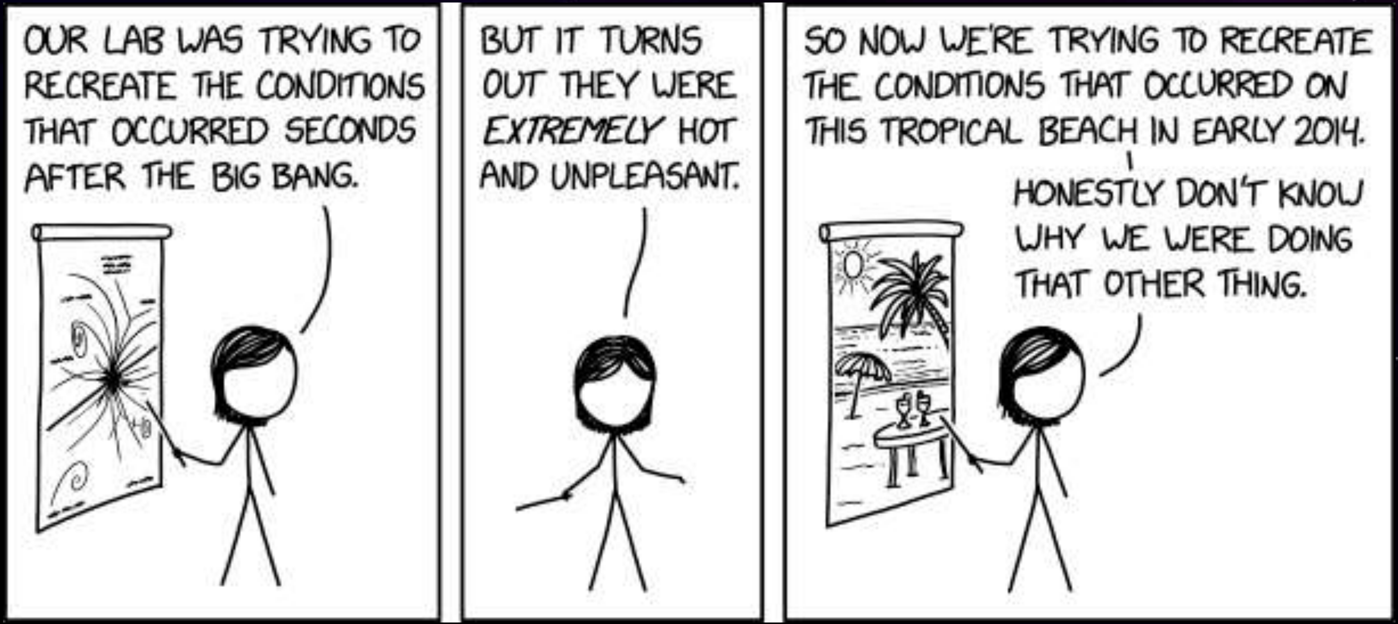
# A universal scaling law for inflationary gravitational waves

**Martin Teuscher** with Ruth Durrer, Aurélien Barrau & Killian Martineau
2nd year PhD; based on arXiv **2510.00869** & **2512.14670**

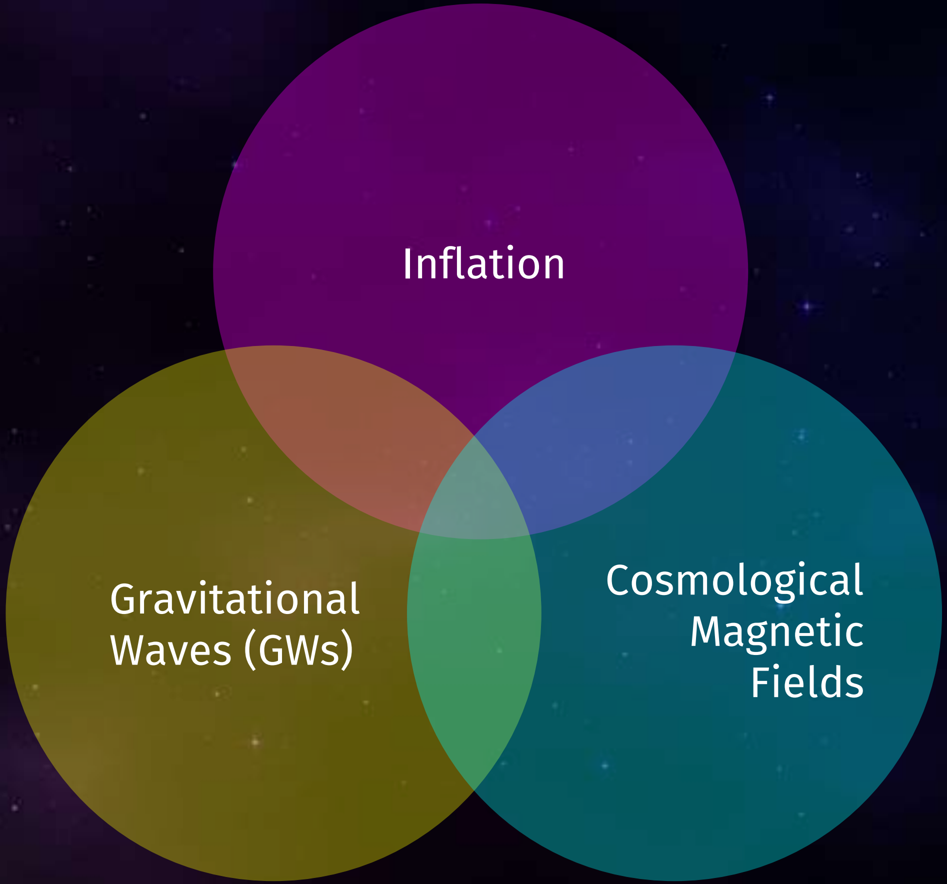



[Credits: xkcd]

# Common thread

*Where should we seek direct probes of early Universe physics ?*

# Table of Contents

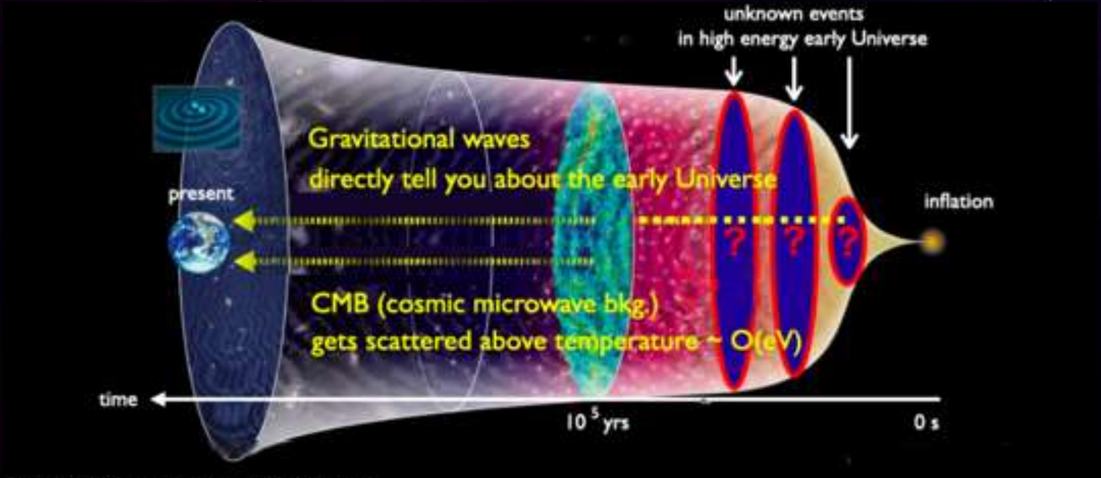

Inflation
Gravitational Waves (GWs)
Cosmological Magnetic Fields

# Why should cosmology care about GWs?

Seeing ONE astrophysical event = Nobel prize… but **worth it to look even harder** ?

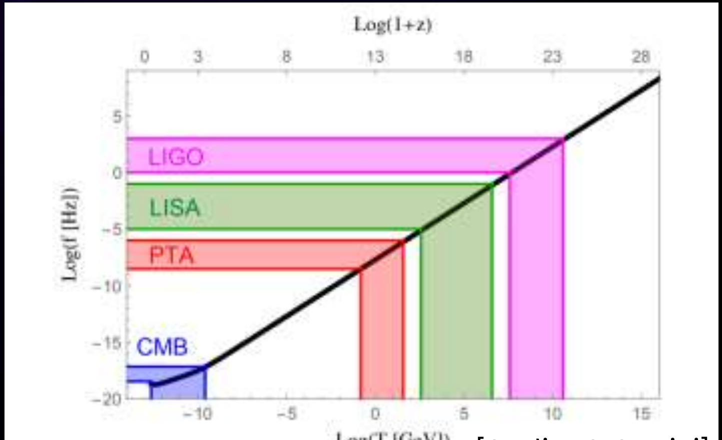
Detection ➔ Access to inflation energy scale

$$\Gamma_{GW}/H \propto (T/M_{pl})^3$$




[ https://gwpo.nao.ac.jp/en/gallery/000061.html ]

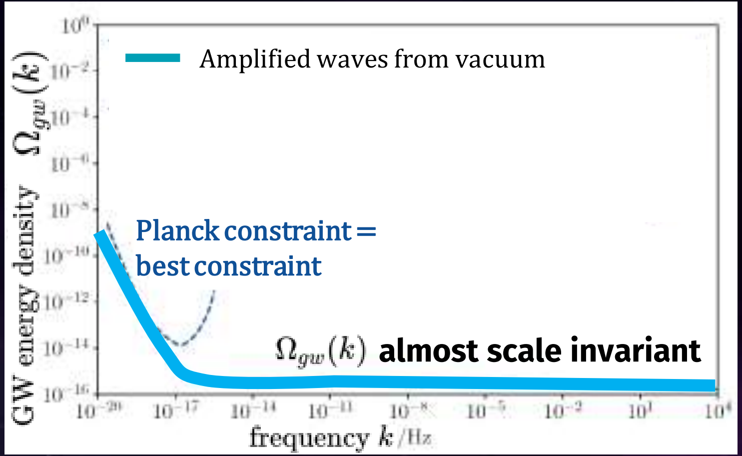
**DIRECT** probe of pre-CMB physics ! (!!!!)




[Credits: C. Caprini]

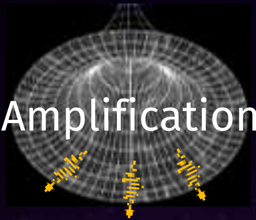
Frequency ⇔ Energy scale to probe
(after inflation)

# Our target?



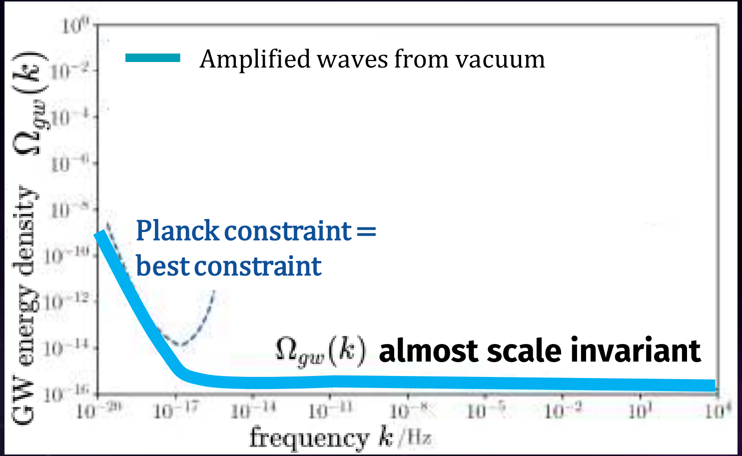

$$\Omega_{gw}^{\text{vacuum}}(k) \propto k^{n_T}\,,\; n_T \simeq 0$$

Inflation




Gravitational wave production

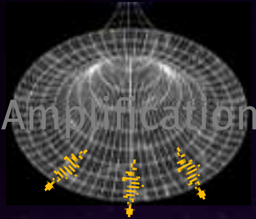
# Our target: *sourced* gravitational waves

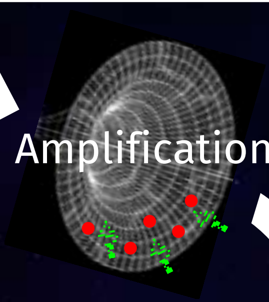



$$\Omega_{gw}^{\text{vacuum}}(k) \propto k^{n_T}, \; n_T \simeq 0$$

- What is the new slope $n_T$ ?
- Can we learn on the content?
- Is this detectable?

Inflation




Density fluctuations of other fields (scalars, spin-1...)

GW equation

$$\Box h_{\mu\nu} \propto T_{\mu\nu}^{\text{source}}$$

Gravitational wave production

# Table of Contents

# Model-independent estimation of $n_T$

$$\Box h_{\mu\nu} \propto T^{\text{source}}_{\mu\nu} \implies \rho^{\text{today}}_{\text{gw}} \propto [\text{transfer func.}] \times \iint \mathrm{d}\tau' \mathrm{d}\tau'' G_1(\tau') G_1(\tau'') [\Pi\text{-correlator}(\tau', \tau'')]$$

Green function

anisotropic stress tensor

Vastly studied [Caprini, Domcke, Domenech...], **but can we claim properties on the resulting GW spectrum with little to no knowledge on the source ?**

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| **Parameter** $q$ | Equation of state $q = 2(1+3w)$ |
| --- | --- |

$\simeq -1$ if slow-roll

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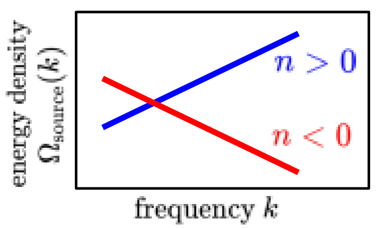
Green function — anisotropic stress tensor

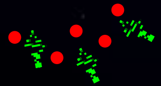
Vastly studied [Caprini, Domcke, Domenech…], **but can we claim properties on the resulting GW spectrum with little to no knowledge on the source ?**

Simple parametrization:

| **Parameter $q$** | Equation of state $q = 2(1+3w)$ ← $\simeq -1$ if slow-roll |
|---|---|

| **Parameter $n$** | Slope of source's spectrum |
|---|---|



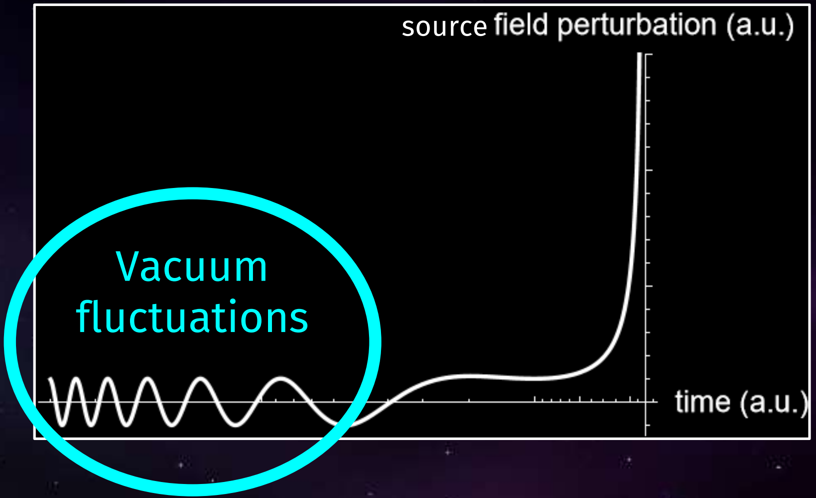
$$\Pi_{ij} \propto \Psi_i \Psi_j,$$

$$\mathcal{P}_\Psi(k) \propto k^n$$

# When can scales source gravitational waves?

**Careful treatment of sub-Hubble modes!**

For each frequency *k*:




*Try:*

$$\Omega_{gw}^{\text{today}} \propto \int_0^\infty \frac{\mathrm{d}k}{k} (\text{source 2-pt. function})_k$$

➔ **Divergent !** (vacuum energy-like problem)

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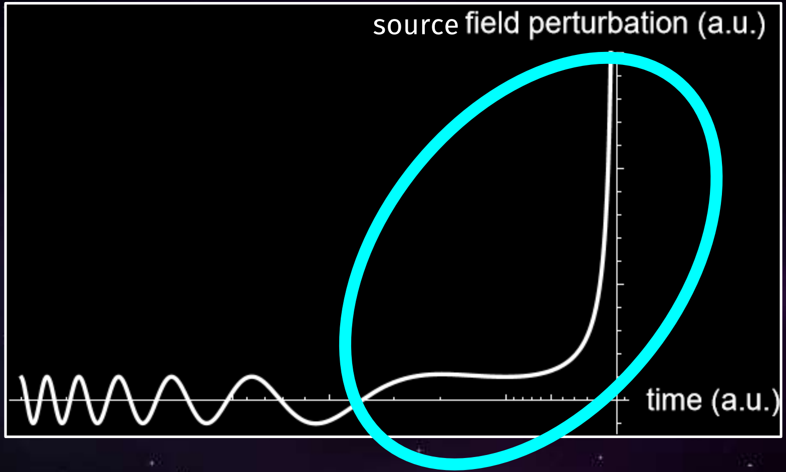


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*Adiabatic regularization scheme* [Parker, Durrer]

$$\Omega_{gw}^{\text{today}} \propto \int_0^{\text{cutoff}(t)} \frac{\mathrm{d}k}{k} (\text{source 2-pt. function})_k$$

➔ Modes need to be populated to source GWs!

(& close to proper renormalization schemes [Marozzi *et al.* ‘11, Braglia&Pinol ’25])

# Model-independent estimation of $n_T$

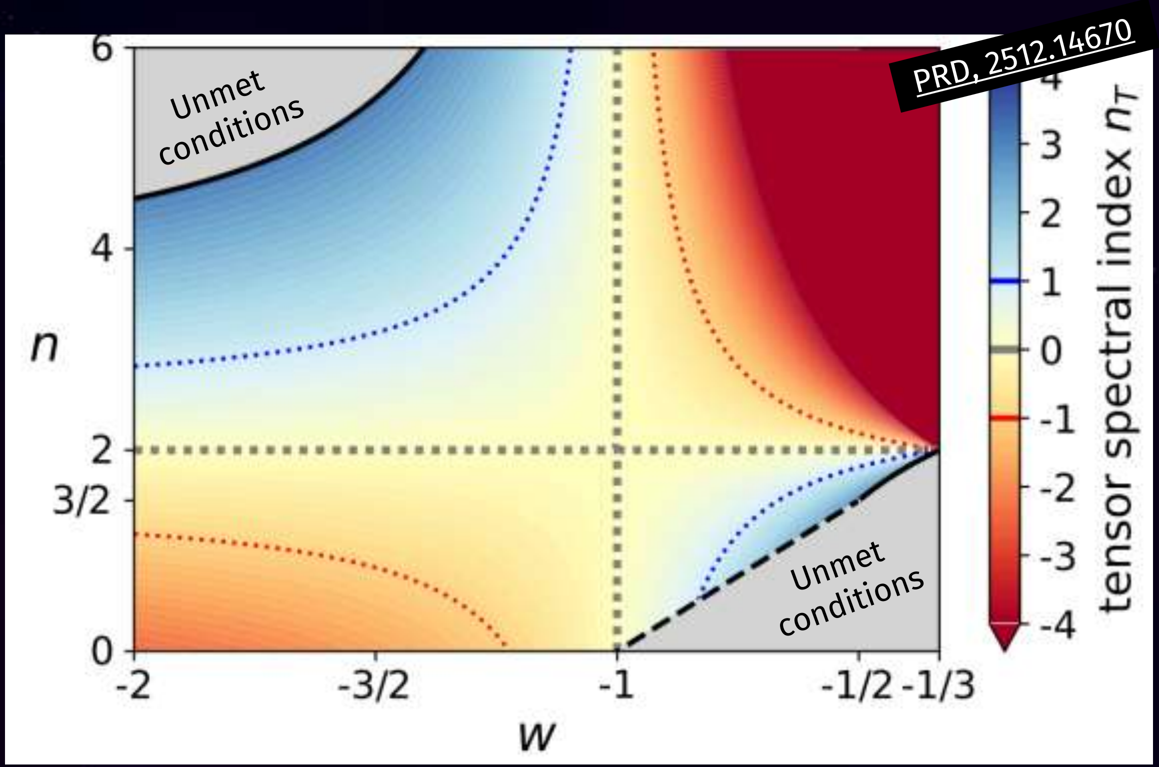
$$\Omega_{gw}(k) \propto k^{n_T}, \quad n_T = 2(n-2)(q+1)$$

$\mathcal{P}_{\rm source}(k) \propto k^n$

$q = 2/(1+3w)$

$\simeq$ const.

& few assumptions



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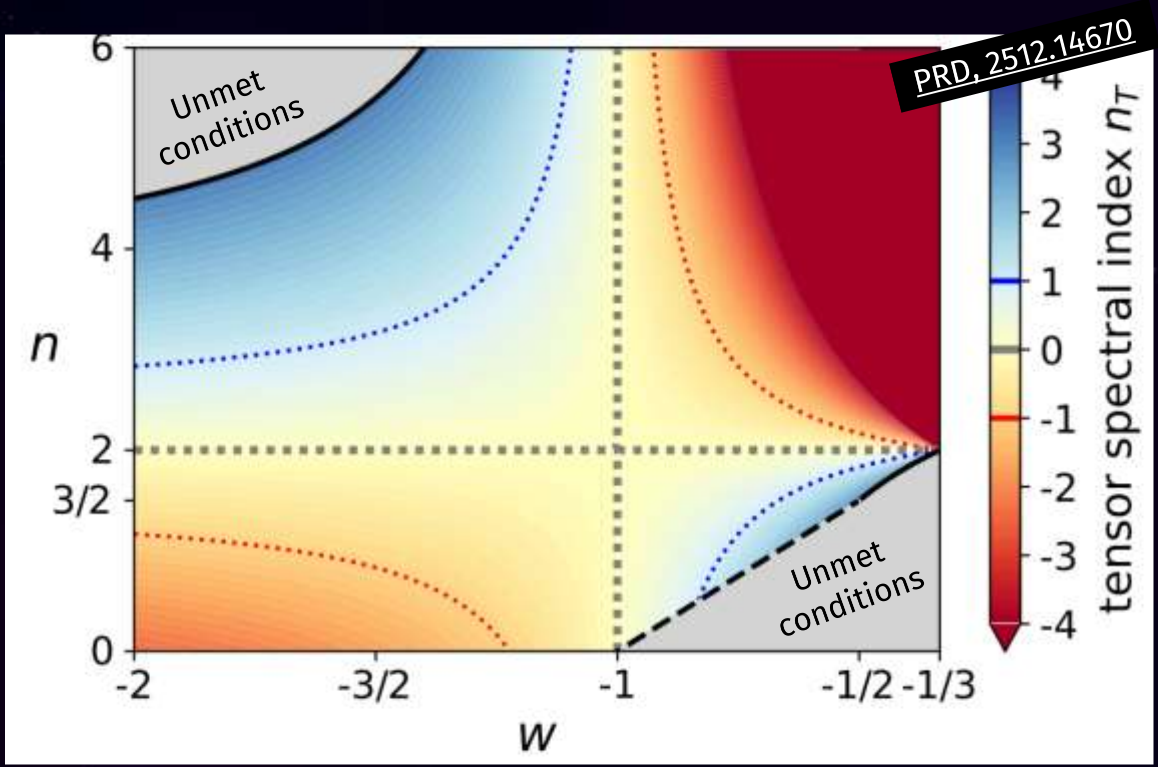


Slow-roll inflation: $q \simeq -1$ so

$$n_T \simeq 0 \; \forall n \, !$$

**GW spectral index almost independent of that of the source**

**(« dilution compensates small scale »)**

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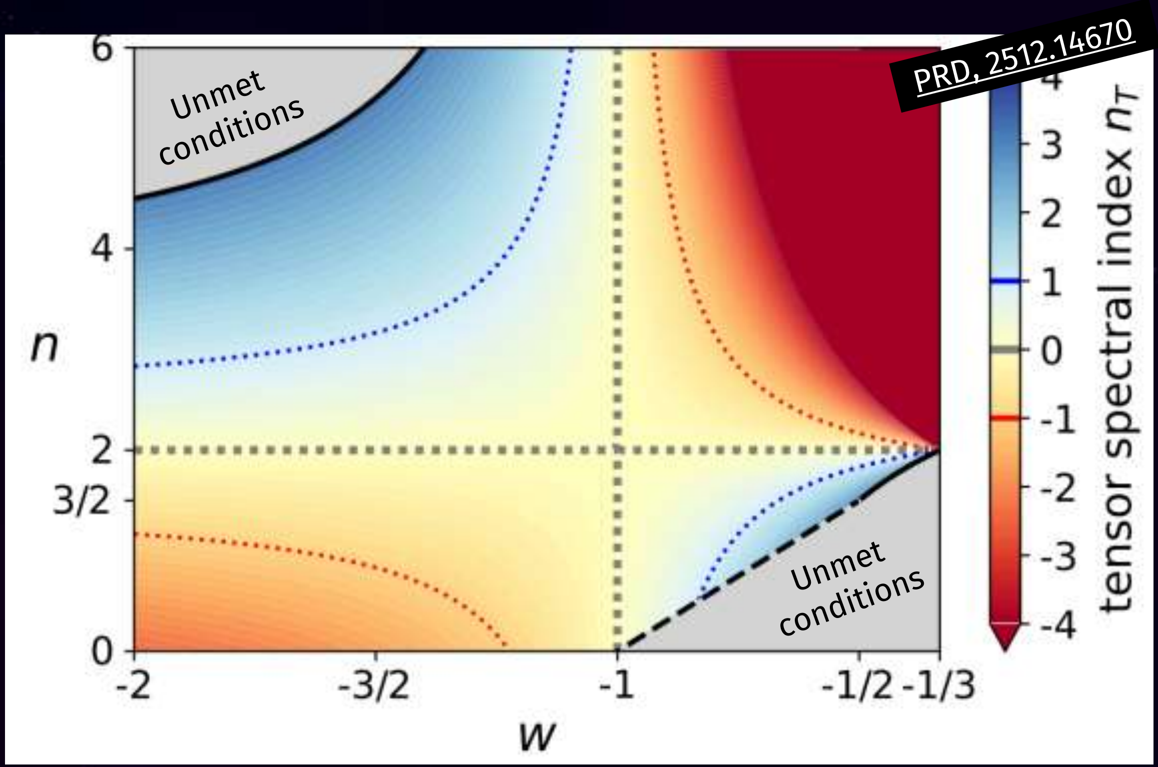


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**GW spectral index almost independent of that of the source**

- $\mathcal{O}(\epsilon_{\rm SR})$ corrections [Caprini *et al.*]
- Use: **fast estimation for model-building**

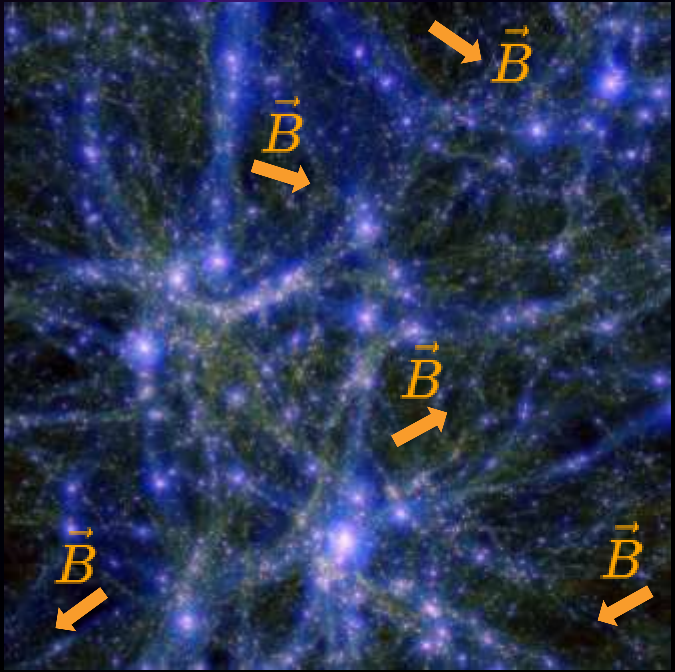
*We have slope.*
*Next: AMPLITUDE* ➔

# Table of Contents

# Cosmological magnetic fields

- Evidence for magnetic fields in the intergalactic ***voids*** (from GeV photon deviation)

  $\Longrightarrow$ primordial origin?




$|B| > 10^{-15}\mathrm{G}$ $\qquad$ $\lambda_B \sim \mathrm{Mpc}$

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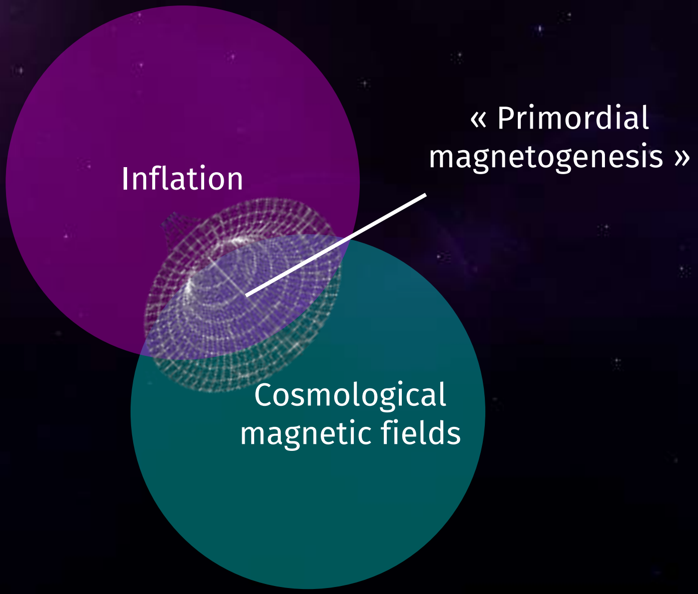


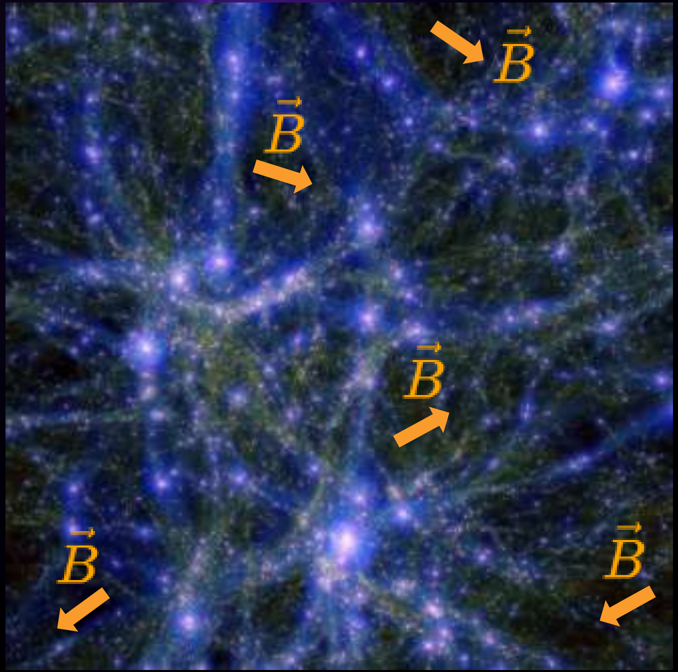


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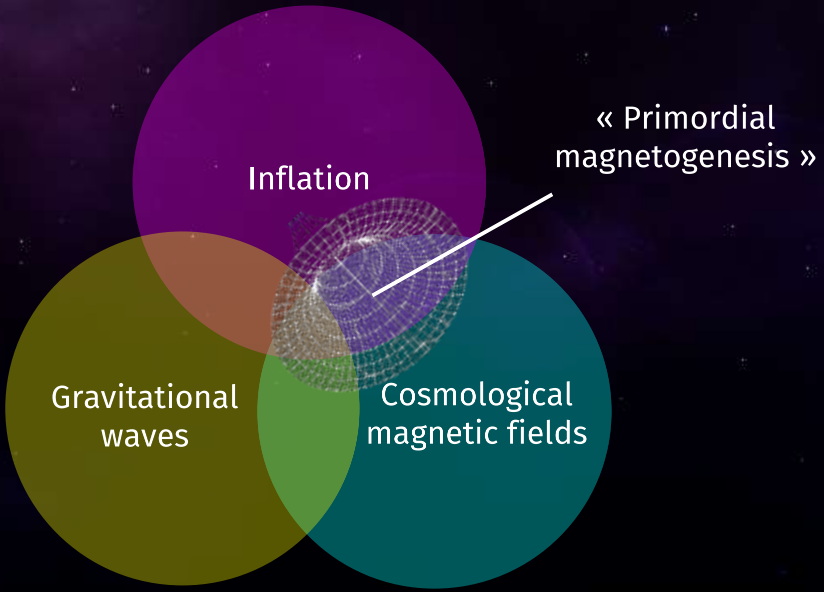


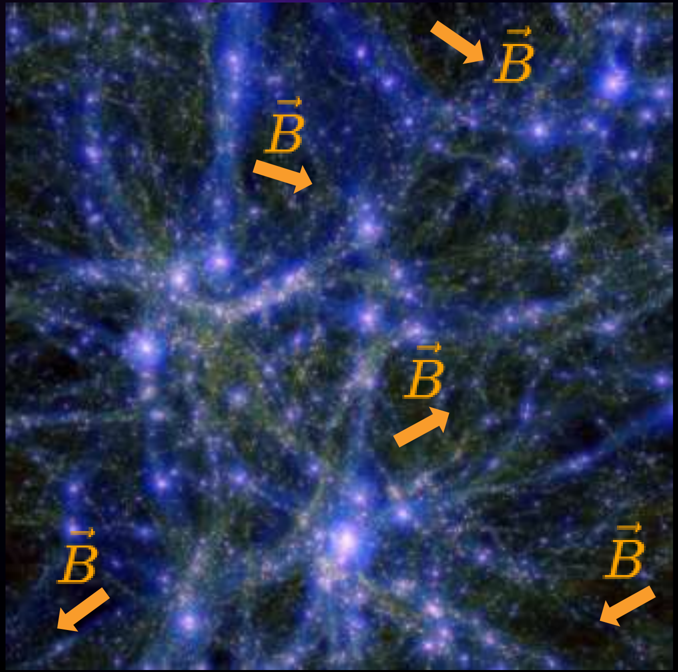


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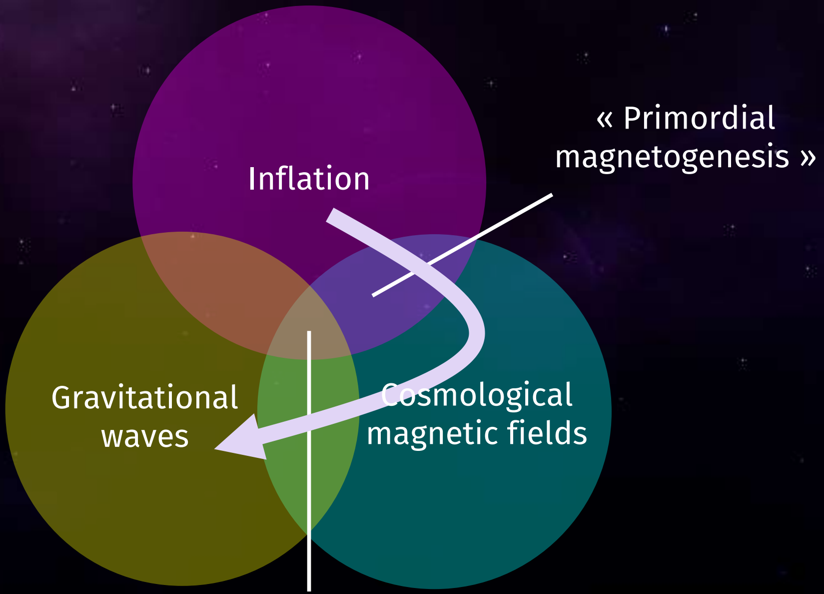
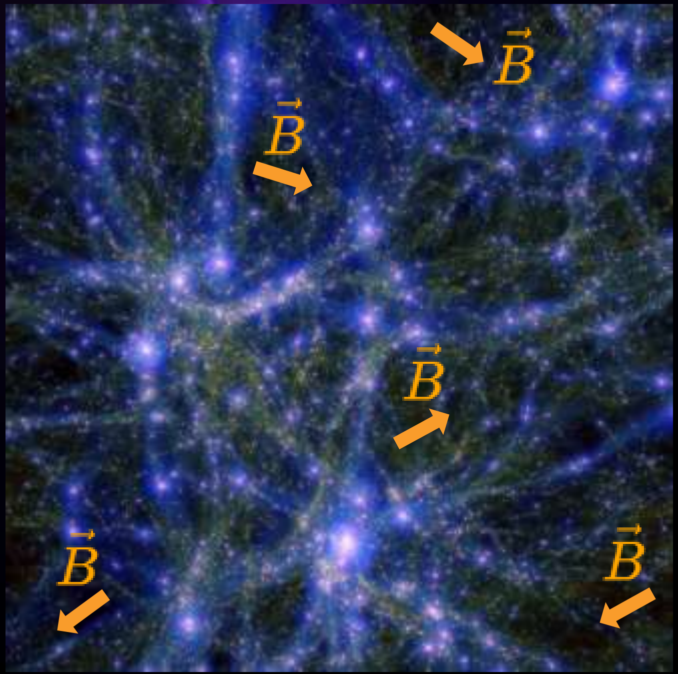



**GW signal produced by primordial magnetic fields ➔ combined direct probes**

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# To amplify or not to amplify, that is the question

$$S = \int \sqrt{-g} \mathrm{d}^4 x \left[ \mathcal{L}(\phi) \right.$$ [redacted] $$\left. \right]$$

spacetime geometry    inflaton

Minimally-coupled scalars:

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0$$

tensors:

$$\chi_k'' + \left(k^2 - \frac{a''}{a}\right) \chi_k = 0$$

➔ **vacuum perturbations get amplified**

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spacetime geometry　inflaton　**electromagnetism**

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Massless gauge field:

$$A_k'' + k^2 A_k = 0$$

**No amplification!**

Conformal invariance (in 4D)

Specific to force carriers

# To amplify or not to amplify, that is the question

[Fleury *et al.* ‘14: most general couplings]

$$S = \int \sqrt{-g} d^4x \left[ \mathcal{L}(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} i_1(\phi) F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} i_2(\phi) (\star F)_{\mu\nu} F^{\mu\nu} \right]$$

spacetime geometry | inflaton | electromagnetism | “kinetic” coupling | “chiral” coupling

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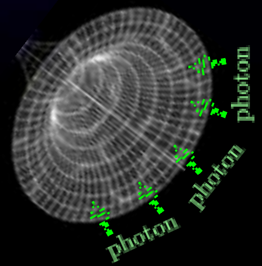


**Coupling-dependent amplification!**

# Model's parameters

Slow-roll: $\partial_\phi i_{1,2}(\phi) \simeq \gamma_{1,2}(1+i_1)\frac{H}{\dot{\phi}}$

| | |
|---|---|
| *Parameter* | |
| *Theory interpretation* | |
| *Phenomenological effect* | |

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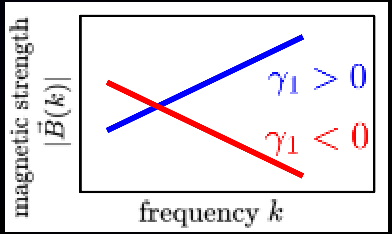
| | |
|---|---|
| *Parameter* | $0 \leqslant \gamma_1 < 4$ |
| *Theory interpretation* | Electric charge renormalization $e \to e' = e/\sqrt{t^{\gamma_1}}$ [Sobol, Durrer] |
| *Phenomenological effect* | **Controls spectral index** <br> **(Blue Fields)** $\lvert\vec{E}(k)\rvert^2 \propto k^{1-\gamma_1}$ <br> $\lvert\vec{B}(k)\rvert^2 \propto k^{1+\gamma_1}$ |



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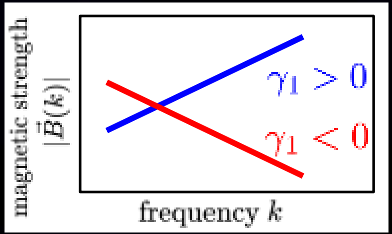
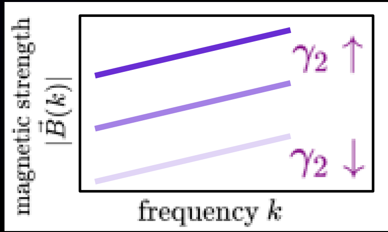
| | | |
|---|---|---|
| *Parameter* | $0 \leqslant \gamma_1 < 4$ | $\gamma_2 \in \mathbb{R}$ |
| *Theory interpretation* | Electric charge renormalization | Axion inflation: $\gamma_2 = \sqrt{2\epsilon_{\rm SR}}\frac{\alpha M_{pl}}{f_a} = \mathcal{O}(1-10)$ [Barnaby et al.] |
| *Phenomenological effect* | **Controls spectral index**   | **Controls polarization (*P*-odd) and amplitude** $\lvert\vec{B}^+(k)\rvert^2 - \lvert\vec{B}^-(k)\rvert^2 \propto \exp(\gamma_2)$   |

# Roadmap

### Interaction model

Action coupling $\varphi$ to the $A_\mu$ field

### Electro-magnetism

Power spectra of $\vec{B}_k$ and $\vec{E}_k$

### Anisotropies

$\Pi_{ij}$ anisotropic part of $T_{\mu\nu}(B, E)$

### Polarized gravity waves

$h_{ij}^{\pm}$ is sourced by $\Pi_{ij}$

### Observables

Tensor power spectrum $P_T$, energy density $\Omega_{gw}$

### Constraints

Allowed coupling parameters from known data

# Table of Contents

# Do you see anything?

1) Nearly scale-invariant GW power spectrum (*a*like std. inflation)

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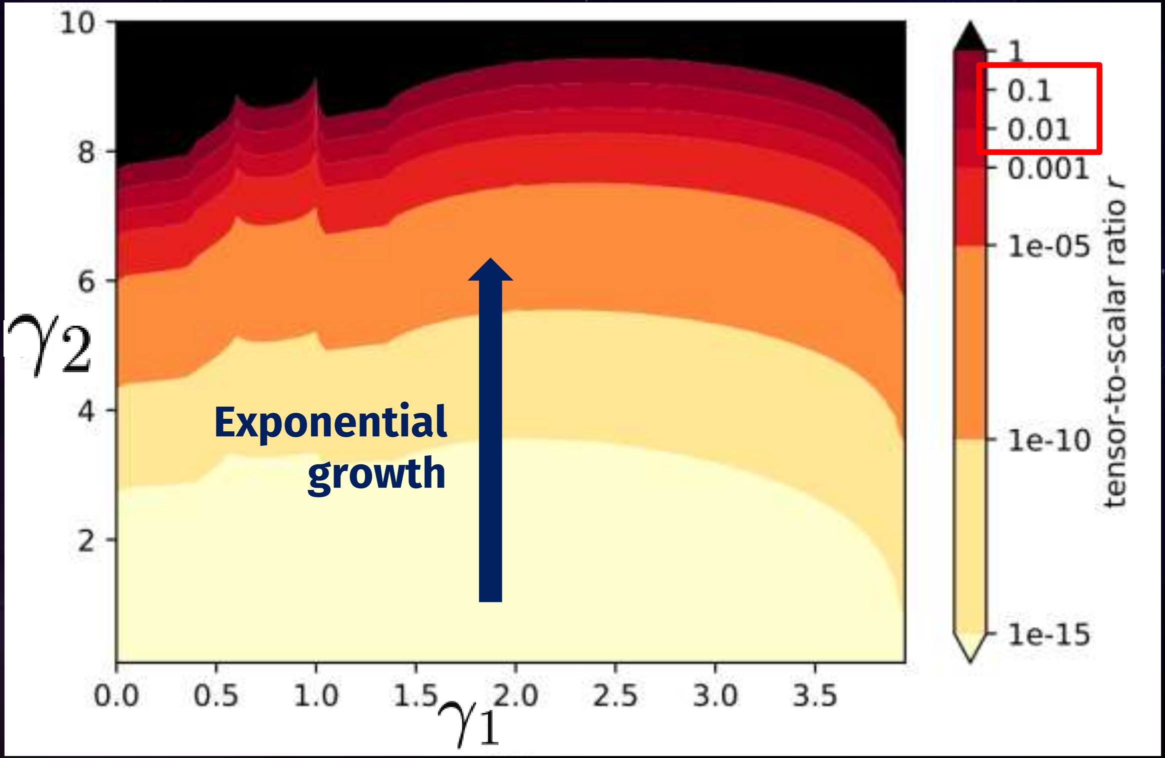
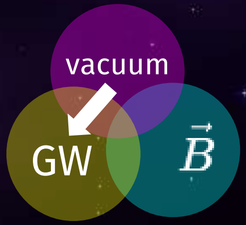



Observationally hot zone

# Secondary v.s. primary background

Standard inflation:




$$\Omega_{gw} \propto \underbrace{(E_{\text{inflation}}/E_{\text{Planck}})^2}_{\ll 1}$$

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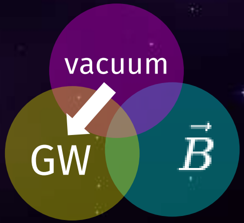


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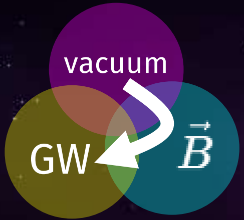
$$\ll 1$$

Here:




$$\Omega_{gw} \propto \left(E_{\text{inflation}}/E_{\text{Planck}}\right)^4$$

$$\lll 1\ ?$$

**Naively less signal, but exponential boost $\Omega_{gw} \propto \exp(\gamma_2)$ can reverse this trend!**

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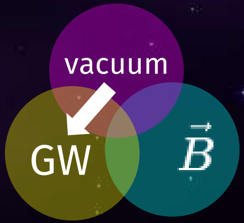


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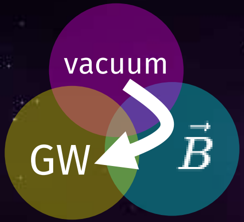


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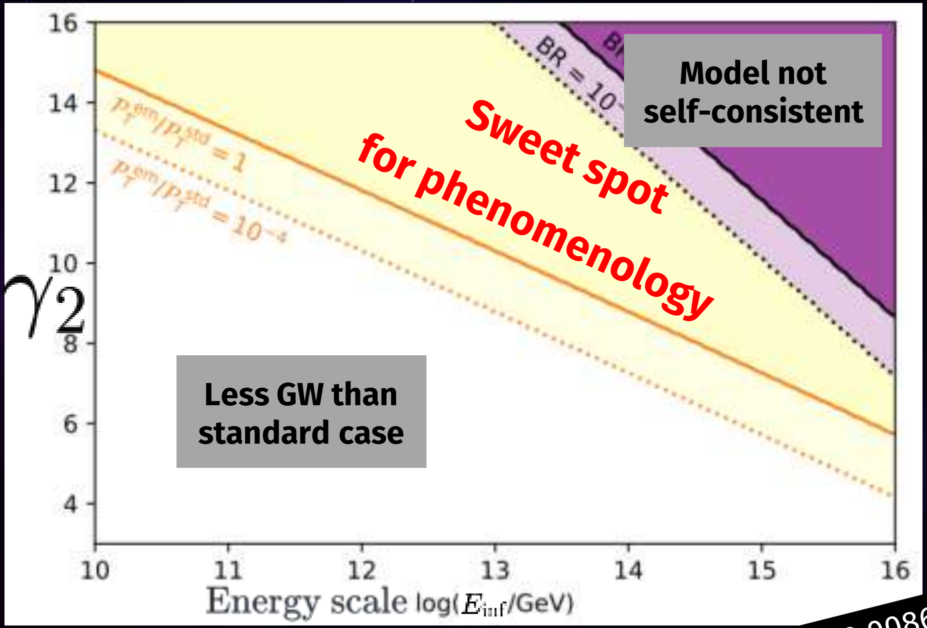


JCAP, 2510.00869

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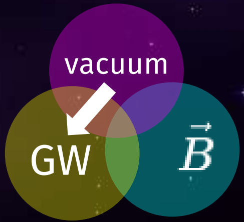


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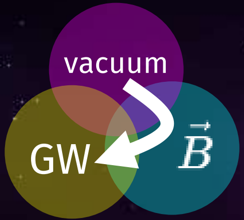


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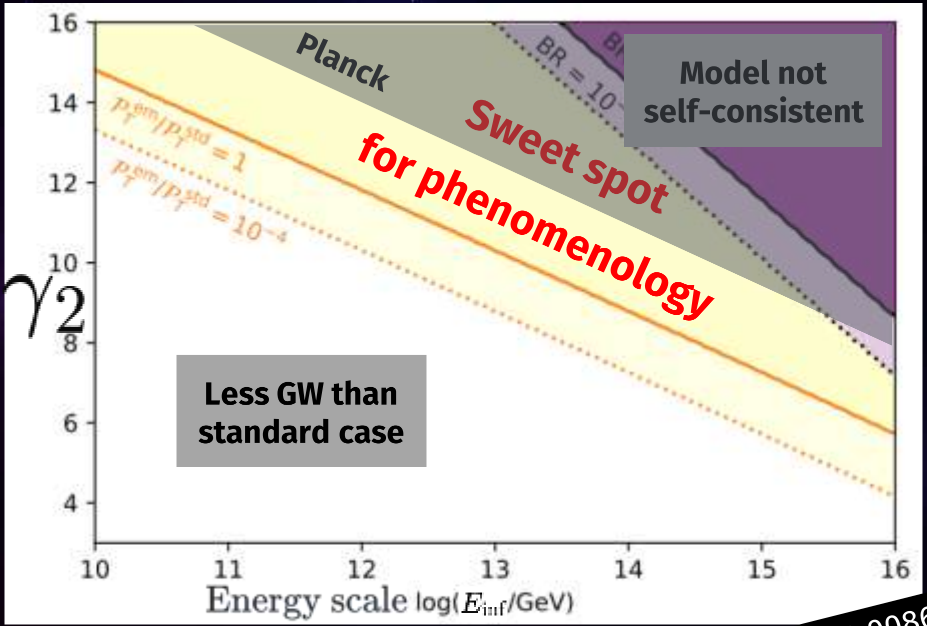


JCAP, 2510.00869

# Conclusion

| *Take-away 1* | Gravitational waves = powerful **direct probes** |
|---|---|

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PRD, 2512.14670

***Take-away 2*** **New formula** to estimate GW slope index

$n_T = 2(n-2)(q+1) \longrightarrow$ great for model building

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PRD, 2512.14670

***Take-away 2*** **New formula** to estimate GW slope index

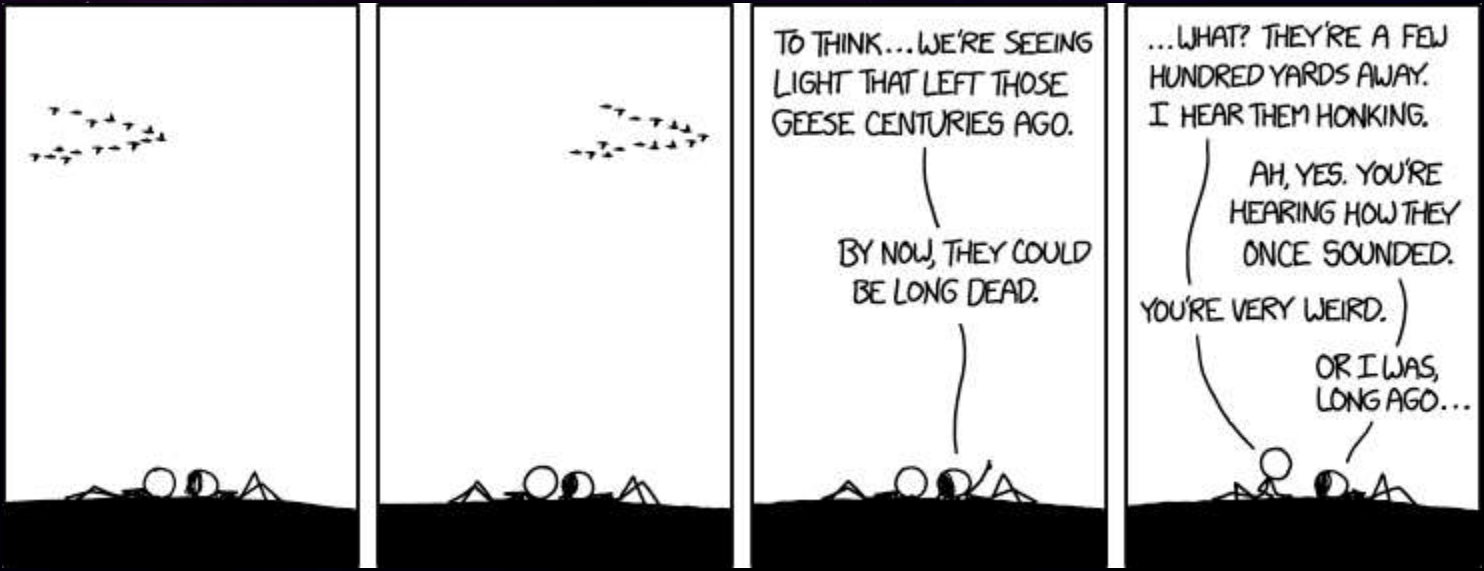
$$n_T = 2(n-2)(q+1) \longrightarrow$$ great for model building

JCAP, 2510.00869

***Take-away 3*** Do not under-estimate **secondary GW**!

Can easily overcome « primary » GW signal

# Thank you!




[Credits: xkcd]

# List of assumptions

Universal scaling of GW power spectrum as soon as:

- ➔ Blue enough source ($n>0$)
- ➔ Accelerated expansion ($w<-1/3$, $w$ constant)
- ➔ Cutoff = Hubble horizon
- ➔ Conclusion: if slow-roll, scale invariant spectrum

# List of assumptions (2)

To summarize, let us recall the assumptions leading to relation (2): *(i)* we consider an inflationary phase with constant equation of state $w < -1/3$; *(ii)* only super-horizon perturbations generate GWs, as is the case if the source field $\Psi_i$ was amplified from the vacuum; *(iii)* $\Psi_i$ is a Gaussian field; *(iv)* its spectrum – a generic power law of $k$, of $a$, and of an arbitrary energy scale – is not red, $n \geqslant 0$; *(v)* a technical but not very restrictive inequality between $n$ and $w$ must be satisfied – see Figure 1.

Would assumption *(i)*, *(ii)* or *(iv)* be relaxed, our derivation may however still proceed, as long as the dependence on $k$ of the bounds dominating integrals (17)

$$\mathcal{P}_T(k,\tau) \propto \frac{C^2}{2\pi^2} \frac{M^{4-2(n-2)-2[C]} H_{\rm end}^{2q(2-n)}}{M_{\rm Pl}^4} k^{2(n-2)(q+1)} .$$

➔ Corrections exist if the amplitude changes with k :

$$\mathcal{P}_h^{(\rm vac)} (1+\Delta\mathcal{P}_h) \simeq \frac{2\,H^2}{\pi^2\,m_{\rm Pl}^2}\left(1+4.3\cdot 10^{-7}\frac{H^2}{m_{\rm Pl}^2}\frac{e^{4\pi\xi}}{\xi^6}\right)$$

[C. Caprini 1801.04268]

➔ If xi cannot be treated as independent of expansion:

$$n_T \simeq -4\epsilon + (4\pi\xi - 6)\,(\epsilon - \eta)$$

**Our formula's prediction**

Corrections, but still generically close to zero!

# Conclusion

- Lots of possible improvements:
  include back-reaction [Durrer, Sobol, Vilchinskii], non-gaussianities [Caprini, Sorbo], post-inflation contributions, numerical treatment [Schober]...

# Polarization: antisymmetric power spectrum

$$\begin{aligned}
\mathcal{P}_T =& \frac{H^4}{M_{\rm Pl}^4}\mathcal{F}(\gamma_1,\gamma_2)\\
=& \frac{H^4}{M_{\rm Pl}^4}\frac{\gamma_3^8}{2\pi^2(4\pi)^2}\left[\frac{\Delta_B^2\gamma_3^{2m_B}}{9(4+m_B)^2}\left(\cosh^2\left(\frac{\pi\gamma_2}{2}\right)C(f_S,m_B)\right.\right.\\
&\left.+\sinh^2\left(\frac{\pi\gamma_2}{2}\right)C(f_A,m_B)\right)\\
&+\frac{\Delta_E^2\gamma_3^{2m_E}}{9(4+m_E)^2}\left(\cosh^2\left(\frac{\pi\gamma_2}{2}\right)C(f_S,m_E)+\sinh^2\left(\frac{\pi\gamma_2}{2}\right)C(f_A,m_E)\right)\\
&+\frac{2\Delta_B\Delta_E\gamma_3^{m_B+m_E}}{9(4+m_B)(4+m_E)}\left(\cosh^2\left(\frac{\pi\gamma_2}{2}\right)C\left(f_S,\frac{m_B+m_E}{2}\right)\right.\\
&\left.\left.+\sinh^2\left(\frac{\pi\gamma_2}{2}\right)C\left(f_A,\frac{m_B+m_E}{2}\right)\right)\right]
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_T^A =& \frac{H^4}{M_{\rm Pl}^4}\mathcal{F}_A(\gamma_1,\gamma_2)\\
=& \frac{H^4}{M_{\rm Pl}^4}\frac{\gamma_3^8}{2\pi^2(4\pi)^2}\frac{\sinh\pi\gamma_2}{2}\left[\frac{\Delta_B^2\gamma_3^{2m_B}}{9(4+m_B)^2}C(g,m_B)\frac{\Delta_E^2\gamma_3^{2m_E}}{9(4+m_E)^2}C(g,m_E)\right.\\
&\left.+\frac{2\Delta_B\Delta_E\gamma_3^{m_B+m_E}}{9(4+m_B)(4+m_E)}C\left(g,\frac{m_B+m_E}{2}\right)\right]
\end{aligned}$$

# Extra equations

$$\widetilde{k}_h(\tau) \equiv \frac{1}{-\tau}\left(\frac{|\gamma_2|}{2} + \frac{1}{2}\sqrt{\gamma_2^2 + |\gamma_1(2-\gamma_1)|}\right)$$

$$\begin{aligned}\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V &= -\frac{1}{4}\partial_\phi i_1 \langle F_{\mu\nu}F^{\mu\nu}\rangle - \frac{1}{4}\partial_\phi i_2 \left\langle F_{\mu\nu}\tilde{F}^{\mu\nu}\right\rangle \\ &= \frac{1}{1+i_1}\left[\frac{1}{2}\partial_\phi i_1 \left\langle \boldsymbol{\mathcal{E}}^2 - \boldsymbol{\mathcal{B}}^2\right\rangle + \partial_\phi i_2 \langle \boldsymbol{\mathcal{B}} \cdot \boldsymbol{\mathcal{E}}\rangle\right]\end{aligned}$$

$$\rho \ll \rho_\phi \qquad \text{i.e.} \qquad \frac{1}{2(1+i_1)}(\boldsymbol{\mathcal{B}}^2 + \boldsymbol{\mathcal{E}}^2) \ll 3M_{\rm Pl}^2 H^2 \,.$$

# Extra equations

$$S_\Pi(k,\tau=\tau')[\mathcal{B},\mathcal{E}=0]=\frac{2}{(4\pi)^3}\int \mathrm{d}^3p\Big[S_\mathcal{B}(|p|)S_\mathcal{B}(|k-p|)(1+\mu^2)(1+\beta^2)$$
$$+A_\mathcal{B}(|p|)A_\mathcal{B}(|k-p|)4\mu\beta\Big]$$
$$A_\Pi(k,\tau=\tau')[\mathcal{B},\mathcal{E}=0]=\frac{2}{(4\pi)^3}\int \mathrm{d}^3p S_\mathcal{B}(|p|)A_\mathcal{B}(|k-p|)4(1+\mu^2)\beta\ ,$$

$$S_\Pi(k,\tau,\tau')[\mathcal{B},\mathcal{E}]=S_\Pi(k,\tau,\tau')[\mathcal{B},0]$$
$$+(\theta^2(\tau)+\theta^2(\tau'))S_\Pi(k,\tau,\tau')[\sqrt{f}\mathcal{B},0]$$
$$+\theta^2(\tau)\theta^2(\tau')S_\Pi(k,\tau,\tau')[f\mathcal{B},0]\ ,$$

# Compact analytical transfer function

Propagates GWs through any arbitrary number of eras with $w_1, w_2, \ldots, w_n$!

$$M_i[\ell_i]\begin{pmatrix} A_k^i \\ B_k^i \end{pmatrix} = M_{i+1}[r_{i+1}]\begin{pmatrix} A_k^{i+1} \\ B_k^{i+1} \end{pmatrix}$$

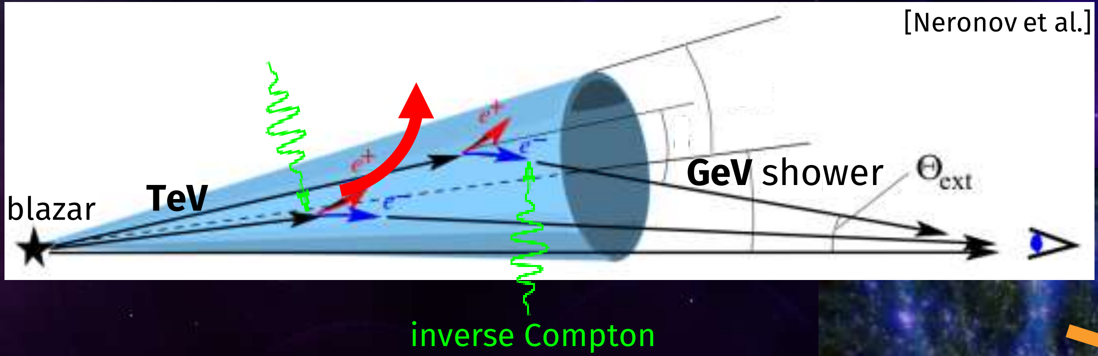
$$M_i[x] \equiv \begin{pmatrix} x j_{\nu_i}(x) & x y_{\nu_i}(x) \\ (\nu_i+1) j_{\nu_i}(x) - x j_{\nu_i+1}(x) & (\nu_i+1) y_{\nu_i}(x) - x y_{\nu_i+1}(x) \end{pmatrix}$$

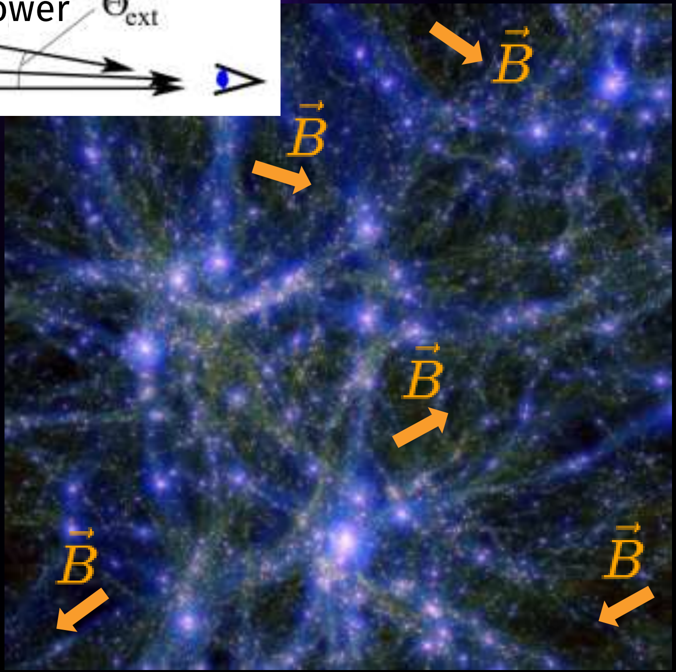
$$T_i \equiv M_{i+1}[r_{i+1}]^{-1} M_i[\ell_i]$$

$$\Omega_{\mathrm{gw},n}^{\mathrm{1pol.}}(k) = \frac{a_{\mathrm{end}}^4 H_{\mathrm{end}}^2}{a_n^4 H_n^2}\frac{1}{24}\left\| T_{n-1}\cdots T_1 M_1[r_1]^{-1}\begin{pmatrix} (k/\mathcal{H}_{\mathrm{end}})\sqrt{\mathcal{P}_{T,\mathrm{end}}^{\mathrm{1pol.}}} \\ \sqrt{12\Omega_{\mathrm{gw,end}}^{\mathrm{1pol.}}} + \sqrt{\mathcal{P}_{T,\mathrm{end}}^{\mathrm{1pol.}}} \end{pmatrix}\right\|^2$$

# Evidence for extra-galactic magnetic fields

POV : you look at a TeV blazar

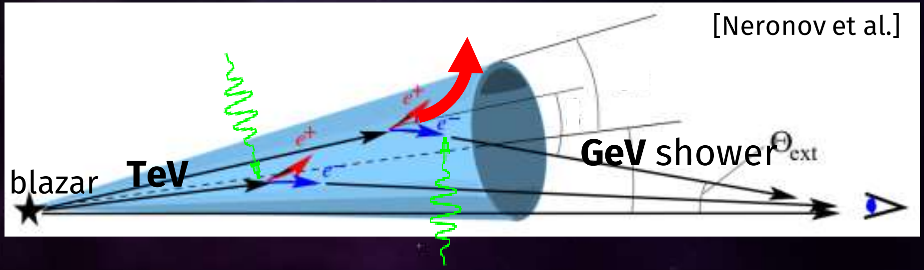



Measure GeV shower depending on line of sight angle
➔ Information on deflection

We see too little abundance of GeV shower
➔ **particles must have been deflected away ➔ B-fields ?**

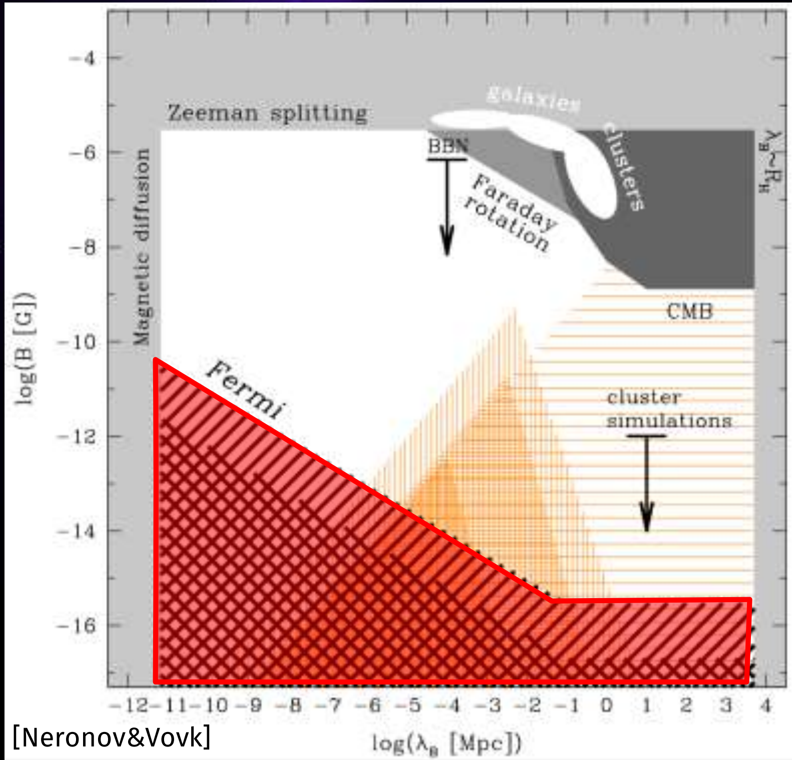
# Evidence for extra-galactic magnetic fields




Measure GeV shower depending on line of sight angle
➔ Information on deflection

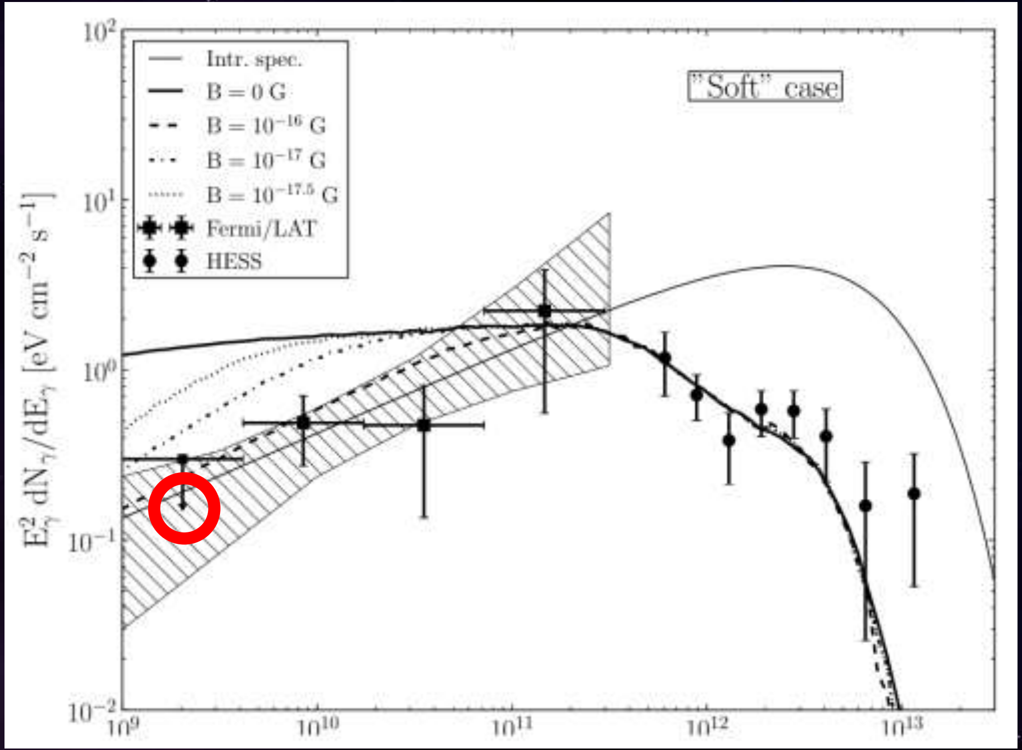
We see too little abundance of GeV shower
➔ **particles must have been deflected away ➔ B-fields ?**

***Hot question:** how can B-fields have been created in the utter void?*



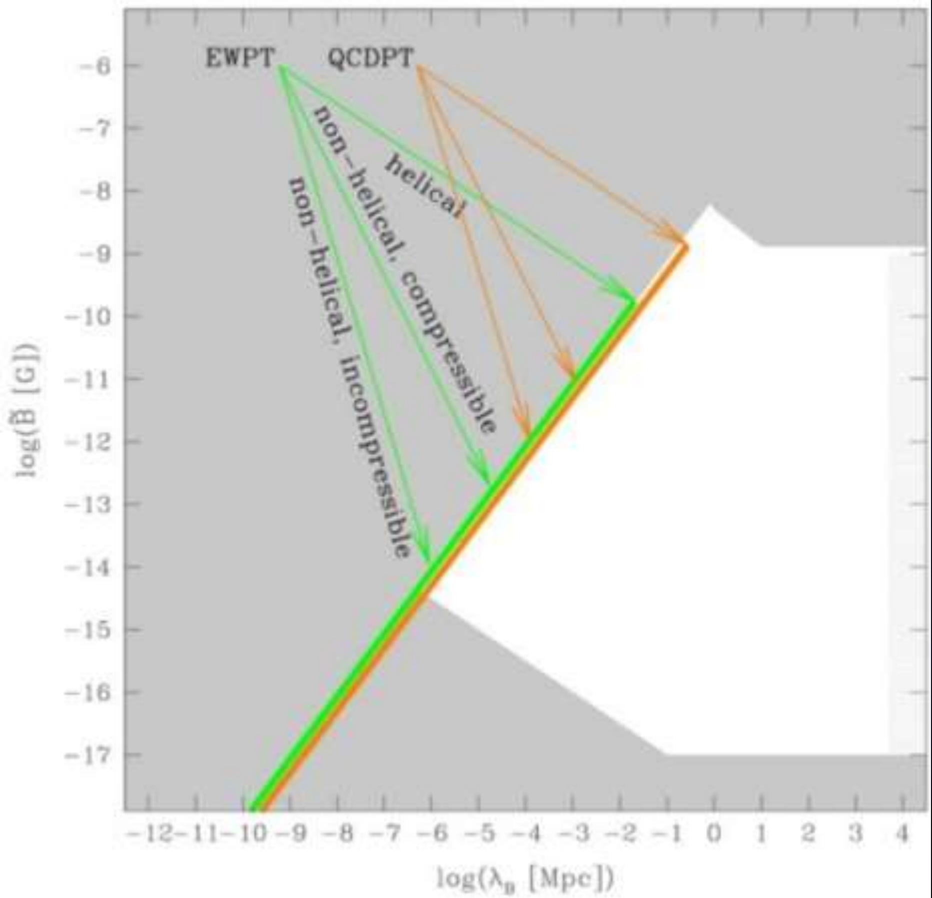

[Neronov&Vovk]

# Measurements extragalactic B-fields




[Credits: Durrer & Neronov]

# Evolution of extragalactic B-fields




[Credits: Jedamzik, Sigl, Kahniashvili, Durrer, Neronov,...]

# Positive kernels

A positive kernel is a symmetric function $K : \mathbb{R}_+^2 \to \mathbb{R}$ satisfying (among other requirements) the Mercer condition [28], namely: for any $I \subset \mathbb{R}_+$ and $f \in L^2(I)$, $\iint_{I^2} f(y)f(z)K(y,z)\mathrm{d}y\mathrm{d}z \geqslant 0$. We show here the following result:

*For any decreasing (resp. increasing) function $\phi : \mathbb{R}_+ \to \mathbb{R}_+$, $K = \phi \circ \max$ (resp. $K = \phi \circ \min$) is a positive kernel.*

The choice of function $\phi : u \mapsto 1/u$ made for $\Lambda_{\mathrm{UV}}$ thus ensures that $\mathcal{P}_T$ given by (7) is always a positive quantity.

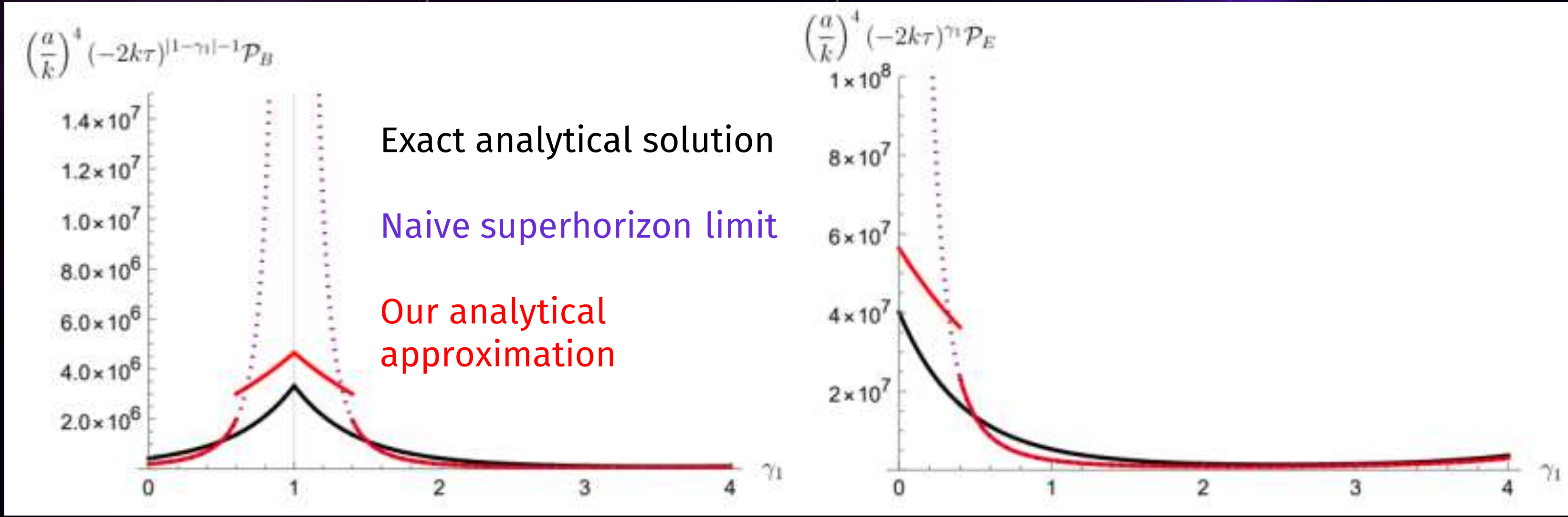
*Proof:* Given $y, z \in \mathbb{R}_+$, we write $\phi(\max(y,z)) = \int_0^\infty \mathbb{1}_{\{s \leqslant \phi(\max(y,z))\}}\mathrm{d}s$, with $\mathbb{1}$ denoting the indicator function. As $\phi$ is monotonically decreasing, $\{s \leqslant \phi(\max(y,z))\} = \{s \leqslant \phi(y)\} \cap \{s \leqslant \phi(z)\}$. Thus, after a permutation,

$$\iint_{I^2} f(y)f(z)K(y,z)\mathrm{d}y\mathrm{d}z = \int_0^\infty \mathrm{d}s \left( \int_I f(y)\mathbb{1}_{\{s \leqslant \phi(y)\}}\mathrm{d}y \right)^2 \geqslant 0\,. \quad (29)$$

A similar proof follows in the case where $\phi$ is increasing.

# Super-horizon limits

Problem: untractable analytical form ➔ super-horizon approximation



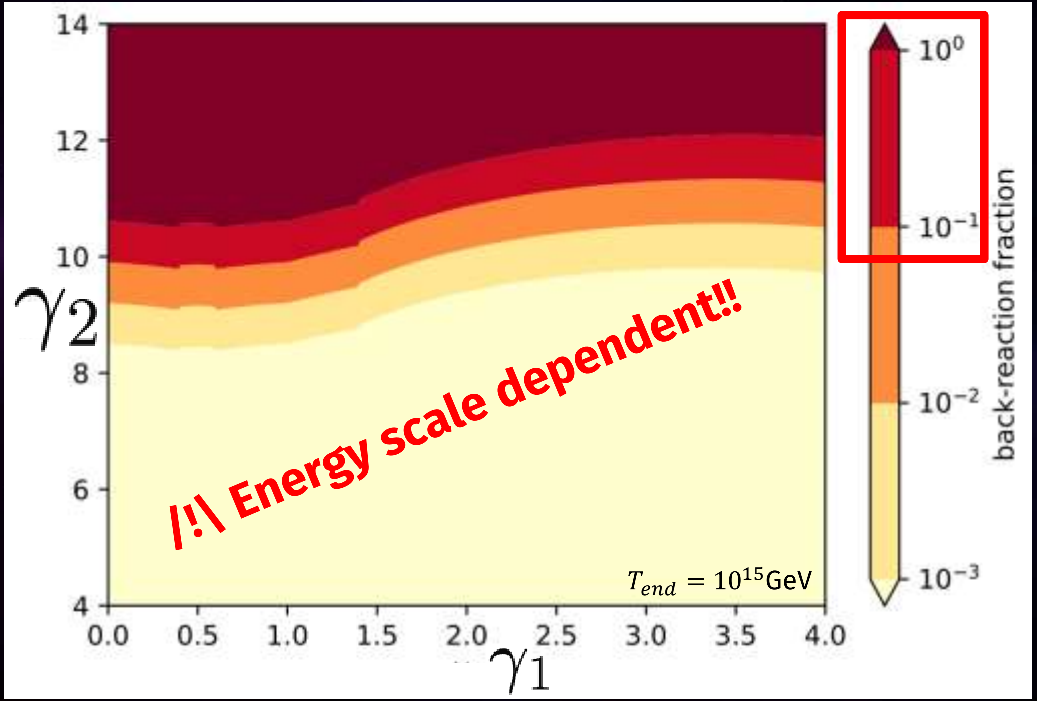
# 1 – Self-consistency constraint

Standard slow-roll evolution was assumed!

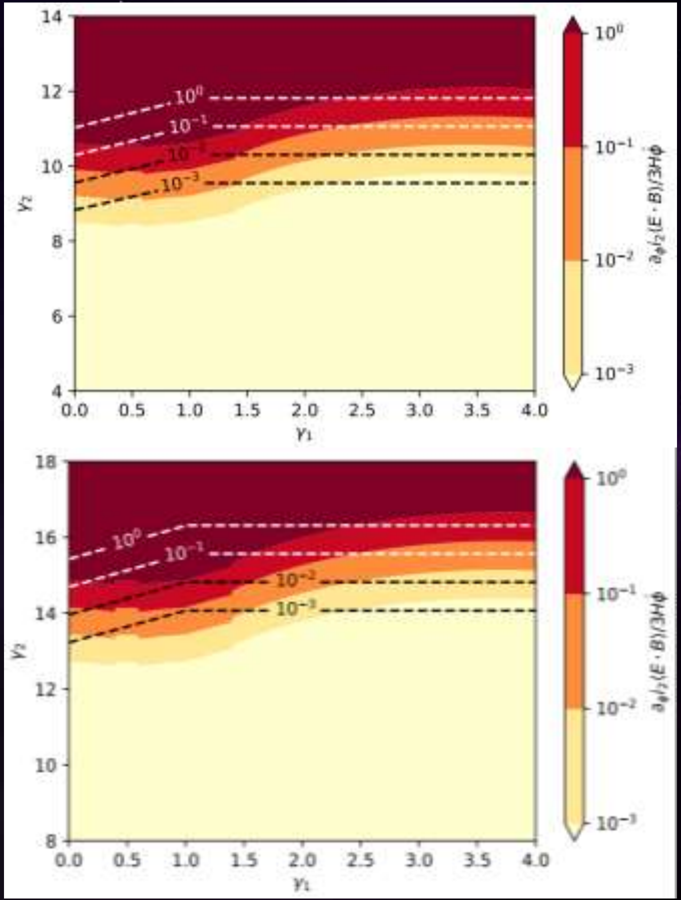
1a – NO change in the Friedmann equation

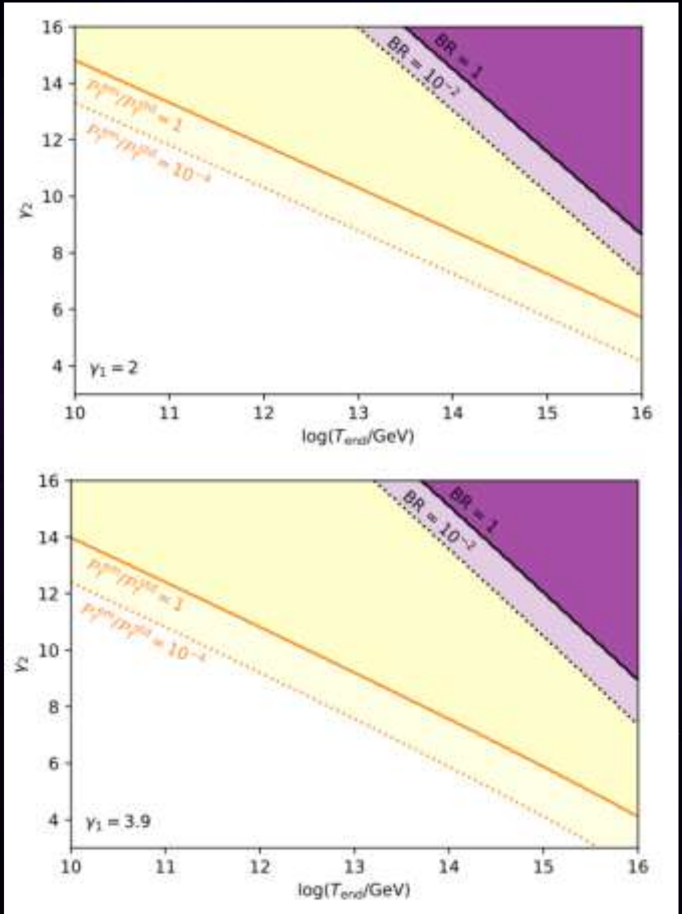
1b – NO change in the e.o.m. for φ

Untrustable analysis



# Energy scale dependence & parameter space



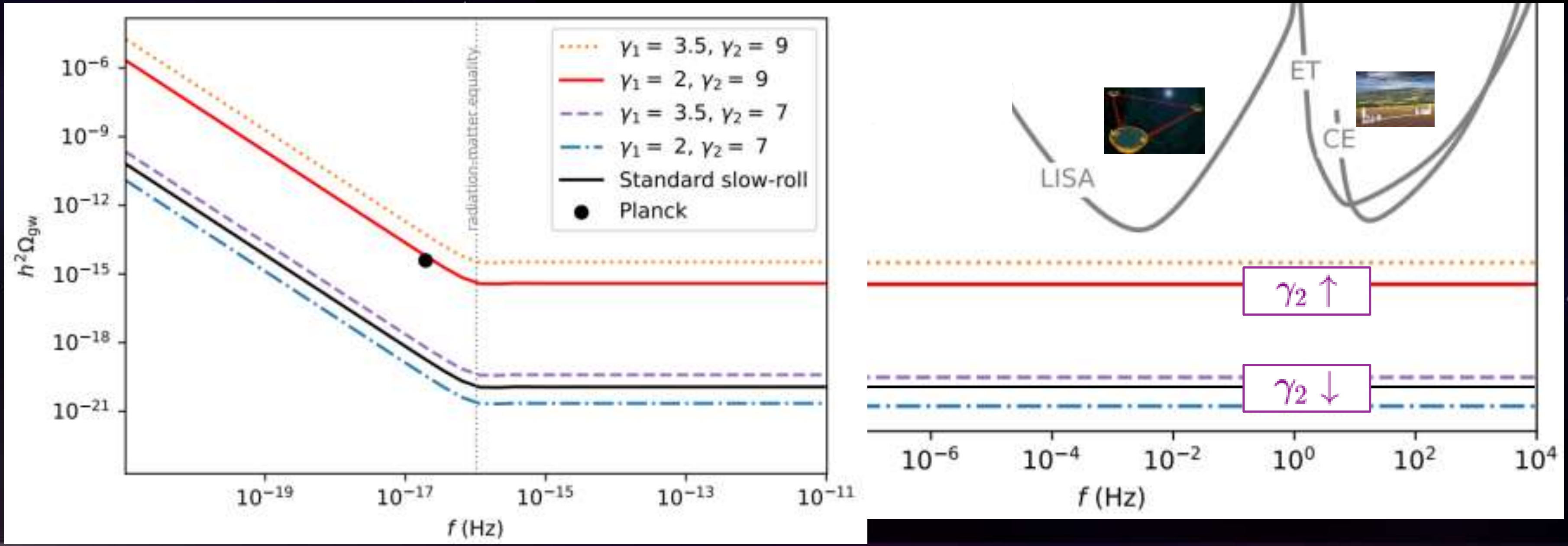
# 2 – Observational constraint

Neat & useful analytical formalism for the transfer function!

$$\Omega^{1\text{pol.}}_{\text{gw},n}(k) = \frac{a^4_{\text{end}} H^2_{\text{end}}}{a^4_n H^2_n} \frac{1}{24} \left\| T_{n-1} \cdots T_1 M_1 [r_1]^{-1} \begin{pmatrix} (k/\mathcal{H}_{\text{end}}) \sqrt{\mathcal{P}^{1\text{pol.}}_{T,\text{end}}} \\ \sqrt{12 \Omega^{1\text{pol.}}_{\text{gw,end}}} + \sqrt{\mathcal{P}^{1\text{pol.}}_{T,\text{end}}} \end{pmatrix} \right\|^2$$

➔ Through arbitrary number of eras $w_1, w_2, \ldots, w_n$!

**Scale-invariant power spectrum ➔ Planck is just the best**



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➔ Through arbitrary number of eras $w_1, w_2, \ldots, w_n$!

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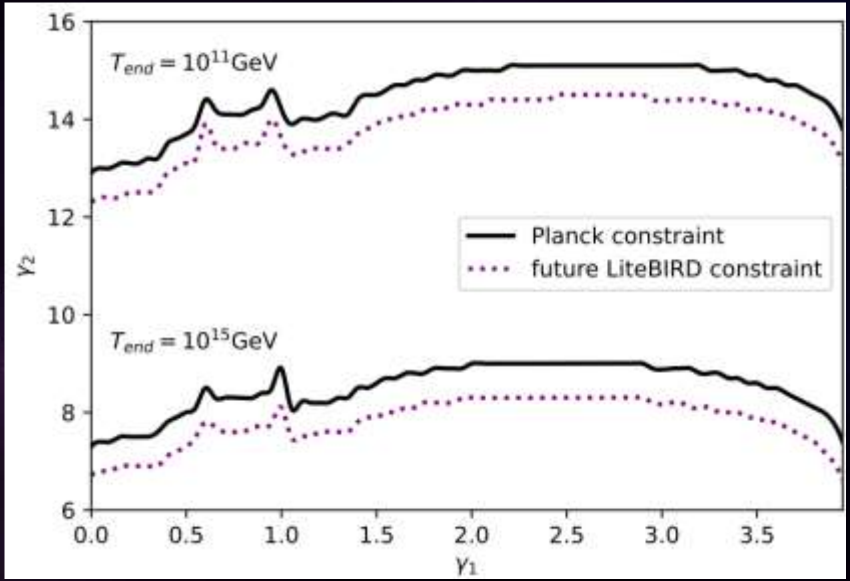


# Analytics v.s. numerics

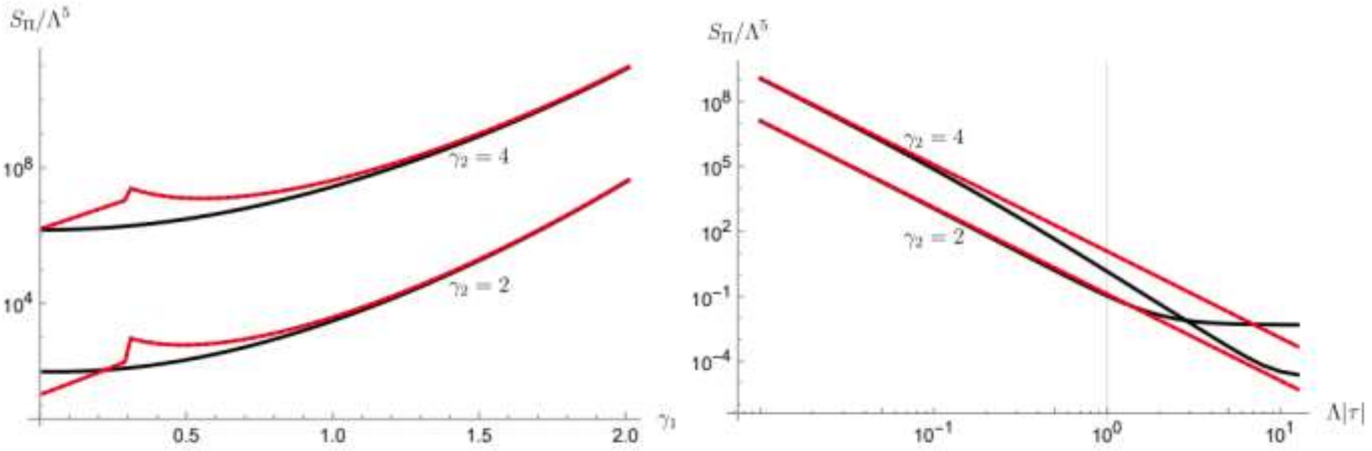
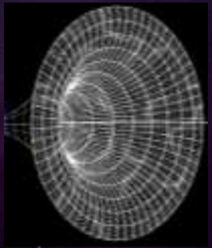
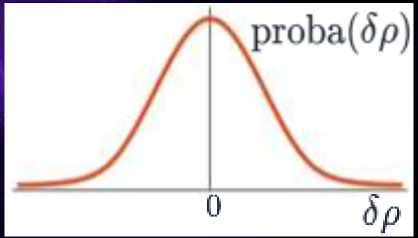



**Figure 8**. The symmetric spectrum $S_\Pi$ for various values of $\gamma_1, \gamma_2$. Solid black lines correspond to the numerical integral of the exact Whittaker functions, while solid red line correspond to our analytical expression (2.59). To obtain the dotted purple lines (that is perfectly covered by the red lines) we expanded the Whittaker function but performed the integral (2.54) numerically, which shows that the formulae (2.56)–(2.57) are correct. Furthermore, one observes an strong dependence on $\gamma_2$ that is due to the exponential enhancement of one polarization when $\gamma_2$ is non-zero. In the left (resp. right) panel we have set $|\tau| = 10^{-2}/\Lambda$ (resp. $\gamma_1 = 2$), and $k = 0$, $\Delta_B = 0$ (see the text). The vertical line marks the value $\Lambda|\tau| = \gamma_3$.

# Cosmic voids from inflation

Inflation  → Initial density fluctuations of matter 


$\delta\rho > 0$

Collapse

$\delta\rho < 0$

**Clusters abundance**

**Voids abundance**