frac{i}{\hbar}S_{\text{bulk}}[\phi,\sigma_{ij}]}$$

Correlation functions framework

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- Effective action $S_{\text{eff}}[\{x_a^\mu\}] \longleftrightarrow$ logarithm of path integral

$$e^{\frac{i}{\hbar} S_{\text{eff}}[\{x_a^\mu\}]} = \frac{1}{Z_0} \int \mathcal{D}\phi\, \mathcal{D}\sigma_{ij}\, \exp\left[\frac{i}{\hbar} S_{\text{bulk}}[\phi, \sigma_{ij}]\right] \underbrace{\exp\left[\frac{i}{\hbar} S_{\text{pp}}[\phi; \{x_a^\mu\}]\right]}_{\mathcal{O}} .$$

$$Z_0 = \int \mathcal{D}\phi\, \mathcal{D}\sigma_{ij}\, e^{\frac{i}{\hbar} S_{\text{bulk}}[\phi, \sigma_{ij}]}$$

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Correlation functions framework

[Brunello, Mandal, Mastrolia, Patil, M.P., Smith, Steinhoff (2026)]

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- Post-Newtonian static potential $\longleftrightarrow$ expectation value in bulk *interacting* theory

$$S_{\text{eff}}[\{x_a^\mu\}] = -i\hbar\log\left(\left\langle \exp\left[\frac{i}{\hbar}S_{\text{pp}}[\phi;\{x_a^\mu\}]\right]\right\rangle_{\text{bulk}}\right)$$

Correlation functions framework

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- Point-particle static action:
  - *localizes* the expectation values at the worldline positions x_1 and x_2;
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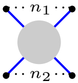
- Post-Newtonian static potential $\longleftrightarrow$ *connected correlation functions* Γ_{n_1,n_2} of the ϕ field:

$$S_{\rm eff} \qquad \longleftrightarrow \qquad \Gamma_{n_1,n_2} = \left(\frac{i}{\hbar}\right)^{n_1+n_2-1} \left\langle \phi_1^{n_1} \phi_2^{n_2} \right\rangle_c =$$



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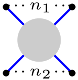

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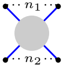
- Bulk expectation value decomposed *algebraically* using generating functional with localized sources.

$$W_{\rm bulk}(J_1, J_2) = \sum_{n_1,n_2 \geq 1} \frac{J_1^{n_1} J_2^{n_2}}{n_1! n_2!}$$



$$, \qquad \langle \Phi_1^a \Phi_2^b \rangle_{\rm bulk} = \exp\left(\frac{i}{\hbar} W_{\rm bulk}\left(-i\hbar\, a, -i\hbar\, b \right) \right)$$

$\mathbb{Z}_2$ symmetry of static bulk theory

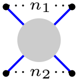
- The static bulk action in GR presents a $\mathbb{Z}_2$ symmetry

  [Kol, Smolkin (2012); Parra-Martinez, Podo (2025)]

$$\phi \longrightarrow -\phi \; ,$$

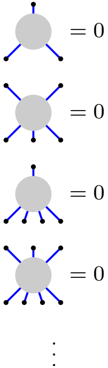
  sub-group of $SL(2, \mathbb{R})$ symmetry.

- All ϕ-correlators with odd number of legs vanish:

$$\Gamma_{n_1, n_2} =$$ 

$$= 0 \qquad \text{for} \qquad n_1 + n_2 \text{ odd.}$$

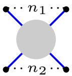
Vanishing correlators



All-order static formula

- The *static post-Newtonian* effective action to *all orders* is:

$$\mathcal{S}_{\text{eff}} = -i\hbar \log \left[\sum_{a,b\geq 0} \left(-\frac{i}{\hbar}\right)^{a+b} \frac{m_1^a}{a!} \frac{m_2^b}{b!} \exp\left(\sum_{\substack{n_1,n_2\geq 1 \\ n_1+n_2 \text{ even}}} (-i\hbar)^{n_1+n_2-1} \frac{a^{n_1}}{n_1!} \frac{b^{n_2}}{n_2!}\right.\right.$$



$$\left.\left.\vphantom{\sum_{a}}\right)\right]$$

All-order static formula

[Brunello, Mandal, Mastrolia, Patil, M.P., Smith, Steinhoff (2026)]

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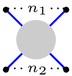


$$\left.\left.\vphantom{\sum}\right)\right]$$

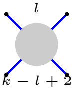
- Classical limit: $\hbar \to 0$, after Taylor expansion in m_1 and m_2.
- Static kPN potential:

$$\mathcal{V}_{k\text{PN}}^{G_N^{k+1}} = \sum_{l=1}^{k+1} m_1^l m_2^{k-l+2} \frac{\widehat{\mathcal{V}}_{l,\,k-l+2}}{l!(k-l+2)!}\,, \qquad \widehat{\mathcal{V}}_{r,s} = -\lim_{\hbar\to 0} \frac{\partial^{r+s}}{\partial m_1^r\, \partial m_2^s} (\mathcal{S}_{\text{eff}})\Big|_{m_1=0,m_2=0}\,.$$

Factorization theorem from $\mathbb{Z}_2$ symmetry

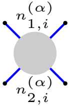
- At fixed kPN order $(1 < l < k+1)$, the potential comprises *prime* and *factorizable* contributions:

$$\widehat{\mathcal{V}}_{l,k-l+2} = c_{l,k-l+2}$$



prime

$$+ \sum_{\alpha} c_{\alpha} \prod_{i=1}^{N_\alpha}$$



factorizable
(Product of lower PN data)

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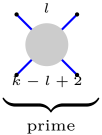
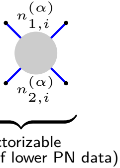
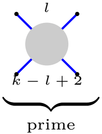


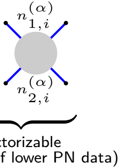
$$+ \sum_{\alpha} c_\alpha \prod_{i=1}^{N_\alpha}$$



- At any odd k-PN order, prime contributions *vanish* due to $\mathbb{Z}_2$ symmetry:



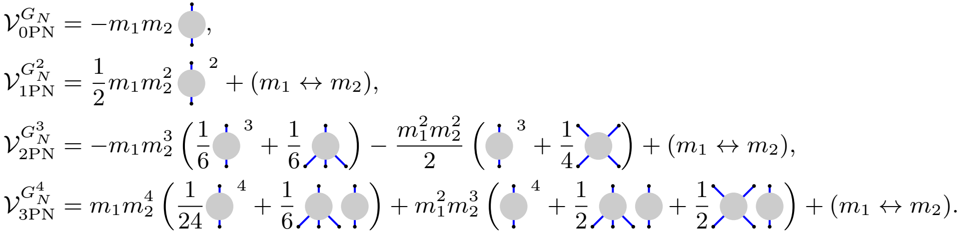
$$= 0 \qquad \Longrightarrow \qquad \widehat{\mathcal{V}}_{\text{oddPN}} = \sum_{\alpha} c_\alpha \prod_{i=1}^{N_\alpha}$$



- $\mathbb{Z}_2$ symmetry $\Longrightarrow$ odd-PN potential *fully determined by lower-order data*.
- Underlying origin of the *factorization theorem* presented in [Foffa, Mastrolia, Sturani, Sturm, Torres Bobadilla (2019)].

Factorization theorem from $\mathbb{Z}_2$ symmetry

- At fixed kPN order $(1 \leq l \leq k+1)$, the potential comprises *prime* and *factorizable* contributions:

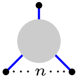


- A
- $\mathbb{Z}$
- Underlying origin of the *factorization theorem* presented in [Foffa, Mastrolia, Sturani, Sturm, Torres Bobadilla (2019)].

Known correlators values

- Matching to point-particle limit fixes *0SF correlators to all orders*:

$$\Gamma_{1,n} =$$



$$= \left(\frac{G_N}{r}\right)^n (n-1)! \quad \text{for } n \text{ odd}\,. \tag{1}$$

- Additional correlators fixed by *matching* to known post-Newtonian results:

PN	0SF	1SF	2SF	3SF
0	$= 1$			
2	$= 2$	$= 8$		
4	$= 24$	$= 160$	$= 504$	
6	$= 720$	$= 6400$	$= 31760$	$= 83456$

(With overall scaling $(G_N/r)^{n_1+n_2-1}$ understood)

[Foffa, Mastrolia, Sturani, Sturm (2017); Foffa, Mastrolia, Sturani, Sturm, Torres Bobadilla (2019); Blümlein, Maier, Marquard (2020); Brunello, Mandal, Mastrolia, Patil, M.P., Ronca, Smith, Steinhoff, Torres Bobadilla (2025)]

- These 10 numbers *fully determine* the static potential up to 7PN order.
- Intriguingly, all known correlators are positive integers: hints of a *deeper structure*.

7PN static potential

$$\mathcal{V}_{\rm 7PN}^{G_N^8} = \frac{G_N^8}{r^8}\left(\frac{35}{128} m_1 m_2^8 + \frac{248}{9} m_1^2 m_2^7 + \frac{1059}{2} m_1^3 m_2^6 + \frac{89383}{36} m_1^4 m_2^5 + (1 \leftrightarrow 2)\right).$$

7PN static potential

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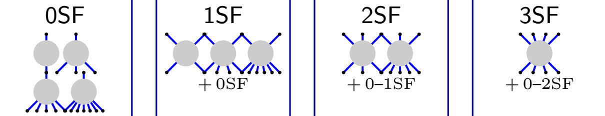

- Complete *static* contribution $(G_N^8 v^0)$ at *seventh post-Newtonian order*.
- Formally requires 7-loop diagram computations.
- Yet, factorization theorem ($\mathbb{Z}_2$ symmetry) ensures no seven-loop master integrals are needed.
- Agrees with independent diagrammatic factorization computation (involving 3842 diagrams).

[PNTHR: Mandal, Mastrolia, Patil, Steinhoff (unpublished); PRISM: Brunello, Mastrolia, Ronca, Smith, Zeng (in progress)]

Beyond 7PN

0SF:  $= 40320$

1SF: 

2SF: 

3SF: 

4SF: 

$$\mathcal{V}_{8\mathrm{PN}}^{G_N^9} = -\frac{G_N^9}{r^9}\Bigg[\frac{35}{128}m_1 m_2^9 + \left(\frac{\bar{\Gamma}_{2,8}}{80640}+\frac{266}{9}\right)m_1^2 m_2^8 + \left(\frac{\bar{\Gamma}_{3,7}}{30240}+\frac{250705}{288}\right)m_1^3 m_2^7$$
$$+\left(\frac{\bar{\Gamma}_{4,6}}{17280}+\frac{20084}{3}\right)m_1^4 m_2^6 + \left(\frac{\bar{\Gamma}_{5,5}}{14400}+\frac{7656577}{576}\right)\frac{m_1^5 m_2^5}{2}\Bigg] + (1 \leftrightarrow 2).$$

- At 8PN, four new prime correlators enter, which correspond to 8-loop diagrams computations.

Beyond 7PN

0SF:  $= 40320$

1SF: 

2SF: 

$$\mathcal{V}^{G_N^9}_{8\mathrm{PN}} = -\frac{G_N^9}{r^9}\Bigg[\frac{35}{128}m_1 m_2^9 + \left(\frac{\bar{\Gamma}_{2,8}}{80640}+\frac{266}{9}\right)m_1^2 m_2^8 + \left(\frac{\bar{\Gamma}_{3,7}}{30240}+\frac{250705}{288}\right)m_1^3 m_2^7$$
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3SF: 

4SF: 

- At 8PN, four new prime correlators enter, which correspond to 8-loop diagrams computations.

0SF, 1SF, 2SF, 3SF, 4SF

$$\mathcal{V}^{G_N^{10}}_{9\mathrm{PN}} = \frac{G_N^{10}}{r^{10}}\Bigg[\frac{63}{256}m_1 m_2^{10} + \left(\frac{\bar{\Gamma}_{2,8}}{40320}+\frac{577}{18}\right)m_1^2 m_2^9 + \left(\frac{\bar{\Gamma}_{2,8}+\bar{\Gamma}_{3,7}}{10080}+\frac{186233}{144}\right)m_1^3 m_2^8$$
$$+\left(\frac{\bar{\Gamma}_{3,7}+\bar{\Gamma}_{4,6}}{4320}+\frac{270781}{18}\right)m_1^4 m_2^7 + \left(\frac{\bar{\Gamma}_{4,6}+\bar{\Gamma}_{5,5}}{2880}+\frac{2501525}{48}\right)m_1^5 m_2^6\Bigg] + (1 \leftrightarrow 2).$$

- At 9PN no additional prime correlators enter: fully determined by data up to 8PN.

Conclusions

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- Next generation observatories require more accurate waveform models, and therefore more precise theoretical predictions for two body dynamics in GR.
- The *6PN static potential* required the breakthrough computation of *six-loop Feynman integral*.
- The *correlation function framework* makes manifest the static GR $\mathbb{Z}_2$ symmetry and explains the odd-PN factorization theorem.
- This framework determines the *7PN static potential* directly from lower-order data.
- These computations solved the *most challenging* sector for integral evaluation, enabling the computation of the post-Newtonian conservative potential up to 7PN.

Università degli Studi di Padova

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Thank you for your attention

Matteo Pegorin

Based on work with Giacomo Brunello, Manoj K. Mandal, Pierpaolo Mastrolia, Raj Patil, Jonathan Ronca, Sid Smith, Jan Steinhoff, William J. Torres Bobadilla

Università degli studi di Padova, INFN Padova, Max Planck Institute for Gravitational Physics (AEI) June 23, 2026

Image credit: Argonne Leadership Computing Facility