

# Discovering New Particles with Cosmic Microwave Background: Cosmological Collider Primordial Non-Gaussianity

**Anish Ghoshal**

University of Sussex, United Kingdom

*A.Ghoshal@sussex.ac.uk*

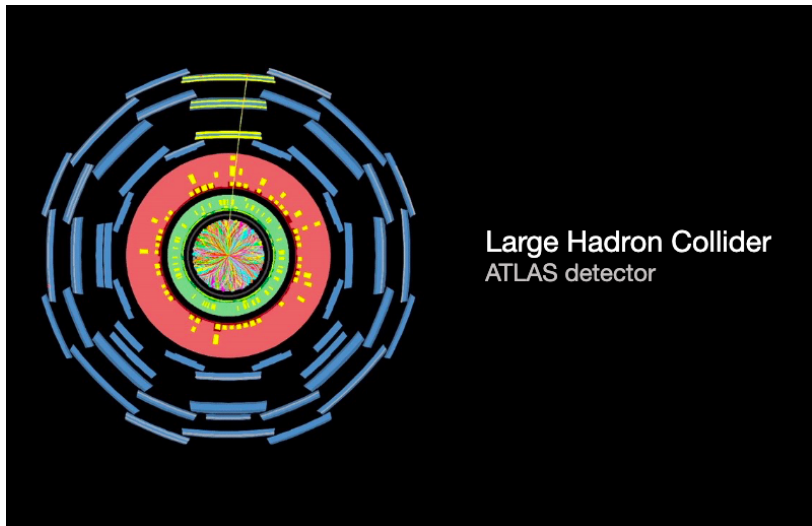
June 2026

Based on JHEP 11 (2024) 009 (2408.07069) & 2606.XXXXX

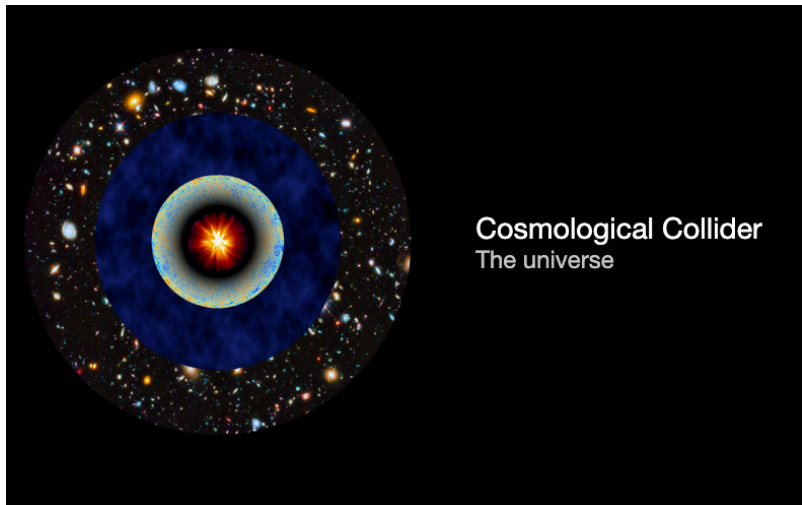
## Outline of the talk:

- ▶ Inflation & Primordial Non-gaussianities: cosmological collider.
- ▶ Imprints of SM and BSM Particles in cosmological collider
- ▶ Cosmological Collider in generic multi-field inflationary framework
- ▶ Cosmological Collider in Higgs- $\mathcal{R}^2$  Framework

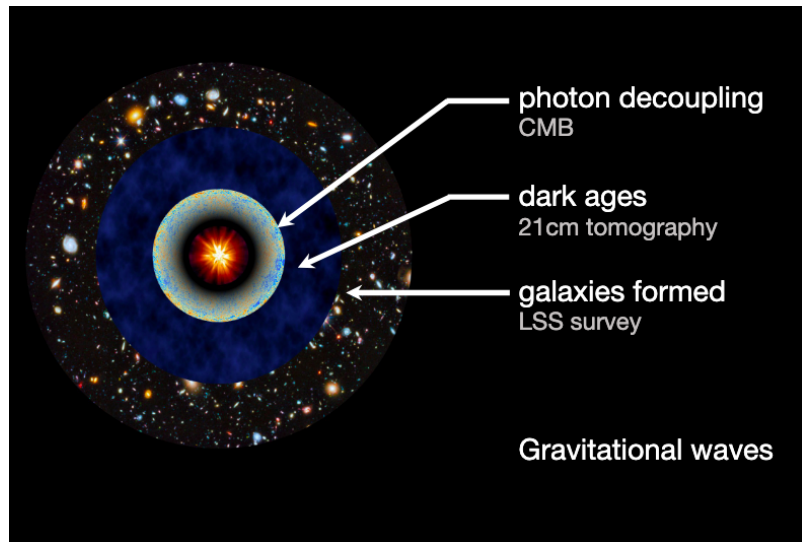
## Cosmological Collider: Introduction and Standard Model



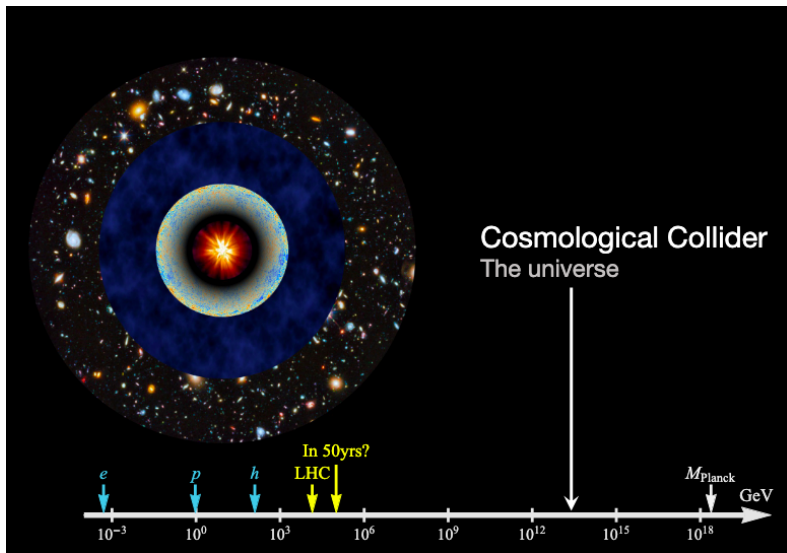
## Cosmological Collider: Introduction and Standard Model



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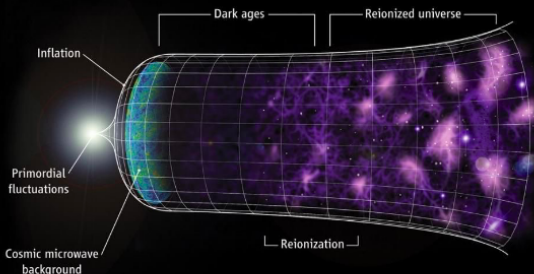


## Cosmological Collider: Introduction and Standard Model



## Cosmological Collider: Introduction and Standard Model

### Cosmic inflation: the engine of cosmic collider



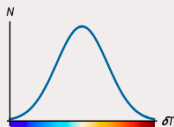
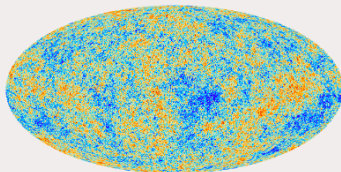
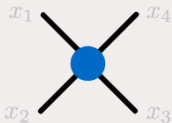
A period of exponentially fast expansion  
Within down to  $10^{-36}$  s, the size increased by up to  $10^{26}$

The quantum fluctuations of spacetime shape us all

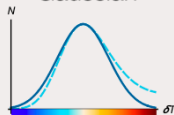
## Cosmological Collider: Introduction and Standard Model

How to extract more information from CMB map?

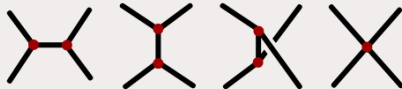
$$\langle \delta T(x_1) \cdots \delta T(x_n) \rangle$$



Gaussian

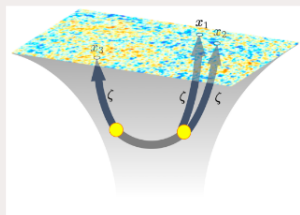


Non-Gaussian



## Cosmological Collider: Introduction and Standard Model

“Non-Gaussianity”

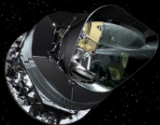


Non-Gaussianity  $\sim$  interaction

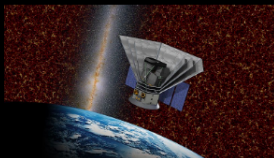
The size measured by a dimensionless number  $f_{\text{NL}}$

## Cosmological Collider: Introduction and Standard Model

## Observational prospects



Planck: final data release in 2018



SPHEREx: selected by NASA in 2019, launching in ~2024

Planck 2018

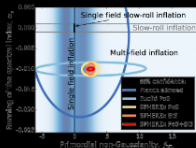
1905.05697

$$f_{NL}^{(\text{local})} = -0.9 \pm 5.1$$

$$f_{NL}^{(\text{equil})} = -26 \pm 47$$

$$f_{NL}^{(\text{ortho})} = -38 \pm 24$$

O(1) in ~10yrs?



SPHEREx, 1412.4872

O(0.01) ultimately  
21cm tomography

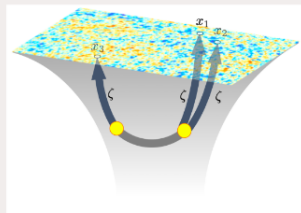
Meerburg, Muñoz, Ali-Haïmoud, Kamionkowski,  
1506.04152; Münchmeyer,  
Muñoz, Chen, 1610.06559;  
Dizgah, Lee, Muñoz, Dvorkin  
1801.07265;

## Cosmological Collider: Introduction and Standard Model

## Discover new heavy particles

When massive particles are produced, the inflation did two things:

1. Dilute the number density
2. Exhaust the momentum, so that the particle quickly becomes nonrelativistic



$$\sigma(t) \sim \left( e^{-imt} + e^{-\pi m/H} e^{+imt} \right) e^{-\frac{3}{2}Ht}$$

Boltzmann factor Negative frequency mode; particle production Comoving dilution

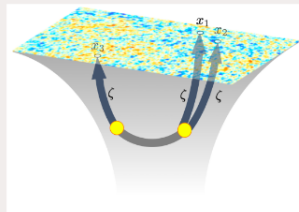
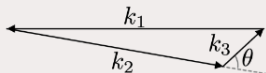
We would be able to measure the mass if we can trace the time dependence.

But we can't. We observe only the final state (CMB)

## Cosmological Collider: Introduction and Standard Model

## Discover new heavy particles

A solution: we try to measure the 3-point correlation in the **squeezed limit**



Small-momentum mode redshifts earlier, and oscillates like a nonrelativistic particle when the other two large-momentum modes are still deeply inside the horizon.

The ratio of long and short momenta is actually a measure of time difference. => Measure the 3pt function at different  $k$  ratio  $\sim$  measure the mode at different time

## Cosmological Collider: Introduction and Standard Model

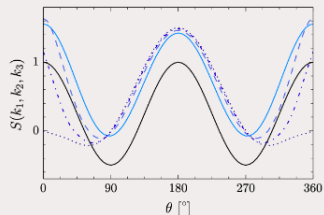
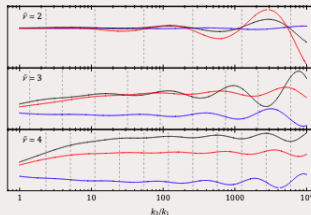
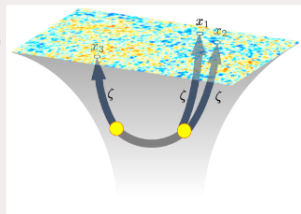
Point to appreciate: probe of masses and energy scales in early universe.

## Discover new heavy particles

$$S(\mathbf{k}_1, \mathbf{k}_3) = A(\lambda, m) \left( \frac{k_3}{k_1} \right)^{1/2 \pm \nu} P_s(\cos \theta)$$

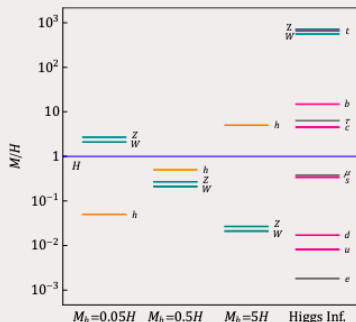
$$\nu = \begin{cases} \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} & s = 0 \\ \sqrt{\left(s - \frac{1}{2}\right)^2 - \frac{m^2}{H^2}} & s \neq 0 \end{cases}$$

Chen, Wang, 0911.3380; 1205.0160  
Arkani-Hamed, Maldacena, 1503.08043



## Cosmological Collider: Introduction and Standard Model

$$S(\mathbf{k}_1, \mathbf{k}_3) = A(\lambda, m) \left( \frac{k_3}{k_1} \right)^{1/2 \pm \nu} P_s(\cos \theta)$$



Example: “SM background”

“Thermal” mass  $\sim$  Hubble

All in loops: spin info lost

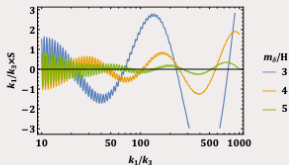
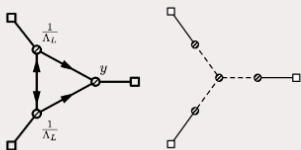
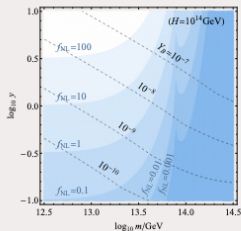
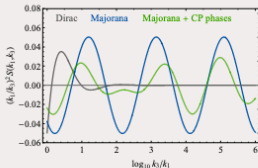
Signal size: tiny unless tuned

## Cosmological Collider: Introduction and Standard Model

## Probing heavy neutrinos

Probing seesaw mechanisms

$$\mathcal{L}_\phi = \frac{\partial_\mu \phi}{\Lambda_L} L^\dagger \bar{\sigma}_\mu L - \frac{\partial_\mu \phi \partial^\mu \phi}{\Lambda_\Delta^2} \mathbf{s} \cdot \mathbf{s}^*$$

Majorana mass and CP phases  
(Probing leptogenesis?)

## Cosmological Collider: Introduction and Standard Model

## CP violation

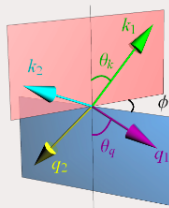
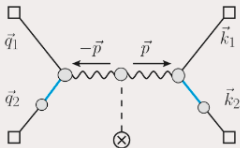
$$\Delta\mathcal{L} = \frac{c_1}{\Lambda} \partial_\mu \phi (\mathcal{H}^\dagger D^\mu \mathcal{H}) + \frac{c_2}{\Lambda^2} (\partial\phi)^2 \mathcal{H}^\dagger \mathcal{H} - \frac{c_0}{4} \theta(t) Z_{\mu\nu} Z_{\rho\sigma} \mathcal{E}^{\mu\nu\rho\sigma}$$

Two types of external legs needed

Odd-angular dependence in imaginary part

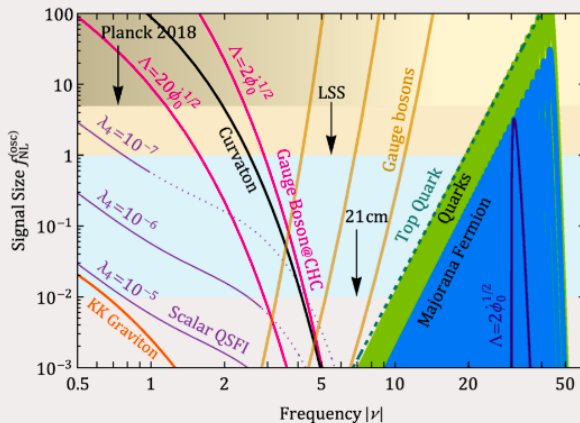
No local CP-odd correlations in dS limit

Chemical potential helps



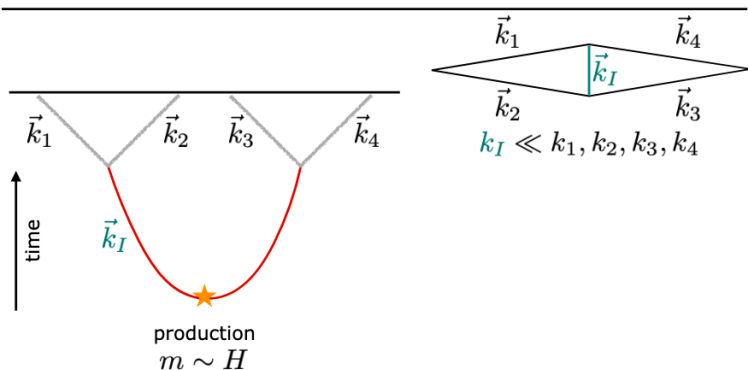
## Cosmological Collider: Introduction and Standard Model

## A status summary



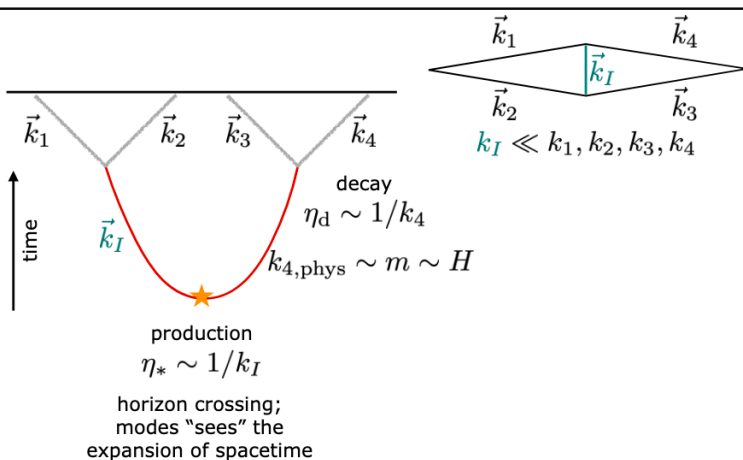
## Cosmological Collider: Key Ideas &amp; Probe of New Physics BSM

## Cosmological Collider



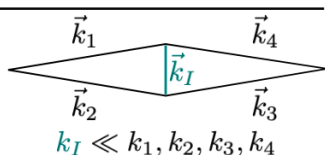
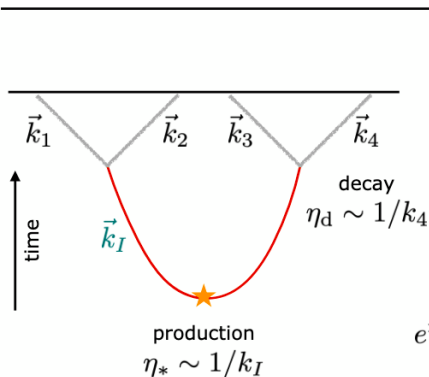
## Cosmological Collider: Key Ideas &amp; Probe of New Physics BSM

## Cosmological Collider



## Cosmological Collider: Key Ideas &amp; Probe of New Physics BSM

## Cosmological Collider



Long oscillation  
of the massive particle

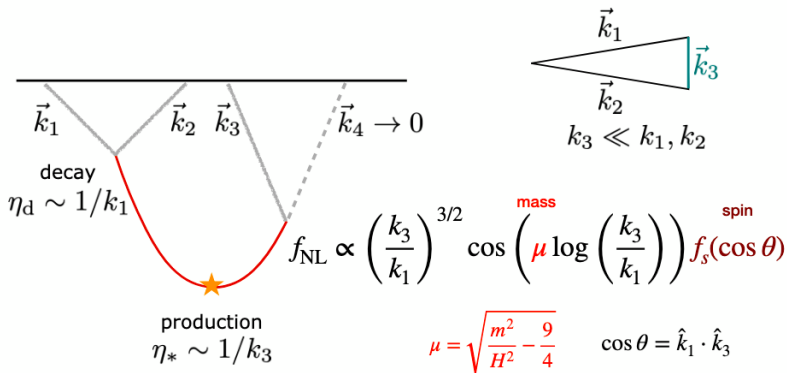
$$e^{\pm im(t_d - t_*)} \rightarrow (\eta_d / \eta_*)^{\pm im/H}$$

$$\rightarrow (k_I / k_4)^{\pm im/H}$$

$$\cos \left( \frac{m}{H} \log \left( \frac{k_I}{k_4} \right) \right)$$

## Cosmological Collider: Key Ideas &amp; Probe of New Physics BSM

## Cosmological Collider



on-shell mass and spin information from bi/trispectrum!

## Cosmological Collider: Key Ideas &amp; Probe of New Physics BSM

## Can We Observe This?

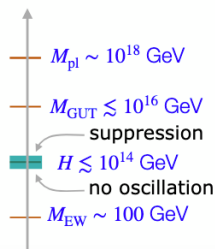
$$f_{\text{NL}} \propto c \left( \frac{k_3}{k_1} \right)^{3/2} \cos \left( \mu \log \left( \frac{k_3}{k_1} \right) \right) f_s(\cos \theta)$$

$\uparrow$   
 $?$

Inflation gives  $H$ -scale energy:  $c \sim \exp \left( -\frac{\pi m}{H} \right)$

Bad news!  
 $m \simeq 3H \rightarrow c \simeq 10^{-4}$

only  
 narrow  
 window?



Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

## Setup

- two scalars  $\phi^a$  ( $a = 1, 2$ )

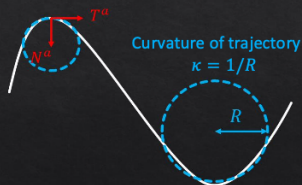
$$S = \int d^4x \sqrt{|\det g|} \left[ -\frac{\bar{M}_{\text{Pl}}^2}{2} R + \frac{K_{ab}(\phi)}{2} (\partial_\mu \phi^a) (\partial^\mu \phi^b) - V(\phi) \right]$$

- Turn: specific to multifield dynamics

$$T^a \equiv \phi_0^{i'a} / \phi_0^{i'}$$

$$N_a \equiv \sqrt{\det K} \epsilon_{ab} T^b$$

$$\phi_0^{i'2} \equiv K_{ab} \phi_0^{i'a} \phi_0^{i'b}$$



Note:  $\phi_0$  is BG. The prime denotes a derivative w.r.t e-folding:  $dN = H dt$

Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

## Turn rate

- Second SR parameter  $\eta^a$

$$\eta^a \equiv -\frac{D_N \phi_0'^a}{\phi_0'} = \eta_T T^a + \eta_N N^a, \quad \eta_T = -\frac{\phi_0''}{\phi_0'}, \quad \eta_N \equiv -N_a D_N T^a.$$

Turn rate

- Other expression:  $\eta_N = \frac{d\phi}{dN} \times \kappa$

A. Achúcarro, J.-O. Gong, S. Hardeman, G.A. Palma, S.P. Patil, 1010.3693

- Perturbation for fluctuation  $\delta\phi^a = \varphi T^a + \sigma N^a$   
( $\varphi$ : adiabatic,  $\sigma$ : isocurvature)

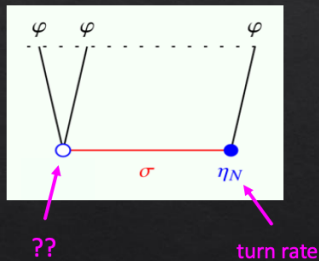
D. Langlois, S. Renaux-Petel, 0801.1085  
J.-O. Gong, T. Tanaka, 1101.480,  
...

$$\mathcal{L}^{(2)} \approx (\text{free } \sim \text{massless } \varphi) + (\text{free massive } \sigma) + 2\eta_N H \sigma \dot{\varphi}.$$

$$\varphi \leftrightarrow \sigma$$

Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

Understanding so far



Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

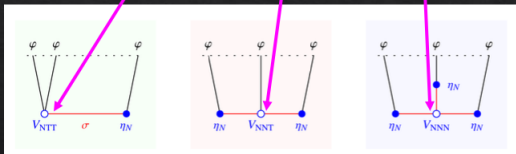
## Dominant cubic interactions

- Many cubic, dominated by potential couplings

$$\mathcal{L}^{(3)} \sim \underbrace{V_{TTT}\varphi^3}_{\text{small}} + V_{NTT}\sigma\varphi^2 + V_{NNT}\sigma^2\varphi + V_{NNN}\sigma^3 + \dots$$

Maldacena '03

- Dominant diagrams



Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

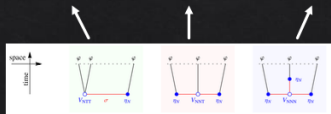
## CC signal

➤ shape function  $S$ :  $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \frac{(2\pi)^7 P_\zeta^2}{(k_1 k_2 k_3)^2} \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) S\left(\frac{k_1}{k_3}, \frac{k_2}{k_3}\right)$  with  $\zeta = -H \frac{\varphi}{\dot{\phi}_0}$ .

➤ Define  $f_{\text{NL}}$  in squeezed limit  $k_3 \equiv k_L \ll k_{1,2} \equiv k_S$

$$S \simeq \frac{9}{10} \left[ f_{\text{NL}}(\nu) \left(\frac{k_L}{k_S}\right)^{1/2-\nu} + f_{\text{NL}}(-\nu) \left(\frac{k_L}{k_S}\right)^{1/2+\nu} \right] \quad \text{where } \nu = \sqrt{\frac{9}{4} - \frac{m_\sigma^2}{H^2}}.$$

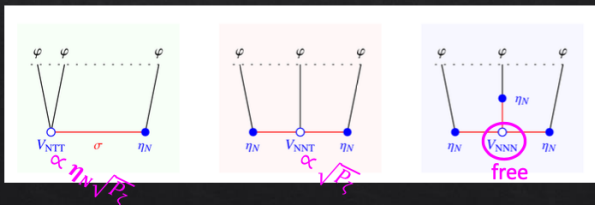
$$f_{\text{NL}} = \frac{10}{9\sqrt{P_\zeta}} \left[ \eta_N C_{\text{NTT}} \frac{V_{\text{NTT}}}{H} + \eta_N^2 C_{\text{NNT}} \frac{V_{\text{NNT}}}{H} + \eta_N^3 C_{\text{NNN}} \frac{V_{\text{NNN}}}{H} \right]$$



Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

## Remark

- Requiring  $m_\sigma \sim H$  for CC, potential derivatives along  $T$ -direction (inflaton) are not independent
  - $V_{NTT}/H \sim \eta_N \times \sqrt{P_\zeta}$  with  $\sqrt{P_\zeta} \sim 10^{-5}$
  - $V_{NNT}/H \sim \sqrt{P_\zeta}$
- However,  $V_{NNN}$  is not constrained (free parameter) in general
- $\eta_N$  determines almost everything



Cosmological Collider: Higgs- $\mathcal{R}^2$  FrameworkScalar  $\phi + R^2$ 

## ➤ J-frame

$$S = \int d^4x \sqrt{|\det g|} \left[ -\frac{1}{2} f(\phi) R + \frac{R^2}{6f_0^2} + \sum_{\phi} \frac{(D_{\mu}\phi)(D^{\mu}\phi)}{2} - V_J(\phi) \right]$$

- $\phi$ : some scalars (specified later)
- $R^2$  gives an additional scalar  $z$  (scalaron)

## ➤ E-frame

$$S = \int d^4x \sqrt{|\det g|} \left[ -\frac{\bar{M}_{\text{Pl}}^2}{2} R + \frac{6\bar{M}_{\text{Pl}}^2}{z^2} \frac{(\partial_{\mu}z)^2 + \sum_{\phi} (D_{\mu}\phi)^2}{2} - V(\phi, z) \right]$$

$$V = \left( \frac{6\bar{M}_{\text{Pl}}^2}{z^2} \right)^2 \left[ V_J(\phi) + \frac{3}{8} f_0^2 (f + \xi_z z^2)^2 \right].$$

$$\xi_z = -1/6,$$

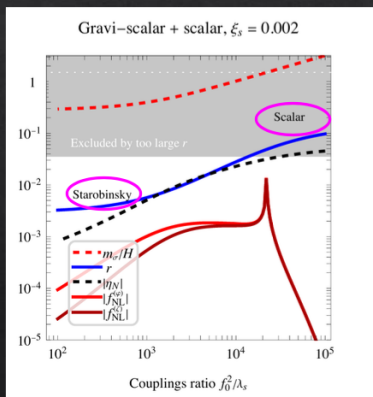
## Cosmological Collider: Higgs- $\mathcal{R}^2$ Framework

Imagine the Higgs being responsible for generating the Electroweak scale dynamically.

### Higgs+ $R^2$

Salvio, Mazumdar '15, Ema '17, Gorbunov, Tokareva '18, Gundhi, Steinwachs '18, ...

- corresponds to the choice:
 
$$\left\{ \begin{array}{l} f = \bar{M}_{\text{Pl}}^2 + \xi_s s^2 \\ V_{\text{J}}(s) = \frac{\lambda_s}{4} s^4 \end{array} \right.$$
- $s$ : Higgs in unitary gauge
- Motivated as a UV completion of Higgs inflation (unitarize Higgs inflation)
- Interpolate Higgs inflation ( $\lambda_s/\xi_s^2 \ll f_0^2$ ) and Starobinsky inflation ( $\lambda_s/\xi_s^2 \gg f_0^2$ )
- $M_{\text{pl}}$  is the only scale

Cosmological Collider: Higgs- $\mathcal{R}^2$  FrameworkCC signal from Higgs+ $R^2$ 

## Cosmological Collider: Higgs- $\mathcal{R}^2$ Framework

Imagine the Higgs being responsible for generating the Planck scale dynamically.

### dilaton+ $R^2$

K. Kannike, G. Hutsi, L. Piza, A. Racioppi, M. Raidal, A. Salvio, A. Strumia, 1502.01334.

- Corresponds to the choice (dimensionless theory):

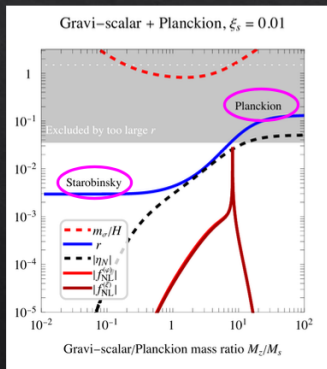
$$\left\{ \begin{array}{l} f = \xi_s s^2 \\ V_J = \lambda_s(s) s^4 / 4 \end{array} \right. \quad \lambda_s(s) \simeq \frac{b}{8} \ln^2 \frac{s^2}{w^2}$$

- $s$ : dilaton, dynamically induce the Planck scale:  $\langle s^2 \rangle = w^2 = M_{\text{pl}}^2 / \xi_s$

- Three parameters ( $f_0, b, \xi_s$ ) but constrained by  $P_\zeta \sim 2 \times 10^{-9}$

$$M_z = f_0 \bar{M}_{\text{pl}} / \sqrt{2}, \quad M_s = \bar{M}_{\text{pl}} \sqrt{4b / \xi_s (1 + 6\xi_s)}$$

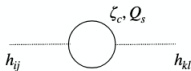
- Only scale  $\sim M_{\text{PL}} \Rightarrow \kappa \propto 1/M_{\text{pl}} \Rightarrow \eta_N \propto \sqrt{\epsilon}$ : Small turn

Cosmological Collider: Higgs- $\mathcal{R}^2$  FrameworkCC signal from dilaton+ $R^2$ 

# Primordial Gravitational Wave Signals

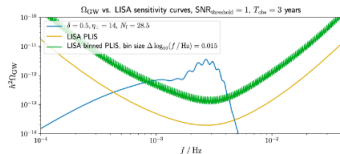
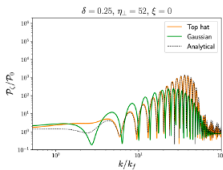
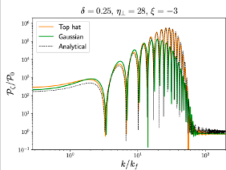
## SGWB

- Primordial scalar perturbations induce a gravitational wave spectrum.



Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290

$$\Omega_{\text{GW}} \sim 10^{-5} P_{\zeta}^2$$



Fumagalli, Renaux-Petel, Witkowski: ArXiv:2012.02761

## Summary:

- ▶ With current Planck data, and upcoming CMB missions like LiteBRD, & 21-cm, LSS experiments, one will measure primordial non-gaussianities and its shape dependence.
- ▶ During inflation, in early universe, there is enough energy to produce all SM particles as well as heavy BSM states like right-handed neutrinos. Any such interactions leave inevitable signal in primordial non-gaussianities. **Particularly in the limit of cosmological collider ( $m \sim \mathcal{O}(H)$ ), they leave a unique oscillatory scale-dependent bi-spectrum in  $\langle \zeta\zeta\zeta \rangle$  correlator in the squeezed limit that can be dug out from observational data.**
- ▶ Higgs Inflation and Starobinsky inflation in the framework of EFT of Higgs- $\mathcal{R}^2$ , having any interactions between the two fields along with matter sector (fermions and gauge bosons) can lead to such detectable oscillatory signals.
- ▶ At much smaller scales of the universe ( $k \gg k_{\text{CMB}}$ ) such Higgs- $\mathcal{R}^2$  framework also give rise to large scalar-induced Gravitational Waves (can be tested in LISA) and lead to formation of Primordial Blackholes as the entire DM candidate.

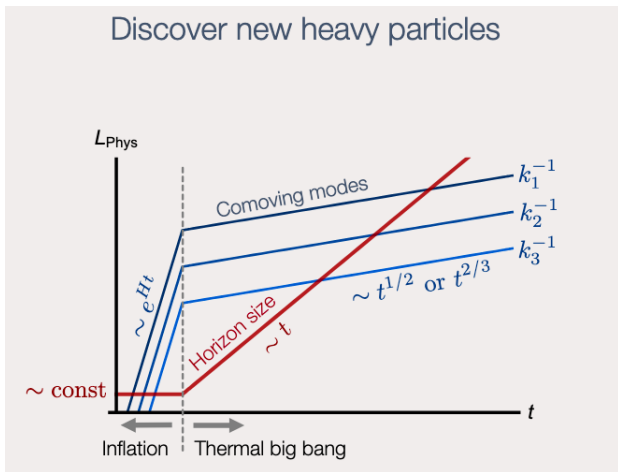
## Future:

- ▶ **Tests of your favorite BSM heavy particle models, CP Violation, mixed bi-spectrum, cross-correlation between scalars and tensors ( $\zeta hh$ ) with CMB  $\mu$ -distortions, 21-cm and GW..**

Thank You

# Backup begins

## Cosmological Collider: Introduction and Standard Model



Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

## Dominant cubic interactions

- Many cubic, dominated by potential couplings

$$\mathcal{L}^{(3)} \sim \underbrace{V_{TTT}\varphi^3}_{\text{small}} + V_{NTT}\sigma\varphi^2 + V_{NNT}\sigma^2\varphi + V_{NNN}\sigma^3 + \dots$$

Maldacena '03

## Cosmological Collider: Higgs- $\mathcal{R}^2$ Framework

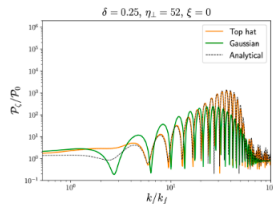
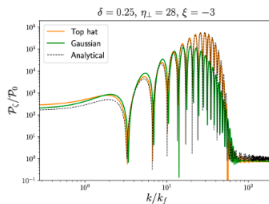
Application to concrete models

## Primordial Gravitational Wave Signals

So far only CMB, now we go to smaller scales of the universe, typically  $k \gg k_{\text{CMB}}$ .

- Sustained turning is hard to achieve, but sporadic turning is pretty common. This, in turn, generates features in the spectrum.
- Observed scale invariance  $10^{-4} \text{Mpc}^{-1} \lesssim k \lesssim 10^{-1} \text{Mpc}^{-1}$
- PBHs if feature with  $k \gtrsim 10^8 \text{Mpc}^{-1}$  and amplification of  $10^7$  larger than CMB. Palma, Sypsas, Zenteno ArXiv:2004.06106
- Generation of SGWB, possibly visible at LISA

Do these models have other phenomenological consequences?



Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

## More interesting possibilities

## ➤ Large constant turn

- A. Achucarro, V. Atal, S. Céspedes, J.-O. Gong, G. A. Palma and S. P. Patil, Heavy fields, reduced speeds of sound and decoupling during inflation, 1205.0710
- P. Christodoulidis, D. Roest and E.I. Sfakianakis, Angular inflation in multi-field  $\alpha$ -attractors, 1803.09841.  $\Rightarrow \kappa \propto 1/\alpha M_{pl}$  for  $\alpha \ll 1$
- Y. Welling, Simple, exact model of quasisingle field inflation, 1907.02951
- ...

## ➤ Sudden turn

- G.A. Palma, S. Sypsas, C. Zenteno, Seeding primordial black holes in multifield inflation, 2004.06106
- Fumagalli, S. Renaux-Petel, J.W. Ronayne, L.T. Witkowski, Turning in the landscape: A new mechanism for generating primordial black holes, 2004.08369
- ...

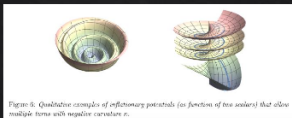
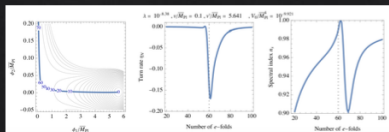


Figure 6: Qualitative examples of refractory potentials (as function of two scalars) that allow multiple turns with negative curvature.



Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

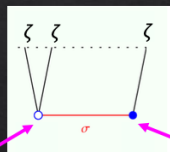
## For large signal

- Large turn rate. Remember  $\eta_N = \frac{d\phi}{d\mathcal{N}} \times \kappa = \sqrt{2\epsilon} M_{\text{PL}} \times \kappa$   
⇒ Large  $\kappa$  ⇒ sub-Planckian physics
- Large  $V_{NNN}$  “Quasi single field inflation” Chen, Wang, '10

Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

## Gauge choice

- So far  $\varphi \neq 0, \zeta = 0$  (flat gauge),  
but one can take  $\varphi = 0, \zeta \neq 0$  (comoving gauge) instead



L. Pinol, SA, S. Renaux-Petel, M. Yamaguchi  
2112.05710

$$\frac{\dot{\phi}_0 \eta_N}{H} \sigma \left[ \dot{\zeta}^2 - \frac{(\partial \zeta)^2}{a^2} \right]$$

$$-2\dot{\phi}_0 \eta_N \dot{\zeta} \sigma$$

- There is no gauge issue

Cosmological Collider: Higgs- $\mathcal{R}^2$  Framework

## Interpolate two inflation scenarios

$$V = \left( \frac{6\bar{M}_{Pl}^2}{z^2} \right)^2 \left[ \underbrace{V_J(\phi)}_{\phi\text{-potential}} + \frac{3}{8} f_0^2 \underbrace{(f + \xi_z z^2)^2}_{z(\text{scalaron})\text{-potential}} \right]. \quad \xi_z = -1/6,$$

$\phi$ -potential     $z$ (scalaron)-potential

- $V_J \gg f_0^2$ : **Starobinsky-like inflation**. Potential is minimized by  $\phi \sim \text{const.}$   
 $\Rightarrow \kappa \sim 0 \Rightarrow \eta_N \sim 0$
- $V_J \ll f_0^2$ :  **$\phi$ -like inflation**. Potential is minimized by  $f(\phi) = -\zeta_z z^2$   
 $\Rightarrow z = z(\phi) \Rightarrow \eta_N \neq 0$
- $V_J \sim f_0^2$ : **Mixed regime**
- **Note:** In extreme limits to single field inflation,  $CC \sim 0$  as  $m_\sigma \gg H$

# Primordial Gravitational Wave Signals

How generic are the potentials and field-space metrics in SUGRA?

Aragam, Chiovoloni, SP, Rosati, Zavala: ArXiv:2110.05516

- Rapid-turn inflation in supergravity is rare and tachyonic
- Large turning rates can be generated in a wide class of models, at the cost of high field space curvature.

$$S = \int d^4x \sqrt{-g} \left[ M_P^2 \frac{R}{2} - K_{ij} \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^{\bar{j}} - V(\Phi^k, \bar{\Phi}^{\bar{k}}) \right]$$

$$K = -3\alpha M_P^2 \log[(\Phi + \bar{\Phi})/M_P] + S\bar{S},$$

$$V = \frac{M_P^{3\alpha} |F|^2}{(\Phi + \bar{\Phi})^{3\alpha}}$$

$$\mathcal{R} = -4/(3\alpha)$$

$$\frac{\omega}{H} \simeq \frac{2\sqrt{\epsilon}}{\sqrt{3\alpha}}$$

In the "large" regime (large volume, large complex structure, weak coupling) this coefficient is certainly  $\alpha \sim \mathcal{O}(1)$ , but its value is unclear away from this limit  
—A. Lukas

# Primordial Gravitational Wave Signals

For a specific choice of masses,

$$D_t \zeta_c + 3HD_t \zeta_c + \frac{k^2}{a^2} \zeta_c = 0$$

$$D_t Q_s + 3HD_t Q_s + \frac{k^2}{a^2} Q_s = -2\omega D_t \zeta_c$$

During a top-hat feature:

$$\frac{q_{\pm}}{H} = \sqrt{\frac{k^2}{k_f^2} \pm 2\frac{\omega}{H} \frac{k}{k_f}}$$

$$k_f = Ha(t_f)$$

