

# Primordial Black Hole Formation from Inflationary Higgs Instability

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**PASCOS 2026**  
in Sheffield  
22–26 June 2026

# HIGGS VACUUM INSTABILITY

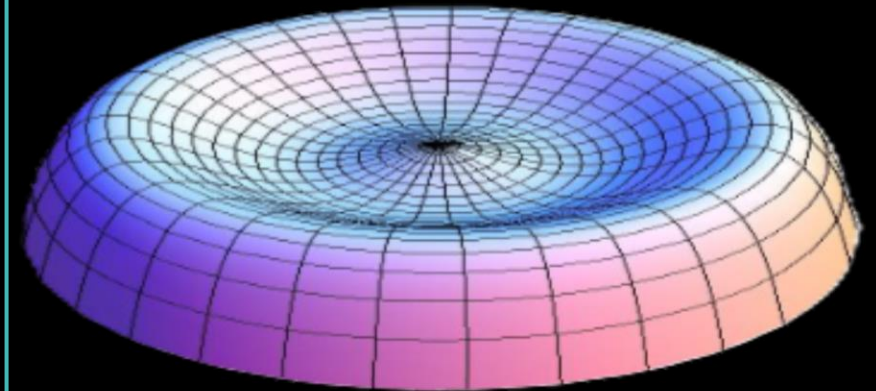
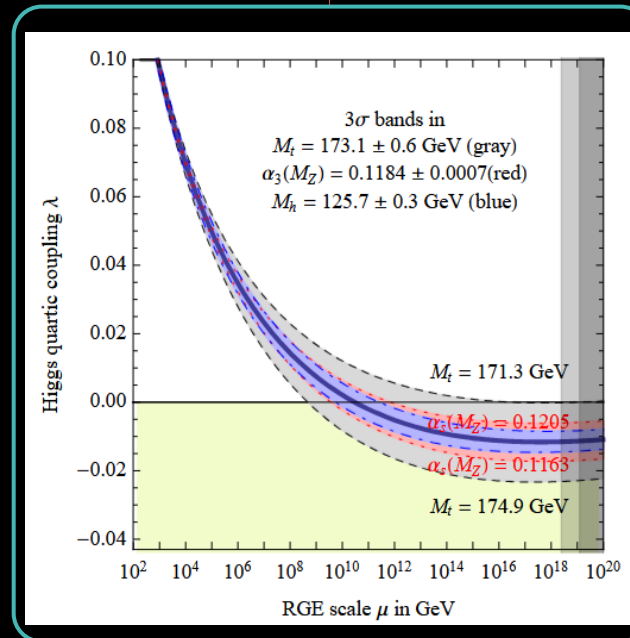
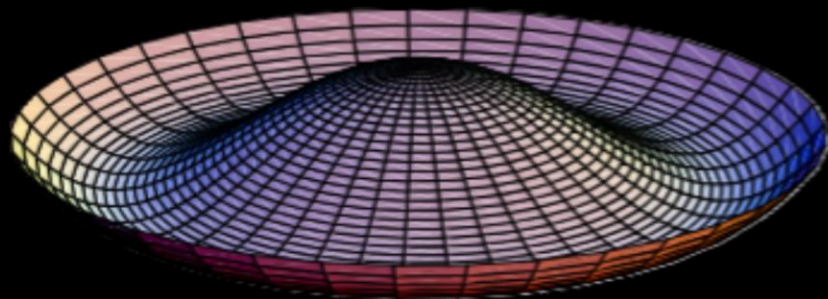


Tree level potential

$$V(h) = \frac{1}{4} \lambda h^4 - \frac{1}{2} \mu^2 h^2$$

RG improved potential

$$V(h) = \frac{1}{4} \lambda(h) h^4 - \underbrace{\frac{1}{2} \mu^2 h^2}_{\text{negligible at } h \gg v}$$





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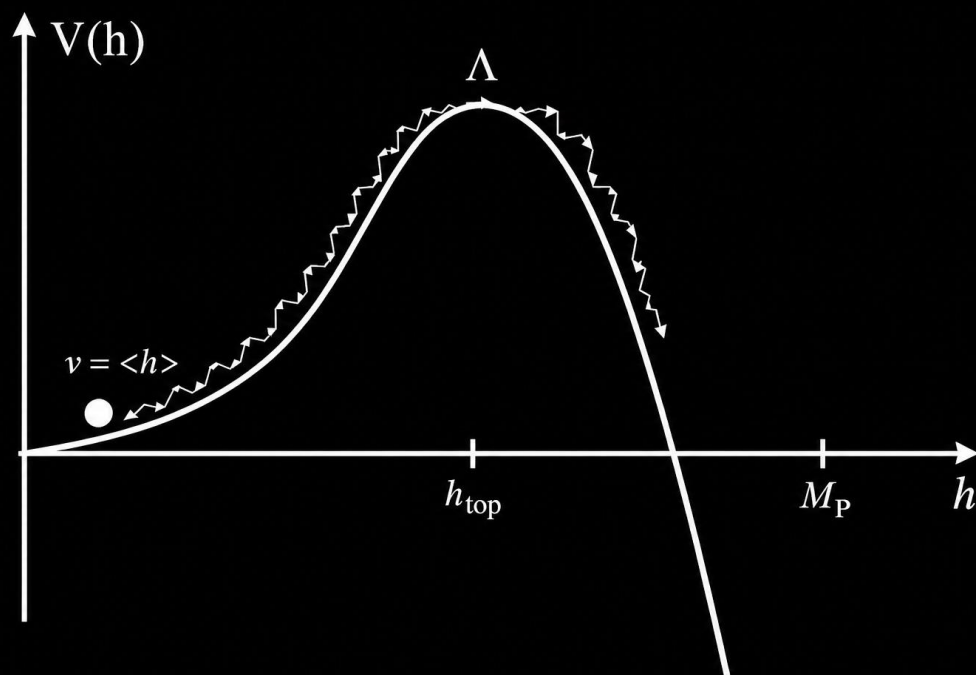
**THE HIGGS METASTABILITY IS NOT  
JUST A PARTICLE PHYSICS  
QUESTION...**

**... IT IS A NONLINEAR SPACETIME  
QUESTION.**

# COSMOLOGICAL IMPLICATIONS & STOCHASTIC KICKS



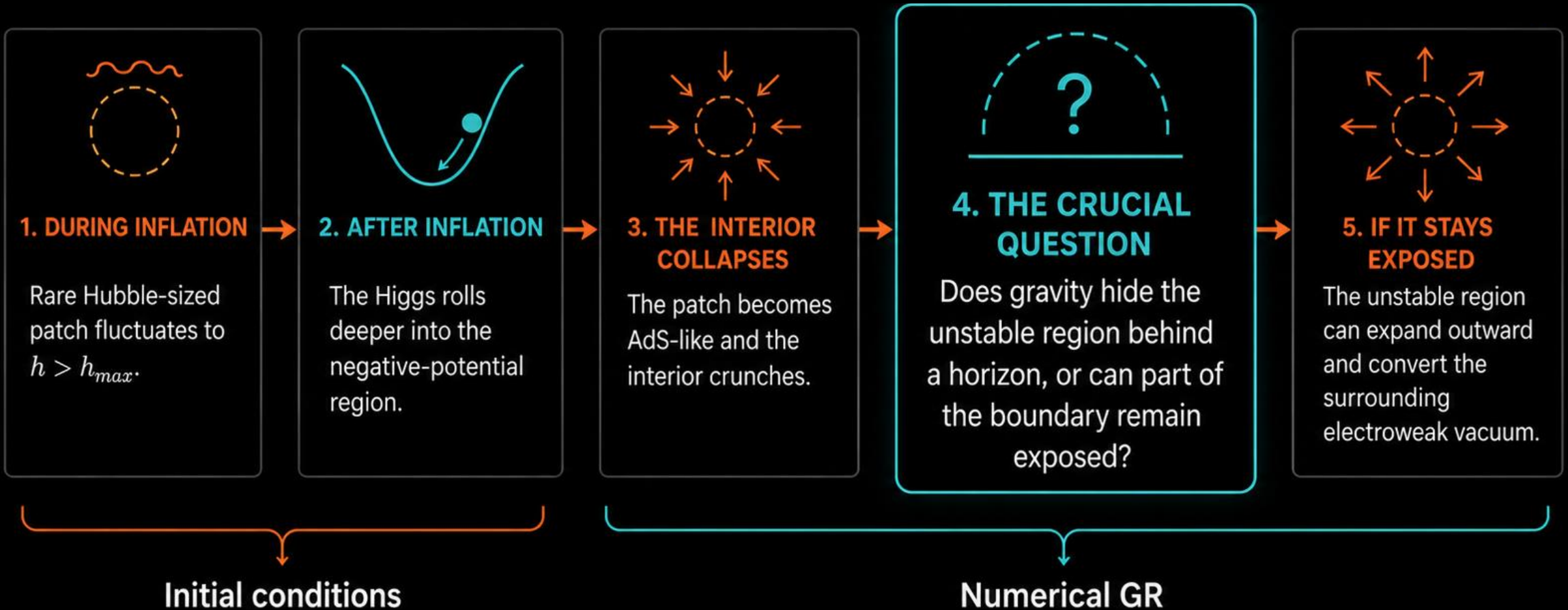
If the Higgs field is a **light spectator** during inflation, stochastic fluctuations  $\delta h$  may push the Higgs field over the top of the hilltop



$$\delta h \sim \pm \frac{H}{2\pi} \gtrsim \Lambda$$

Being over the hilltop is not enough. The field must reach the regime where the classical drift dominates the quantum diffusion

# HOW CAN ONE PATCH THREATEN THE UNIVERSE?

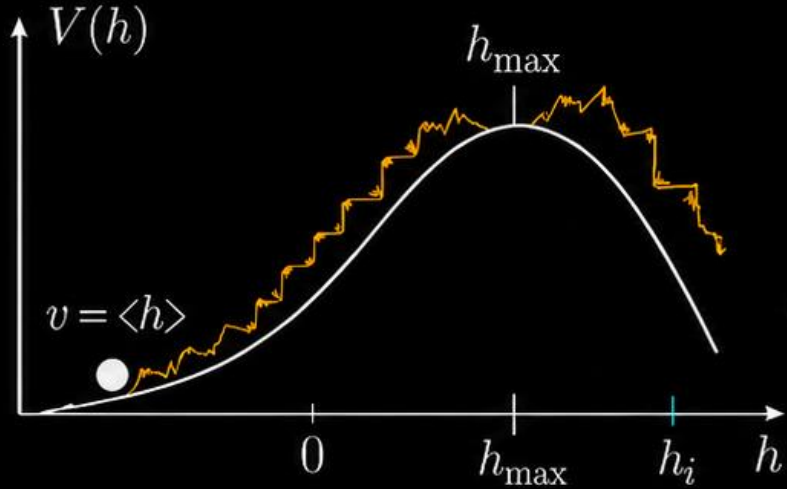


**Our work aims to answer this question.**

# SCALAR FIELD INITIAL CONDITIONS



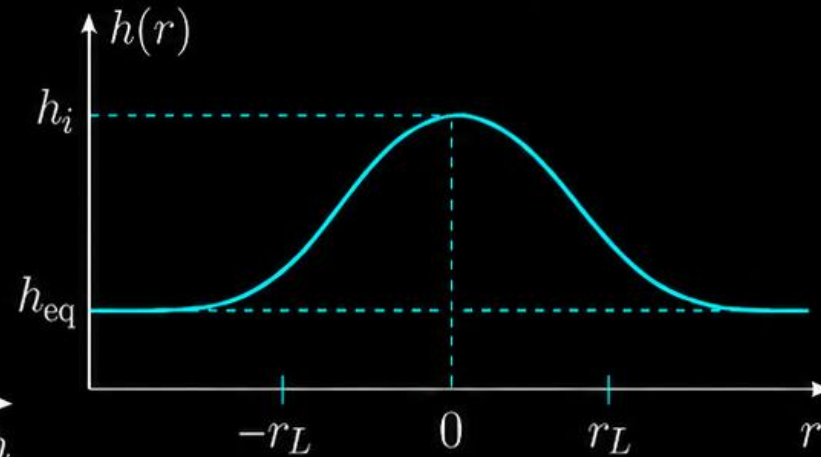
## 1. Over the barrier



$$h_i > h_{\max}$$

A rare inflationary fluctuations pushes the Higgs over the hilltop

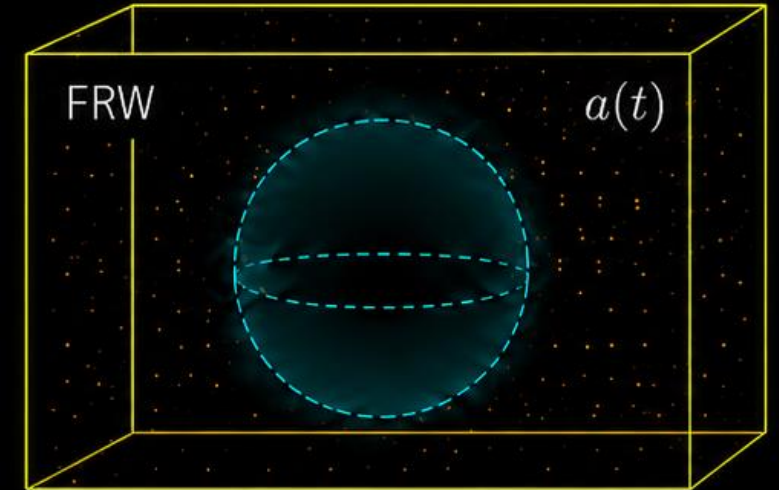
## 2. Gaussian Spatial Profile



$$h(r) = (h_i - h_{\text{eq}}) \exp\left[-\frac{r^2}{2r_L^2}\right] + h_{\text{eq}}$$

A Gaussian spatial profile models the post stochastic Higgs configuration

## 3. Embedded in radiation



$$r_L \gg H_i^{-1}$$

The unstable region is initially superhorizon and sits inside a radiation dominated FRW universe

# COLLAPSE PHASES

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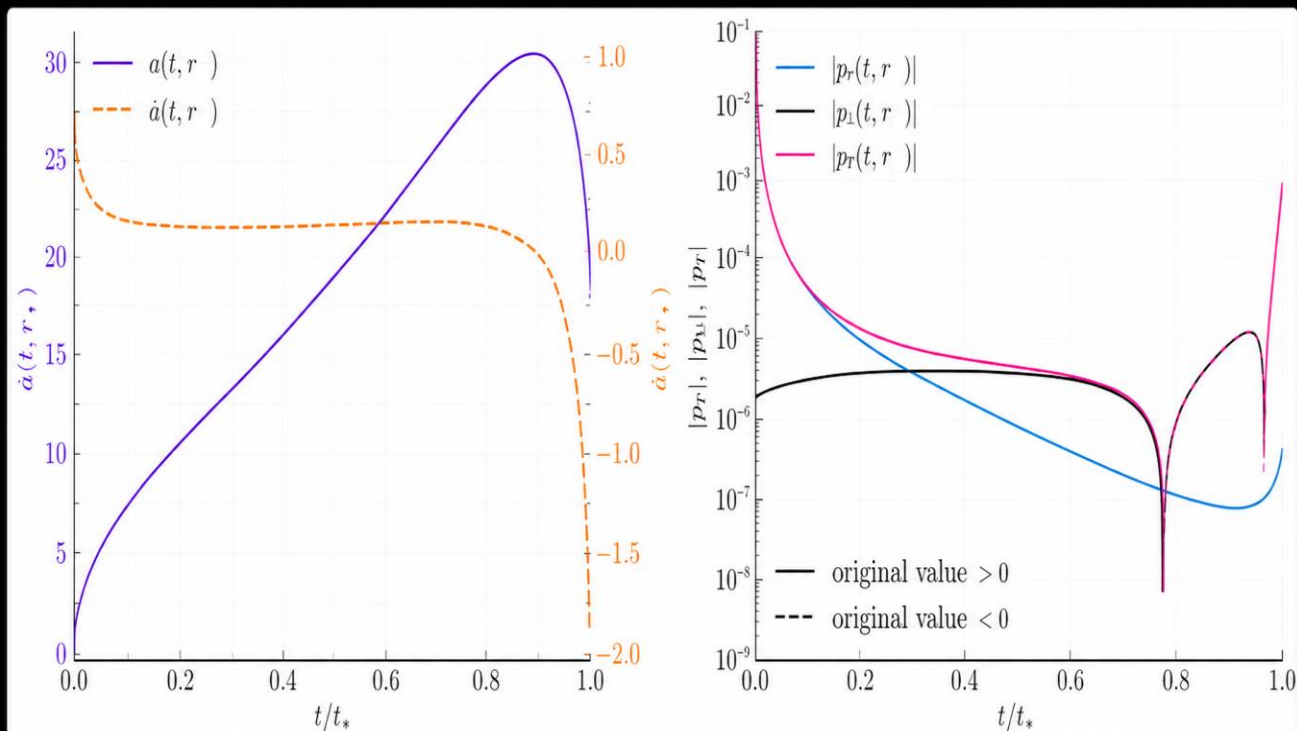
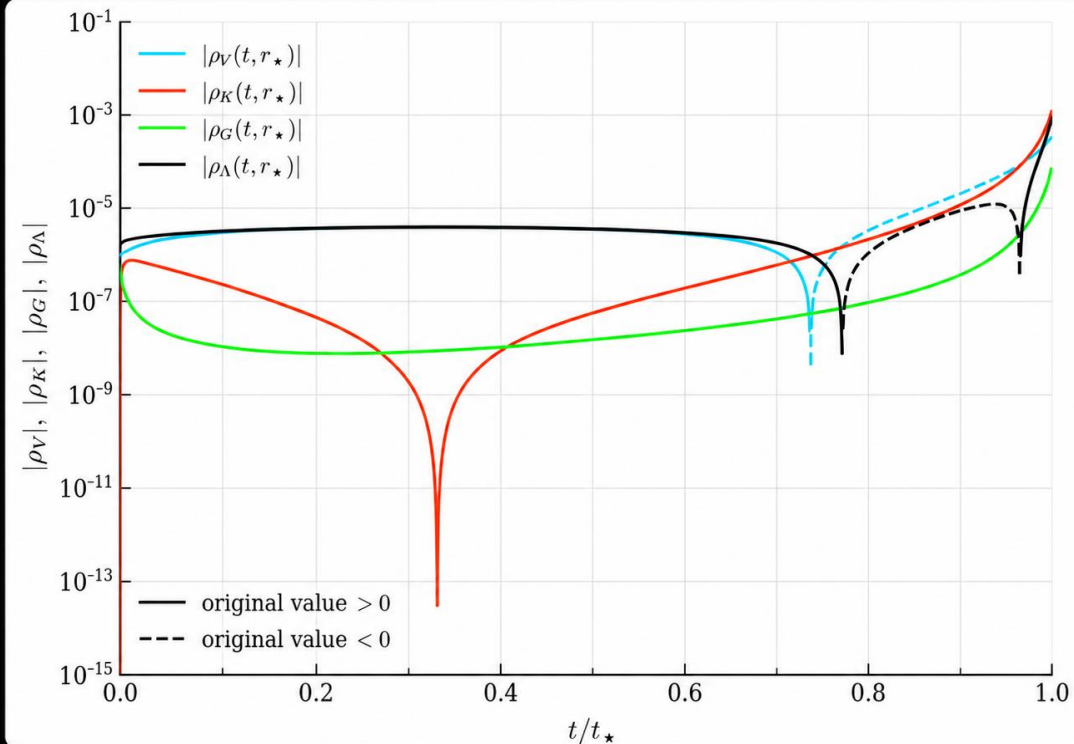


## PHASE 1: INITIAL COLLAPSE

## PHASE 2: SUPERCRITICAL AND SUBCRITICAL COLLAPSE

*Later time evolution*

# PHASE 1: INITIAL COLLAPSE



- **Expansion:** Kinetic energy is Hubble redshifted and gradient energy is subdominant
- **Turnaround:** Negative Higgs potential locally overcomes the radiation density
- **Contraction:** Kinetic energy blueshifts and collapse proceeds toward horizon formation

# COLLAPSE PHASES

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## PHASE 1: INITIAL COLLAPSE

## PHASE 2: SUPERCRITICAL AND SUBCRITICAL COLLAPSE

Later time evolution

# PHASE 2: THE ROLE OF THE HIGGS BARRIER

 $R_w$ 

**BARRIER SCALE  
(PHYSICAL LENGTH)**

The physical size (radius) of the Higgs barrier region.

It sets the size of the patch in physical units.

 $l_\sigma$ 

**SELF-GRAVITY SCALE**

The gravitational length set by the barrier tension  $\sigma$ .

It does not depend on the size of the patch.

$$l_\sigma = \frac{1}{2\pi\sigma}$$

Characteristic gravitational length set by  $\sigma$ .

$$\frac{R_w}{l_\sigma}$$

The ratio  $R_w / l_\sigma$  determines whether the barrier is subcritical or supercritical.

$R_w / l_\sigma < 1$   
: subcritical

$R_w / l_\sigma > 1$   
: supercritical

# PHASE 2: NULL GEODESIC EXPANSIONS



## SIGN INTERPRETATION

- $\Theta^+ > 0$  : outgoing null rays diverge
- $\Theta^+ < 0$  : outgoing null rays converge
- $\Theta^- < 0$  : ingoing null rays converge
- $\Theta^- > 0$  : ingoing null rays diverge
- Normal surface:  $(\Theta^+, \Theta^-) = (+, -)$
- Trapped surface:  $\Theta^+ < 0$  and  $\Theta^- < 0$
- Anti-trapped surface:  $\Theta^+ > 0$  and  $\Theta^- > 0$

Normal surfaces occur in flat spacetime; trapped surfaces occur inside black holes; anti-trapped surfaces occur in expanding universes.

## REGIONS AND HORIZONS

Normal exterior:	$(\Theta^+, \Theta^-) = (+, -)$
Trapped:	$(\Theta^+, \Theta^-) = (-, -)$
Anti-trapped / expanding:	$(\Theta^+, \Theta^-) = (+, +)$
Normal surface:	$(\Theta^+, \Theta^-) = (-, +)$

- Marginally trapped surface:  $\Theta^- < 0$  and  $\Theta^+ = 0$  (apparent horizon)
- Marginally anti-trapped surface:  $\Theta^+ > 0$  and  $\Theta^- = 0$  (cosmological or white-hole horizon)
- Bifurcating trapping horizon:  $\Theta^+ = \Theta^- = 0$  simultaneously.

Apparent horizon condition:

$$\Theta^+ \Theta^- = 0 \Rightarrow 2M = R$$

# PHASE 2: LATE TIME CAUSAL STRUCTURE

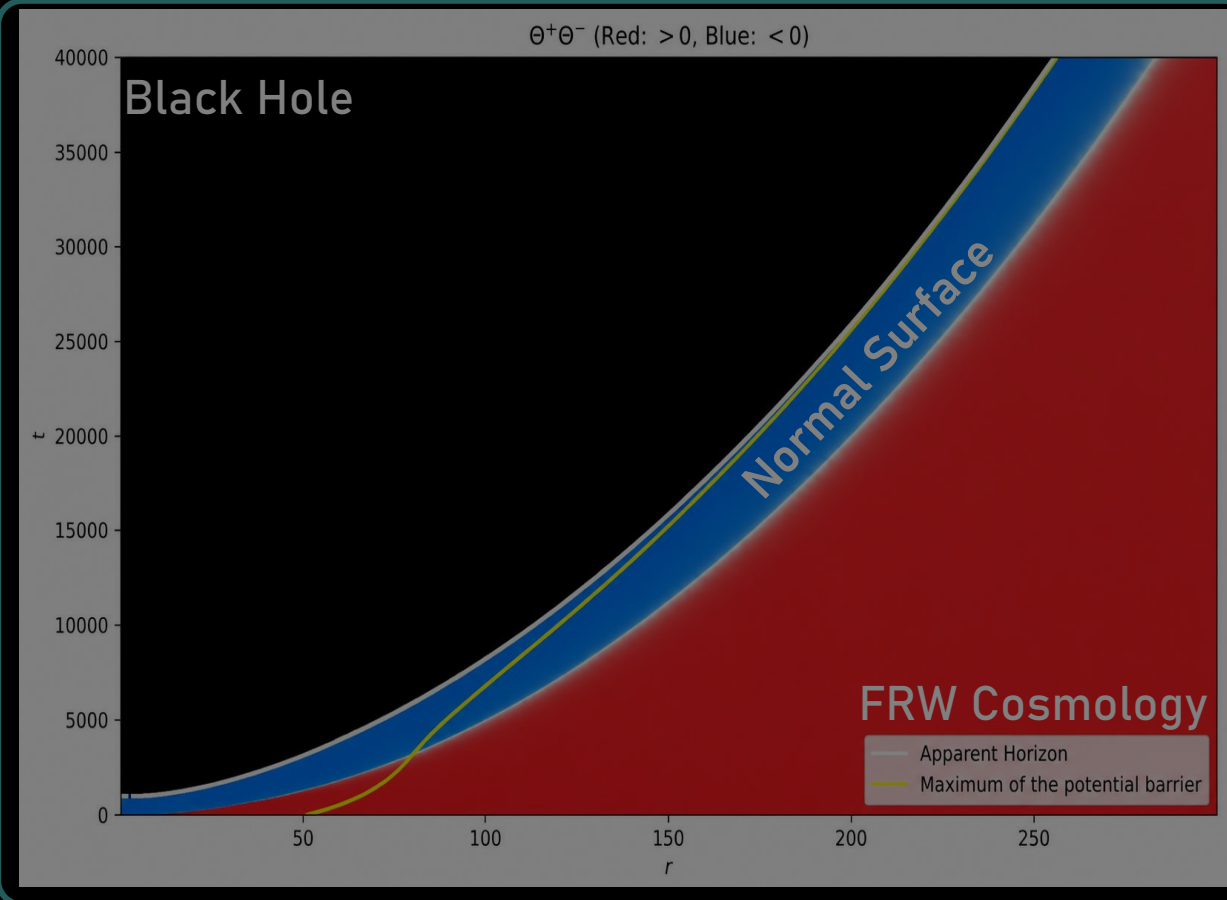
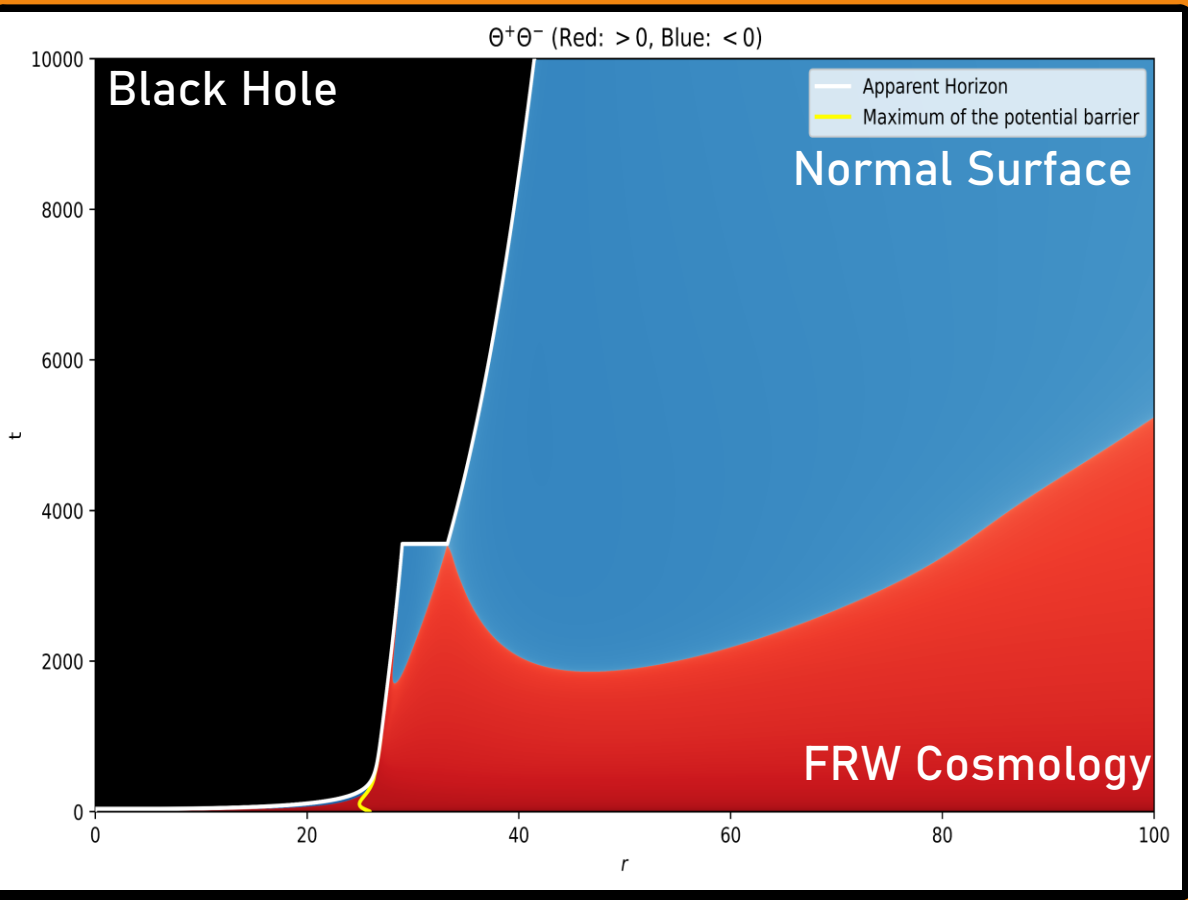


$R_w / l_\sigma > 1$   
: supercritical

$R_w / l_\sigma < 1$   
: subcritical

Large radial excursion signifies **secondary apparent horizon formation**.

Single apparent horizon, gradual growth of the initial PBH.



# PHASE 2: LATE TIME CAUSAL STRUCTURE

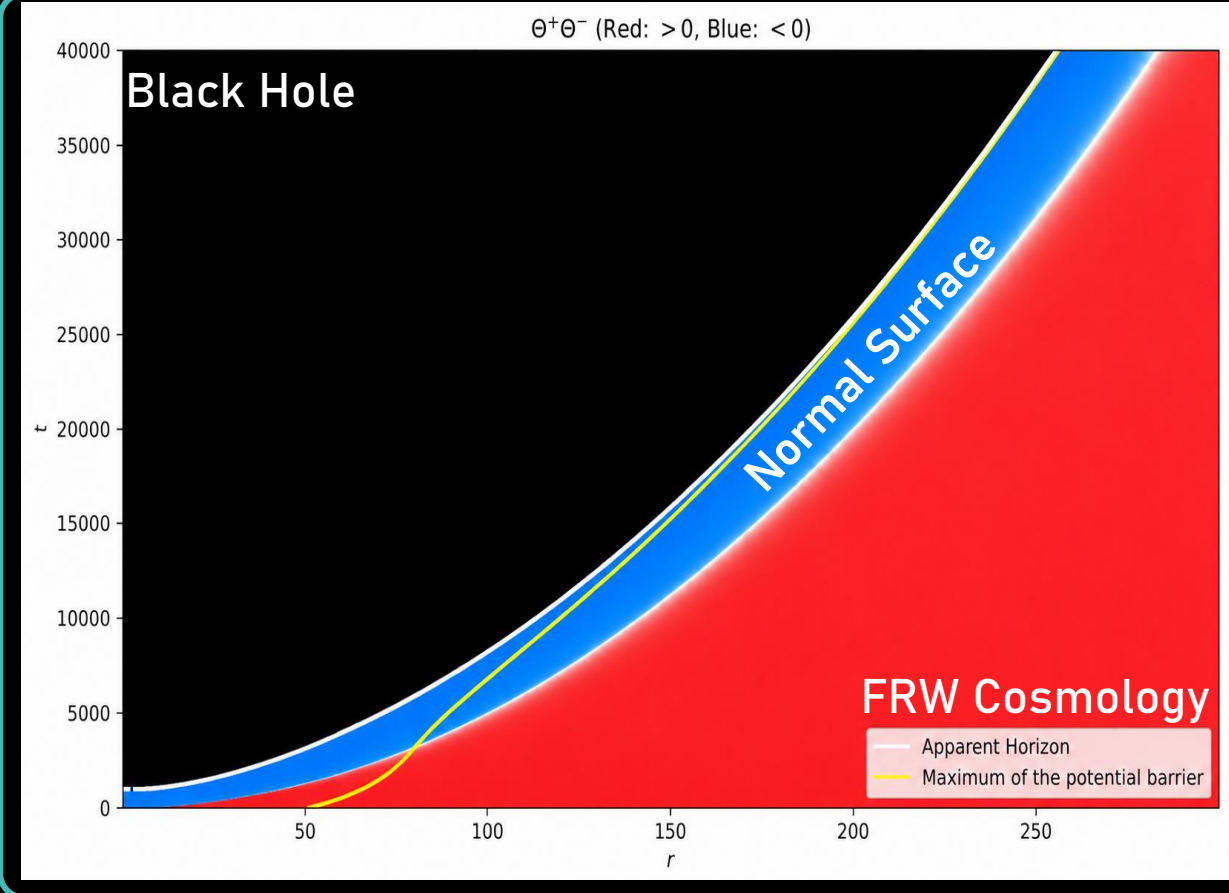
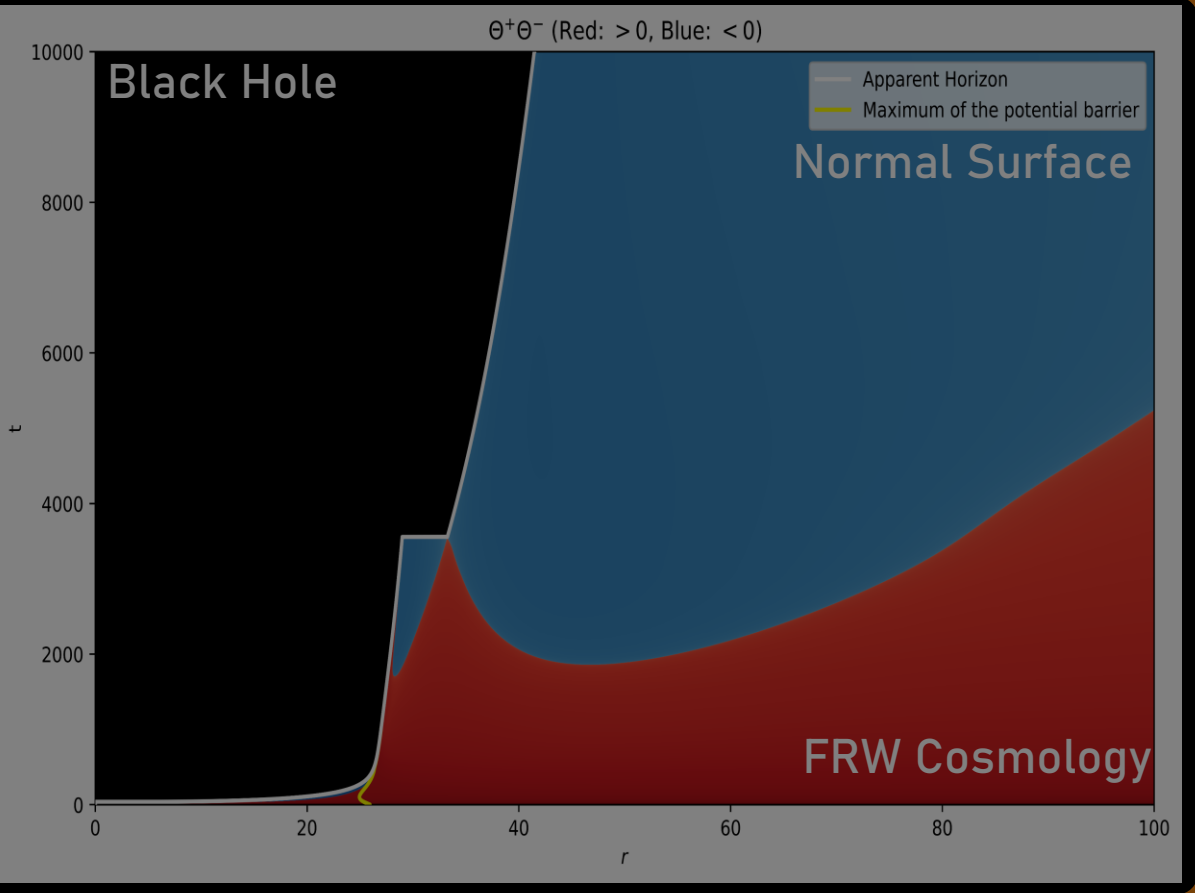


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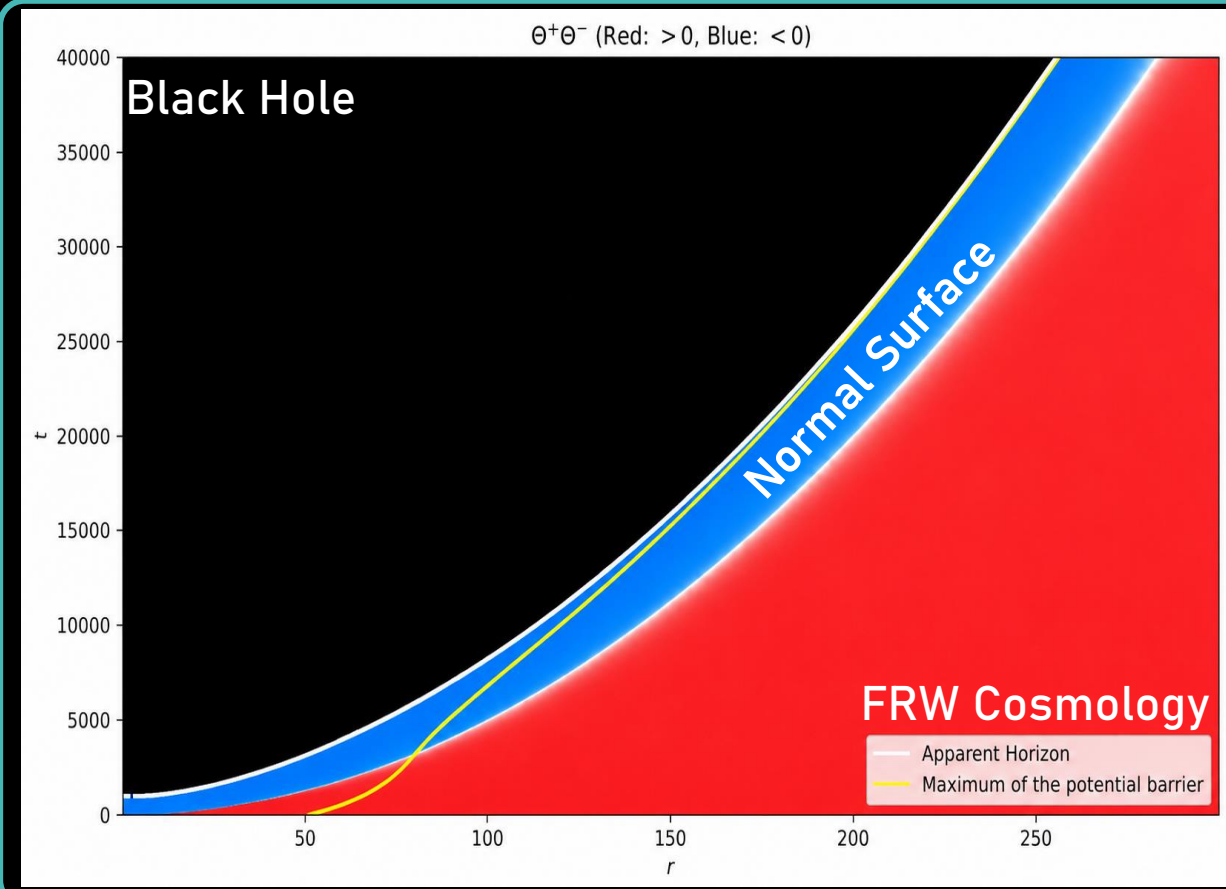
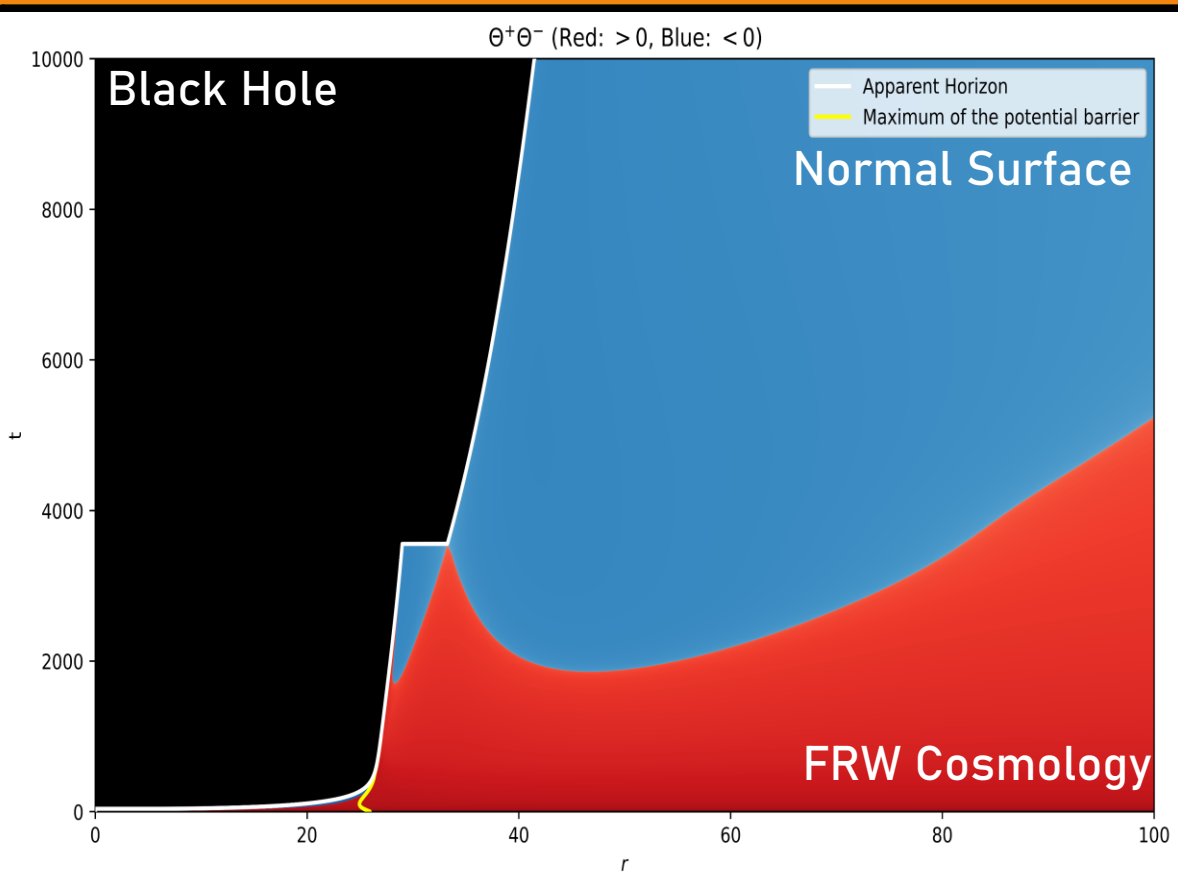


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Large radial excursion signifies **secondary apparent horizon formation**.

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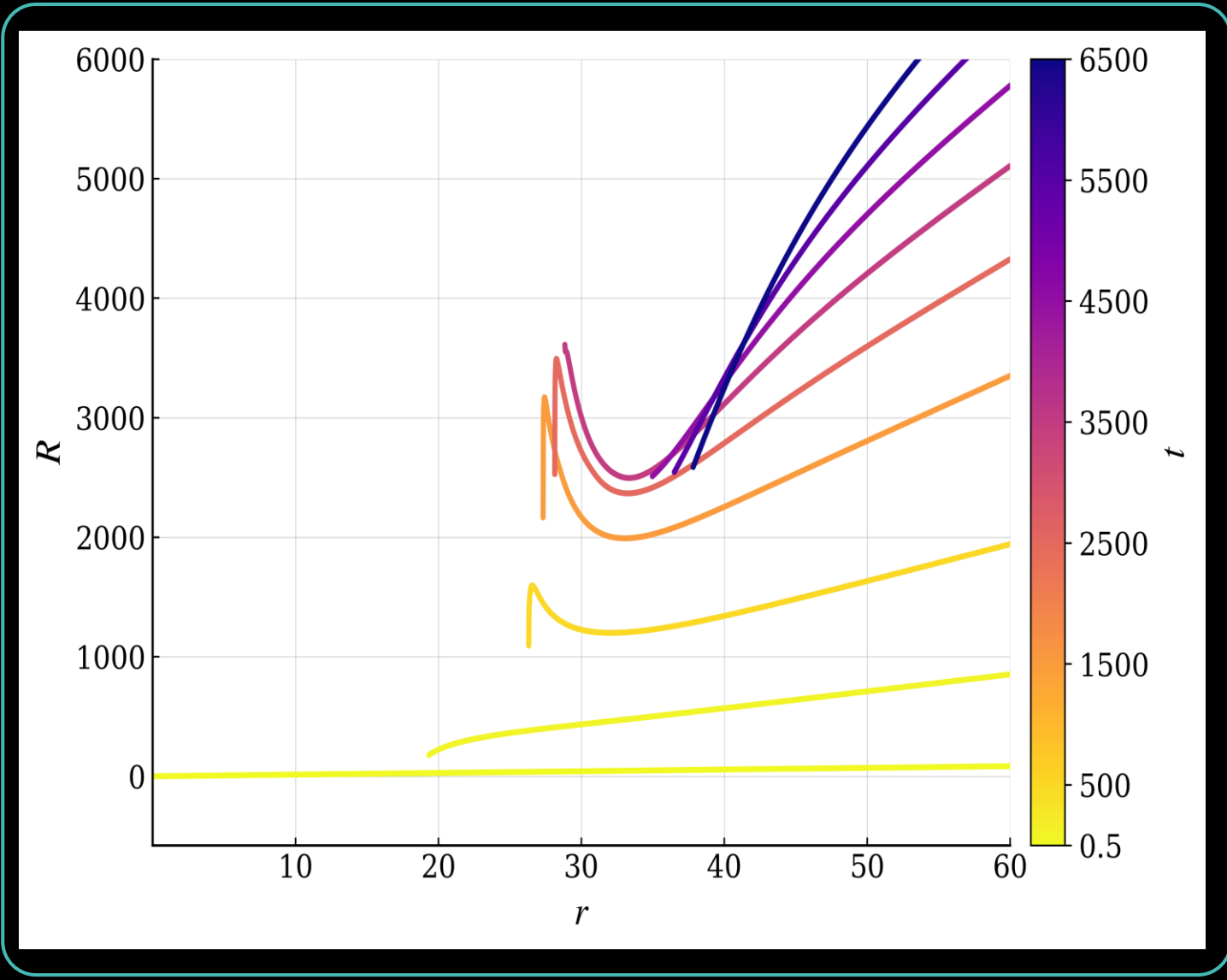


Both **supercritical** and **subcritical** configurations result in the over-the-barrier region hidden behind horizons.

# PHASE 2: THROAT STRUCTURE



- A spatial slice passing through this region experiences the 3-geometry curves back on itself.
- The **minima of  $R$  ( $\partial R/\partial r = 0$ )** corresponds to the **throat connecting the black hole interior to the exterior FRW.**
- The throat pinches off at the **bifurcating horizon.**

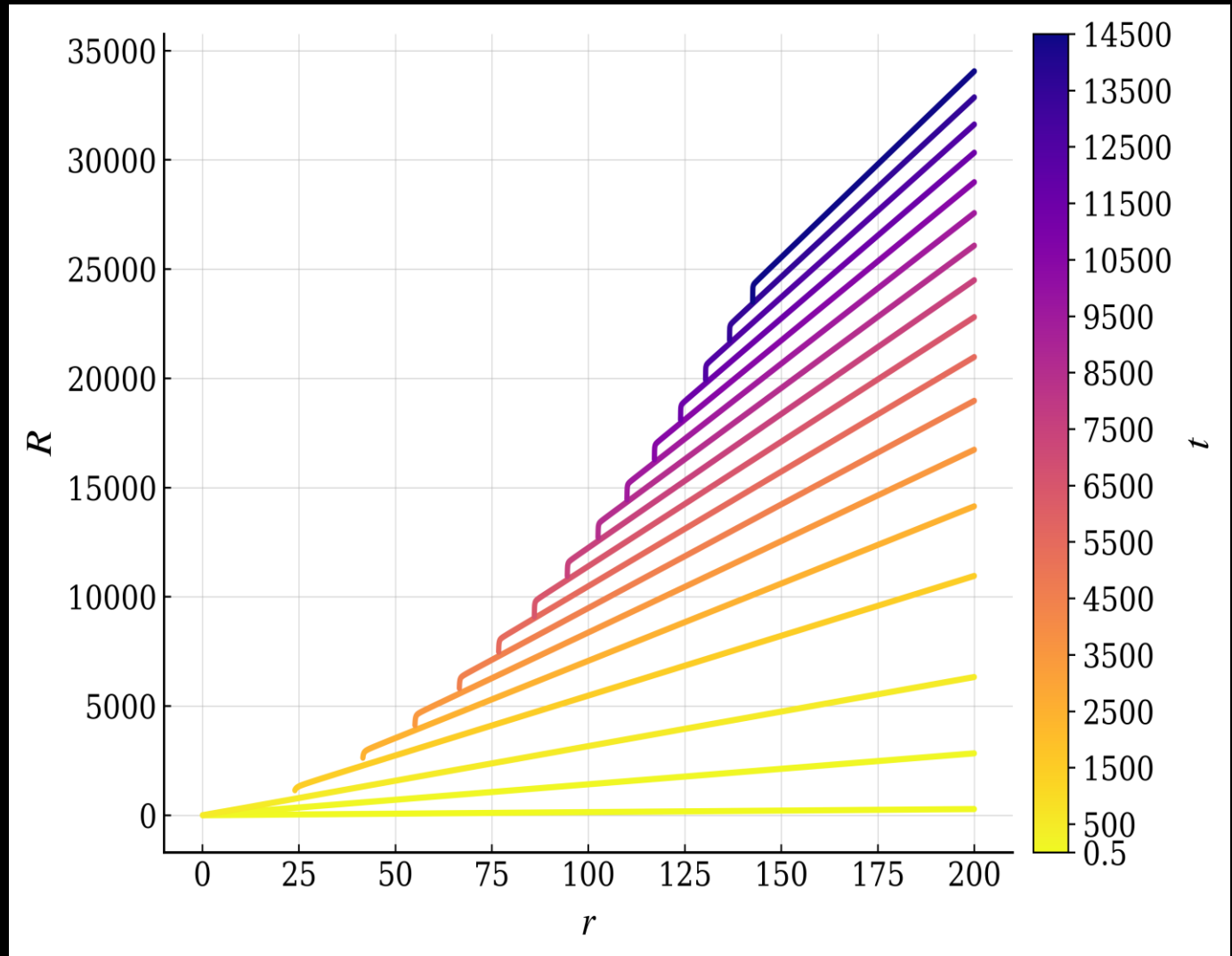


# PHASE 2: ABSENCE OF THROAT FORMATION



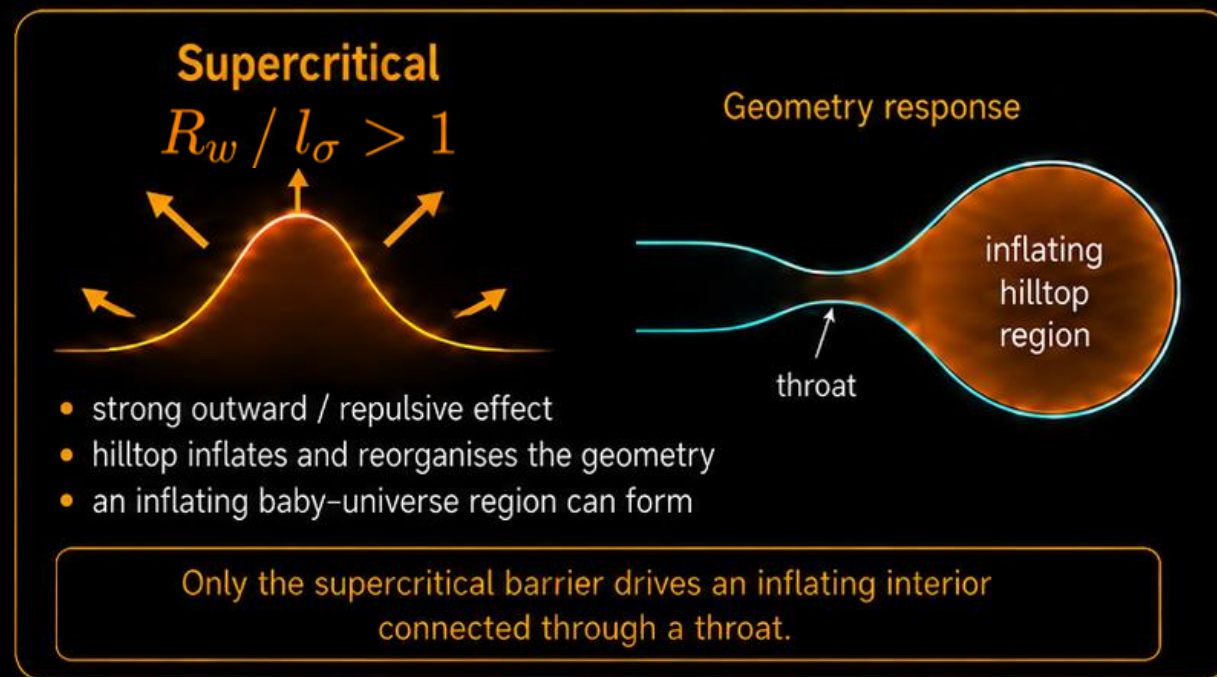
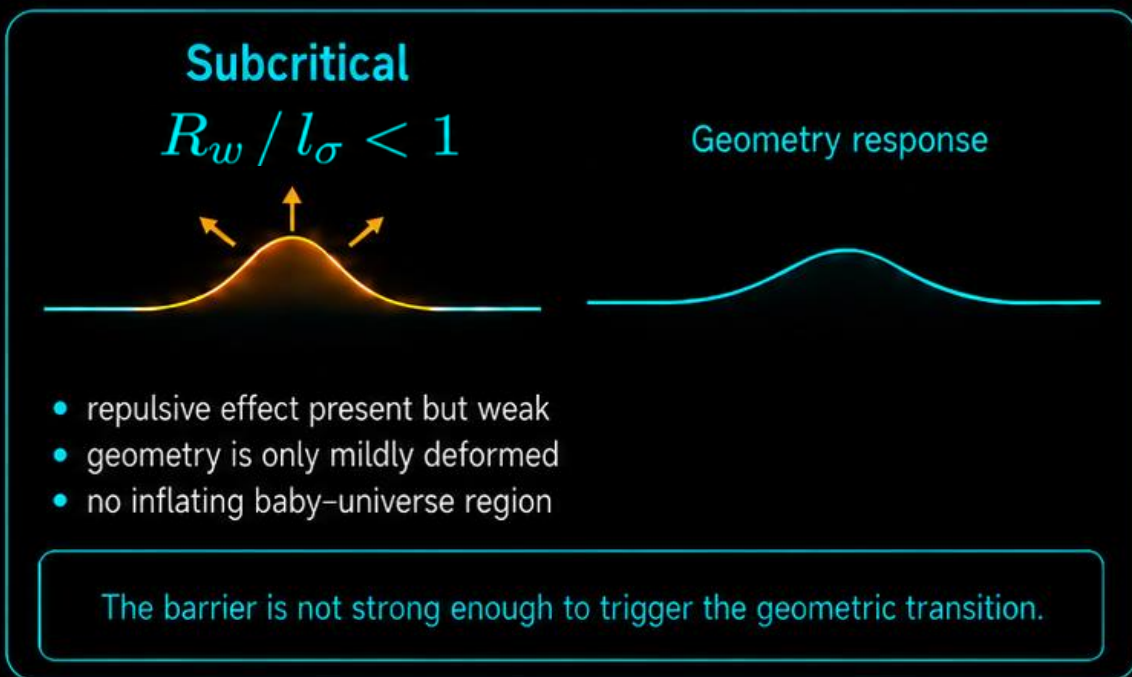
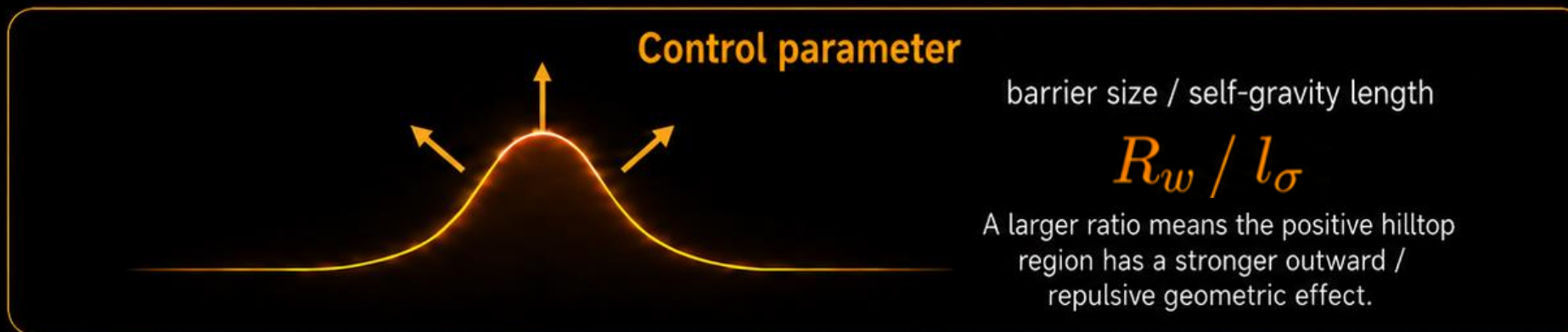
- The areal radius remains **monotonic**
- The initial true vacuum black hole deforms the areal radius away from FRW near the horizon
- The areal radius continues to always label a **single connected exterior**

Subcritical collapse remains a **single horizon spacetime**



# Barrier compactness controls the hilltop response

How the barrier size / self-gravity length determines whether the hilltop becomes repulsive enough to inflate

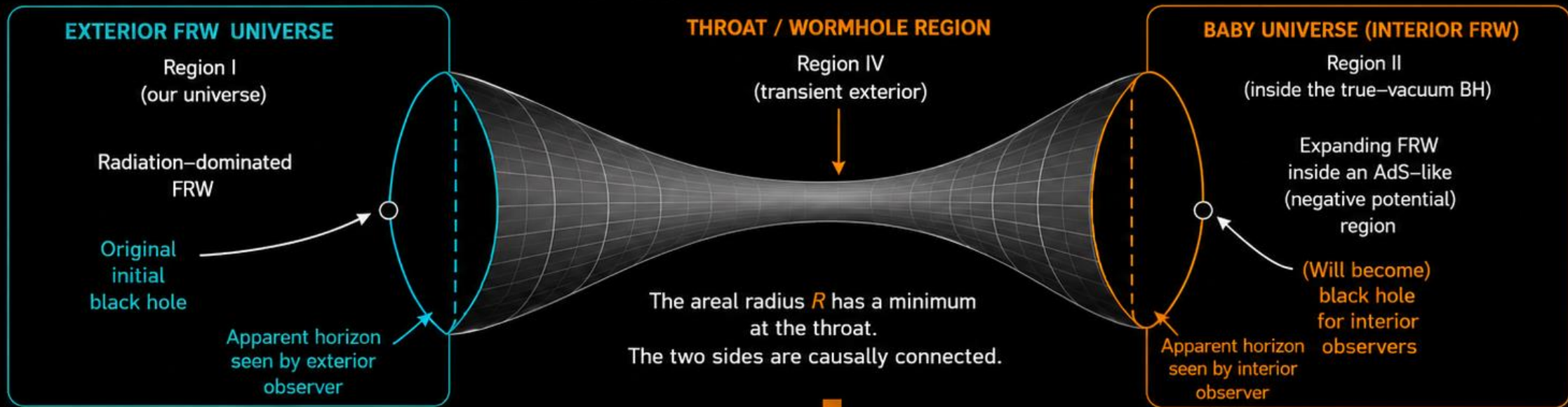


orange = positive barrier / hilltop region

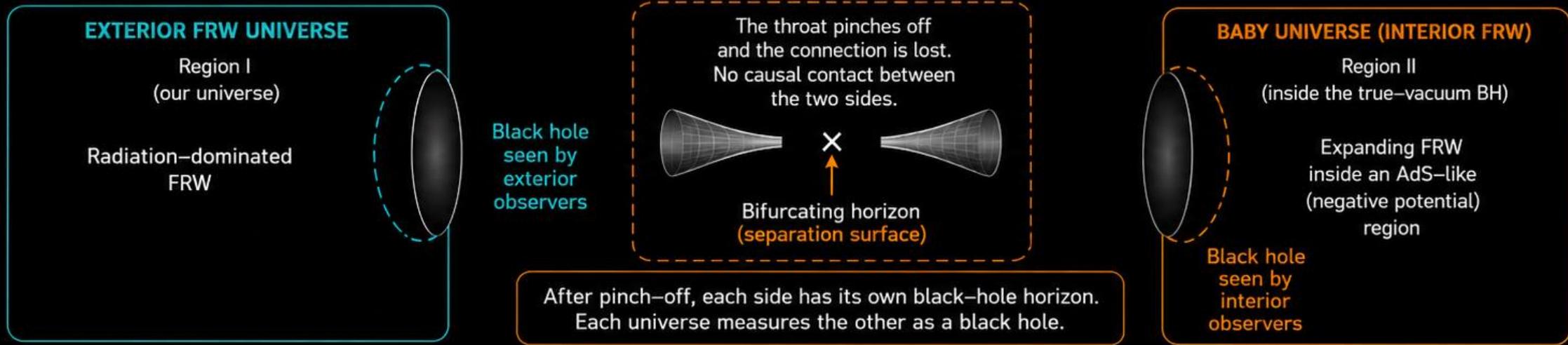
cyan = radiation-dominated FRW exterior

arrows = outward / repulsive effect

# 1. BEFORE PINCH-OFF: ONE CONNECTED GEOMETRY



# 2. AFTER PINCH-OFF: TWO DISCONNECTED BLACK HOLES



## LEGEND (matching earlier slide)

**Region I:** Normal exterior (our FRW universe)

**Region II:** Interior FRW / baby universe (inside true-vacuum BH)

**Region III:** Not shown here (far exterior FRW, causally separated from the BH interior by the outer horizon in full evolution)

**Region IV:** Transient exterior / throat region (exists only before pinch-off)

# CONCLUSIONS

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The final state is always a **standard black hole embedded in a radiation dominated FRW background**, with any unstable over the barrier region **entirely hidden behind horizons**

- The early evolution is controlled by the true vacuum
- The late time evolution is controlled by the potential barrier

The question of whether inflation produces fluctuations that probe **negative regions of field space through stochastic fluctuations** is not only relevant to the Standard Model Higgs field but is a question **of relevance to many multi-field models.**

THANK YOU!

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**BACK UP SLIDES**

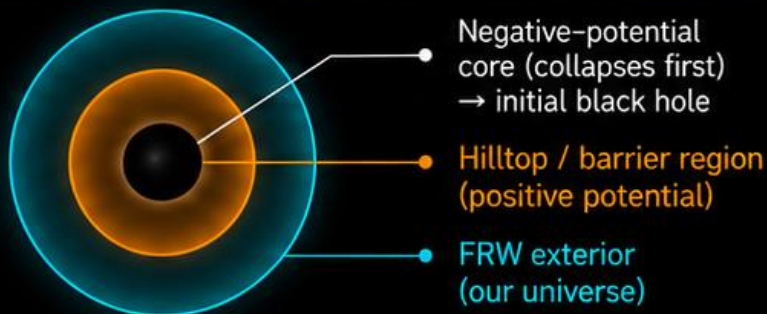
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# Hilltop-driven throat formation

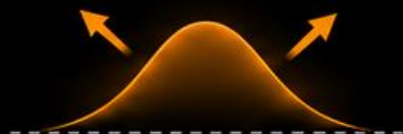
Why supercritical configurations pinch off, while subcritical ones do not

## PHYSICAL PICTURE



- 1 The negative-potential core collapses first and forms the initial black hole.
- 2 The hilltop region controls the late-time geometry.
- 3 Its effect depends on barrier size / self-gravity length.

## SUBCRITICAL HILLTOP



Energetically present, but too weak to drive a secondary geometric transition.

barrier size / self-gravity length  $< 1$

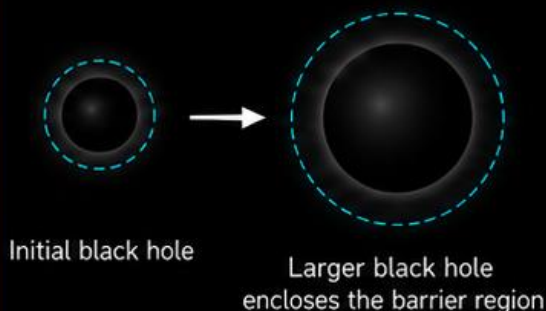
## SUPERCritical HILLTOP



Strong enough to reorganise the geometry and drive throat formation.

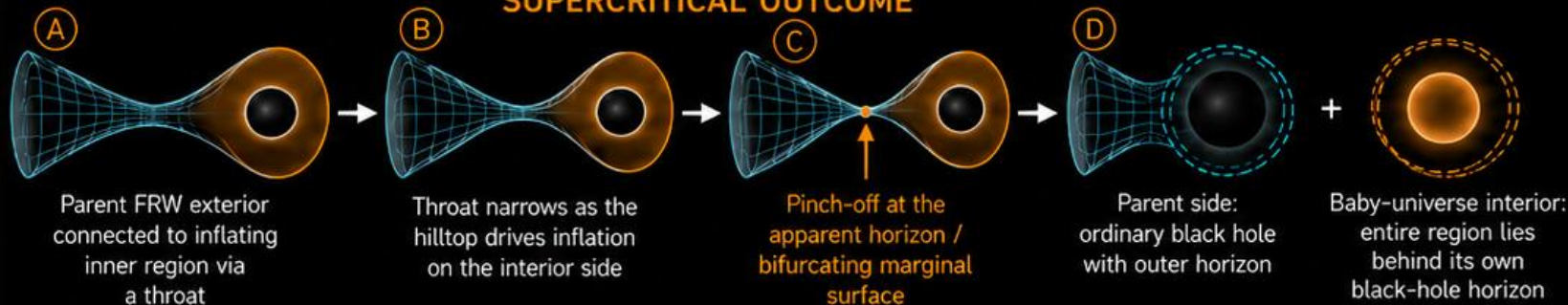
barrier size / self-gravity length  $> 1$

## SUBCRITICAL OUTCOME



- ✓ no throat
- ✓ no second trapped surface
- ✓ type I-like

## SUPERCritical OUTCOME

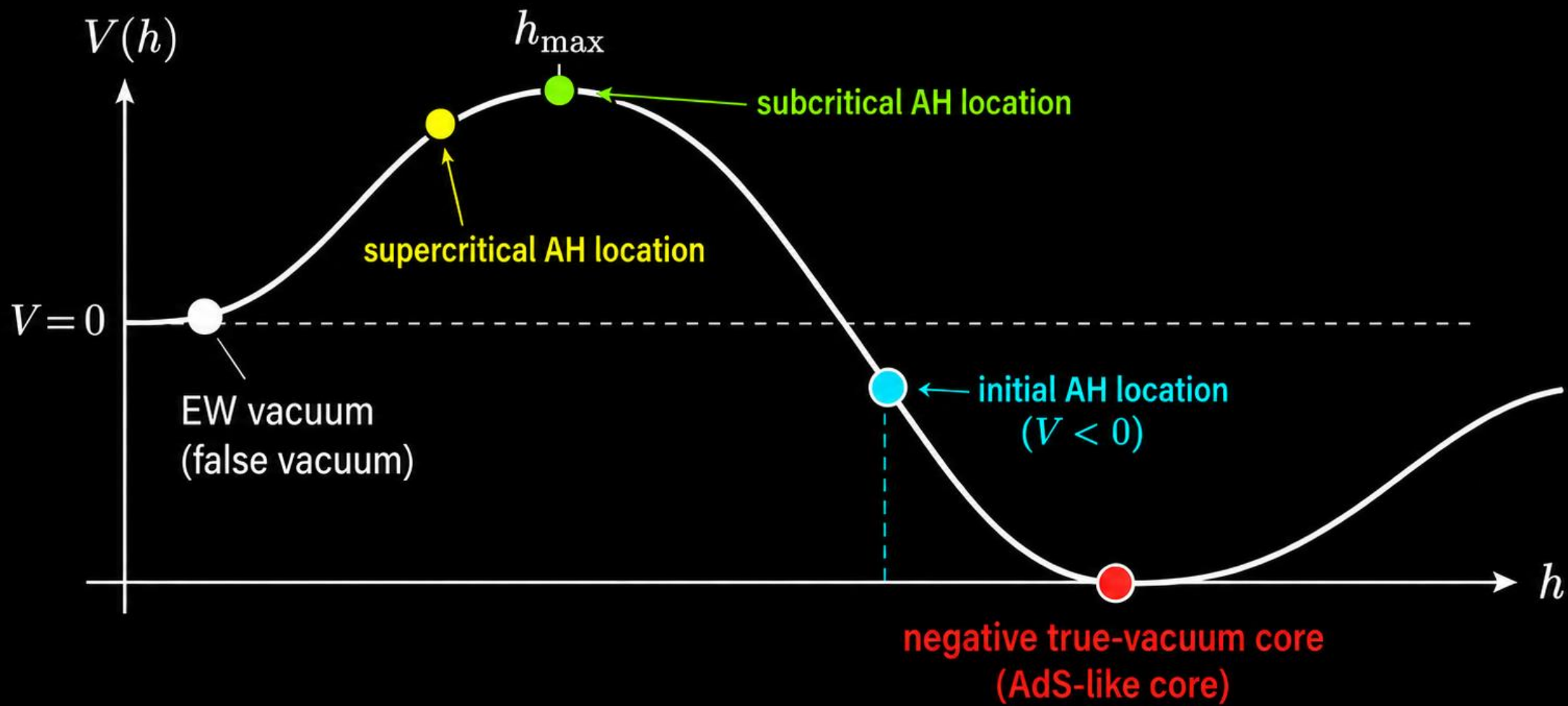


Pinch-off at the apparent horizon disconnects the baby universe.  
The parent FRW exterior sees only an ordinary black hole, while the baby-universe interior lies entirely behind its own black-hole horizon.

**CYAN:** Parent FRW exterior (our universe)

**ORANGE:** Hilltop / barrier region (positive potential)

**BLACK:** Black holes

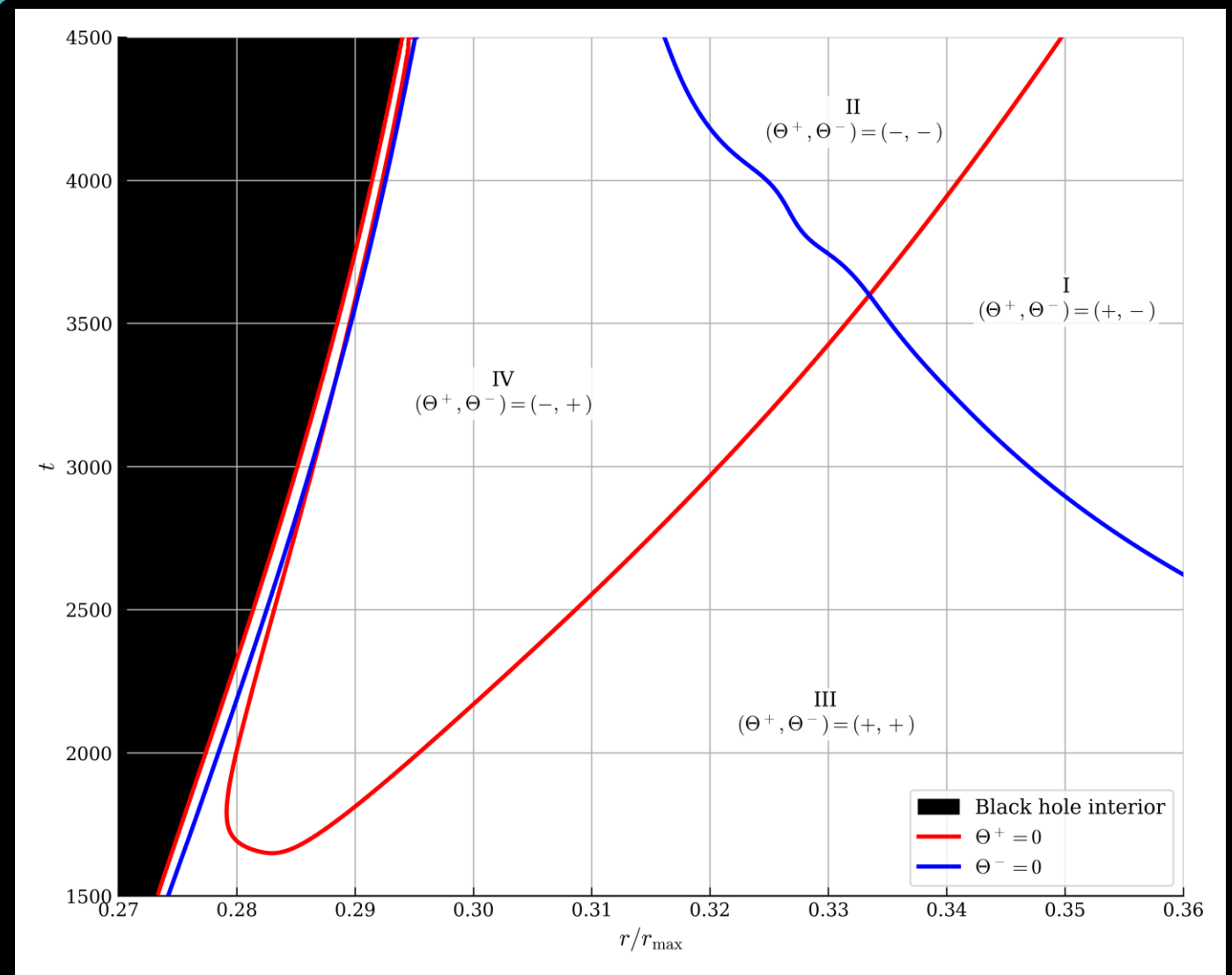


# PHASE 2: THROAT/WORMHOLE STRUCTURE



- Region I: Normal causal exterior.
- Region II: Fully trapped black hole interior with a baby universe inside.
- Region III: FRW cosmology, causally separated from the black hole and accessible to an external FRW observer.
- Region IV: Transient causal exterior to the true vacuum black hole and the supercritical black hole.

The location where the expansions intersect corresponds to a bifurcating horizon.



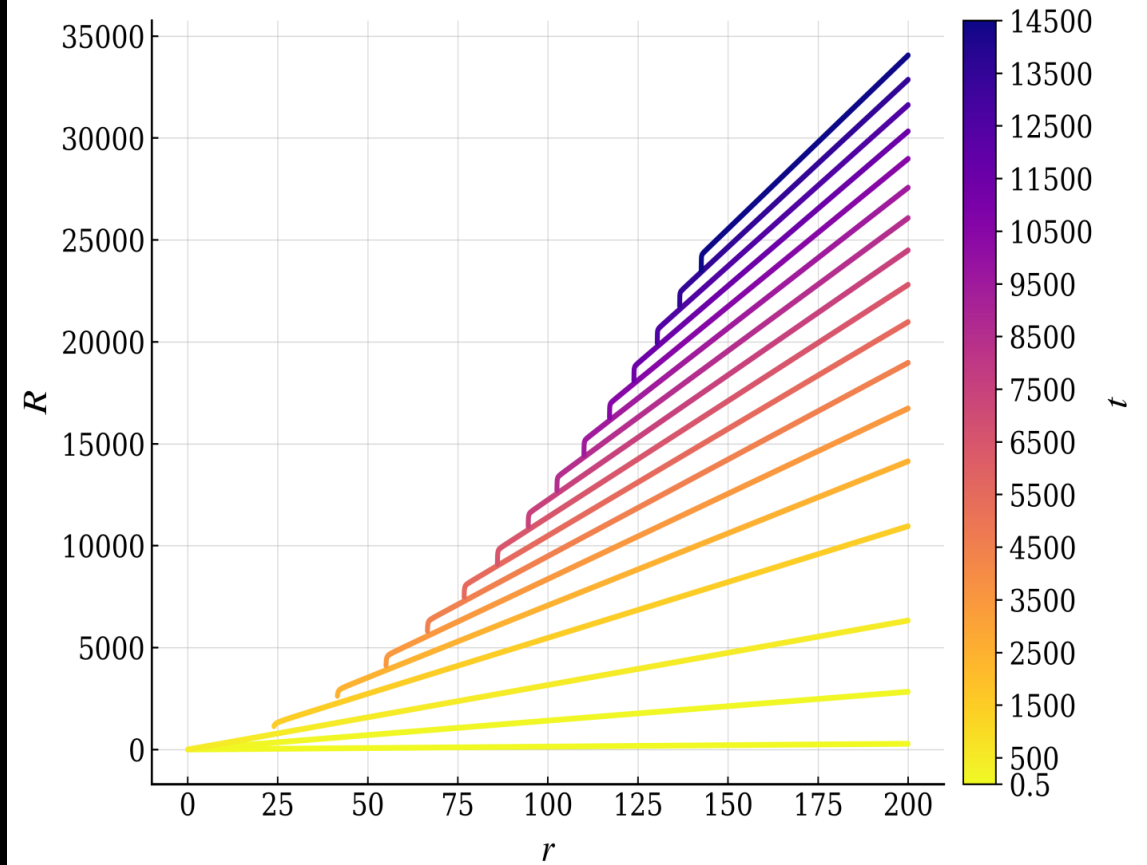
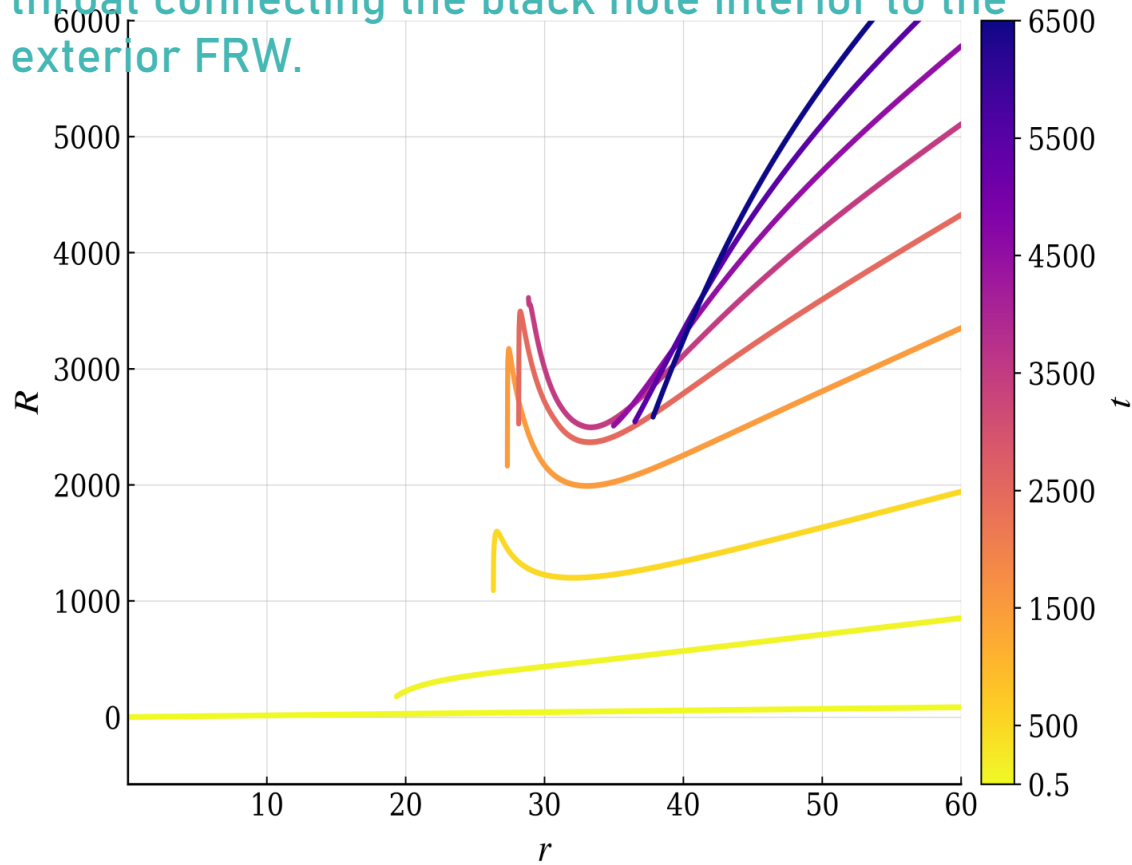
# PHASE 2: THROAT STRUCTURE



$R_w / l_\sigma > 1$   
: supercritical

$R_w / l_\sigma < 1$   
: subcritical

The minima of  $R$  ( $\partial R / \partial r = 0$ ) corresponds to the throat connecting the black hole interior to the exterior FRW.



# COSMOLOGICAL IMPLICATIONS & STOCHASTIC KICKS



The classical motion will beat the quantum motion if:

$$\delta h_{\text{classical}} \sim \frac{\lambda(h)h^3}{3H^2} \gtrsim \delta h_{\text{quantum}} \sim \frac{H}{2\pi}$$

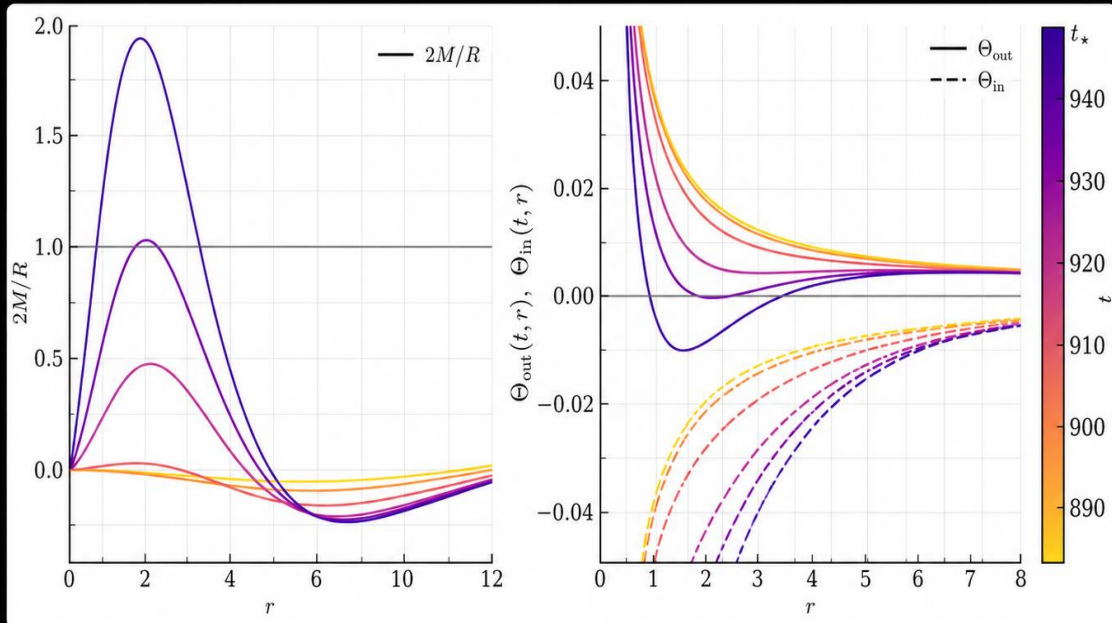
e.g. when the slow-roll evolution due to the potential exceeds the stochastic evolution due to the inflationary fluctuations.

Then, only after the fluctuation becomes very large, that the SR approximation breaks down, the fluctuation rapidly diverges to the true vacuum.

**What is the fate of these resulting rare patches?**

# PHASE 1: TRUE VACUUM PBH

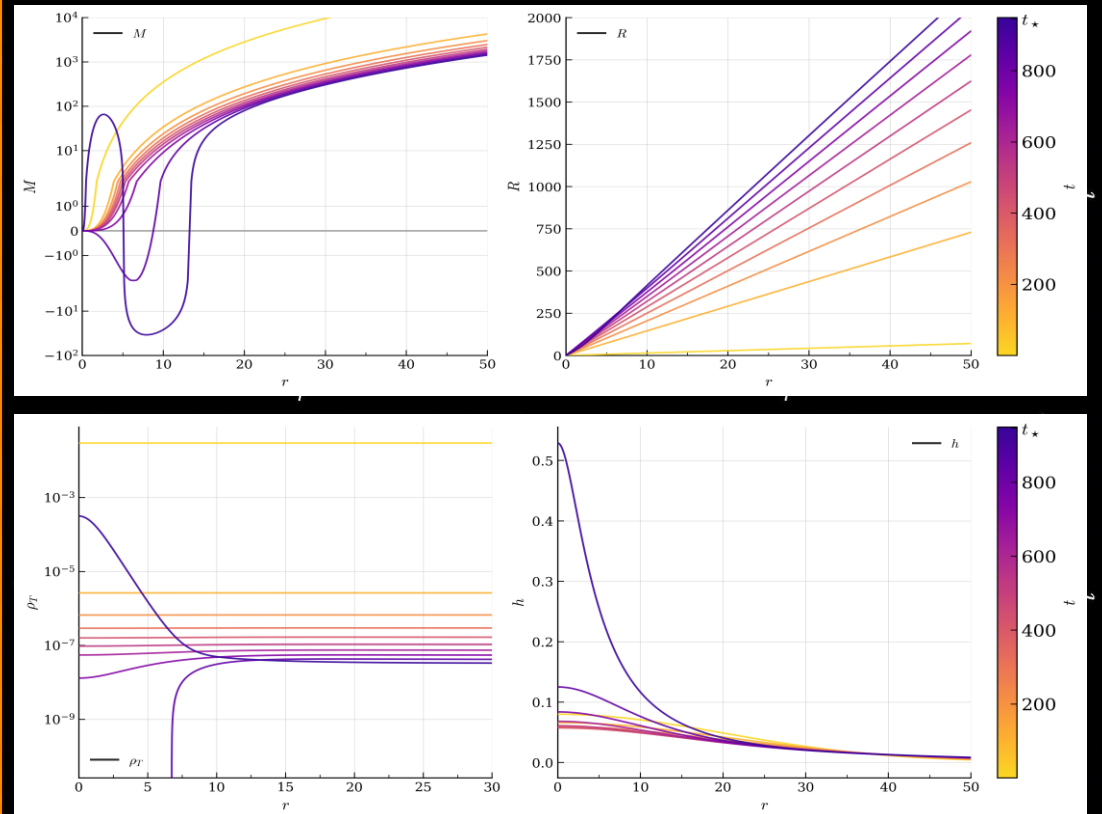
## HORIZON FORMATION



$\Theta_{out}$ : outgoing null expansion     $\Theta_{in}$ : ingoing null expansion

$t_*$ : first apparent horizon time

## PROFILE EVOLUTION



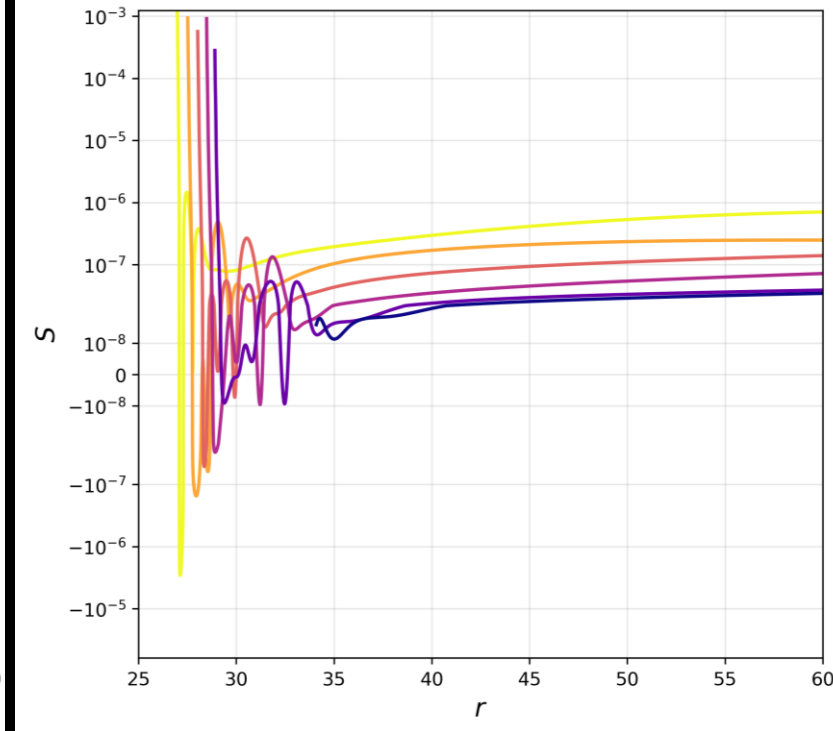
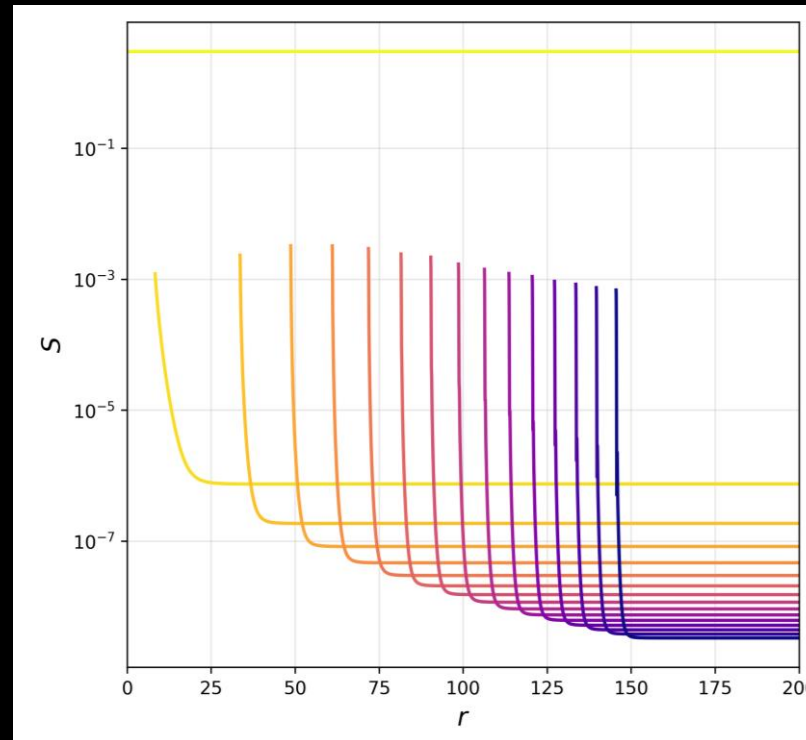
- Compactness rises through  $2M/R = 1.0$  while  $\Theta_{out}$  vanishes and  $\Theta_{in}$  remains negative, indicating an apparent horizon has formed
- **Areal radius is monotonic** and the **Misner Sharp mass** develops a **negative region**

# PHASE 2: ENERGY DRIVING LATE TIME EVOLUTION



$$S \equiv 4\pi (T_{00} + T_1^1 + 2T_2^2) = 2\dot{h}^2 - 2V(h) + \text{fluid terms}$$

- **Negative potential** (True Vacuum) contributes a “focusing”
- **Positive potential** (Higgs Hilltop) contributes a “repulsion”
- The gravitational source is related to the trace of the extrinsic curvature



If the **positive potential barrier dominates**, it reorganises the exterior geometry and dictates the supercritical or subcritical evolution

# NUMERICAL GR SETUP

## GRAVITY-HIGGS-RADIATION SYSTEM

We evolve the coupled Einstein-Klein-Gordon system with a radiation fluid in spherical symmetry.

$$G_{\mu\nu} = 8\pi G (T_{\mu\nu}^{(h)} + T_{\mu\nu}^{(\text{fluid})})$$

$$\square h = \partial_h V$$

$$\nabla_\mu T_{(\text{fluid})}^{\mu\nu} = 0$$

Equation of state:  $p = \omega\rho$ , with  $\omega = 1/3$  (radiation).

### HIGGS (SCALAR) STRESS-ENERGY TENSOR

$$T_{\mu\nu}^{(h)} = \nabla_\mu h \nabla_\nu h - g_{\mu\nu} \left( \frac{1}{2} \nabla_\alpha h \nabla^\alpha h + V(h) \right)$$

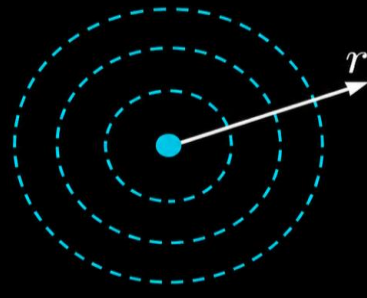
### RADIATION FLUID STRESS-ENERGY TENSOR

$$T_{\mu\nu}^{(\text{fluid})} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$$

## COORDINATES & METRIC

$$ds^2 = -A(r, t) dt^2 + B(r, t)^2 dr^2 + R(r, t)^2 d\Omega^2$$

- Spherical symmetry
- Areal radius  $R(r, t)$
- Radial metric  $B(r, t)$
- Geodesic slicing:  $A(r, t) = 1$
- Fluid 3-velocity  $v(r, t)$



## NUMERICAL IMPLEMENTATION

- Method of lines
- Fourth-order finite differences in  $r$
- Fourth-order Runge-Kutta time integration
- Cell-centred radial grid with AMR
- Ghost cells and parity conditions at  $r = 0$
- Outer boundary conditions
- Kreiss-Oliger dissipation for numerical stability
- FRW in the outer parts of the domain

# WHAT DOES THE LITERATURE SUGGEST?



## ORIGINAL FEAR

### AdS-like bubble

Rare inflationary Higgs fluctuations can push the field over the barrier. If the field rolls into the negative-potential region, the patch behaves like an AdS-like bubble.

- boundary can expand outwards
- interior crunches
- one patch in our past light cone could be catastrophic

(1505.04825, 1607.00381, 1910.13430, ...)

## THIN-WALL BLACK-HOLE PICTURE

### Collapse + horizon

A collapsing unstable patch may instead form a black hole, with the dangerous region hidden behind a horizon.

- collapse precedes outward expansion
- unstable region becomes cloaked
- suggests these patches may be harmless

(2205.10240, Phys. Rev. D 35 (1987) 1747, ...)

## HIGGSTORY CHALLENGE

### Thick-wall danger

Higgs patches from stochastic inflation are not thin walls. Numerical-relativity studies suggested that part of the unstable region can remain exposed.

- realistic patches are broad / thick-walled
- horizon cloaking may be incomplete
- outward danger may survive

(2207.00299, 1607.00381, 1505.04825, ...)

# HIGGS VACUUM INSTABILITY



- The Higgs quartic term receives the largest corrections at large energies from the top quark
  - Metastability favoured at about  $2\sigma$

