Model-independent tests of gravity with the Weyl potential evolution

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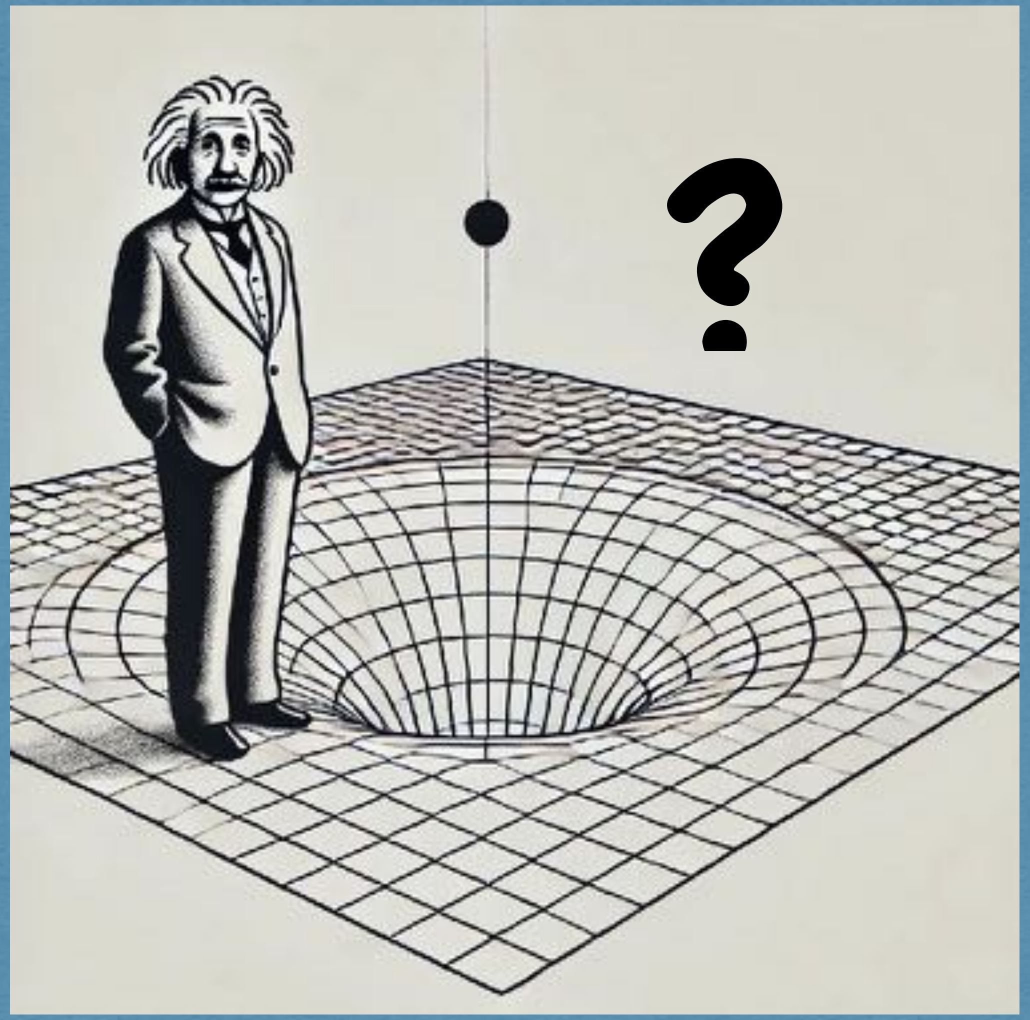


Q1: What is the Weyl potential evolution?

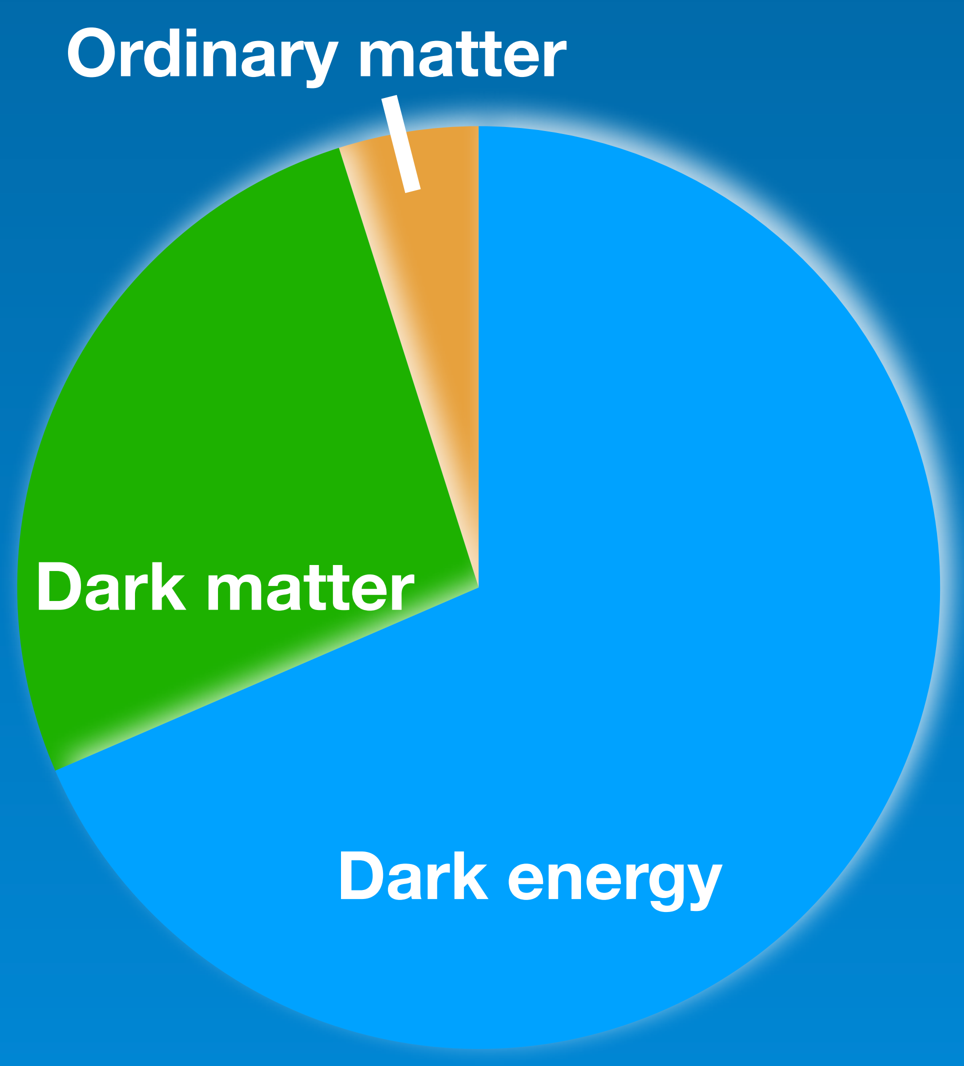
Q2: How can we use it in combination with redshift-space distortions?

Q3: How is this observable affected by evolving dark energy?

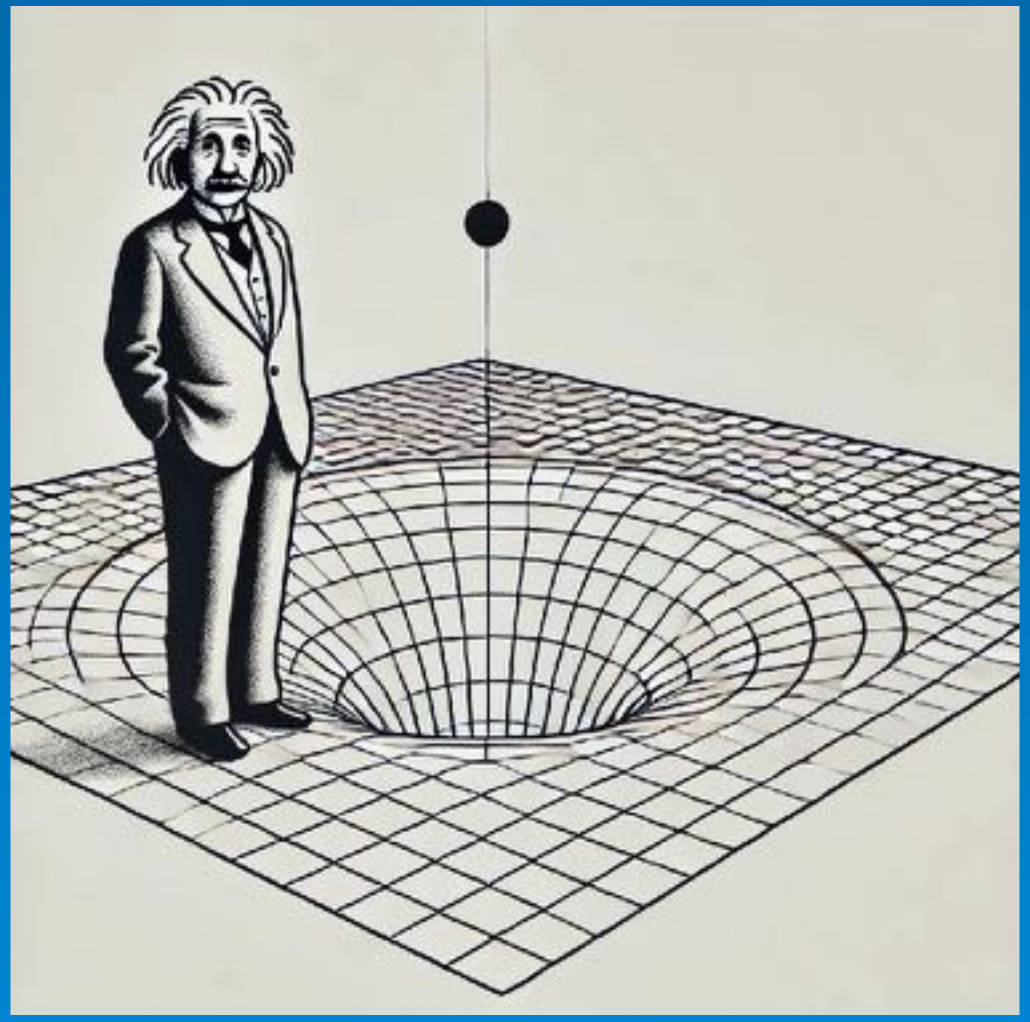
Why should we test gravity?

The ΛCDM model of our Universe:

Energy content of the Universe



Theory of gravity: General Relativity



Many alternative models are possible:

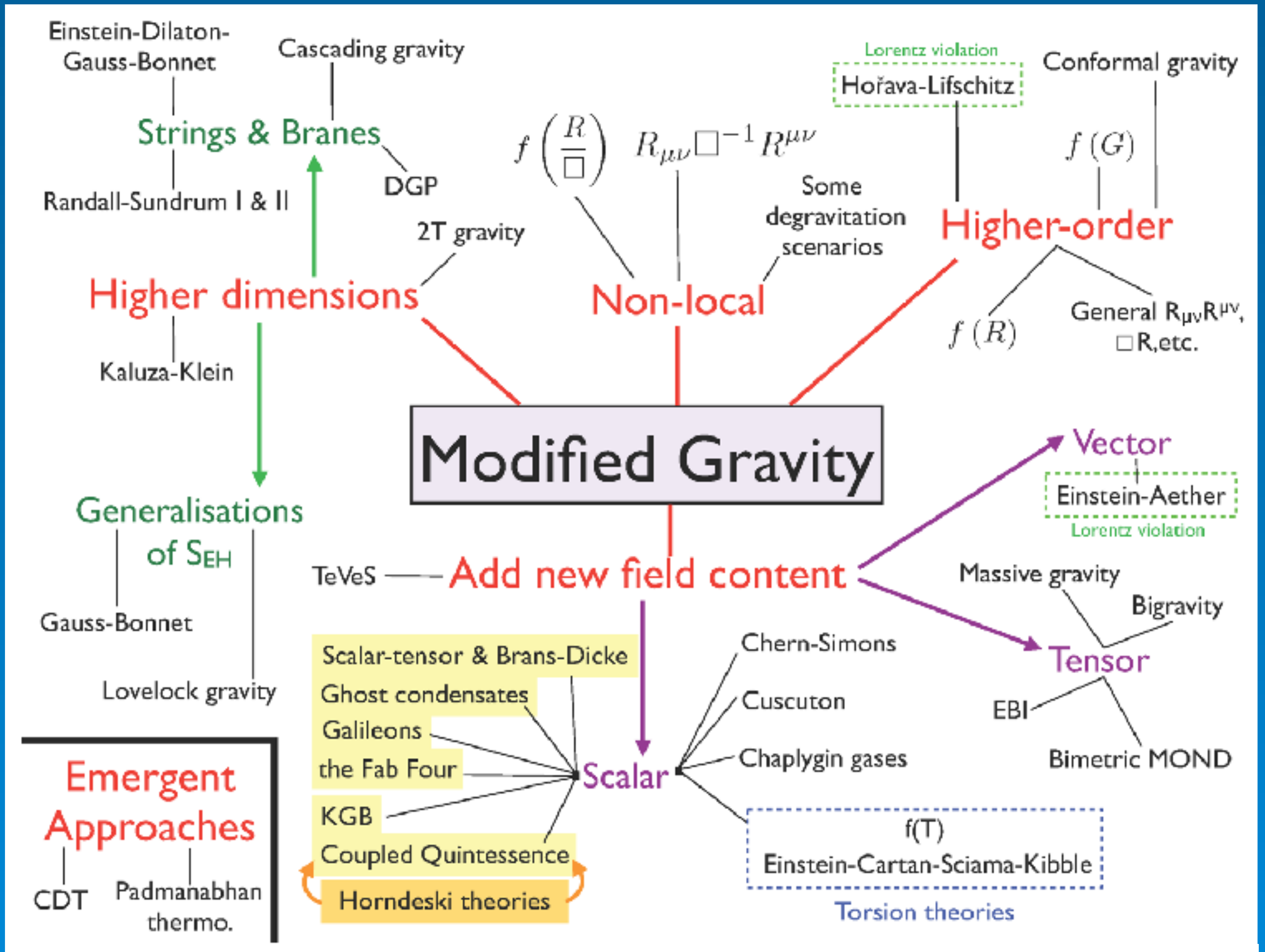


Image credit: Tessa Baker

Problems:

- Dark energy is not well understood.
- Cosmological tensions exist in various data sets.

How can we test modified gravity?
Possibility 1

Pick a modified gravity model → Data analysis with additional parameters → Obtain constraints for one model

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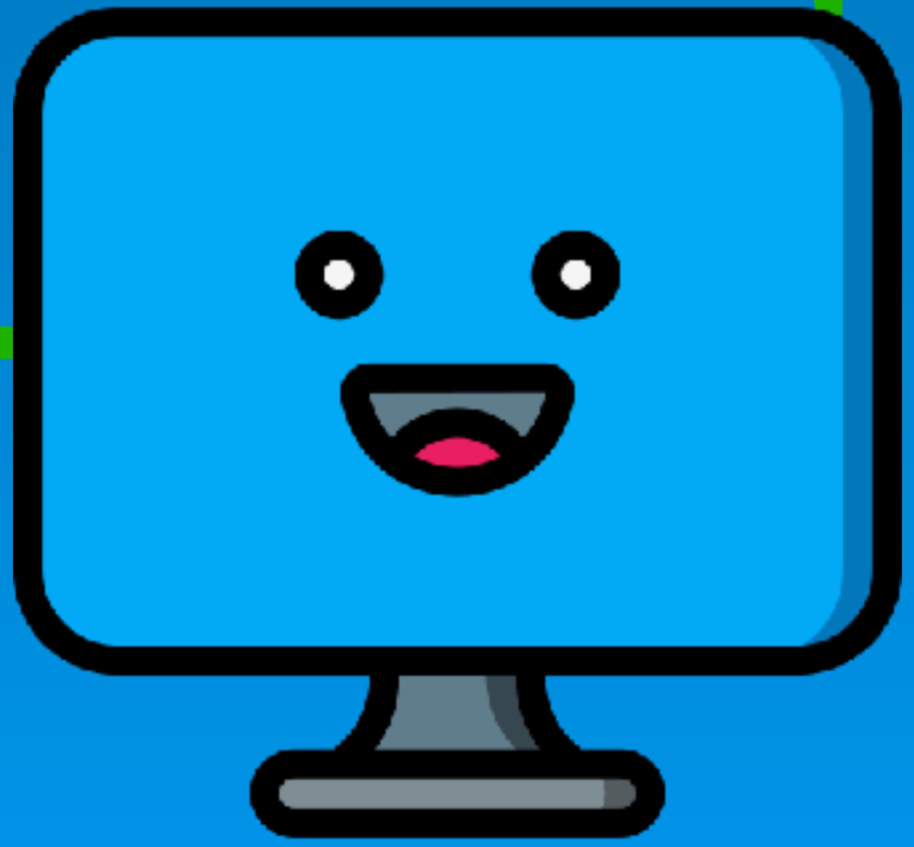

Model-independent observable for gravitational lensing?

Lensing is directly sensitive to the perturbed geometry of the Universe:

$$\mathrm{d}s^2 = -\,a^2(\tau)(1 + 2\Psi)\,\mathrm{d}\tau^2 + a^2(\tau)(1 - 2\Phi)\mathrm{d}\mathbf{x}^2$$

Lensing $\propto$ $\Psi_W = (\Phi + \Psi)/2$ $\longrightarrow$ Weyl potential

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In General Relativity: $\Psi_W \propto D_1(z)\,\Omega_m(z)$

Growth of matter perturbations

Matter content in the Universe

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I. Tutusaus, D. Sobral-Blanco & C. Bonvin (2022), arXiv:2209.08987

Galaxy-galaxy lensing angular power spectrum

Galaxy-galaxy lensing: The correlation of galaxy positions in the foreground with galaxy shapes in the background.

$$C_\ell^{\Delta\kappa}(z_i, z_j) = \frac{3}{2}\int \mathrm{d}z\, n_i(z)\mathscr{H}^2(z)\boxed{\hat{b}_i(z)\hat{J}(z)}B\left(k_\ell, \chi\right)\frac{P_{\delta\delta}^{\mathrm{lin}}\left(k_\ell, z_*\right)}{\sigma_8^2(z_*)}\int \mathrm{d}z' n_j(z')\frac{\chi'(z') - \chi(z)}{\chi(z)\chi'(z')}$$

lens bin source bin

$$\hat{J}(z) \equiv \frac{J(z)\sigma_8(z)}{D_1(z)} \quad \text{(Weyl evolution)}\,, \qquad \hat{b}_i(z) \equiv b_i(z)\sigma_8(z)\,.$$

Galaxy clustering: Depends on $\hat{b}_i(z)\hat{b}_j(z)$

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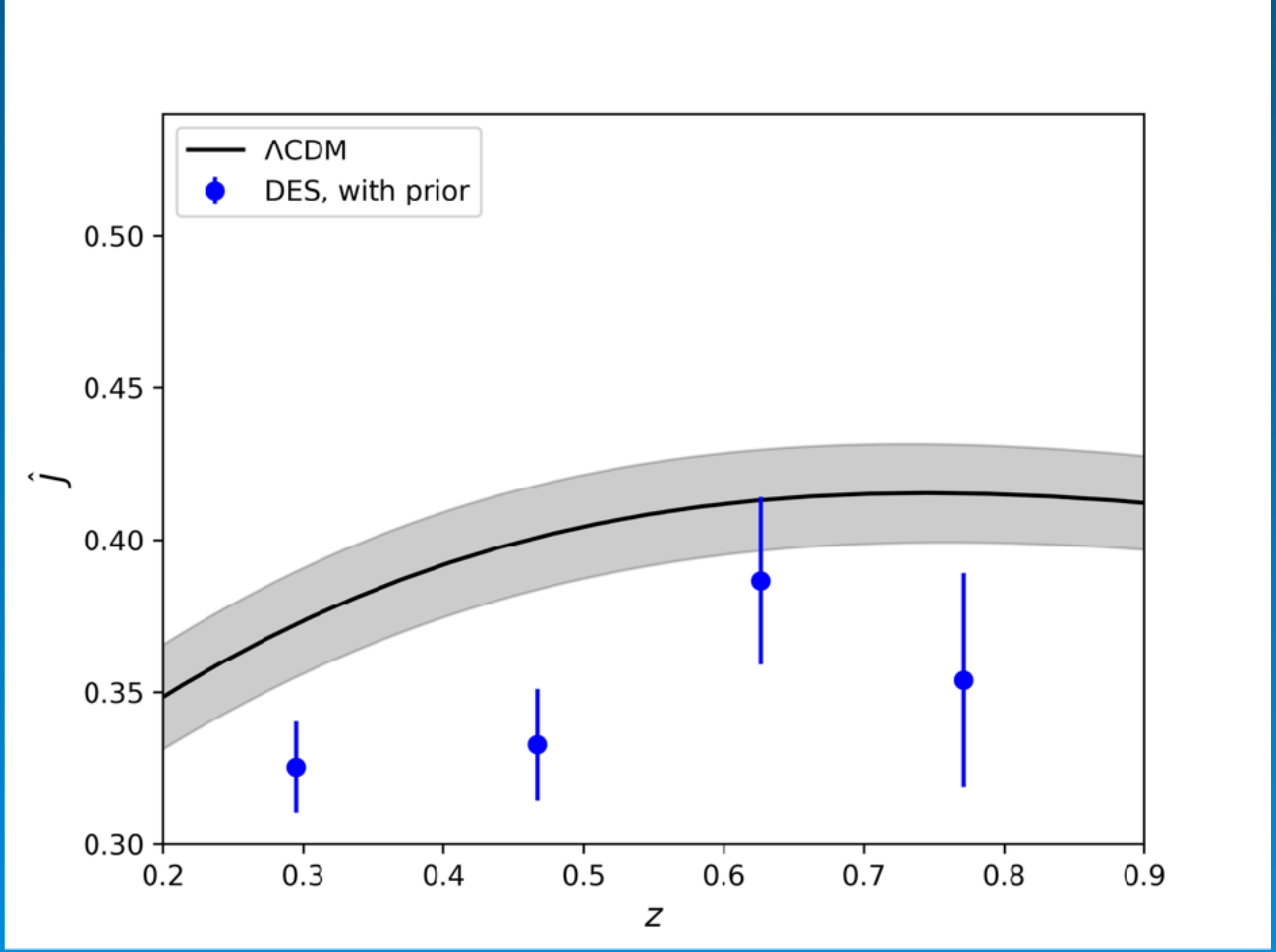
Combining galaxy-galaxy lensing and galaxy clustering → Measurement of $\hat{J}(z)$

Measurement of $\hat{J}(z)$ from Dark Energy Survey data



I. Tutusaus, C. Bonvin & NG, arXiv:2312.06434, *Nature Commun. 15 (2024)* 9295

Measurement in 4 bins of the MagLim sample, 3σ Planck priors

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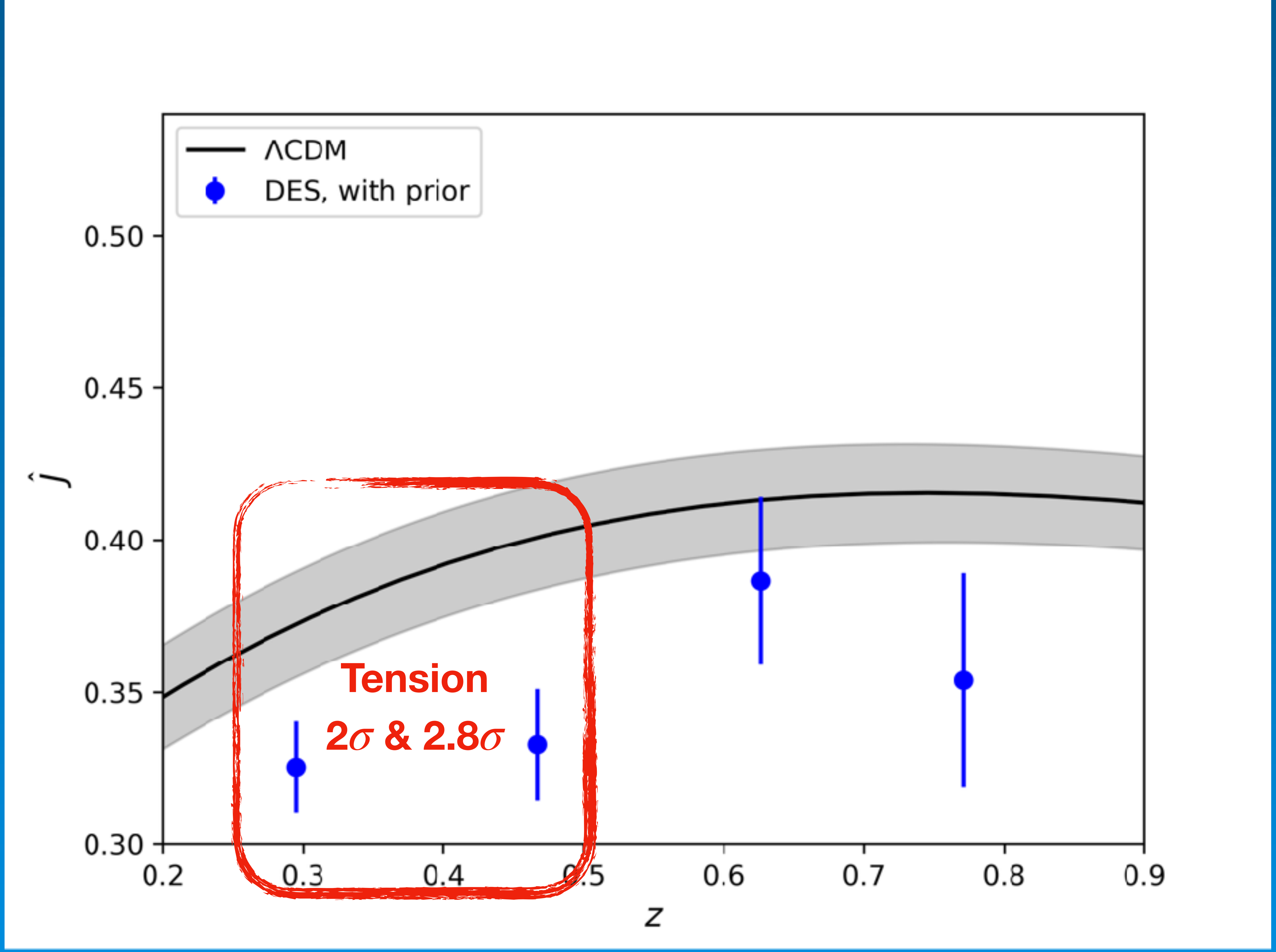

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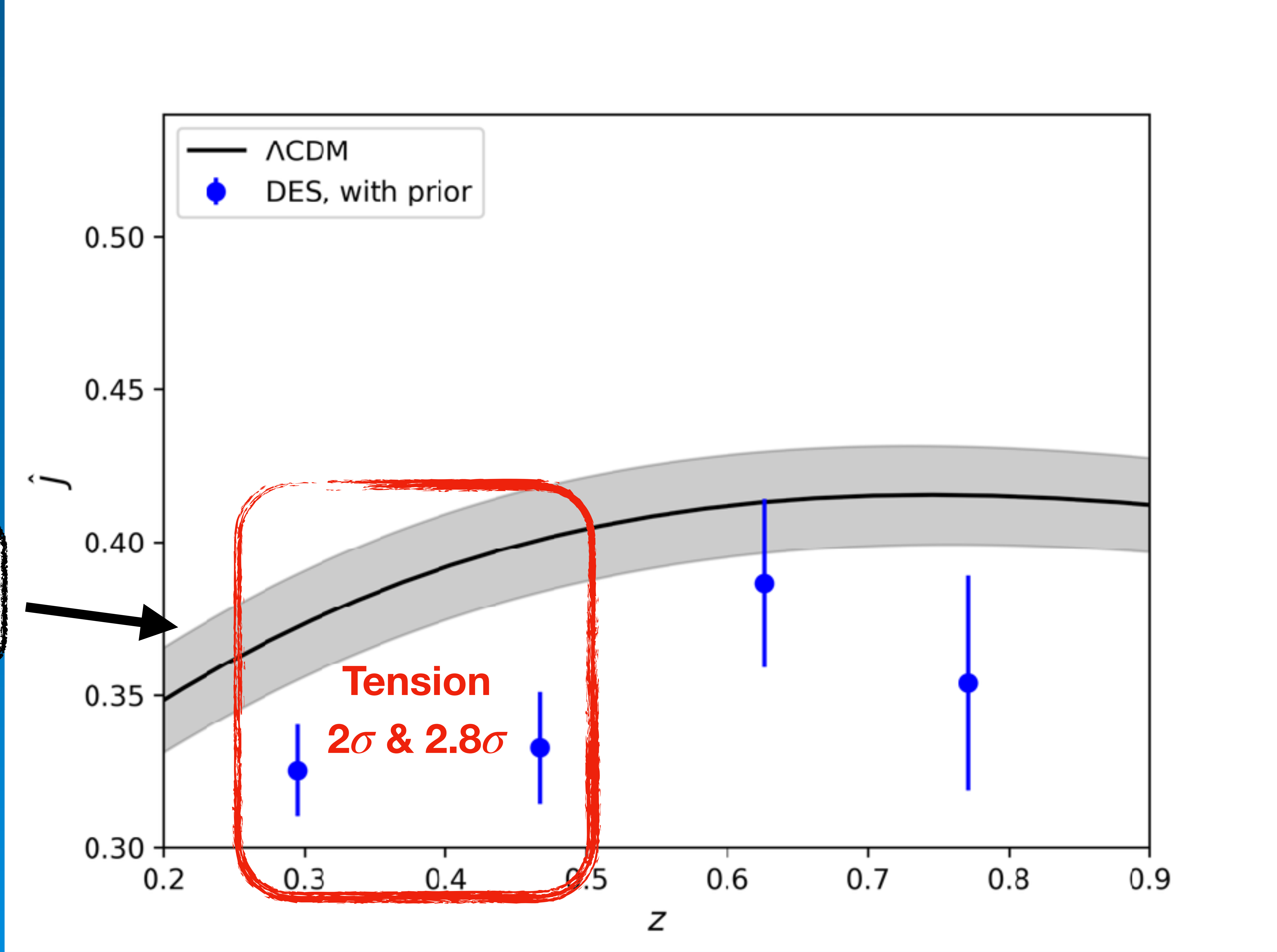

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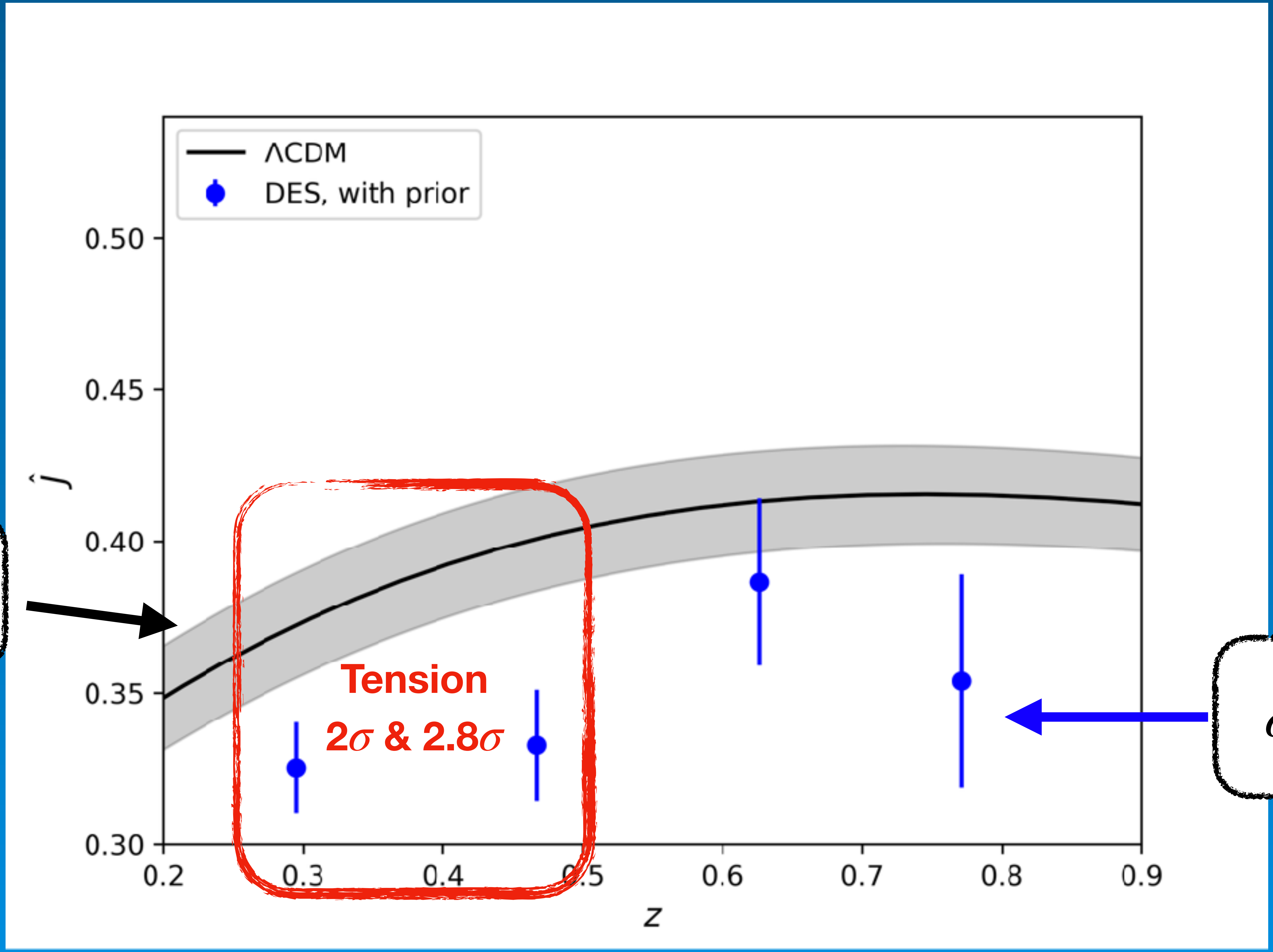

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$\sigma_8(z=0) = 0.74 \pm 0.04$

Measurement in 4 bins of the MagLim sample, 3σ Planck priors

$\hat{J}$ combined with other model-independent observables?

Weak gravitational lensing

$\downarrow$

$\hat{J}(z)$

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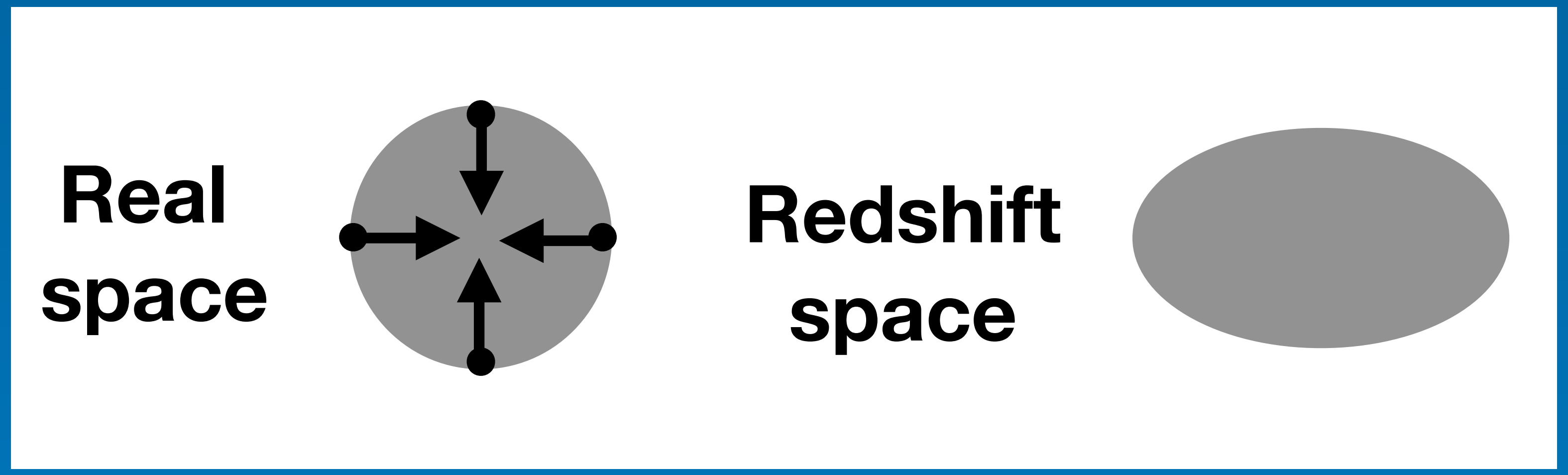
Redshift-space distortions



$\hat{f}(z) = f\sigma_8$

$f\ldots$ growth rate of structure
$\sigma_8\ldots$ clustering amplitude

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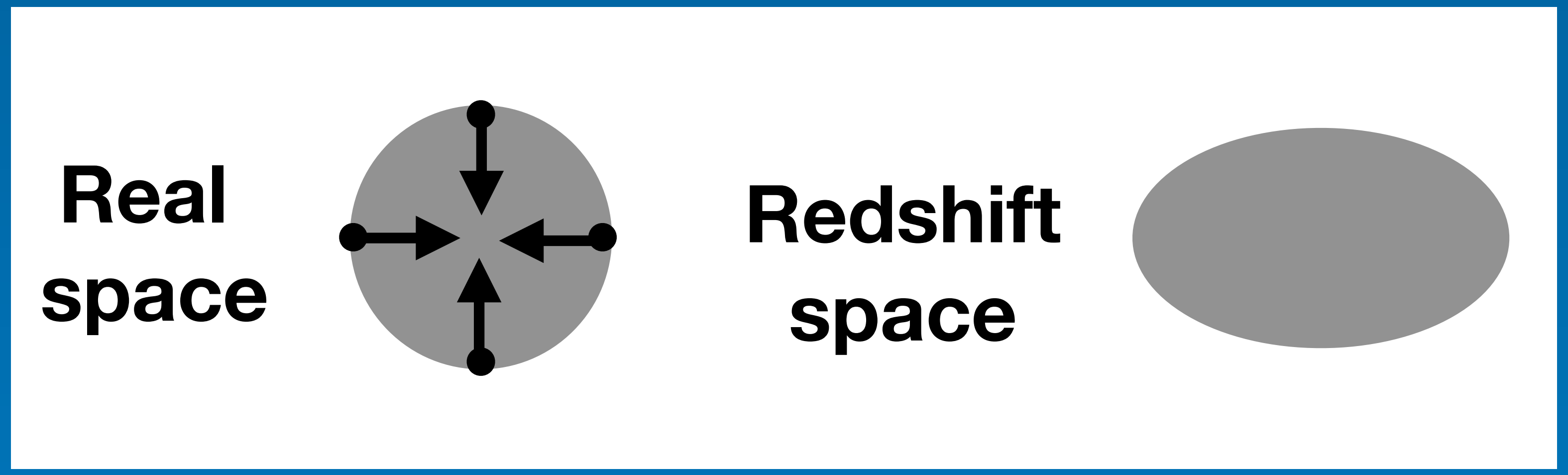

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$\mathbf{E_G}$ statistic
(2403.13709, *PRL 133 (2024) 21*)

Test of the Fifth Force
(2502.12843, *Nature Comm.* 16 (2025) 9399)

Null test: Geometry vs. density growth
(2506.04387, *accepted in A&A*)

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$$\boxed{\mathcal{N}^{\mathrm{grow}}(z) \equiv \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\hat{J}(z)}{\Omega_m(z)}\right) + \frac{\hat{f}(z)}{1+z}}$$

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In GR: $\hat{J}(z) = \Omega_m(z)\sigma_8(z)$. $\rightarrow$ $$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\hat{J}(z)}{\Omega_m(z)}\right) = \frac{\mathrm{d}\sigma_8}{\mathrm{d}z} = -\frac{1}{1+z}\frac{\mathrm{d}\ln D_1(a)}{\mathrm{d}\ln(a)}\sigma_8(z) = -\frac{\hat{f}(z)}{1+z}.$$

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What is this null test sensitive to?

- Effect on the background evolution ($w_0 - w_a$) $\rightarrow$ Does NOT affect $\mathcal{N}^{\text{grow}}$!
- Effect on the growth of structure (parameter μ) $\rightarrow$ Does NOT affect $\mathcal{N}^{\text{grow}}$!
- Effect on the evolution of $\Phi + \Psi$ (parameter Σ) $\rightarrow$ $\mathcal{N}^{\text{grow}} \neq 0$!

Null test: Geometry vs. matter density growth

Problem:

- We need to take derivatives of $\hat{J}(z)$…
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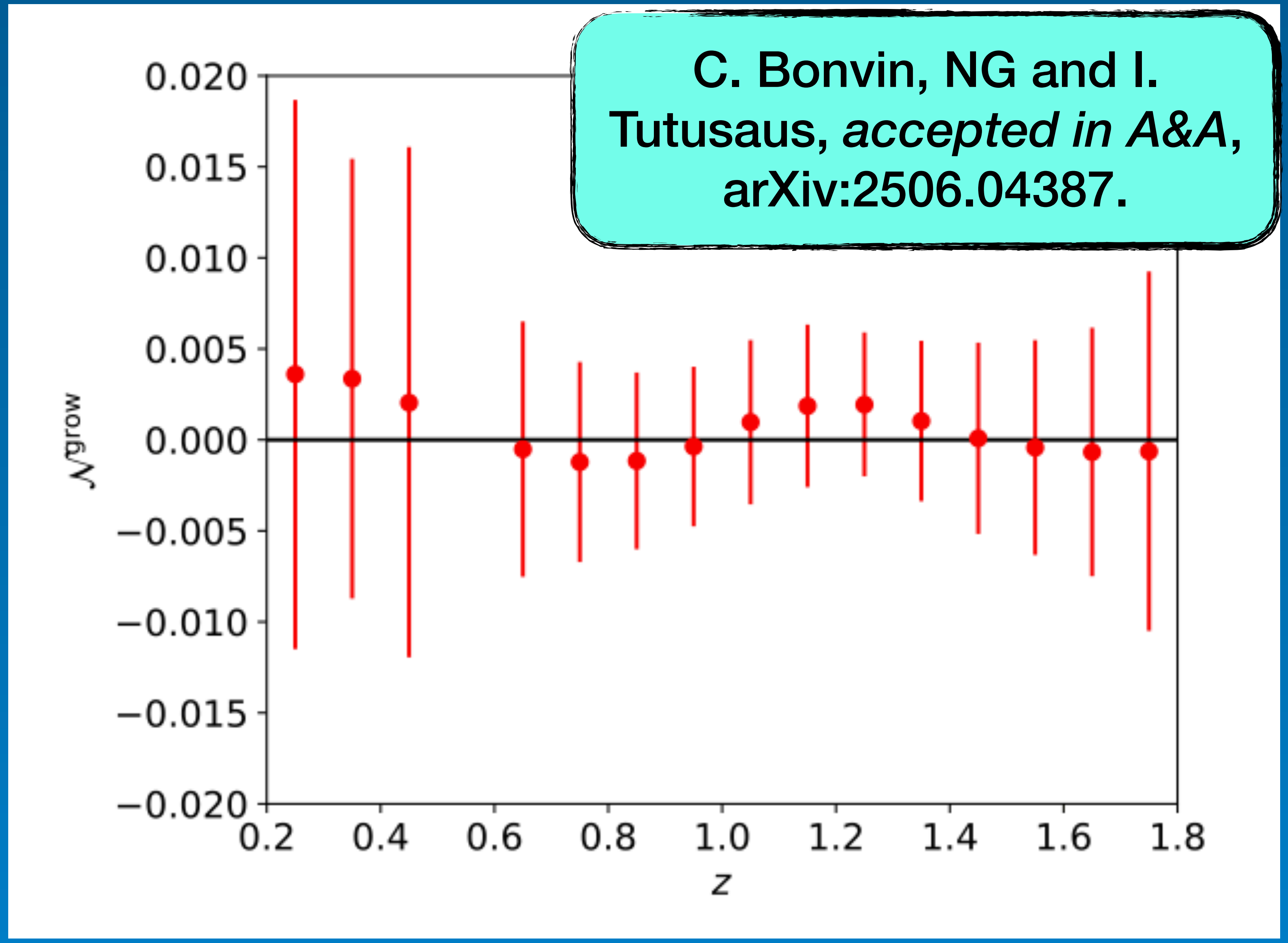

Forecast for $\hat{J}(z)$ from LSST and $\hat{f}(z)$ from DESI (final data release):

- $\mathcal{N}^{\rm grow}$ constrained at the level of 0.005.
- Sensitive to $2-4\,\%$ difference between the evolution of $\Phi+\Psi$ and δ.

One caveat:

In our 2023 paper with DES measurements of $\hat{J}$, we have assumed that the background evolution would be ΛCDM.

Impact of Evolving Dark Energy on the Weyl potential

Observable $\hat{J}(z)$: Model-independent; prediction $\hat{J}(z)_{\mathrm{GR}} = \Omega_m(z)\sigma_8(z)$ depends on specific theory (at background and perturbation level).

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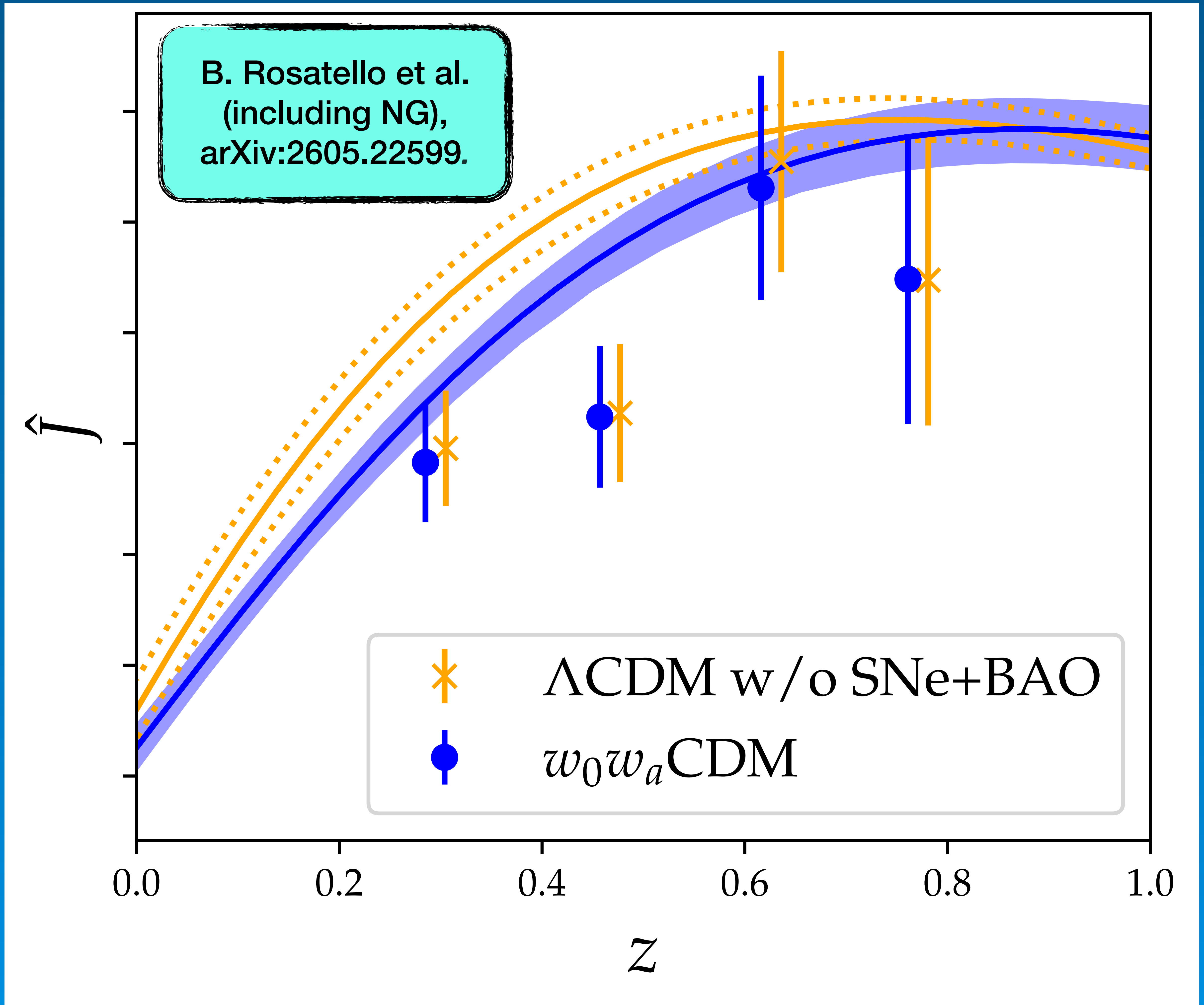
Recent project on impact of evolving DE on the Weyl potential:

Benedetta Rosatello, Gen Ye, Maria Berti, Isaac Tutusaus, NG & Camille Bonvin; arXiv:2605.22599.

- Redo previous DES analysis, but with evolving dark energy (w0-wa).
- Combining with Planck CMB, DESI DR2 BAO, and SNIa data.

Impact of Evolving Dark Energy on the Weyl potential



Different cases considered:

- **Orange:** LCDM, w/o SNe+BAO data (DES Y3 + full CMB likelihood)
- **Blue:** w0waCDM, using full data set

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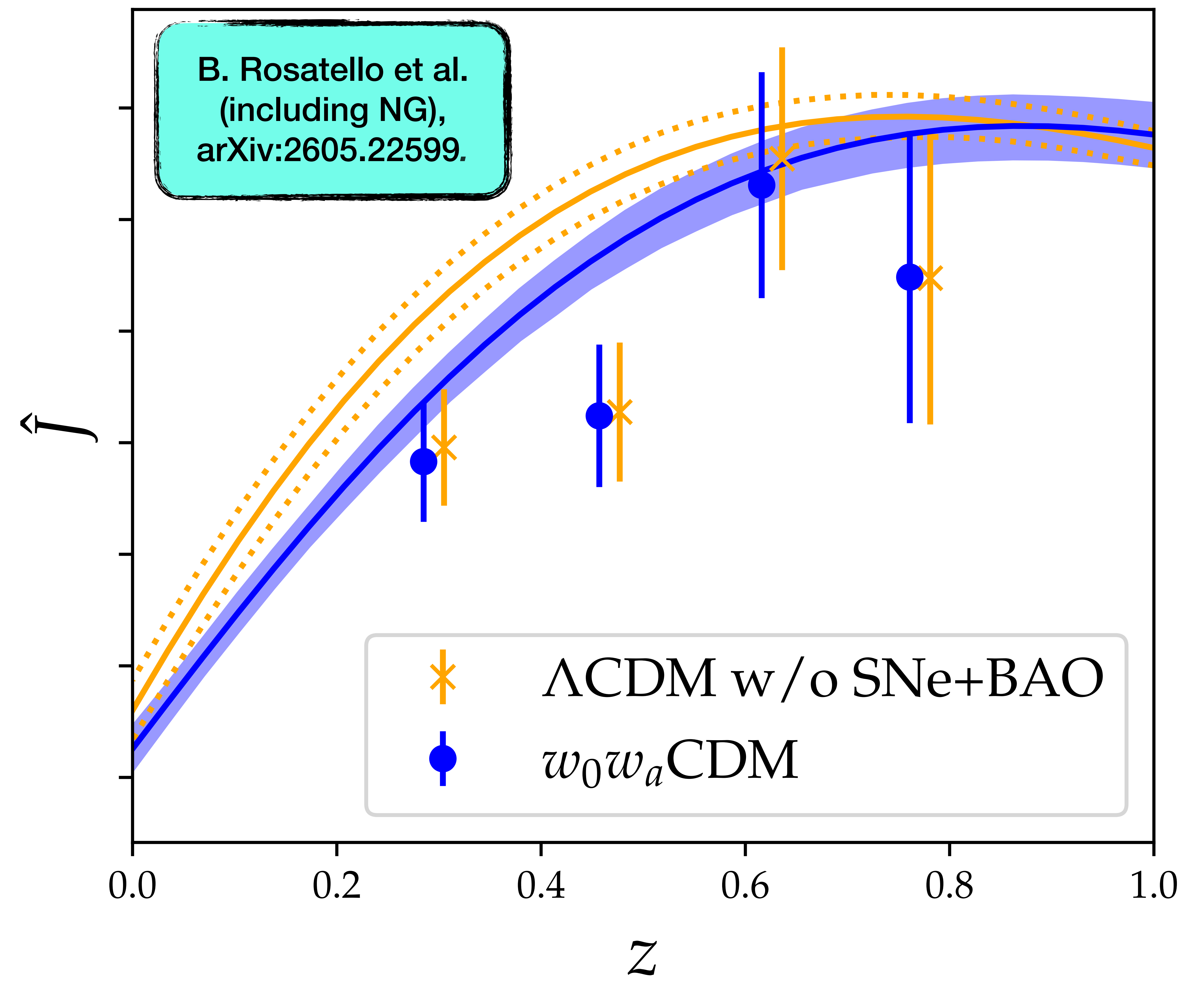

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For both cases, we have:

- 4 measurements of Jhat
- Theory prediction, depends on background model and cosmological parameters

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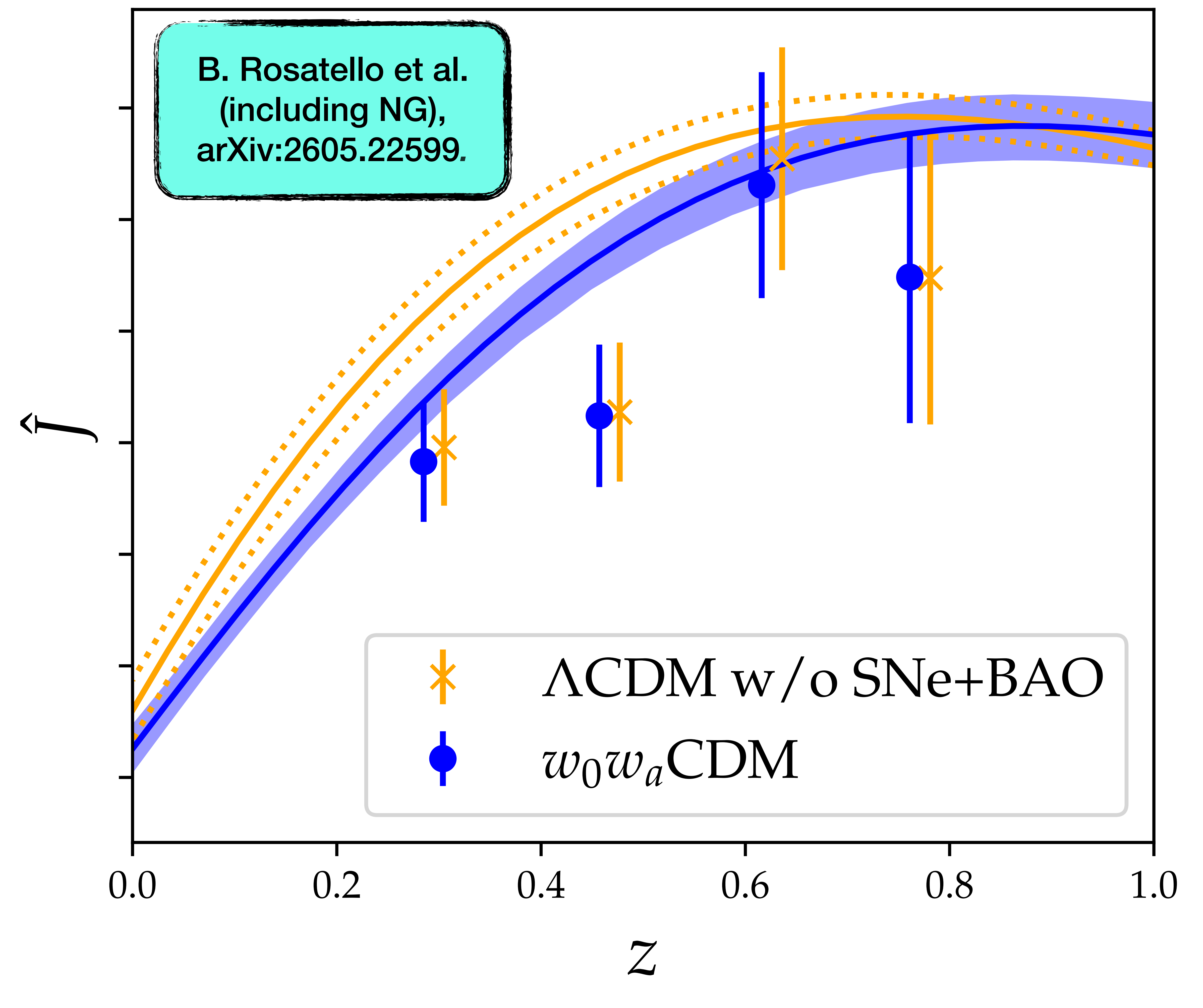

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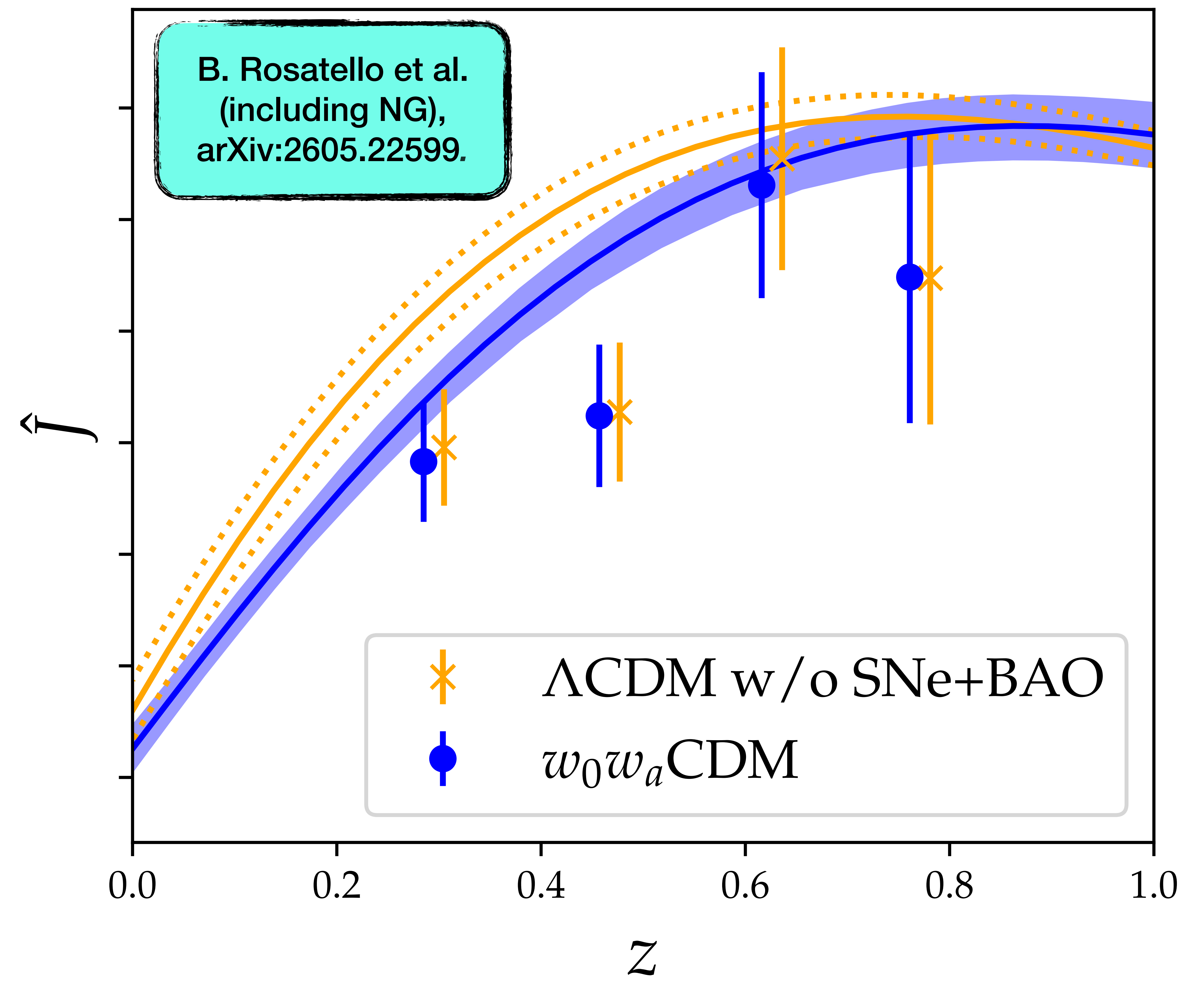
Tension in second bin:

- 2.8σ: old analysis (LCDM, CMB priors)
- 3.1σ: **orange** case (LCDM, full CMB likelihood)
- 2.2σ: **blue** case (w0waCDM, full data set)

Impact of Evolving Dark Energy on the Weyl potential



Conclusion:

- The measured points vary little between the different cases.
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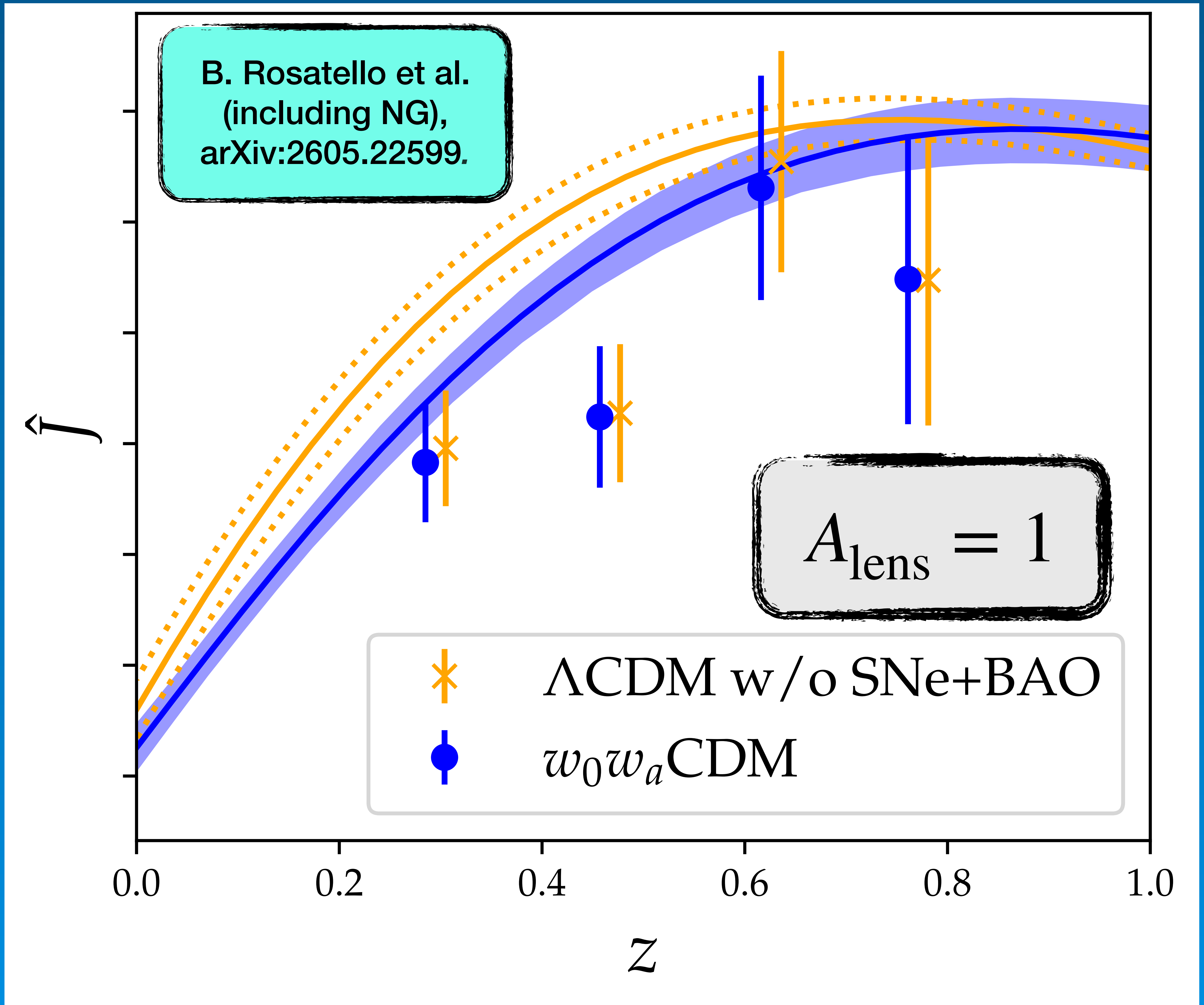

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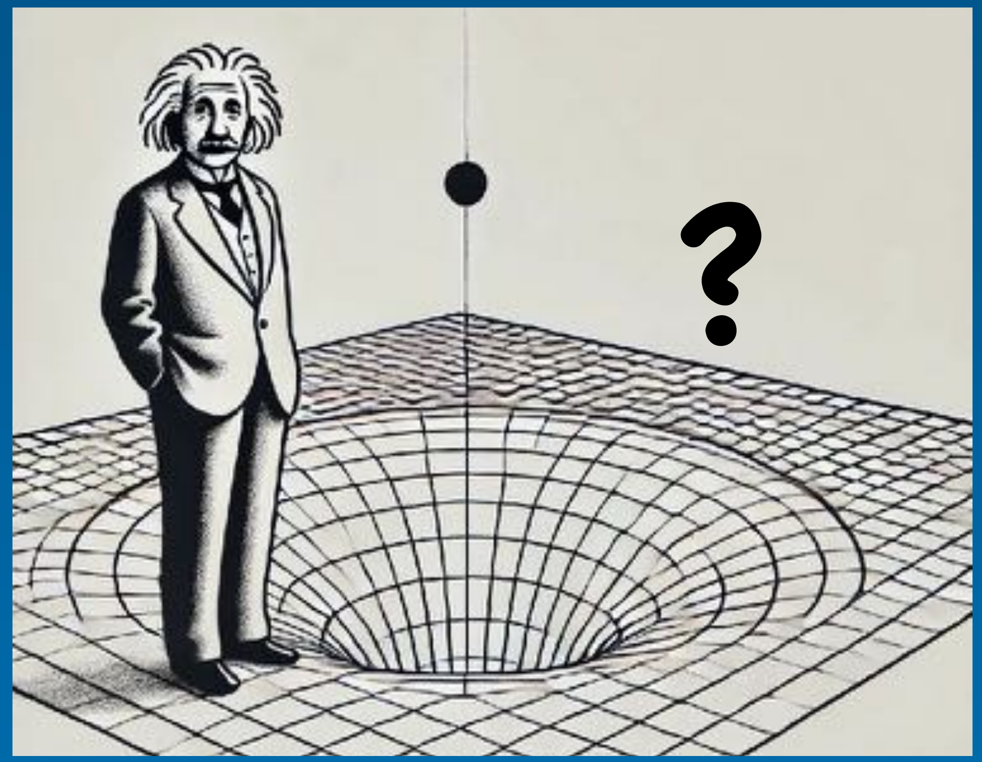
- In this plot: CMB lensing described by GR.
- Varying $A_{\rm lens}$ → degrades constraints.

Conclusions



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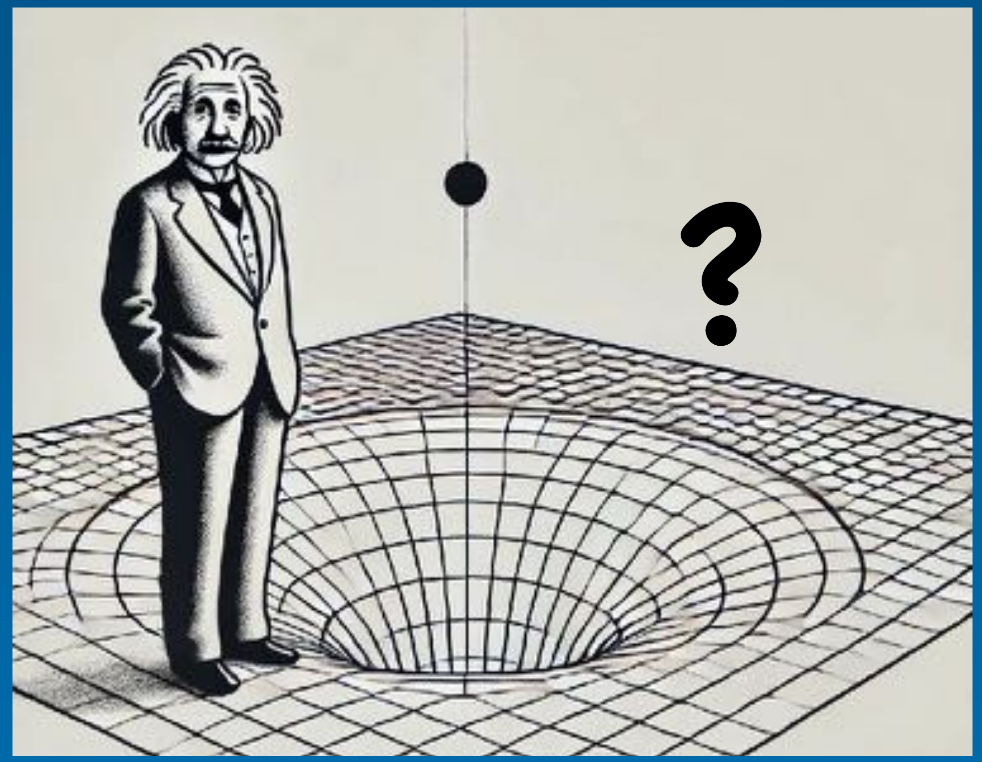
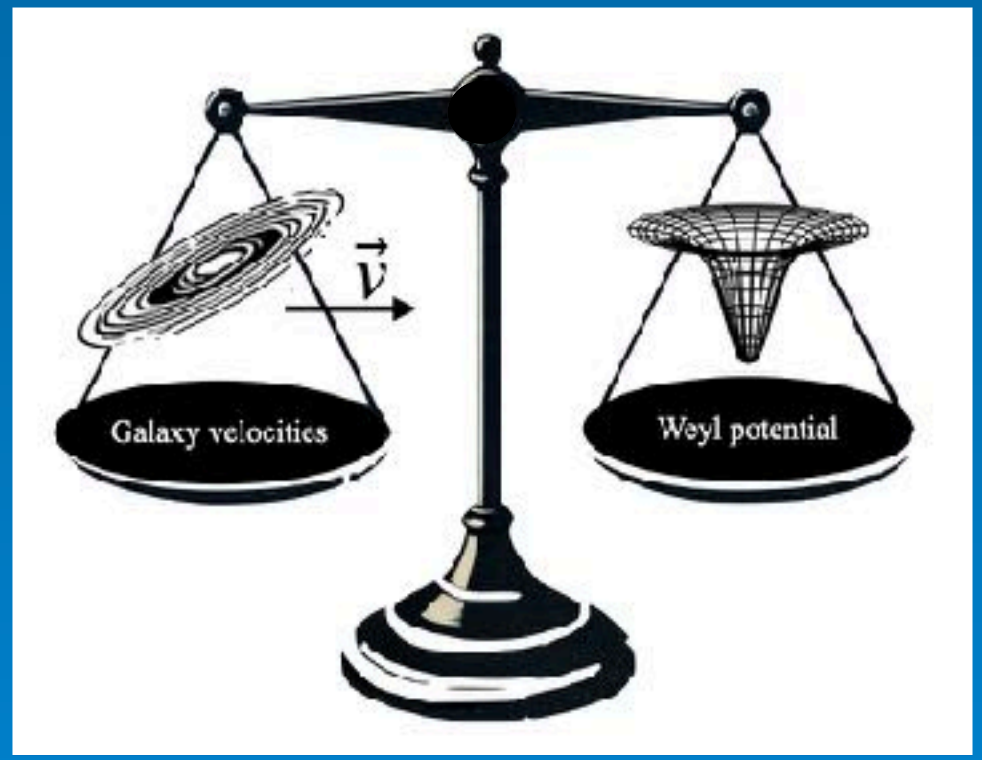


Null test with $\hat{J}$ + RSD:

- Sensitive to changes on light propagation, not sensitive to changes on structure growth.
- Will be a highly interesting test with future data.



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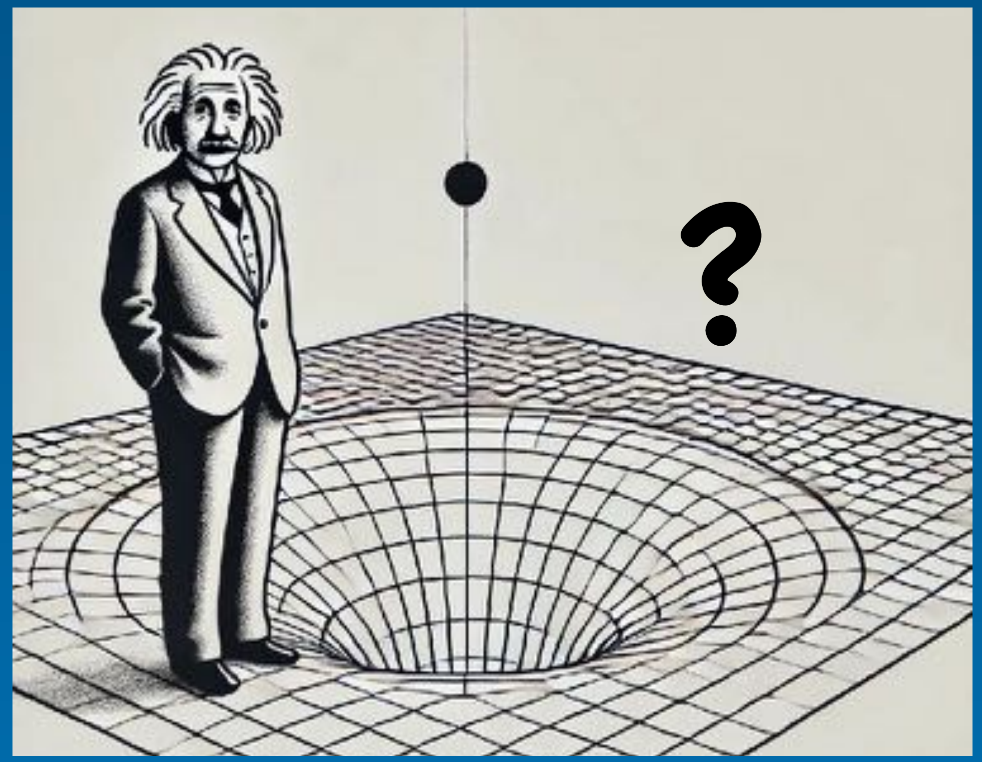
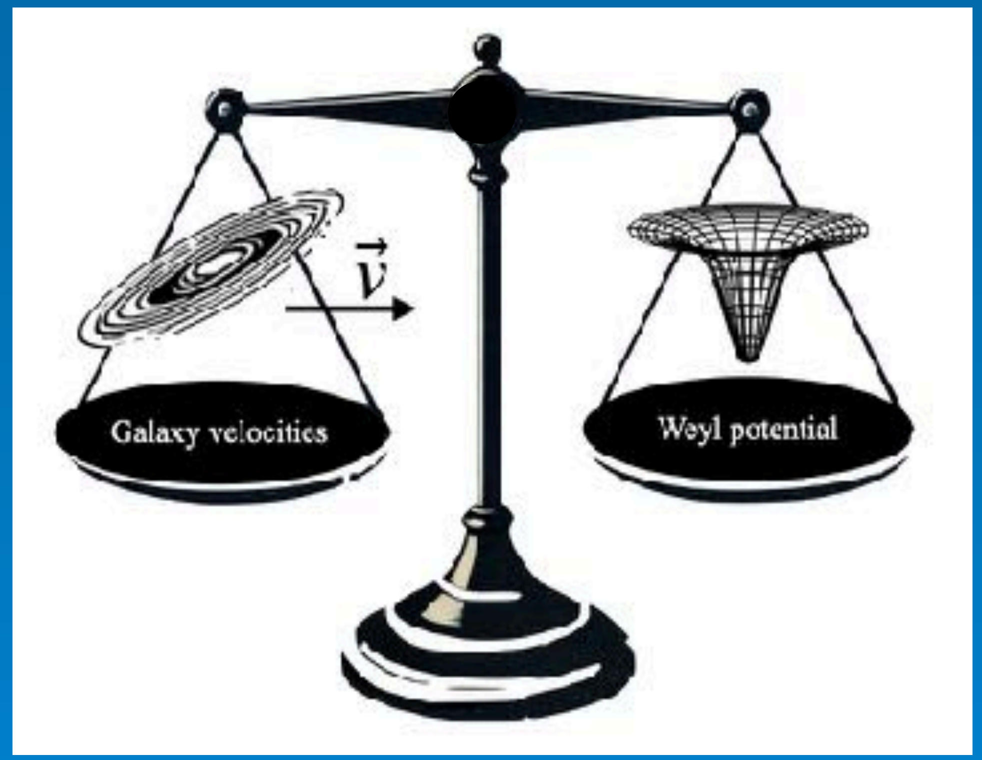
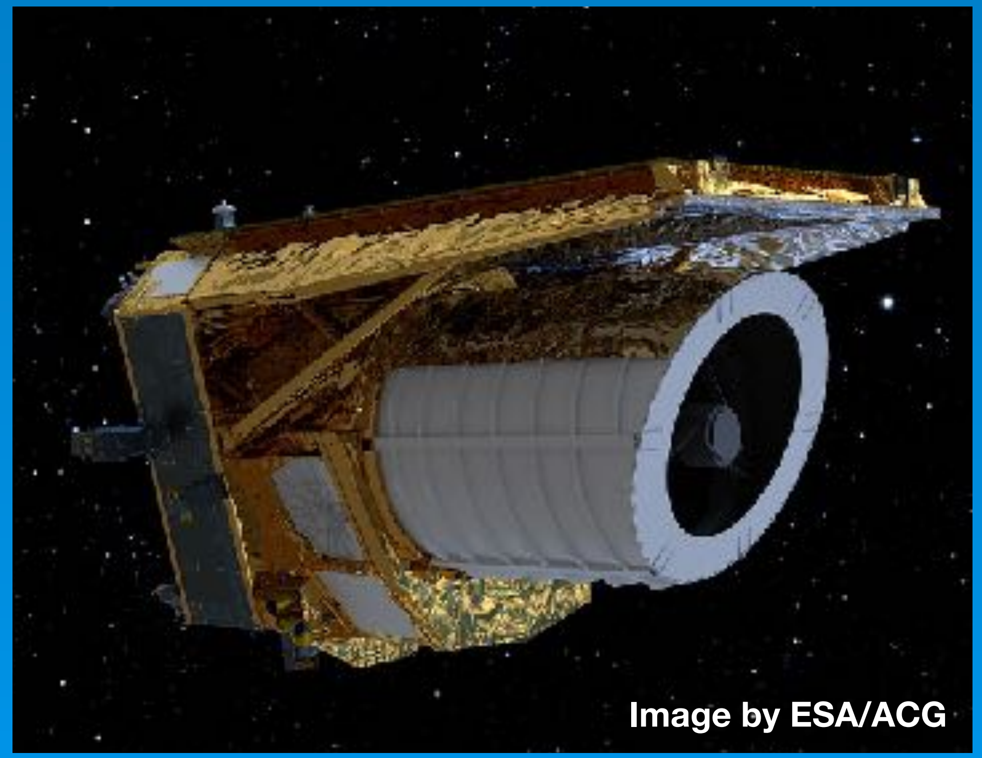
Ongoing work:

- We need more measurements $\rightarrow$ Euclid!
- Constrain $\hat{J}$ using CMB lensing.







Back-up slides Weyl potential

$\hat{J}(z)$ measurements: some technical details

DES analysis in LCDM:

- 2 x 2pt analysis: galaxy clustering and galaxy-galaxy lensing
- Modified CosmoSIS version with Polychord sampler
- CMB priors (baseline case)
- MagLim sample (lenses), Metacalibration (sources)
- We follow closely the official DES analysis (Porredon et al. 2022):
    - Nuisance parameters for the width and shift of the lenses and shift of the sources
    - Shear calibration bias
    - Intrinsic alignments: NLA model in GR
    - Magnification effects for the lenses: modelled as in GR

w0waCDM analysis with extra data:

- Planck 2018 CMB temperature and polarisation spectra, plik_lite version of likelihood
- BAO measurements from DESI DR2
- Luminosity distance measurements from type Ia supernova, Pantheon+ sample

$\hat{J}(z)$ measurements: results

DES results in LCDM:

	CMB prior		No CMB prior	
	Standard cuts	Pessimistic cuts	Standard cuts	Pessimistic cuts
$\hat{J}(z_1)$	0.325 ± 0.015	0.329 ± 0.028	0.327 ± 0.017	0.335 ± 0.030
$\hat{J}(z_2)$	$0.333^{+0.017}_{-0.019}$	0.324 ± 0.035	$0.328^{+0.019}_{-0.023}$	0.326 ± 0.036
$\hat{J}(z_3)$	$0.387^{+0.026}_{-0.029}$	0.428 ± 0.061	0.388 ± 0.032	0.433 ± 0.063
$\hat{J}(z_4)$	0.354 ± 0.035	0.368 ± 0.075	0.345 ± 0.040	0.370 ± 0.080

New paper with varying w0wa and extra datasets:

Model		$\hat{J}(z=0.295)$	$\hat{J}(z=0.467)$	$\hat{J}(z=0.626)$	$\hat{J}(z=0.771)$	Tension
Varying $A_{\rm lens}$	ΛCDM	0.330 ± 0.014	0.337 ± 0.016	0.387 ± 0.025	$0.364^{+0.029}_{-0.033}$	2.1σ
	$w_0 w_a$CDM	$0.325^{+0.014}_{-0.016}$	0.332 ± 0.016	0.386 ± 0.023	$0.365^{+0.029}_{-0.034}$	1.6σ
$A_{\rm lens} = 1$	ΛCDM	0.324 ± 0.013	0.332 ± 0.016	0.389 ± 0.025	0.362 ± 0.033	3.1σ
	$w_0 w_a$CDM	0.321 ± 0.013	0.331 ± 0.016	0.383 ± 0.025	0.362 ± 0.033	2.2σ
Ref. [6]		0.325 ± 0.015	$0.333^{+0.017}_{-0.019}$	$0.387^{+0.026}_{-0.029}$	0.354 ± 0.035	2.8σ

Results for w0-wa:

Varying $A_{\rm lens}$:

$$w_0 = -0.855^{+0.054}_{-0.047}\,,\quad w_a = -0.48 \pm 0.20\,.$$

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

$A_{\rm lens}$ fixed to 1:

$$w_0 = -0.830^{+0.051}_{-0.060}\,,\quad w_a = -0.67^{+0.22}_{-0.19}\,.$$

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

Back-up slides Weyl + RSD

The E_G statistic

Theoretically described in Zhang et al. (2007):

$$E_G(l) = \frac{a}{3H_0^2}\frac{C_{\kappa g}(l)}{C_{gv}(l)}$$

Galaxy-galaxy lensing

Galaxy-velocity correlations

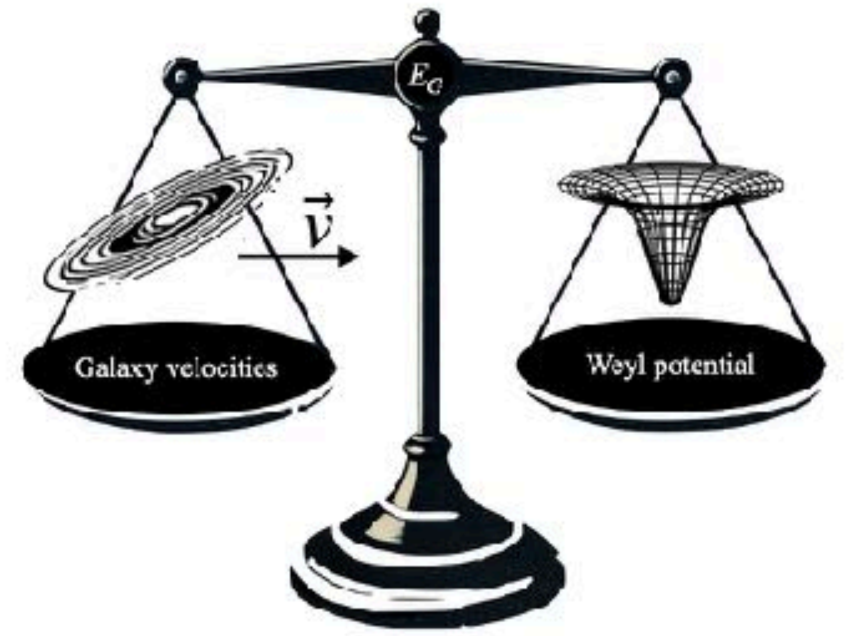
In practise: $E_G(l) \propto \dfrac{C_{\kappa g}(l)}{(\,f/b\,)\cdot C_{gg}(l)}$

Why is it useful?

- Galaxy bias b cancels out.
- Measurements of E_G can be compared to their ΛCDM prediction, $E_G = \Omega_{m,0}/f(z)$.

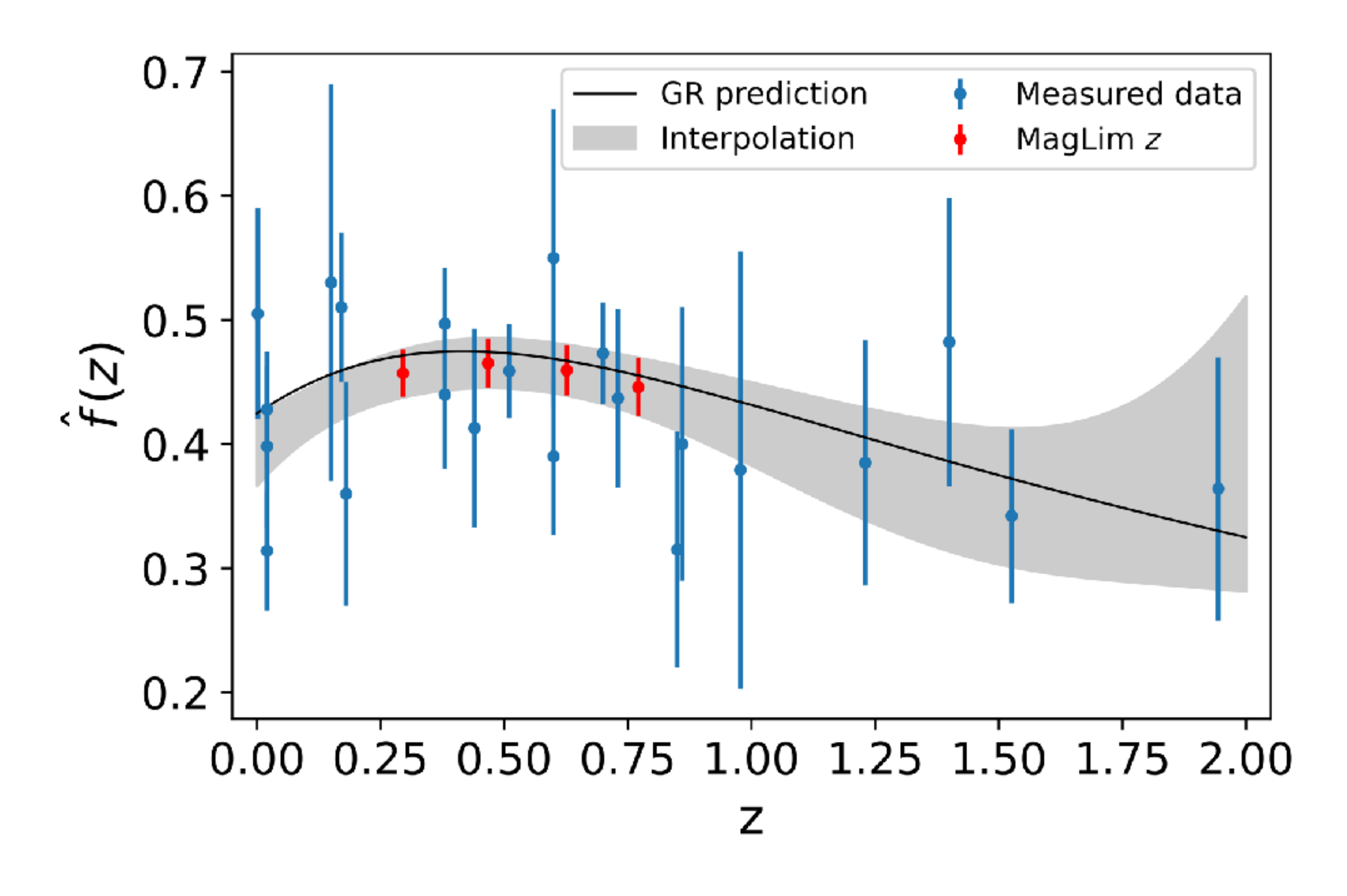
New method E_G written as ratio of model-independent observables:

$$E_G(z) = \left(\frac{H(z)}{H_0}\right)^2 \frac{1}{1+z}\frac{\hat{J}(z)}{\hat{f}(z)}$$



NG, C. Bonvin and I. Tutusaus, arXiv:2403.13709, *Phys. Rev. Lett.* 133 (2024) 9295

Spline interpolation of $\hat{f}(z)$ data



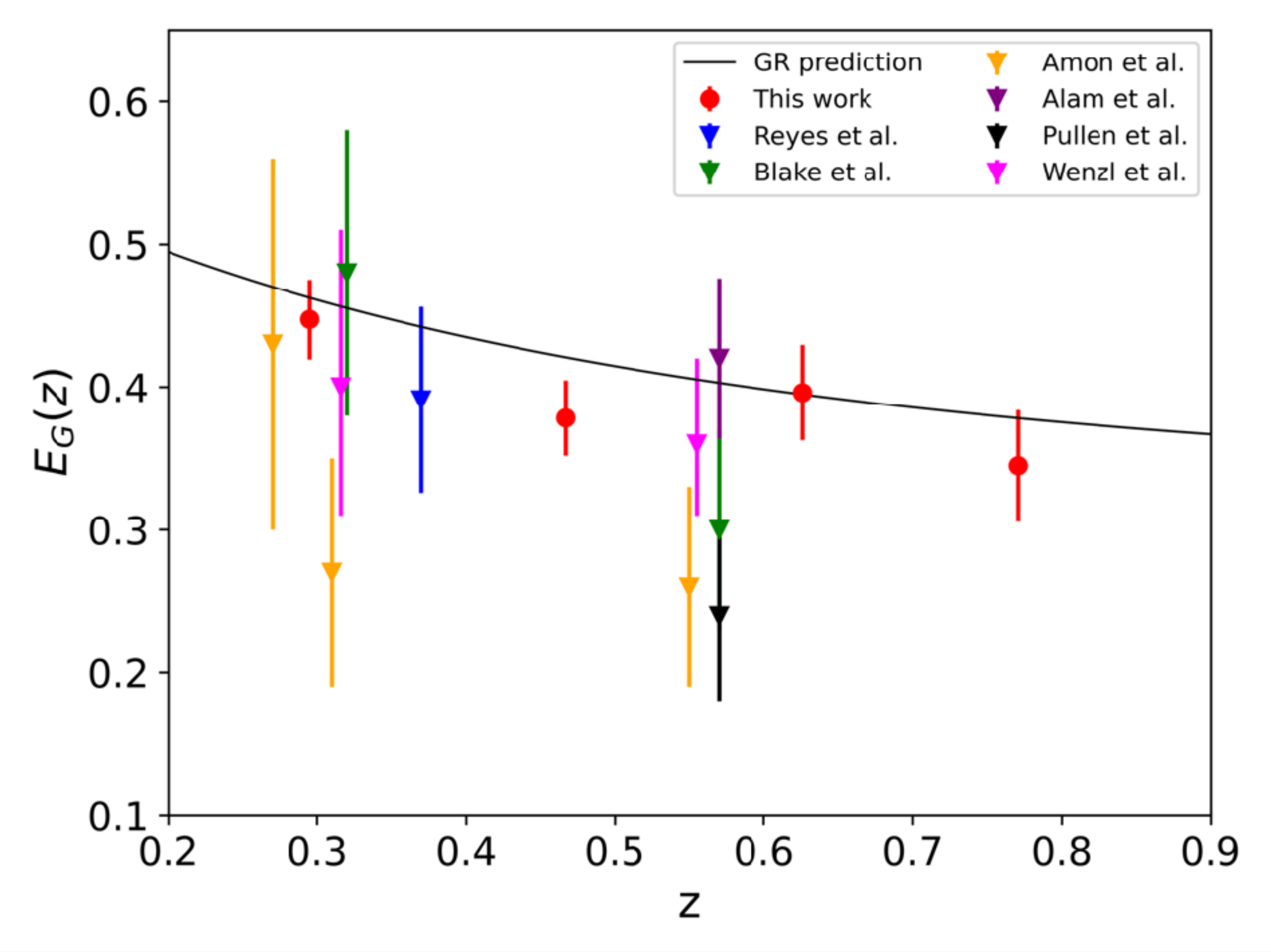
Data:

- 22 measurements of $\hat{f}$ between $z = 0.001$ and $z = 1.944$.
- Based on the Gold-2017 compilation (Nesseris et al. 2017), with updated data from SDSS-IV.

Spline interpolation:

Measurements of $\hat{f}(z)$ at the 4 MagLim effective redshifts with 4-5% precision.

New E_G measurement



Result:

- We can measure E_G with 6—11% precision (compared to 13 – 30% for previous measurements.)
- Agreement with GR (1.6σ deviation in the second bin)

NG, C. Bonvin and I. Tutusaus, arXiv:2403.13709, *Phys. Rev. Lett.* 133 (2024) 9295

Constraining Euler's equation for dark matter

RSD: Depend on the way galaxies fall into gravitational potentials.

Euler's equation: $$V' + V - \frac{k}{\mathcal{H}}\boxed{(1+\Gamma)\Psi} = 0\,. \qquad \Gamma\ldots$$ **Fifth force term**

The quantity $\hat{J}$ describes the evolution of $\Phi + \Psi$ (=2Ψ without anisotropic stress).

$$\Rightarrow \quad 1 + \Gamma(z) = \frac{2\hat{f}(z)}{3\hat{J}(z)}\left(1 - \frac{\mathrm{d}\ln\mathcal{H}(z)}{\mathrm{d}\ln(1+z)} - \frac{\mathrm{d}\ln\hat{f}(z)}{\mathrm{d}\ln(1+z)}\right)$$

NG, C. Bonvin and I. Tutusaus, arXiv:2502.12843, *Nature Comm.* 16 (2025) 9399