Detecting black hole microstates

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PRD, ArXiv: 2510.06309

Date: 25 June 2026

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A contributed talk to PASCOS 2026, hosted by the University of Sheffield

Microscopic origin of black hole entropy

Begin with the Bekenstein-Hawking entropy: [J.D. Bekenstein - 1973], [S.W. Hawking - 1976]

$$S_{BH} = \frac{A}{4G_N}$$

The black hole has a finite dimensional Hilbert space that is spanned by $e^{S_{BH}}$ $\underbrace{\text{orthogonal quantum states}}_{\text{“black hole microstates”}}$.

Statistical mechanical interpretation:

$$S_{BH} = \log(\dim(\mathcal{H}_{BH}))$$

Old problem: **understanding the statistical origin and finiteness of the Bekenstein-Hawking entropy**.

What are these black hole microstates?

String theory:
- Precise microstate counting [A. Strominger, C. Vafa - 1996]
- Fuzzballs [I. Bena, E.J. Martinez, S.D. Mathur, N.P. Warner - 2022]

Approximate approaches:
- Interpretation as entanglement entropy [L. Bombelli, R.K. Koul, J. Lee, R.D. Sorkin - 1986], [M. Srednicki - 1993]
- Associating black hole entropy with the entropy of near-horizon thermally excited quantum fields [G. 't Hooft - 1985]

Need a solution that addresses universality of the Bekenstein-Hawking entropy formula and that produces a finite number of microstates

Proposed solution: a **shell state construction** to build a family of geometric microstates. [M. Sasieta – 2022], [V. Balasubramanian, A. Lawrence, J.M. Magan, M. Sasieta - 2023]

A construction using shell states

No hair theorem [M. Heusler - 1996],[N. Gurlebeck - 2015]

All stationary black hole solutions of the Einstein equations are completely characterized by only a few independent, externally observable parameters, e.g. mass, electric charge, angular momentum.

Many different initial states can evolve into the same black hole as it appears to an external observer, and these black holes may have different late-time interiors.

Distinct interior = Black hole microstate

Naively this seems to produce an infinite number of microstates, but it was shown that these form a complete basis of exactly $e^{S_{BH}}$ states. [V. Balasubramanian, A. Lawrence, J.M. Magan, M. Sasieta - 2023], [A. Climent, R. Emparan, J.M. Magan, M. Sasieta, A. Villar Lopez - 2024]

Our question: can we detect these microstates?

We consider a construction for a family of geometric black hole microstates of the eternal AdS black hole and ask:

What does this construction of microstates unlock for us?

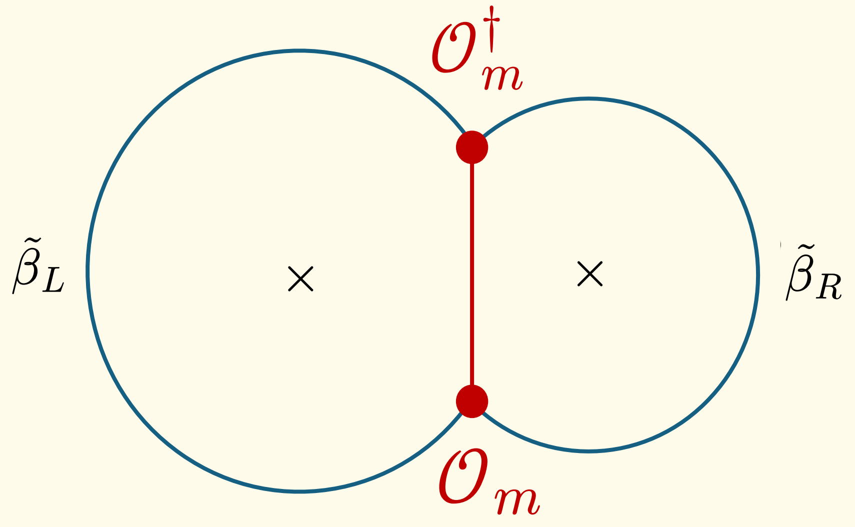
Classically, there is no measurement at asymptotic infinity one can make to detect the microstates that count the Bekenstein-Hawking entropy of black holes.

Is there such a signal for the microstate at infinity that we might detect quantum mechanically?

We claim the answer is <u>yes</u> and show this explicitly in the shell state construction. [V. Balasubramanian, WKLC, C. Murdia - 2025]

Outline

1. Shell states from the Euclidean path integral
2. Junction conditions for single shell states
3. Detecting the black hole microstates
4. Conclusions and outlook

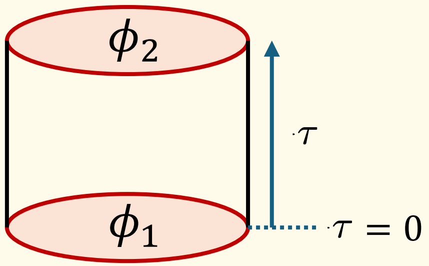


Shell states from the Euclidean path integral

States via the Euclidean path integral

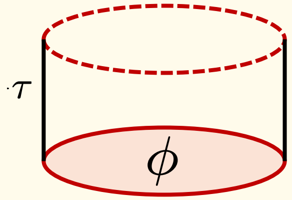
Preparing states via the Euclidean path integral:

$$\langle \phi_2 | e^{-\tau H} | \phi_1 \rangle = \int_{\phi(0)=\phi_1}^{\phi(\tau)=\phi_2} \mathcal{D}\phi e^{-I_E[\phi]}$$

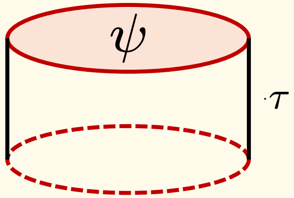


Cutting the path integral gives us ket and bra states:

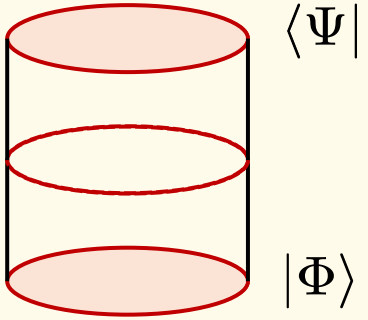
$$|\Phi\rangle = e^{-\tau H}|\phi\rangle$$



$$\langle \Psi| = \langle \psi | e^{-\tau H}$$



Overlaps $\langle \Psi | \Phi \rangle$

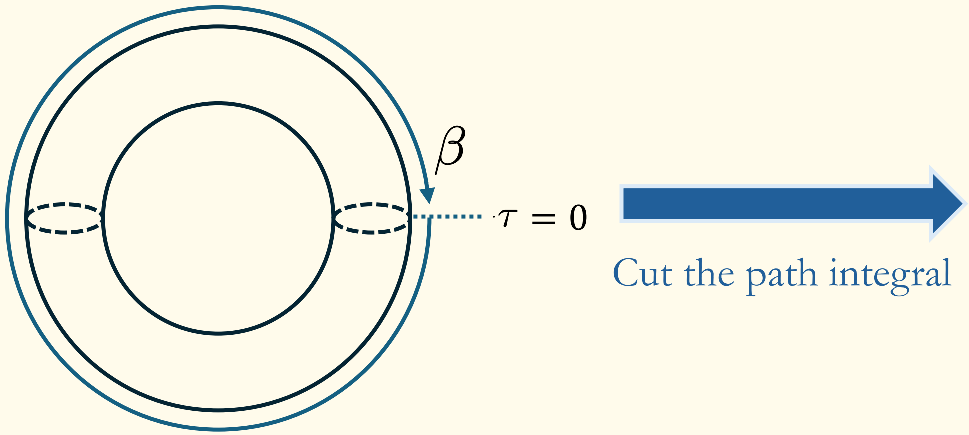


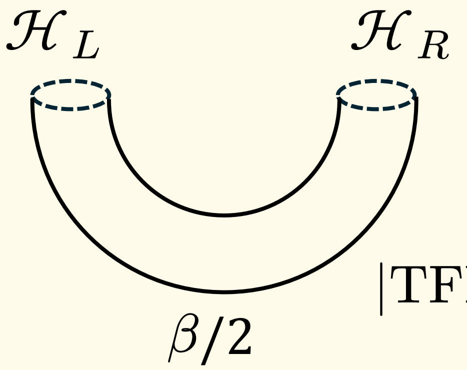
Preparing thermal states from the path integral

For the thermal state we recognize that the Euclidean time is periodic with period β

$$\int_{\phi(0)=\phi(\beta)} \mathcal{D}\phi e^{-I_E[\phi]} = \sum_i \langle E_i | e^{-\beta H} | E_i \rangle = \mathrm{Tr}(e^{-\beta H}), \qquad \tau \sim \tau + \beta$$

Obtain the thermal partition function: $Z(\beta) = \mathrm{Tr}(e^{-\beta H})$



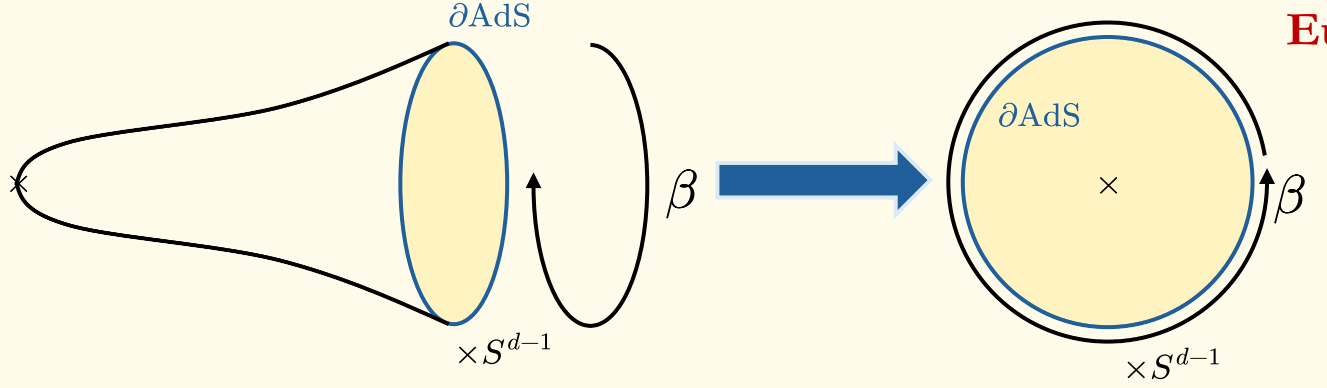


$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

Thermal gravitational path integral [G.W. Gibbons, S.W. Hawking - 1976]

We can perform a similar procedure in gravity, AdS_{d+1}

$$\int_{g(0)=g(\beta)} \mathcal{D}g e^{-I_{EH}[g]}$$



Euclidean AdS black hole

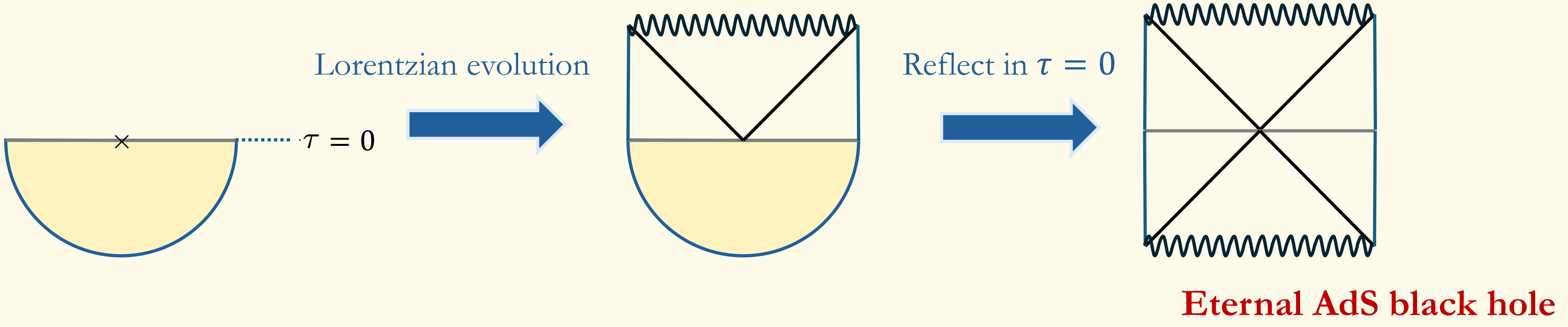
Inverse physical temperature

We cut the gravitational path integral and obtain the TFD state (in gravity)

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

Lorentzian continuation

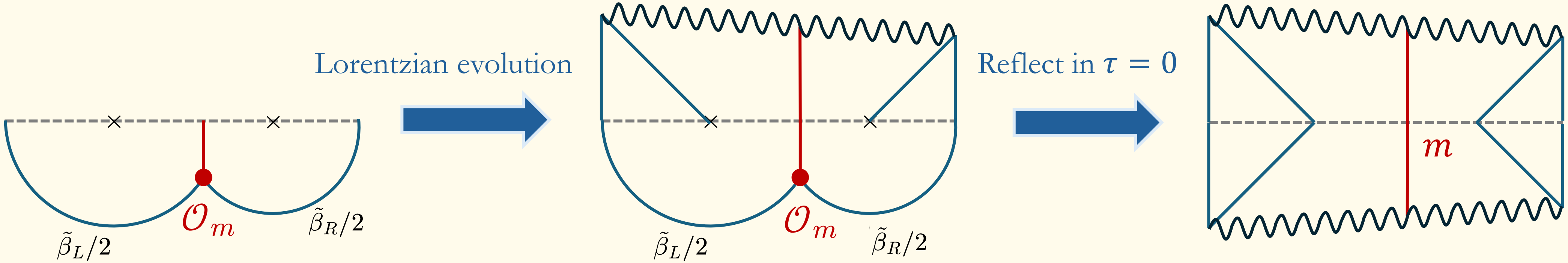
Lorentzian evolution from the point of time reflection symmetry yields the eternal AdS black hole – this is also dual to the TFD state in the boundary theory via AdS/CFT. [J. Maldacena - 2001]

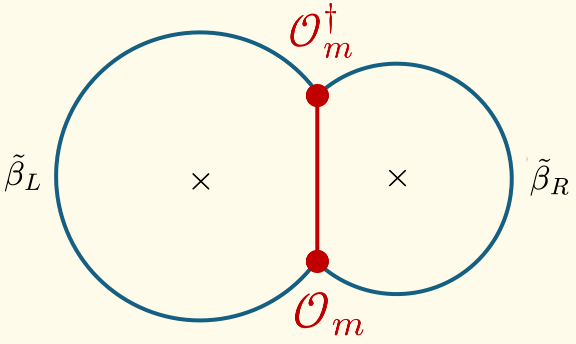


This is one state – we would like to create more to form a basis for the Hilbert space. **Do this by adding matter.**

Adding shells

Now we add matter in the form of a shell of dust particles with rest mass m





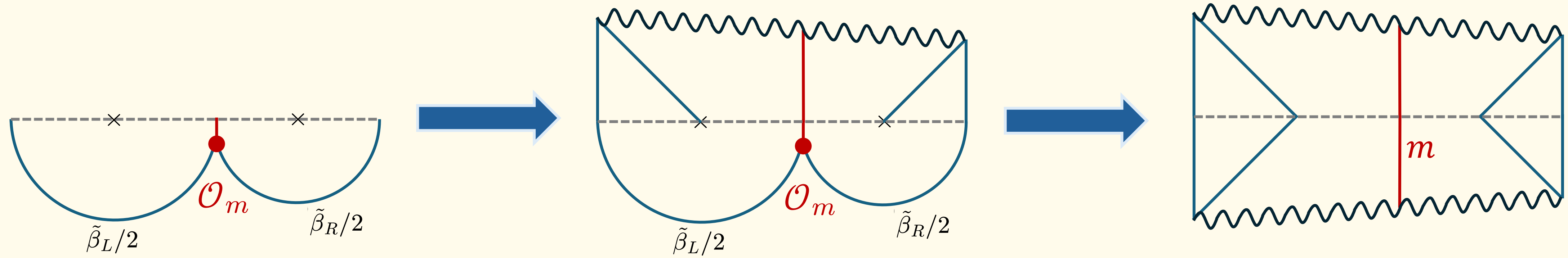
Bulk gravitational theory:
GR with $\Lambda < 0$ + matter

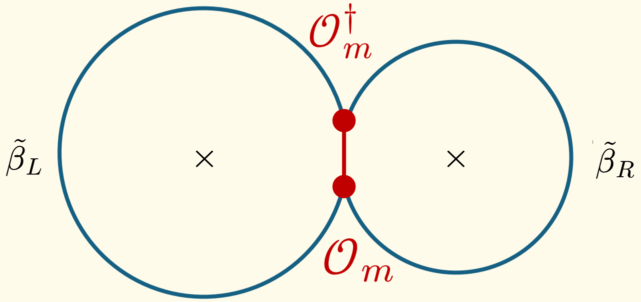
$\longleftrightarrow$ AdS / CFT

Dual boundary theory:
Two copies of a CFT

Heavy shells

Adding a **heavy** shell of dust particles with rest mass m widens the wormhole in the interior





As the rest mass of the shell increases, the Euclidean disks begin to pinch off.

This does not change the view of the black hole to the external observer.

Euclidean shell states

CFTs have Hamiltonians governing their time evolution $H = H_L = H_R$ and we have the total Hilbert space:

$$\mathcal{H} = \mathcal{H}_L^{\mathrm{CFT}} \otimes \mathcal{H}_R^{\mathrm{CFT}}$$

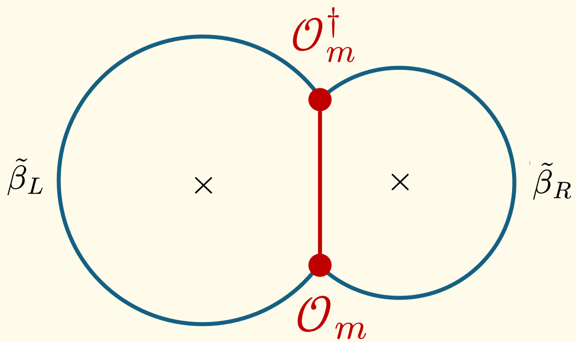
Energy eigenstates:

$$|a, b\rangle : \qquad H_L|a, b\rangle = E_a|a, b\rangle \qquad H_R|a, b\rangle = E_b|a, b\rangle$$

Thermofield double state (dual to the eternal Schwarzschild-AdS black hole):

$$|TFD\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_a e^{-\beta E_a/2}|a, a\rangle, \qquad Z(\beta) = \sum_a e^{-\beta E_a}$$

Euclidean shell states



The shell state obtained for our black hole microstates:

$$|\Psi\rangle = |\rho_{\tilde{\beta}_L/2}\mathcal{O}\rho_{\tilde{\beta}_R/2}\rangle = \frac{1}{\sqrt{Z_1}}\sum_{a,b} e^{-\frac{1}{2}(\tilde{\beta}_L E_a + \tilde{\beta}_R E_b)}\mathcal{O}_{ab}|a,b\rangle$$

Where the matrix elements and normalization are given by:

$$\mathcal{O}_{ab} = \langle a|\mathcal{O}|b\rangle, \qquad Z_1 = \mathrm{Tr}(\mathcal{O}^\dagger e^{-\tilde{\beta}_L H}\mathcal{O}e^{-\tilde{\beta}_R H})$$

Candidate black hole microstates look like the TFD state from the exterior geometries on each side but with varying interior geometries.

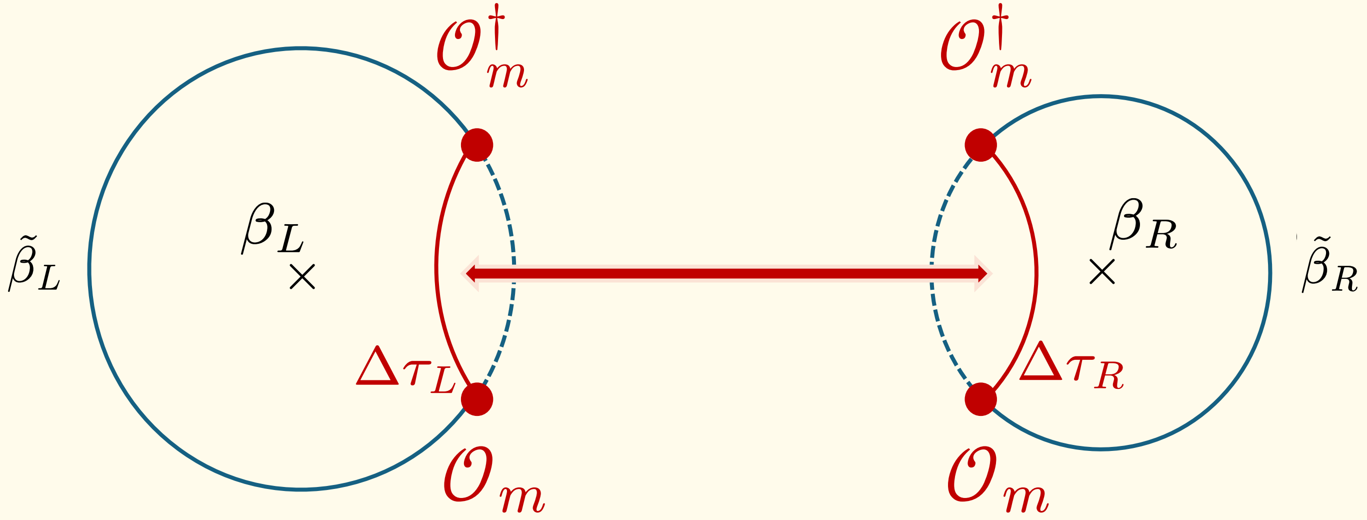
Euclidean shell state construction

It was shown that these states form a complete basis of the Hilbert space with the correct microstate counting – that the dimension of the Hilbert space spanned by this basis is $e^{S_{BH}}$. [V. Balasubramanian, A. Lawrence, J.M. Magan, M. Sasieta - 2023], [A. Climent, R. Emparan, J.M. Magan, M. Sasieta, A. Villar Lopez - 2024]

The microcanonical and single-sided black hole microstates of this kind have also been studied. [V. Balasubramanian, T. Yildirim - 2025]

For these microstates to be geometrically well-controlled in this construction there are requirements that ensure the shell backreacts on the geometry appropriately. [M. Sasieta - 2022]

This construction and the counting in extended examples is explained very nicely in a talk by Roberto Emparan at ICBS 2024 that can be found online.



Junction conditions for single shell states

Geometry of single-shell states

Each region of the Euclidean manifold is a piece of the Euclidean Schwarzschild-AdS black hole:

$$ds^2_{L,R} = f_{L,R}(r)d\tau^2_{\pm} + \frac{dr^2}{f_{L,R}(r)} + r^2 d\Omega^2_{d-1}$$

With the emblackening factor for $\mathrm{d} > 2$ defined as:

$$f_{L,R}(r) = \frac{r^2}{l^2} + 1 - \frac{16\pi G M_{L,R}}{(d-1)V_\Omega r^{d-2}}$$

The angular coordinate is known to be periodic with a period proportional to the Hawking temperature:

$$\tau_{L,R} \sim \tau_{L,R} + \beta_{L,R}, \qquad \beta_{L,R} = \frac{4\pi}{f'_{L,R}(r_{L,R})}$$

Junction conditions

Full geometry is two glued Euclidean regions along the shell worldvolume using the junction conditions.
[W. Israel - 1966]

Junction conditions reduce the shell's dynamics to that of a non-relativistic effective particle with zero total energy: [E. Poisson - 2004]

$$\left(\frac{dR}{dT}\right) + V_{\text{eff}}(R) = 0, \qquad V_{\text{eff}}(R) = -f_R(R) + \left(\frac{M_R - M_L}{m} - \frac{4\pi G m}{(d-1)V_\Omega R^{d-2}}\right)$$

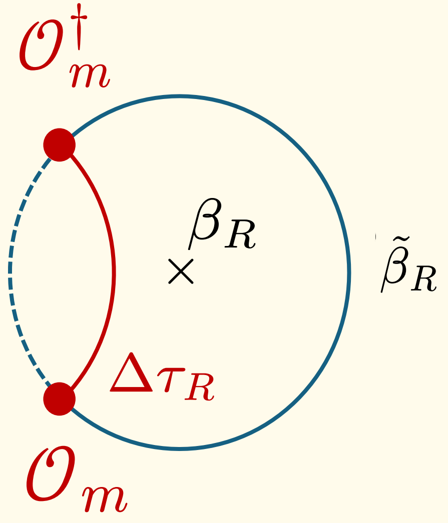
Shell proper time

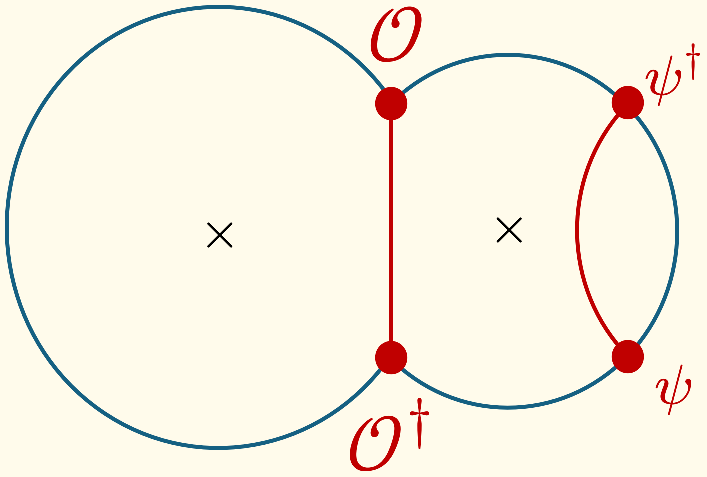
Shell enters the bulk on each side and turns at a minimum radius R_*

The elapsed propagation time is then:

$$\Delta\tau_{L,R} = 2\int_{R_*}^{\infty} \frac{dR}{f_{L,R}(R)}\sqrt{\frac{f_{L,R}(R) + V_{\text{eff}(R)}}{-V_{\text{eff}}(R)}}$$

$$\boxed{\beta_{L,R} = \tilde{\beta}_{L,R} + \Delta\tau_{L,R}}$$





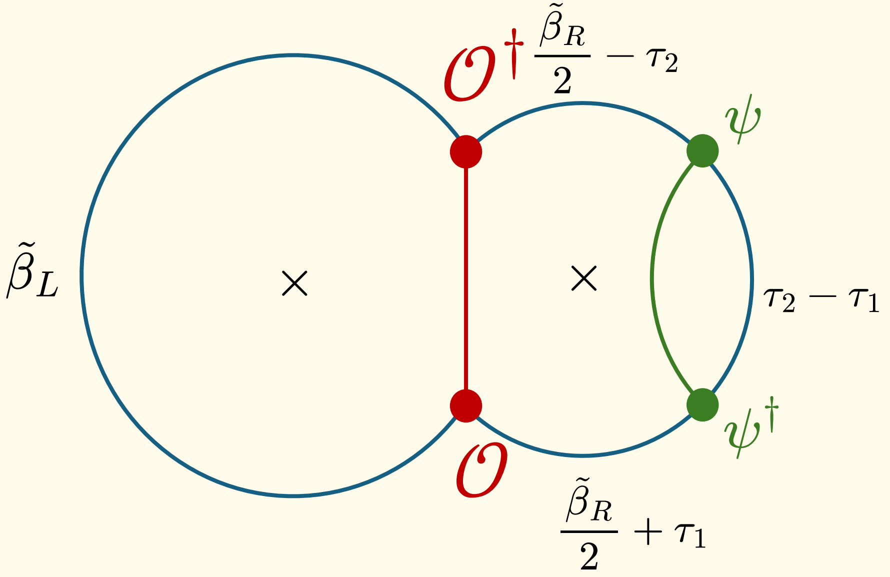
Detecting black hole microstates

Construct a probe operator

To begin, we construct a probe

$$\psi = w_{\mathcal{O}}\mathcal{O} + \sum_i w_i \phi_i, \qquad w_{\mathcal{O}}^2 + \sum_i w_i^2 = 1$$

The insertion times are $\tau_1 < 0$ and $\tau_2 > 0$



Computing the two-point function

The relevant quantity for us is the **single-sided Euclidean two-point function**

$$\langle \Psi_{\mathcal{O}} |\, \mathbb{1}_L \otimes \psi(\tau_2)\psi^\dagger(\tau_1)_R | \Psi_{\mathcal{O}} \rangle = \frac{1}{Z_1} \mathrm{Tr} \left(\mathcal{O}^\dagger e^{-(\tilde{\beta}_R/2)H} \psi(\tau_2)\psi^\dagger(\tau_1) e^{-(\tilde{\beta}_R/2)H} \mathcal{O} e^{-\tilde{\beta}_L H} \right)$$

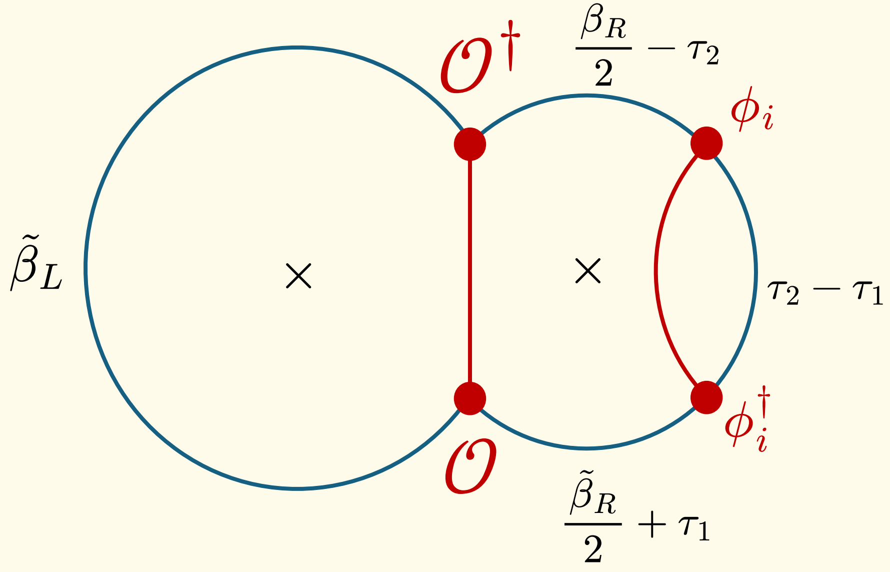
Plug in the probe definition and only consider the numerator:

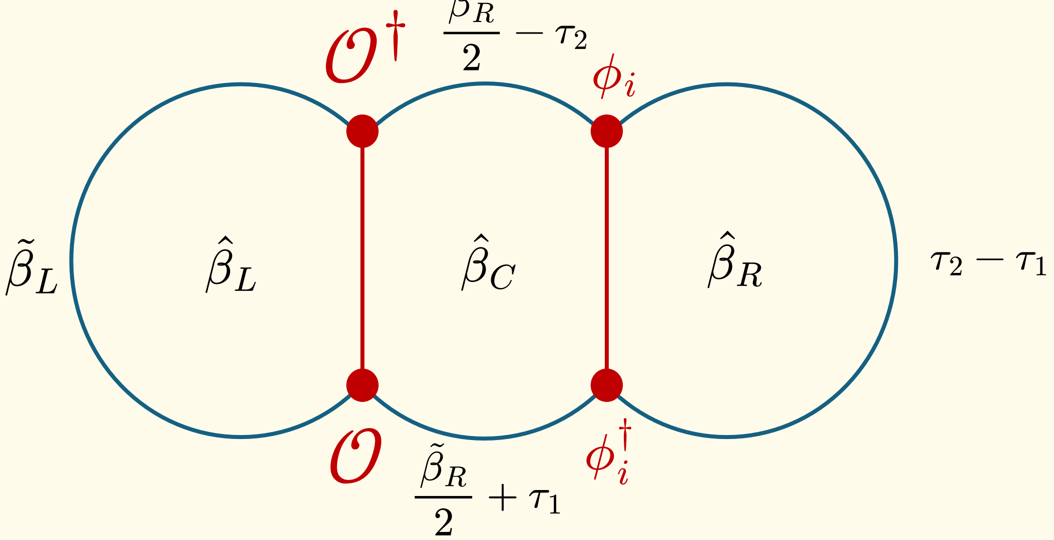
$$\begin{aligned} \mathcal{T}_1 &= w_{\mathcal{O}}^2 \mathrm{Tr} \left(\mathcal{O}^\dagger e^{-\tilde{\beta}_R H/2} \mathcal{O}(\tau_2) \mathcal{O}^\dagger(\tau_1) e^{-\tilde{\beta}_R H/2} \mathcal{O} e^{-\tilde{\beta}_L H} \right) \\ &+ \sum_i w_i^2 \mathrm{Tr} \left(\mathcal{O}^\dagger e^{-\tilde{\beta}_R H/2} \phi_i(\tau_2) \phi_i^\dagger(\tau_1) e^{-\tilde{\beta}_R H/2} \mathcal{O} e^{-\tilde{\beta}_L H} \right) \end{aligned}$$

We evaluate this using gravitational saddle-point analysis, and we obtain saddles for each of the above terms.

The propagation saddle

$$\sum_i w_i^2 \mathrm{Tr}\left(\mathcal{O}^\dagger e^{-\tilde{\beta}_R H/2}\phi_i(\tau_2)\phi_i^\dagger(\tau_1)e^{-\tilde{\beta}_R H/2}\mathcal{O}e^{-\tilde{\beta}_L H}\right)$$



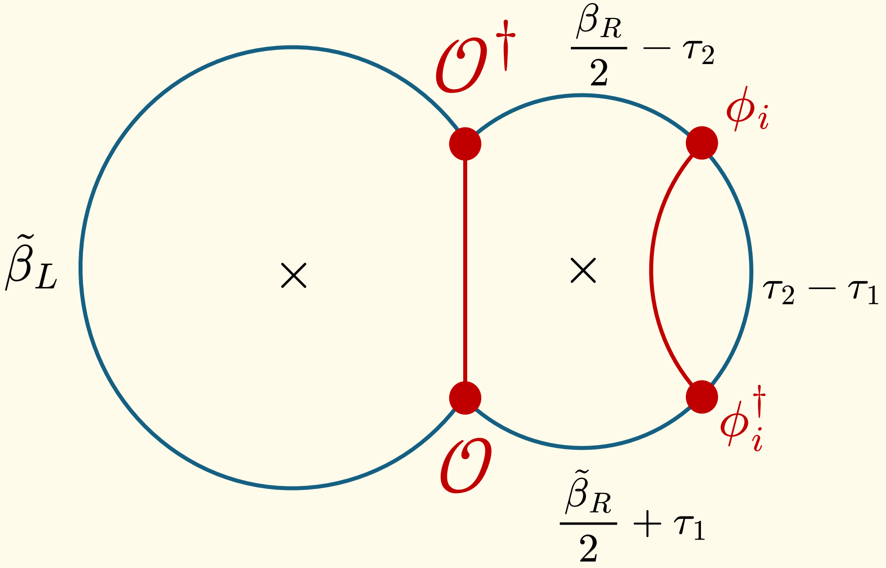


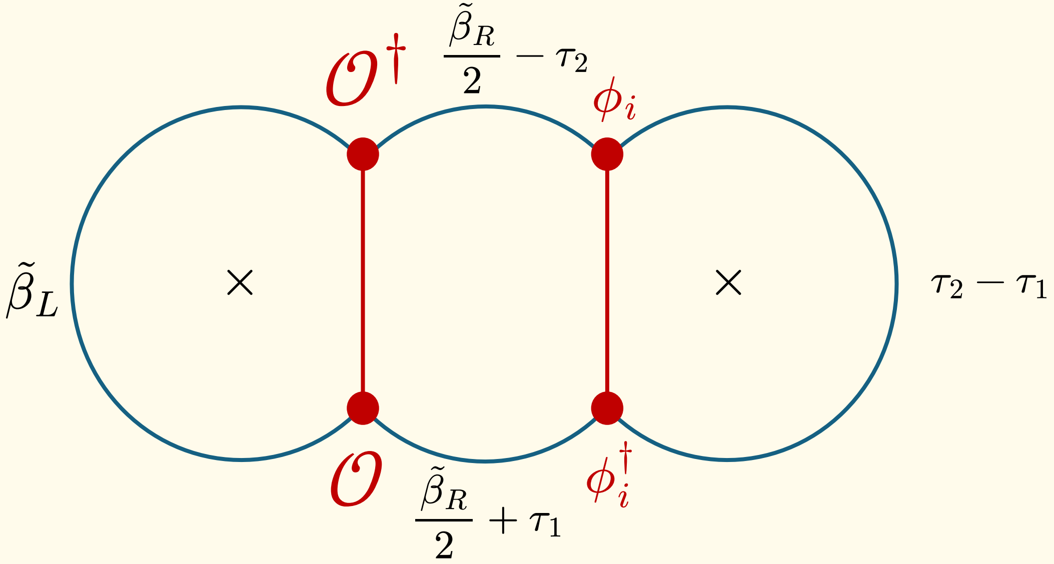
Saddle point equations

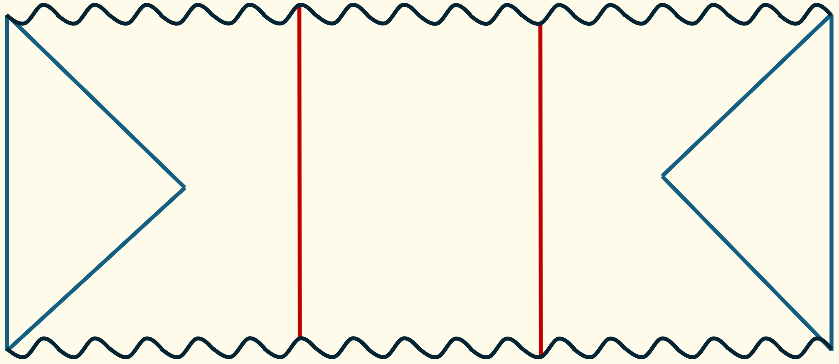
$$\hat{\beta}_L = \tilde{\beta}_L + \Delta\tau(\hat{\beta}_L;\hat{\beta}_C)\,,$$
$$\hat{\beta}_C = \tilde{\beta}_R - (\tau_2 - \tau_1) + \Delta\tau(\hat{\beta}_C;\hat{\beta}_L) + \Delta\tau(\hat{\beta}_C;\hat{\beta}_R)\,,$$
$$\hat{\beta}_R = \tau_2 - \tau_1 + \Delta\tau(\hat{\beta}_R;\hat{\beta}_C)\,,$$

The propagation saddle

$$\sum_i w_i^2 \mathrm{Tr}\left(\mathcal{O}^\dagger e^{-\tilde{\beta}_R H/2}\phi_i(\tau_2)\phi_i^\dagger(\tau_1)e^{-\tilde{\beta}_R H/2}\mathcal{O}e^{-\tilde{\beta}_L H}\right)$$



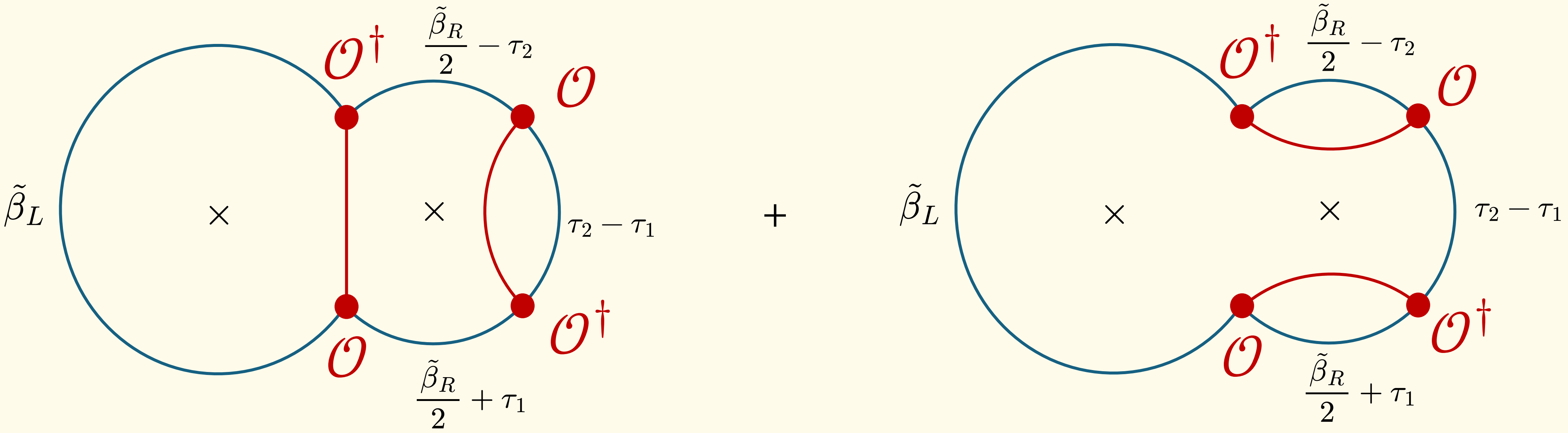




Propagation saddle

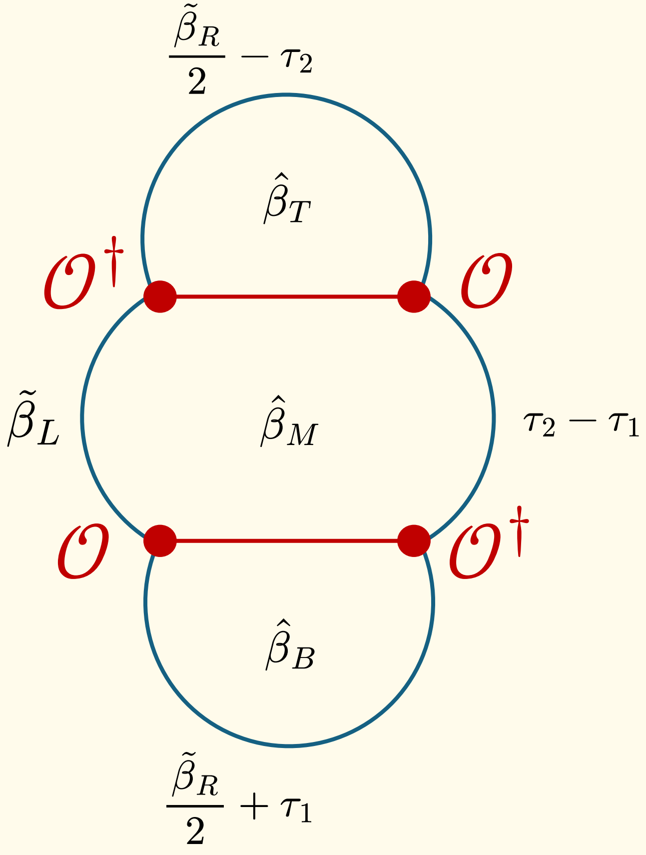
The annihilation saddle

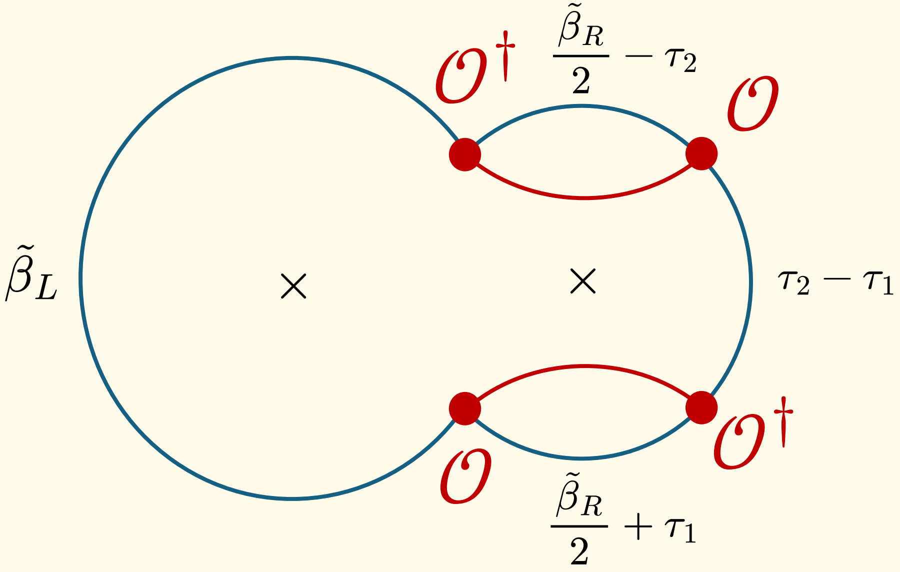
$$w_{\mathcal{O}}^2 \mathrm{Tr}\left(\mathcal{O}^\dagger e^{-\tilde{\beta}_R H/2}\mathcal{O}(\tau_2)\mathcal{O}^\dagger(\tau_1)e^{-\tilde{\beta}_R H/2}\mathcal{O}e^{-\tilde{\beta}_L H}\right)$$



The annihilation saddle

$$w_{\mathcal{O}}^2 \mathrm{Tr}\left(\mathcal{O}^\dagger e^{-\tilde{\beta}_R H/2}\mathcal{O}(\tau_2)\mathcal{O}^\dagger(\tau_1)e^{-\tilde{\beta}_R H/2}\mathcal{O}e^{-\tilde{\beta}_L H}\right)$$





Saddle point equations

$$\begin{aligned}
\hat{\beta}_T &= \tilde{\beta}_R/2 - \tau_2 + \Delta\tau(\hat{\beta}_T;\hat{\beta}_M)\,,\\
\hat{\beta}_M &= \tilde{\beta}_L + \tau_2 - \tau_1 + \Delta\tau(\hat{\beta}_M;\hat{\beta}_T) + \Delta\tau(\hat{\beta}_M;\hat{\beta}_B)\,,\\
\hat{\beta}_B &= \tilde{\beta}_R/2 + \tau_1 + \Delta\tau(\hat{\beta}_B;\hat{\beta}_M)\,.
\end{aligned}$$

Detecting microstates

The physical temperatures of each geometry determine each free energy that contribute to the actions:

$$I_P = \tilde{\beta}_L F(\hat{\beta}_L) + \tilde{\beta}_C F(\hat{\beta}_C) + \tilde{\beta}_R F(\hat{\beta}_R) + I_s(\hat{\beta}_L, \hat{\beta}_C) + I_s(\hat{\beta}_C, \hat{\beta}_R)$$
$$I_A = \tilde{\beta}_T F(\hat{\beta}_T) + \tilde{\beta}_M F(\hat{\beta}_M) + \tilde{\beta}_B F(\hat{\beta}_B) + I_s(\hat{\beta}_T, \hat{\beta}_M) + I_s(\hat{\beta}_M, \hat{\beta}_B)$$

Detecting the interior shell microstate requires that the annihilation saddle dominates over the propagation saddle. This can be achieved by tuning the preparation times τ_1 and τ_2.

The large mass limit

In this limit, the shell action no longer depends on the inverse temperatures of the Euclidean geometries

$$I_s(\beta_R, \beta_L) \approx I_s(m) = 2m \log R_*, \qquad R_* \sim Gm, \qquad \Delta\tau(\beta_R; \beta_L) \approx 0$$

The geometries in this limit become disconnected:

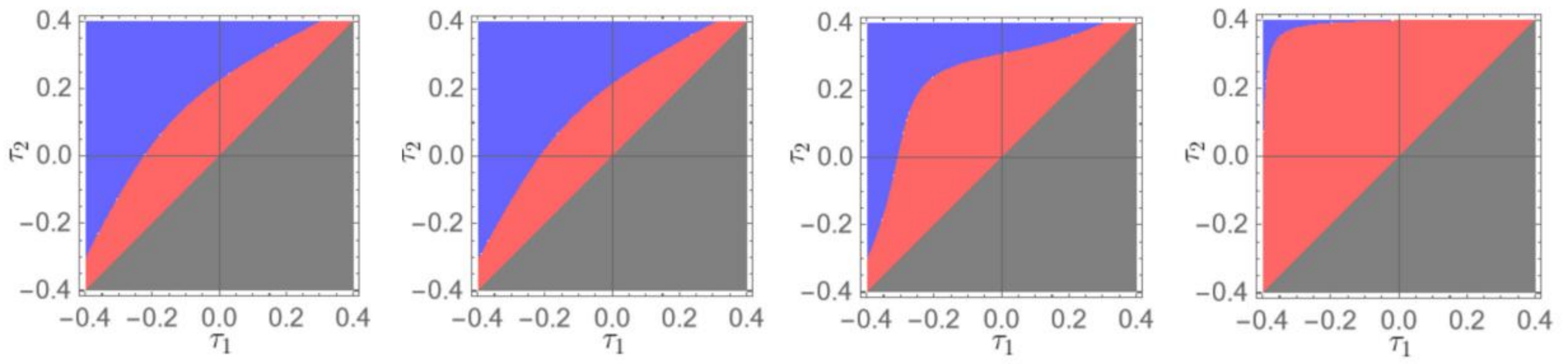
$$Z_1 \approx Z(\tilde{\beta}_L) Z(\tilde{\beta}_R) e^{-I_s(m)}, \qquad \mathcal{T}_1^P \approx Z(\hat{\beta}_L) Z(\hat{\beta}_C) Z(\hat{\beta}_R) e^{-2I_s(m)}, \qquad \mathcal{T}_1^A \approx Z(\hat{\beta}_T) Z(\hat{\beta}_M) Z(\hat{\beta}_B) e^{-2I_s(m)}$$

In d+1 dimensions we can find the explicit answers for the numerators of the two-point functions.

$$\begin{aligned}
\log \mathcal{T}_1^P \approx \frac{V_\Omega}{8G} \Bigg[& \left(\frac{2\pi}{\tilde{\beta}_L}\right)^{d-1} - \left(\frac{2\pi}{\tilde{\beta}_L}\right)^{d-3} + \left(\frac{2\pi}{\tilde{\beta}_R + \tau_1 - \tau_2}\right)^{d-1} - \left(\frac{2\pi}{\tilde{\beta}_R + \tau_1 - \tau_2}\right)^{d-3} \\
& + \left(\frac{2\pi}{\tau_2 - \tau_1}\right)^{d-1} - \left(\frac{2\pi}{\tau_2 - \tau_1}\right)^{d-3} + 2\pi c_d (\tilde{\beta}_L + \tilde{\beta}_R) \Bigg] - 2I_s(m)\,, \\
\log \mathcal{T}_1^A \approx \frac{V_\Omega}{8G} \Bigg[& \left(\frac{2\pi}{\frac{\tilde{\beta}_R}{2} - \tau_2}\right)^{d-1} - \left(\frac{2\pi}{\frac{\tilde{\beta}_R}{2} - \tau_2}\right)^{d-3} + \left(\frac{2\pi}{\tilde{\beta}_L + \tau_2 - \tau_1}\right)^{d-1} - \left(\frac{2\pi}{\tilde{\beta}_L + \tau_2 - \tau_1}\right)^{d-3} \\
& + \left(\frac{2\pi}{\frac{\tilde{\beta}_R}{2} + \tau_1}\right)^{d-1} - \left(\frac{2\pi}{\frac{\tilde{\beta}_R}{2} + \tau_1}\right)^{d-3} + 2\pi c_d (\tilde{\beta}_L + \tilde{\beta}_R) \Bigg] - 2I_s(m)\,,
\end{aligned}$$

The large mass limit

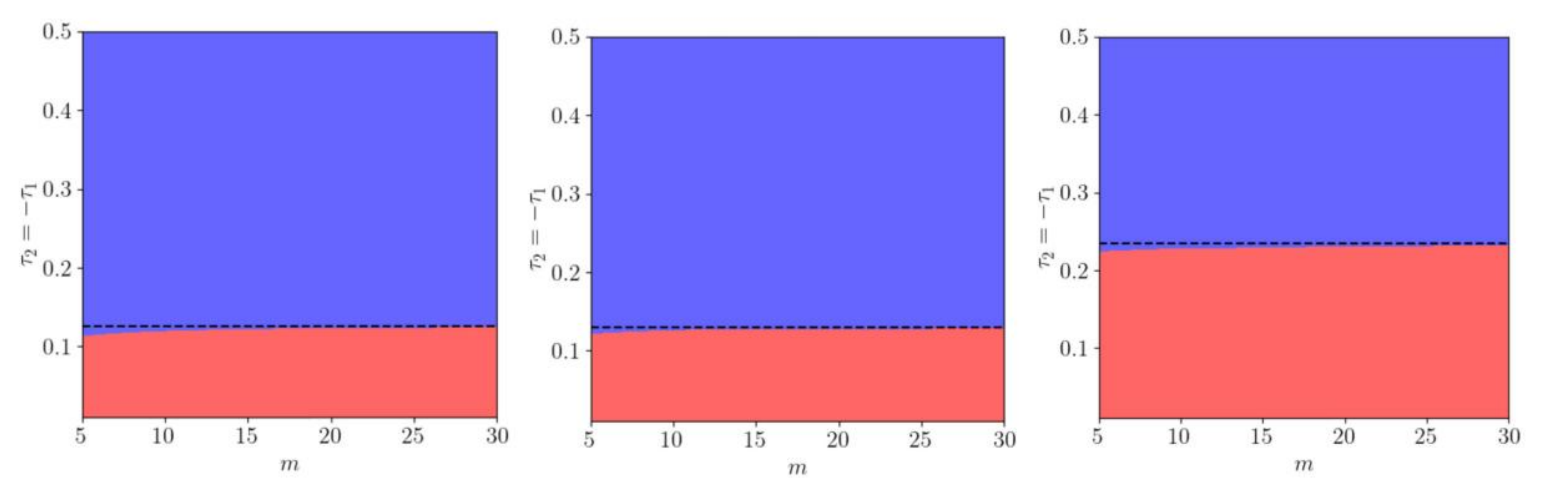
Having seen that we can tune the Euclidean insertion times τ_1, τ_2 such that the annihilation saddle is dominant, we check in 2+1 dimensions numerically if this can always be done



(a) $\tilde{\beta}_{\mathrm{L}} = 10$ (b) $\tilde{\beta}_{\mathrm{L}} = 1$ (c) $\tilde{\beta}_{\mathrm{L}} = 0.2$ (d) $\tilde{\beta}_{\mathrm{L}} = 0.1$

Heavy but finite mass in (2+1) dimensions

We also check when the mass is finite but sufficiently heavy in 2+1 dimensions. We see that as the mass increases, the switchover value asymptotes to that of the large mass limit, shown by the dotted line.



(a) $\tilde{\beta}_{\mathrm{R}} = 1,\ \tilde{\beta}_{\mathrm{L}} = 5$ (b) $\tilde{\beta}_{\mathrm{R}} = 1,\ \tilde{\beta}_{\mathrm{L}} = 1$ (c) $\tilde{\beta}_{\mathrm{R}} = 1,\ \tilde{\beta}_{\mathrm{L}} = 0.2$

Conclusions

We found a new gravitational saddle – the annihilation saddle – that signals the presence of a particular black hole microstate, defined by the shell operator.

This saddle can be made to dominate over the other saddles by tuning the insertion times of a suitable probe operator.

We showed this is always possible for the large mass limit in generic dimensions, confirming this numerically in (2+1)d, in addition to the heavy but finite mass case in (2+1)d.

Not shown here:

1. We perform the computation from the boundary CFT perspective using the Eigenstate Thermalization Hypothesis (ETH) and our result precisely matches the gravitational result.
2. We check the contributions from the sub-leading saddles by computing the variance of the Euclidean two-point function and show these do not dominate over the annihilation saddle, as long as the probe used is appropriately constructed.

Outlook and future work

1. We are checking that this detection holds in other black hole geometries – ongoing work with R. Isichei for Reissner-Nordstrom black holes and the near-extremal limit.

2. We would like to understand if the two-point function can be related to the Hawking radiation – specifically, we speculate that this may be related to the grey-body factor.

3. An open question is how one might detect the microstate in this way if the black hole spacetime is, e.g., in an equal superposition of two shell states.

Thank you for your attention