Non-Gaussian statistics to break degeneracies in Modified Gravity

Alessandro Vadalà
3rd yr. PhD
Joint program INAF-OAR – Sapienza U. – Tor Vergata U.

PhD advisors:
Dr. Vincenzo F. Cardone
Prof. Roberto Maoli

Collaborators:
Simone Vinciguerra, Filippo Bouchè,
Carlo Giocoli, Marco Baldi

alessandro.vadala@inaf.it

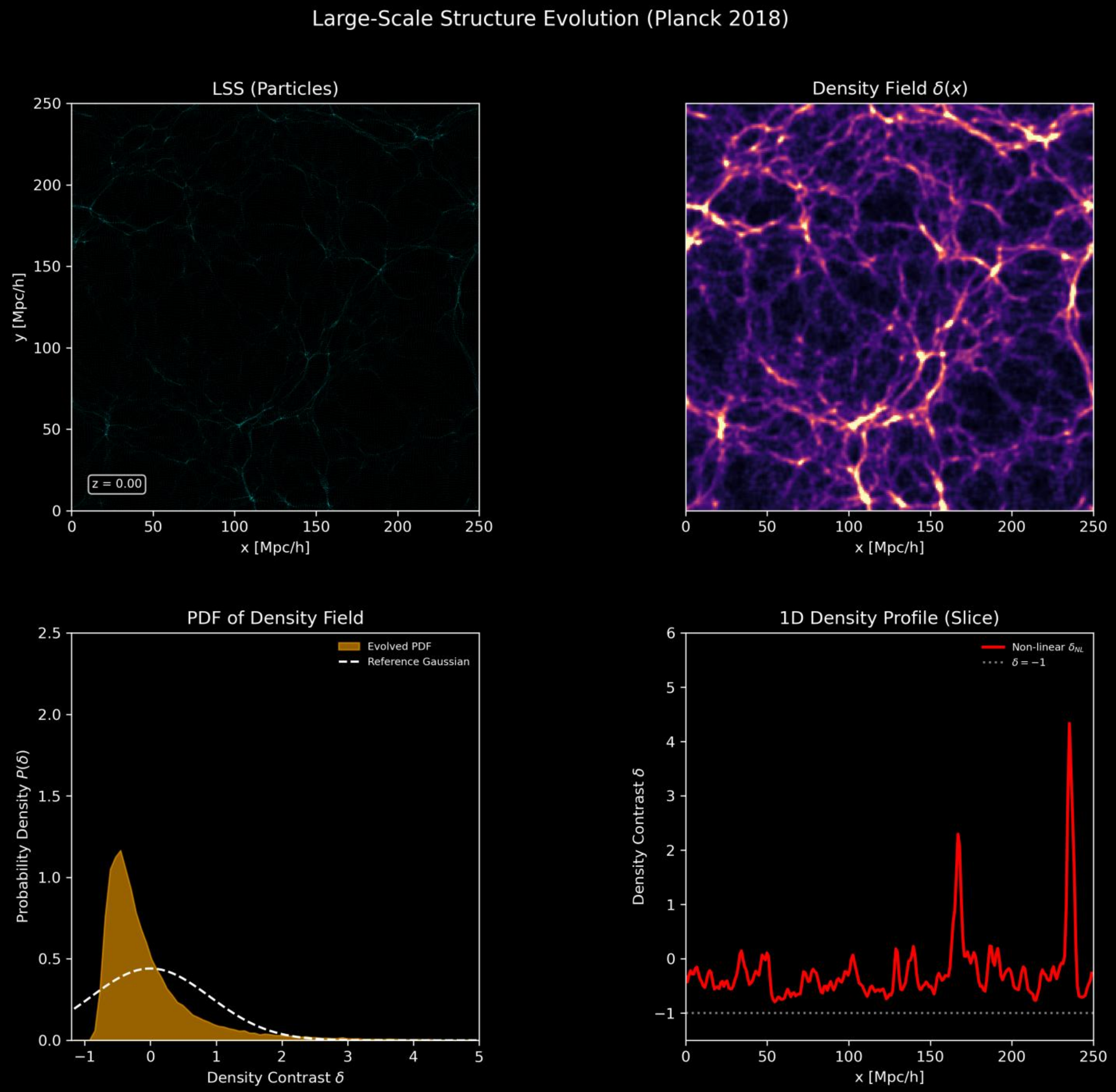
Non-Gaussianities in LSS

Gravitational evolution

→ non-Gaussian features

Tight high-density peaks

Larger underdensities (voids)



See "Coles, Lucchin" 2nd ed, chap. 16

Fourier transform of δ

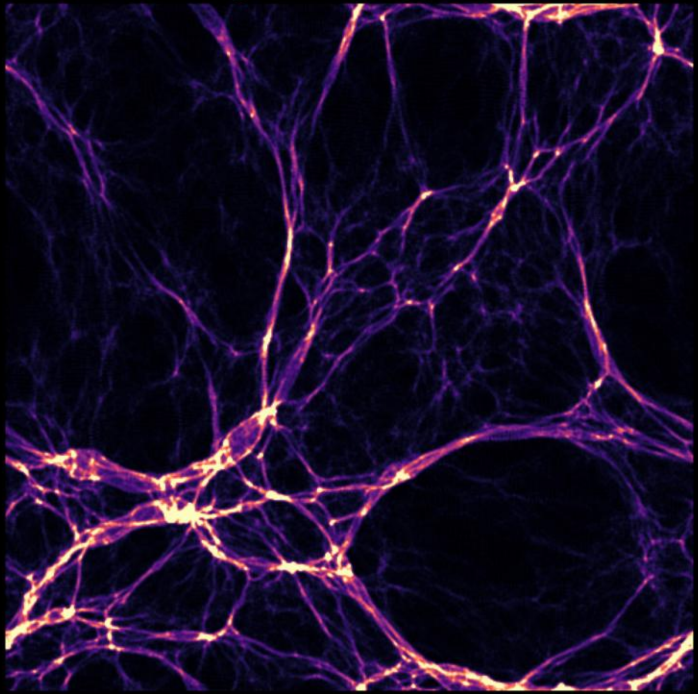
complex

→ amplitude and phase

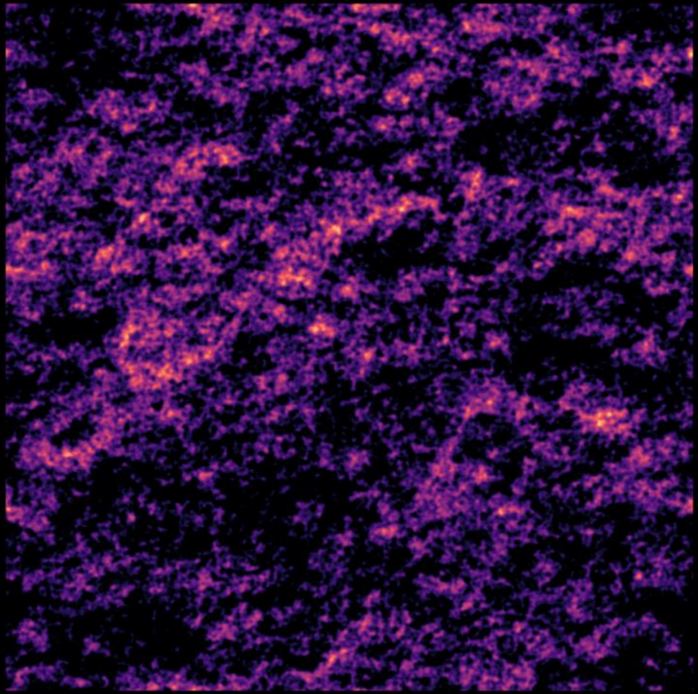
Phase reshuffling

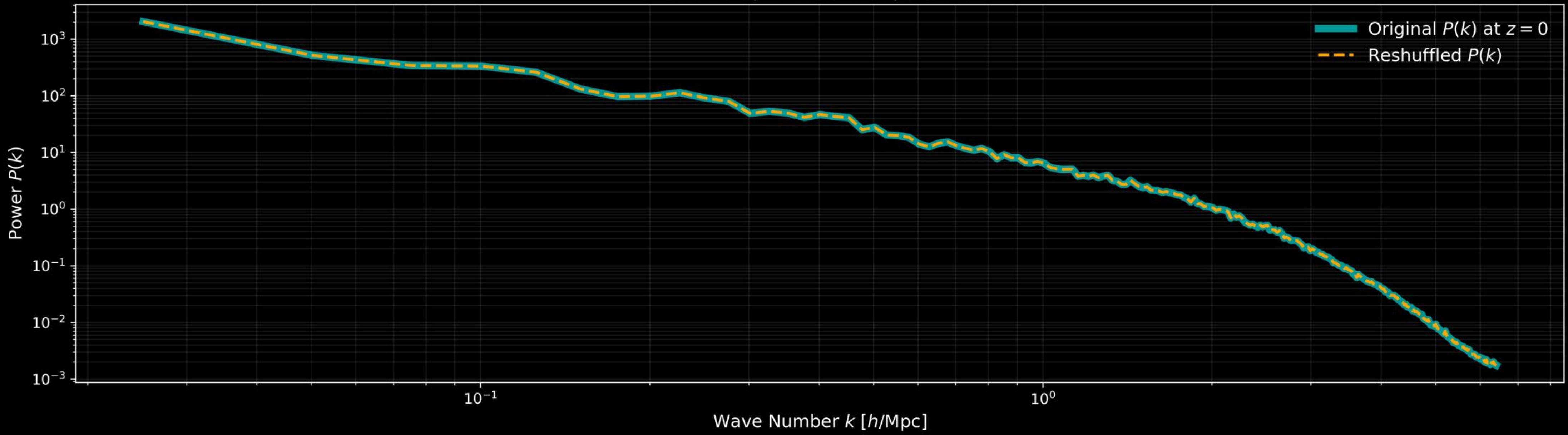
→ Same amplitude

Galaxy Clustering Simulation ($z = 0$)
High Phase Coupling

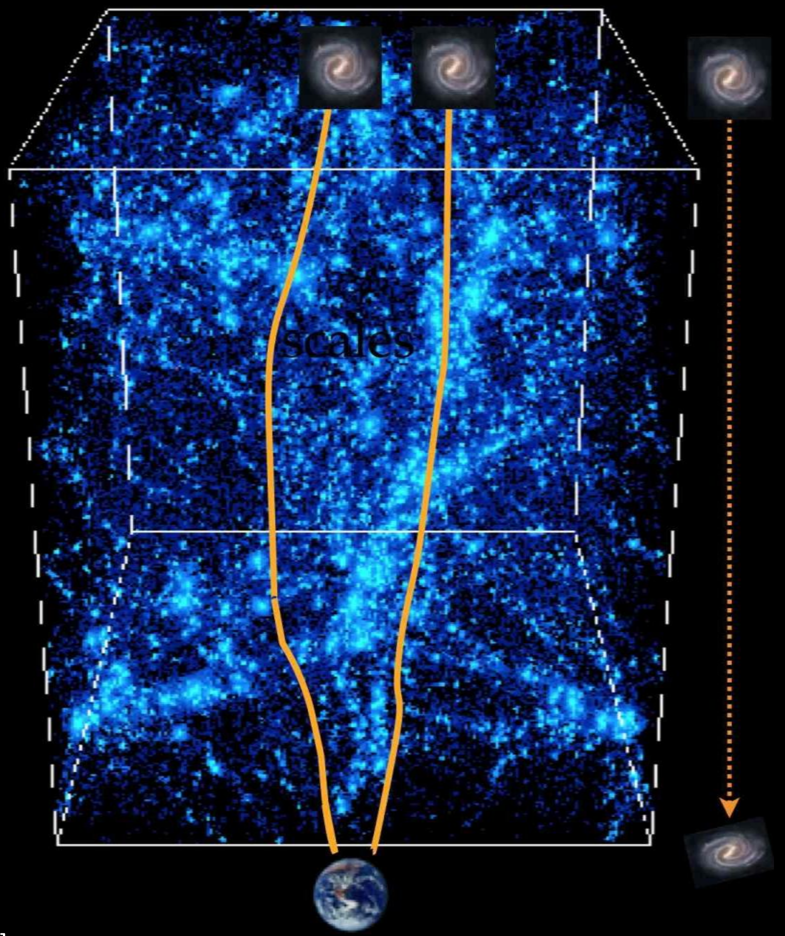


Phase-Reshuffled
Uniformly Random Phases





From shear to convergence



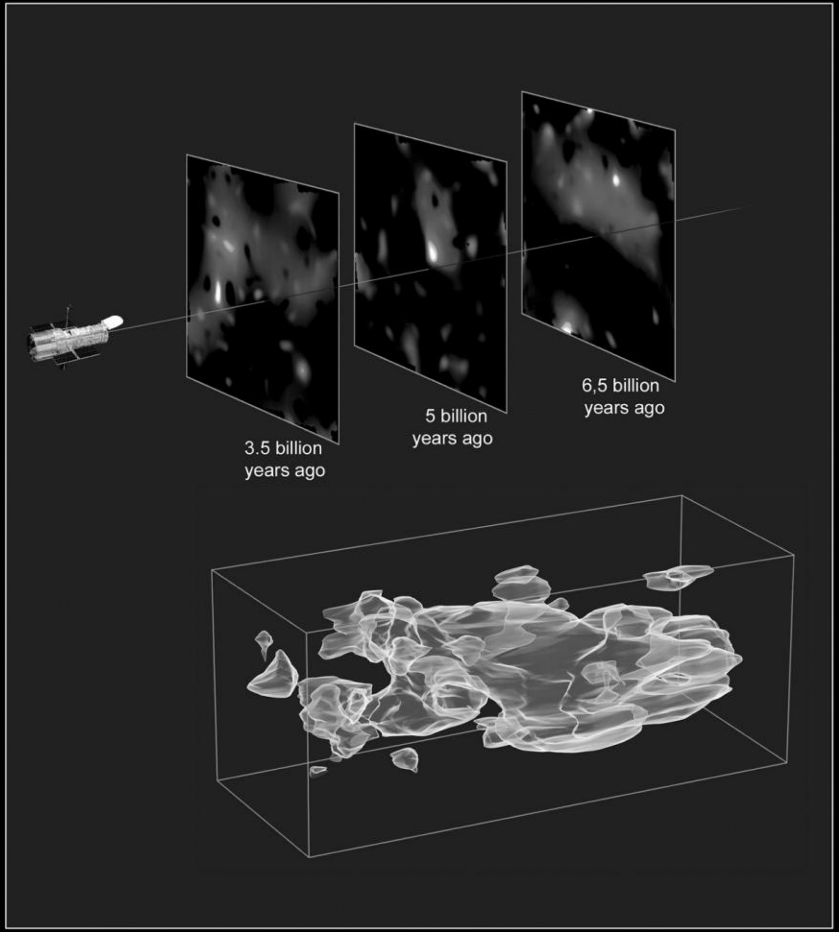
[CEA]

$$\gamma_1 = \frac{1}{2}(\partial_1^2 - \partial_2^2)\psi$$

$$\gamma_2 = \partial_1\partial_2\psi$$

Mass-mapping

$$\kappa = \frac{1}{2}(\partial_1^2 + \partial_2^2)\psi$$

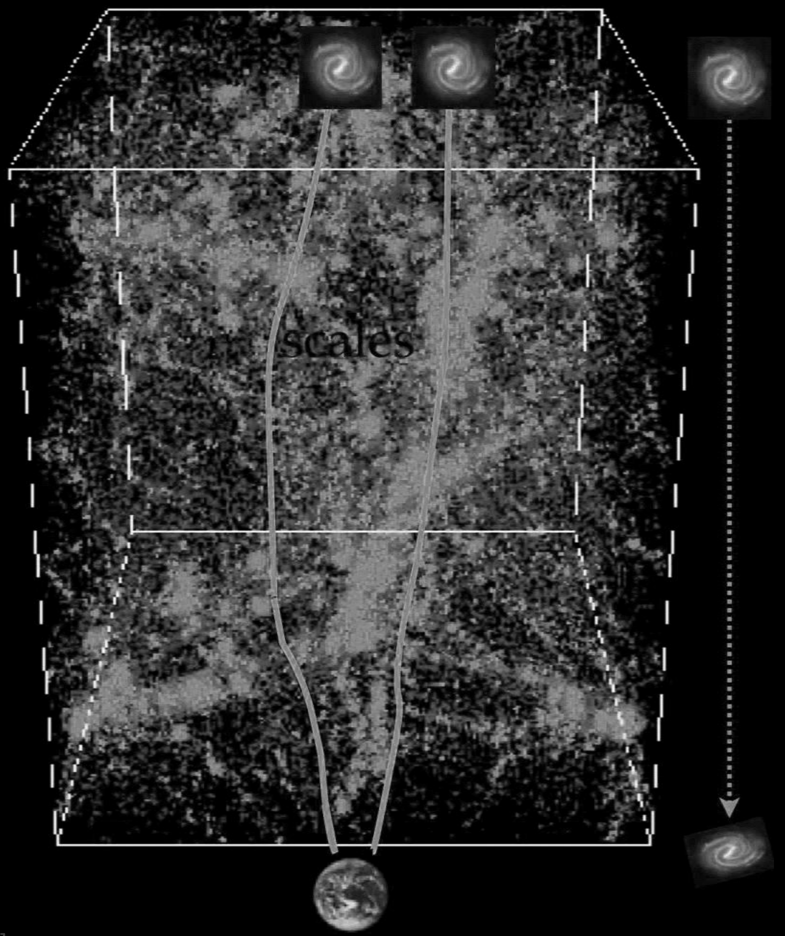


[Euclid/Weak lensing]

Cosmic shear
(coherent deformation)

Convergence maps
(projected mass)

From shear to convergence



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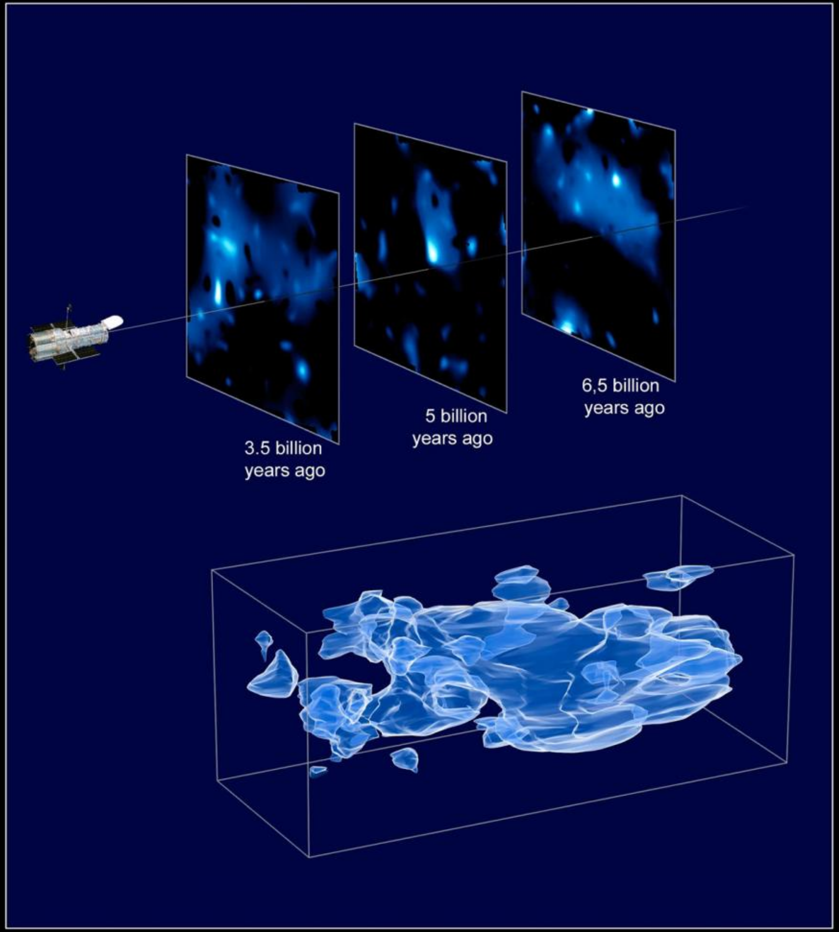

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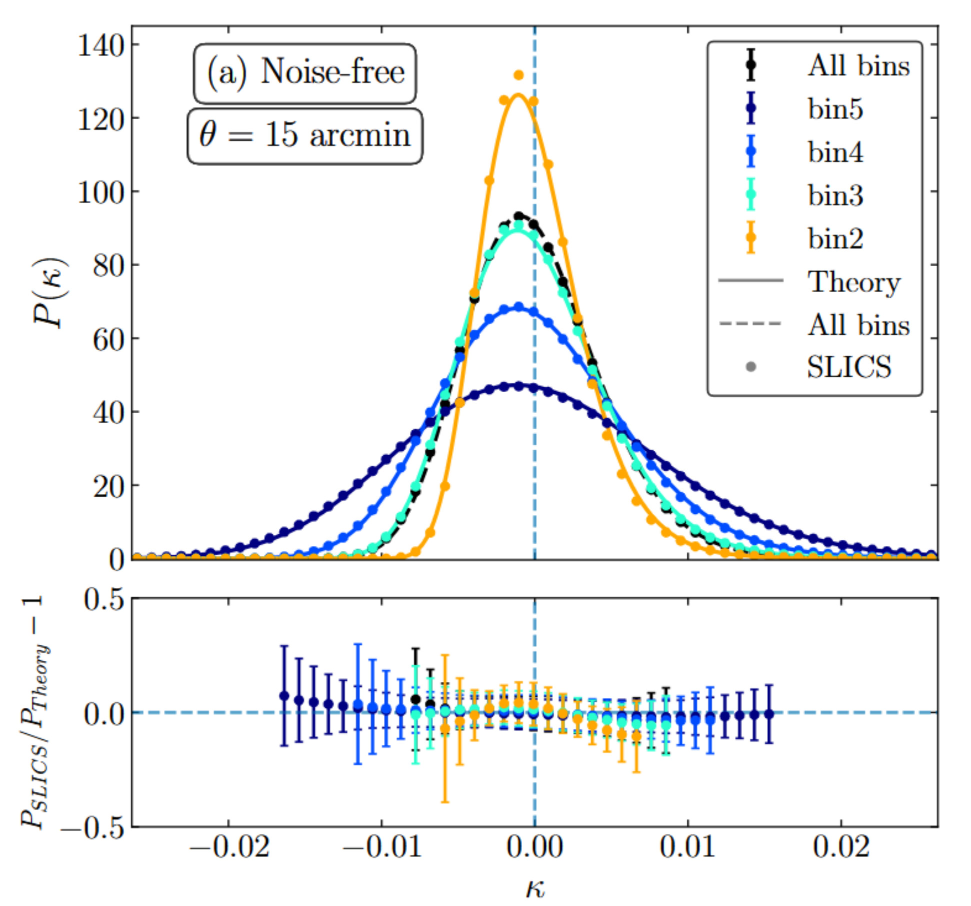
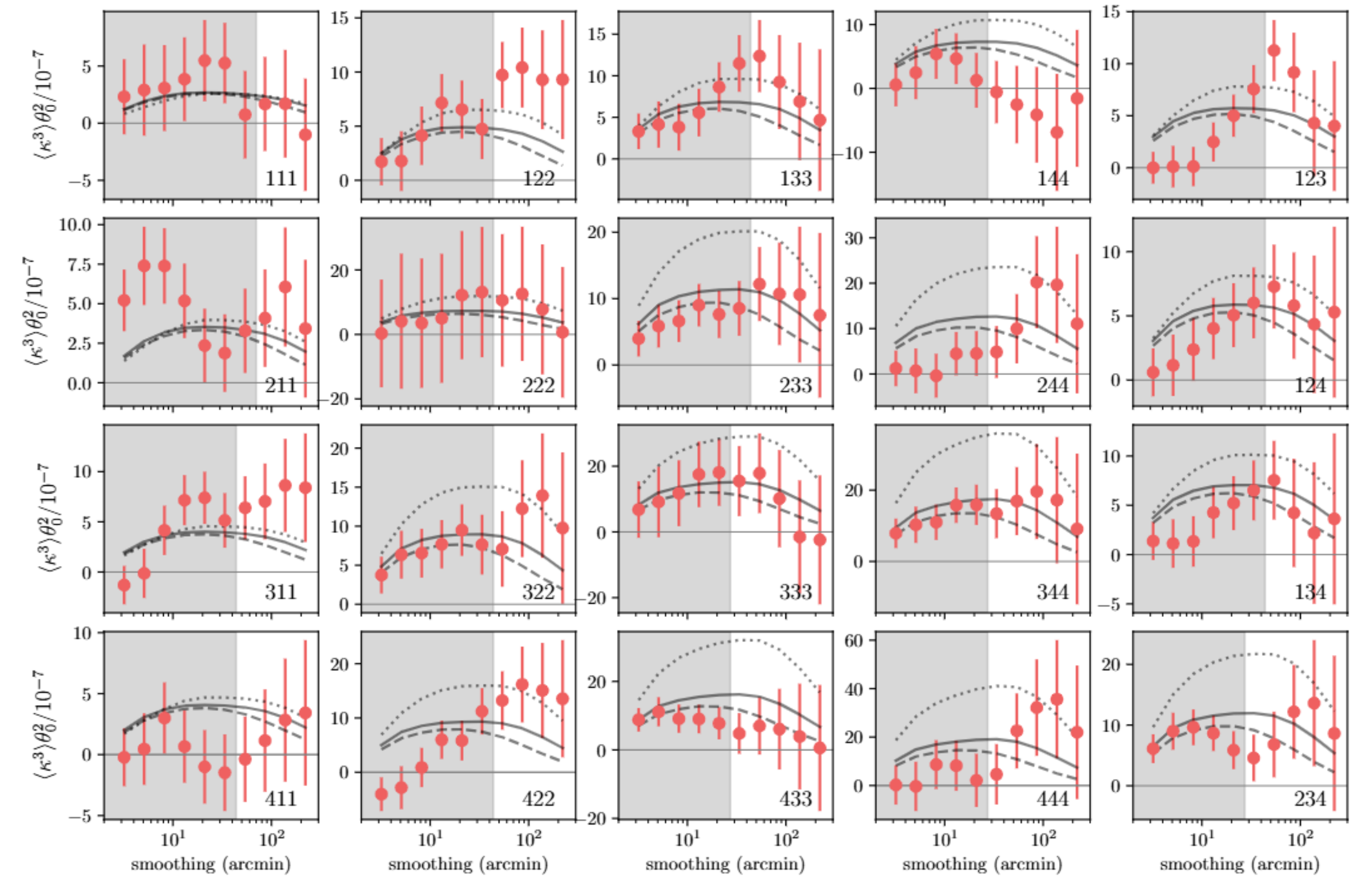
Cosmic shear
(coherent deformation)

Convergence maps
(projected mass)

Higher-order statistics – one-point



1pt **κ** - PDF



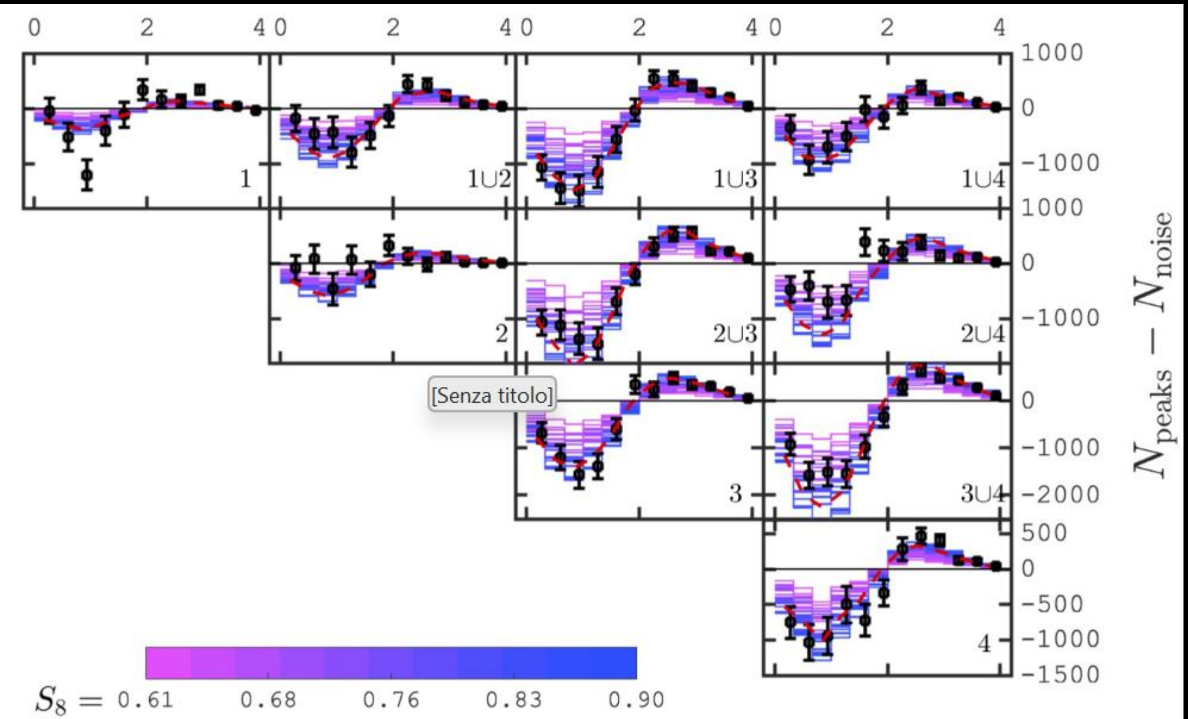
κ - higher-order moments

[2405.09651]

[2110.10141]

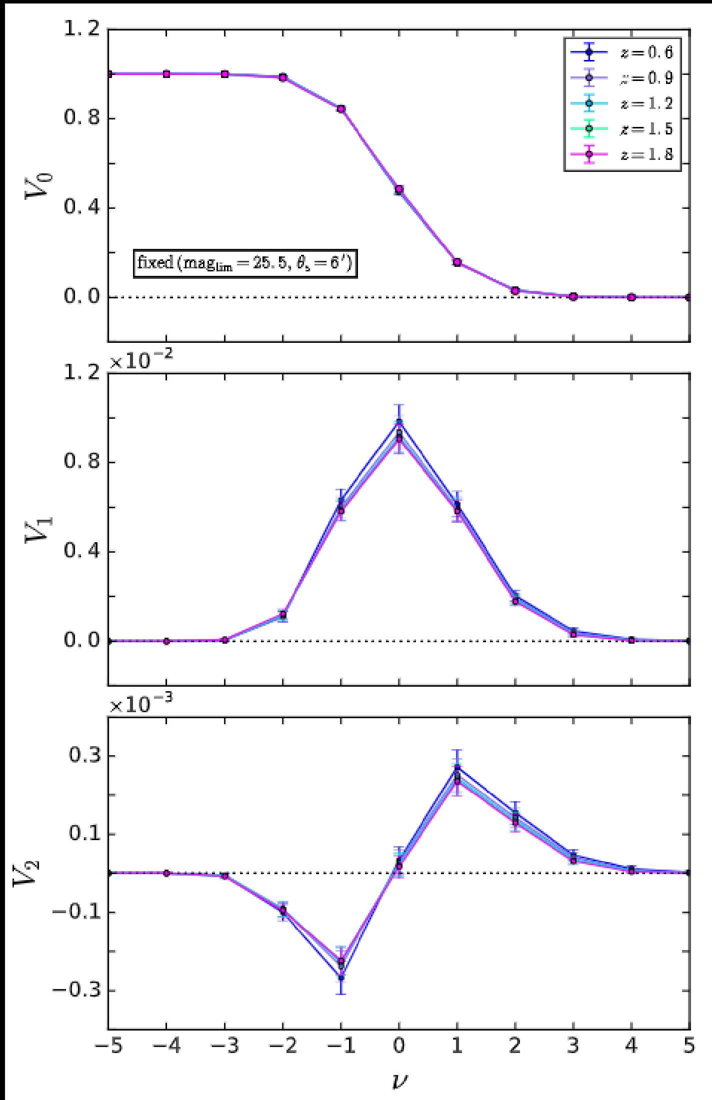
Higher-order statistics – topological

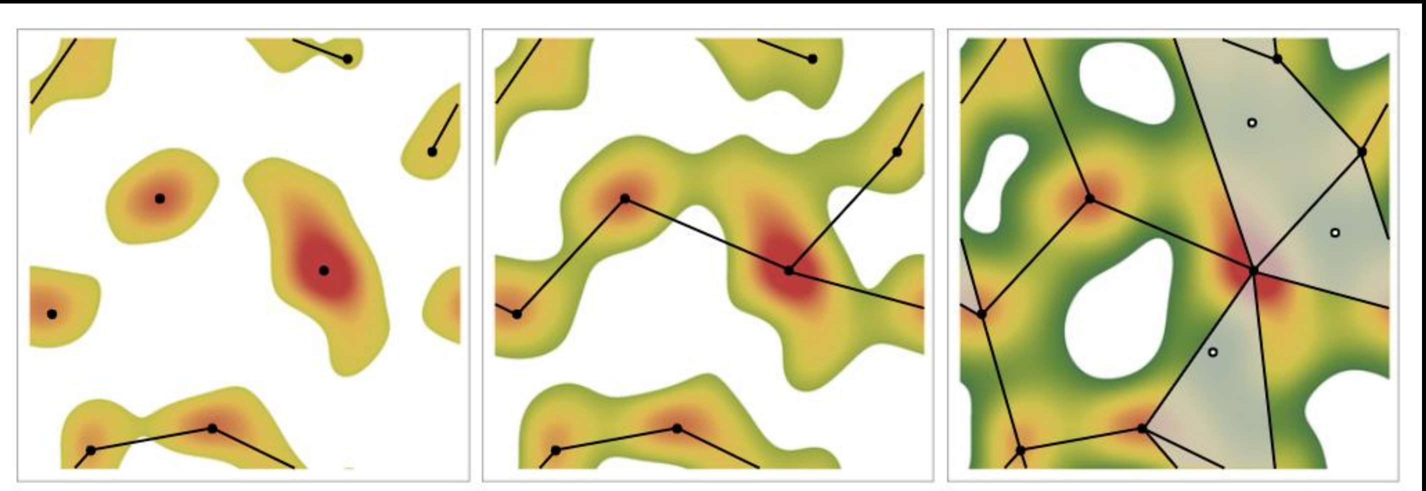
κ – Minkowski functionals



κ - peak counts

[2012.02777]





[1908.01619]

κ - Betti numbers

[1911.06243]

Break modified gravity degeneracy

Weak lensing higher-order statistics to disentangle modified gravity and massive neutrinos

A. Vadalà[a,b,c,d] V.F. Cardone[a,b] S. Vinciguerra[e] F. Bouchè[f,g] M. Baldi[h,i,j] C. Giocoli[i,j]

[a]INAF - Osservatorio Astronomico di Roma, via Frascati 33, 00078 Monte Porzio Catone (Roma), Italy

[b]INFN - Sezione di Roma, Piazzale Aldo Moro, 2 - c/o Dipartimento di Fisica, G.Marconi, I-00185 Rome, Italy

[c]Tor Vergata University, Via della Ricerca Scientifica, 1 - c/o Dipartimento di Fisica, Rome, Italy

[d]Sapienza Università di Roma, Piazzale Aldo Moro, 2 - c/o Dipartimento di Fisica, E. Fermi, I-00185 Rome, Italy

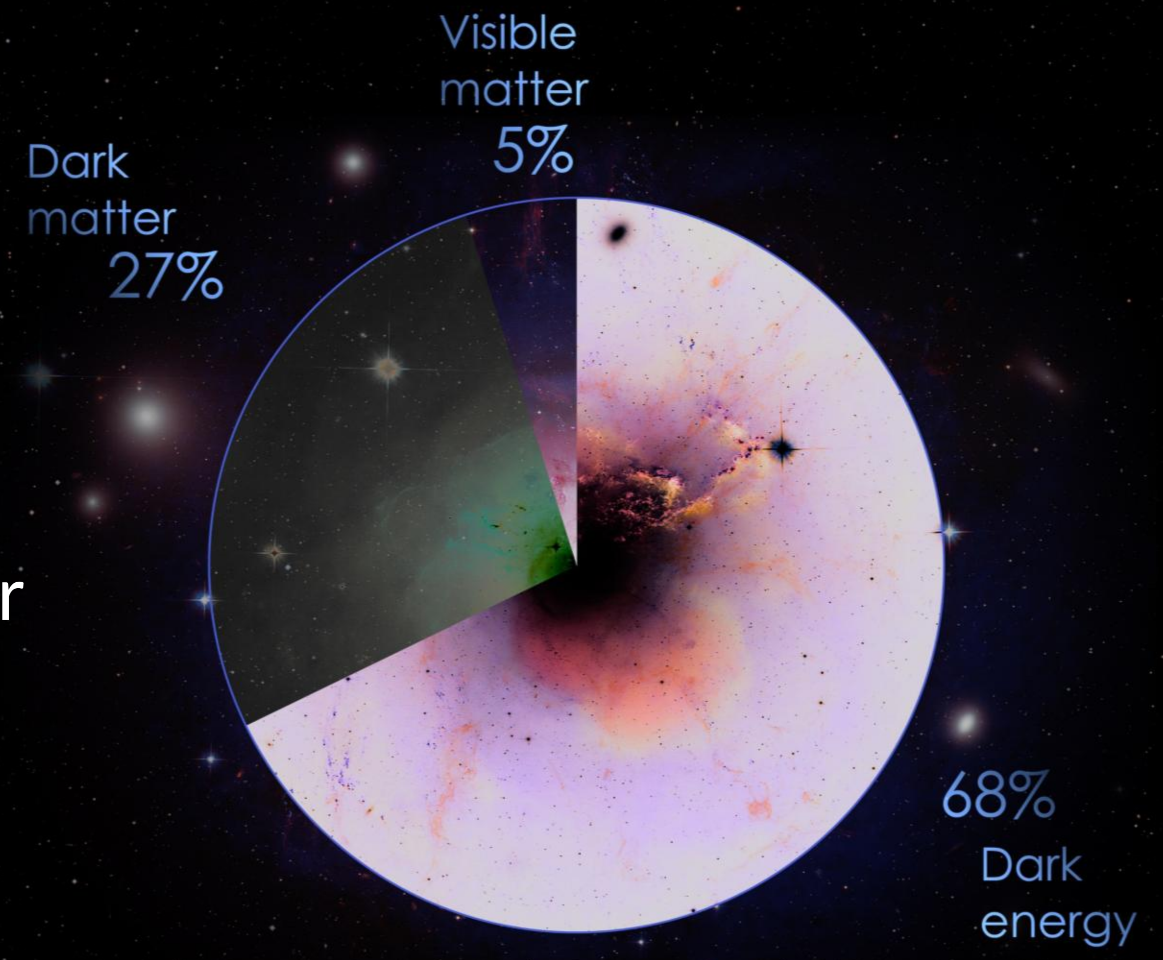
Standard cosmological model - ΛCDM

Phenomenological model

General Relativity (GR)

Λ: Dark Energy–**CDM**: Cold Dark Matter

Cosmological tensions (H_0, S_8)



[Image credits: NASA]

$f(R)$ gravity

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\,[R + f(R)]$$

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Hu & Sawicki

$$f(R) = -m^2 \frac{c_1\left(\frac{R}{m^2}\right)^n}{c_2\left(\frac{R}{m^2}\right)^n + 1}$$

~ ΛCDM at background

different at perturbations

$$f_R = \mathrm{d}f/\mathrm{d}R$$

$$f_{R0}$$

[0705.1158]

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$$f_{R0} \longleftrightarrow m_\nu = \sum_i m_{\nu,i}$$

enhanced

suppressed

[0705.1158]

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$$f_R = \mathrm{d}f/\mathrm{d}R$$

$$f_{R0}$$

HOS?

$$f_{R0} \longleftrightarrow m_\nu = \sum_i m_{\nu,i}$$

enhanced

suppressed

[0705.1158]

Simulations setup

DUSTGRAIN-*pathfinder* simulations

Run in GR and *f(R)* gravity

Different combinations of f_{R0} and m_ν

Fixed redshift planes - $z \in [0.0, 1.0, 2.0, 4.0]$

$|f_{R0}| = 10^{-4}$ strongest

$|f_{R0}| = 10^{-6}$ weakest

+ Shape noise

Simulation name	Gravity type	f_{R0}	m_v(eV)	Ω_{CDM}	Ω_v	σ_8
ΛCDM	GR	–	0	0.31345	0	0.847
fR4	$f(R)$	-1×10^{-4}	0	0.31345	0	0.967
fR5	$f(R)$	-1×10^{-5}	0	0.31345	0	0.903
fR6	$f(R)$	-1×10^{-6}	0	0.31345	0	0.861
fR4_0.3eV	$f(R)$	-1×10^{-4}	0.3	0.30630	0.00715	0.893
fR5_0.15eV	$f(R)$	-1×10^{-5}	0.15	0.30987	0.00358	0.864
fR5_0.1eV	$f(R)$	-1×10^{-5}	0.1	0.31107	0.00238	0.878
fR6_0.1eV	$f(R)$	-1×10^{-6}	0.1	0.31107	0.00238	0.836
fR6_0.06eV	$f(R)$	-1×10^{-6}	0.06	0.31202	0.00143	0.847

Discriminate ΛCDM - f(R) gravity with HOS

Distance metrics

SMAPE

Hellinger

Mahalanobis

Wasserstein

Gaussianity requirements

Nonparametric hypothesis testing

Kolmogorov-Smirnov

Cramér-von Mises

Ansari-Bradley

Mann-Whitney U

Point-like differences in samples

Metrics-based method

$$|\Delta\mathcal{M}| > 3\sigma(\Delta\mathcal{M}) \simeq 3\left[\sigma^2\left(\mathcal{M}_{GR}\right) + \sigma^2\left(\mathcal{M}_{MG}\right)\right]^{1/2}$$

M_{GR} - σM_{GR}, M_{MG} - σM_{MG}

→ median - 68% confidence range, from 500 random realizations sampled from multivariate Gaussian

Must verify HOS Gaussian distributed

→ Gaussianity test (SMAPE-test) [2510.04953]

→ No HOM

Hypothesis rejection-based method

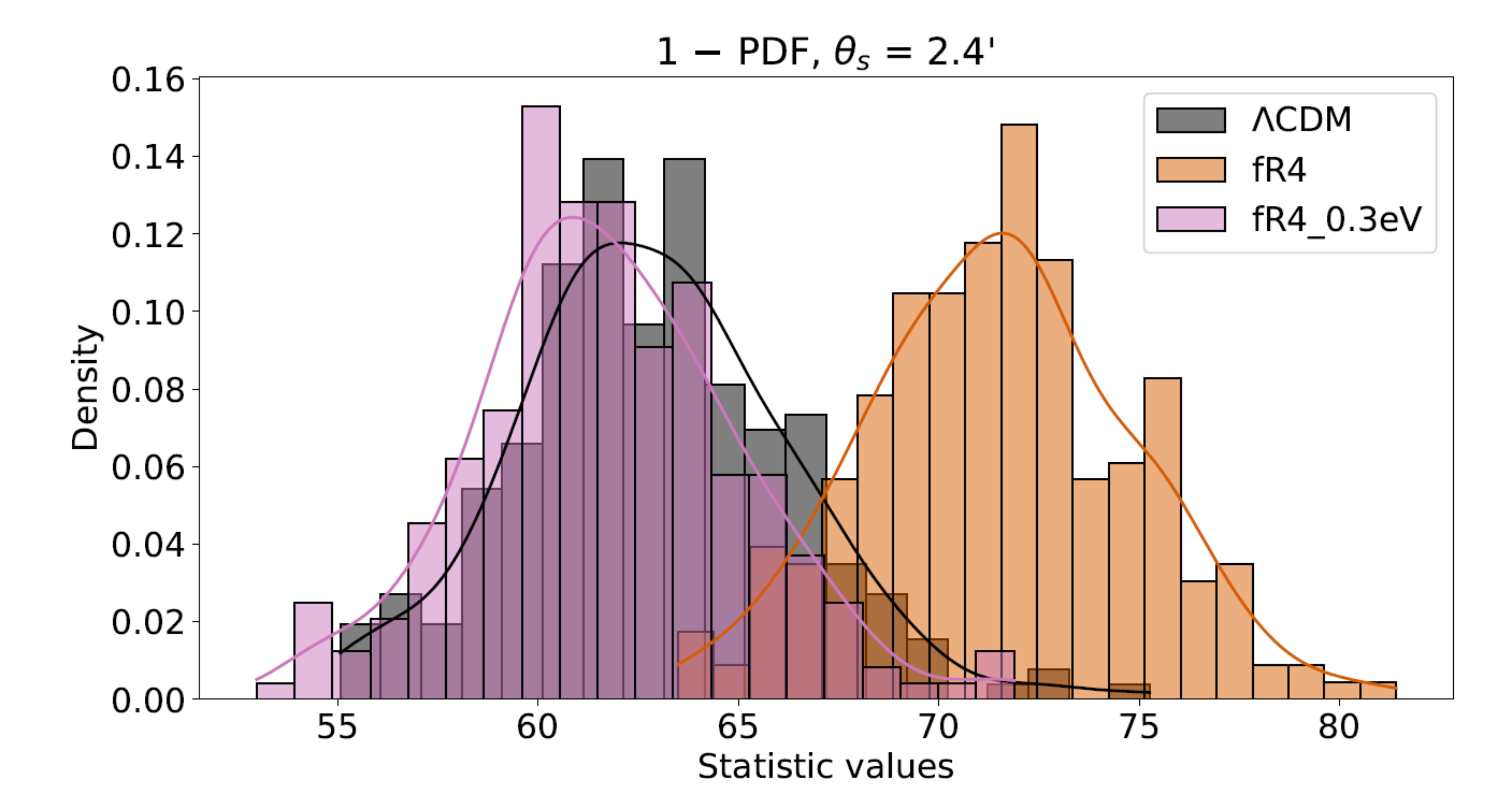
1. For probe and configuration (i.e., smoothing, range, redshift), split range in 20 bins.
2. For bin, histogram of the 256 realizations of that bin → 20 different histograms
3. Gaussian smoothing with kernel density estimation (KDE)

h0 : same distribution

α = 0.001

p < α → reject h0, different

p > α → fail to reject h0



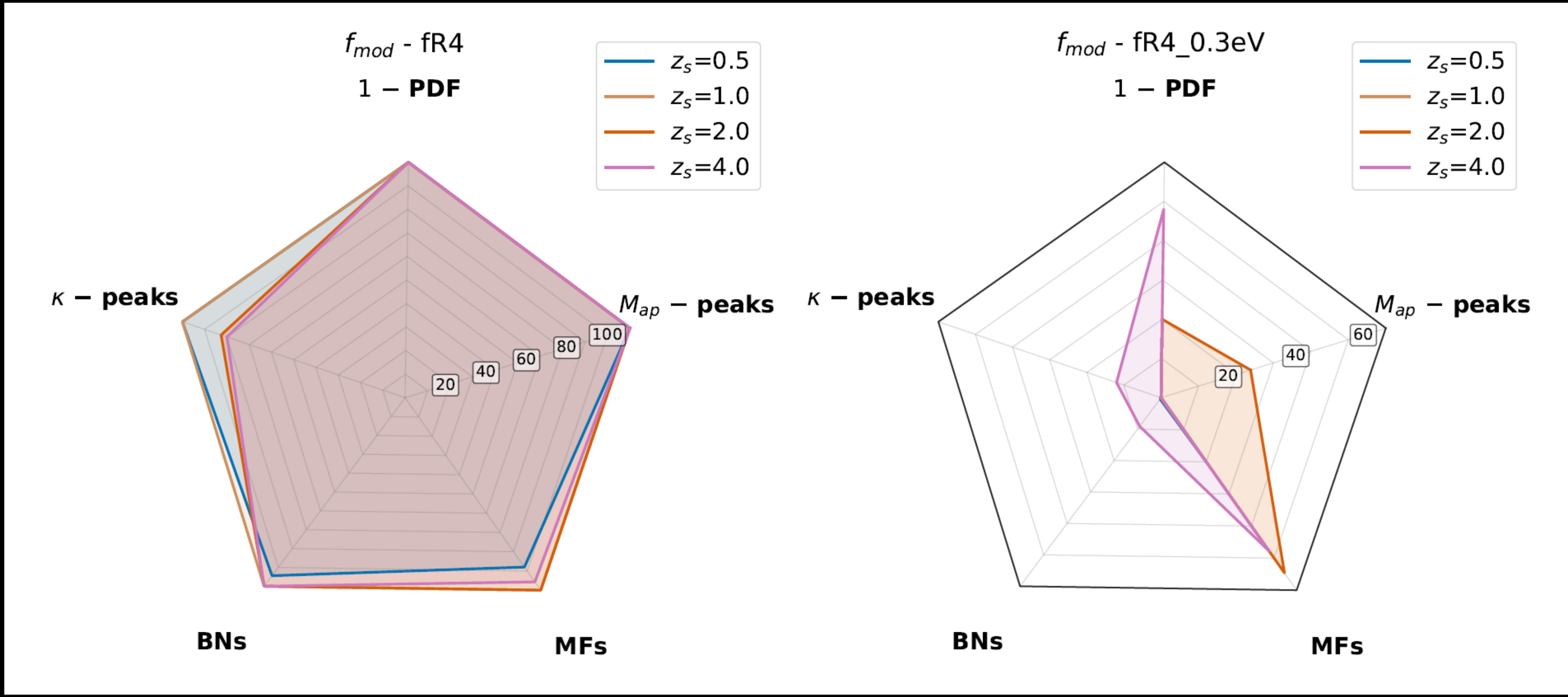
Metric-based method - Results

Combinations range+binning+smoothing → large number of cases

Selected cases with higher S/N → 100 cases per configuration

$$f_{mod}(\mathcal{O}, \mathcal{M})$$

$$\Delta\mathcal{M}/\sigma(\mathcal{M}) > 3$$



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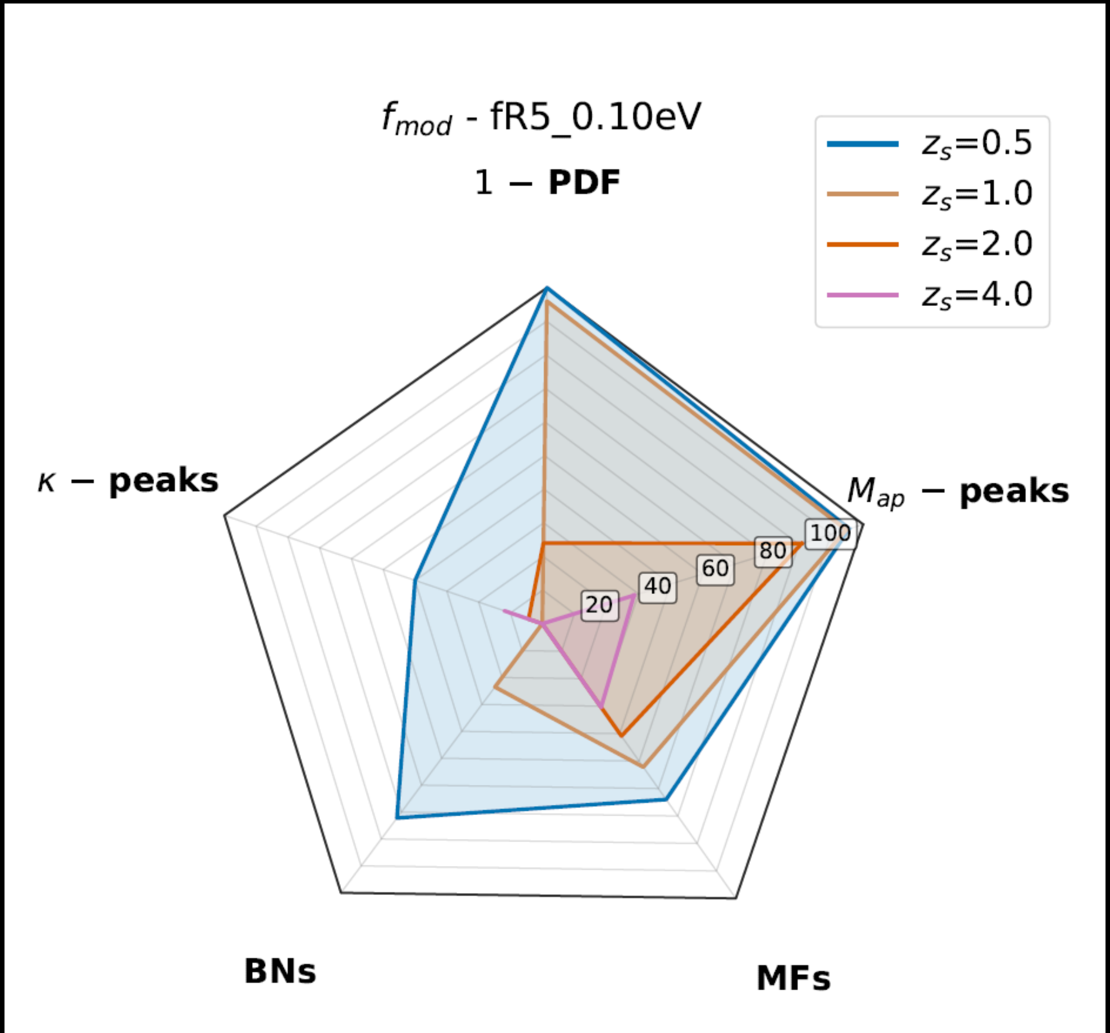

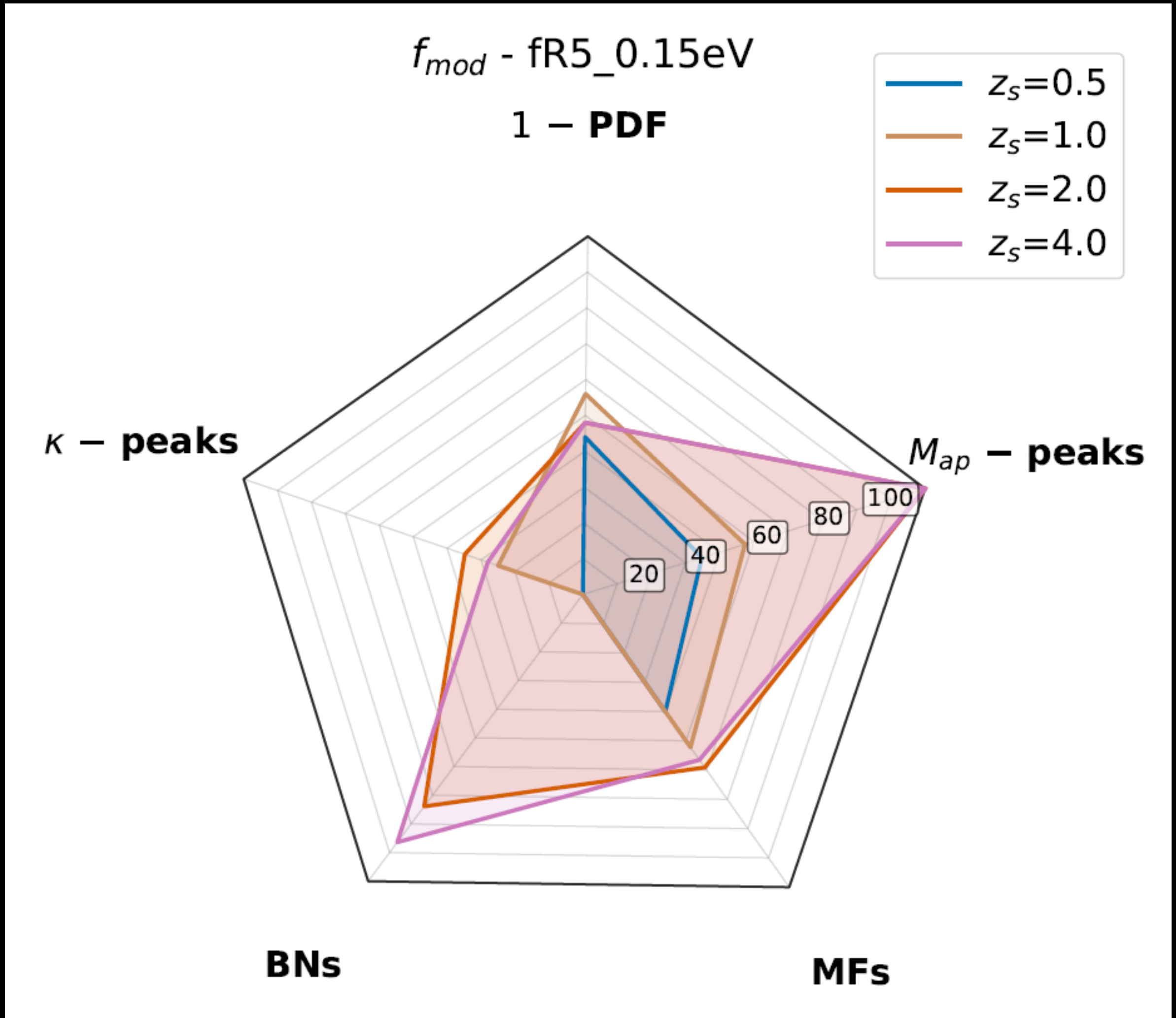

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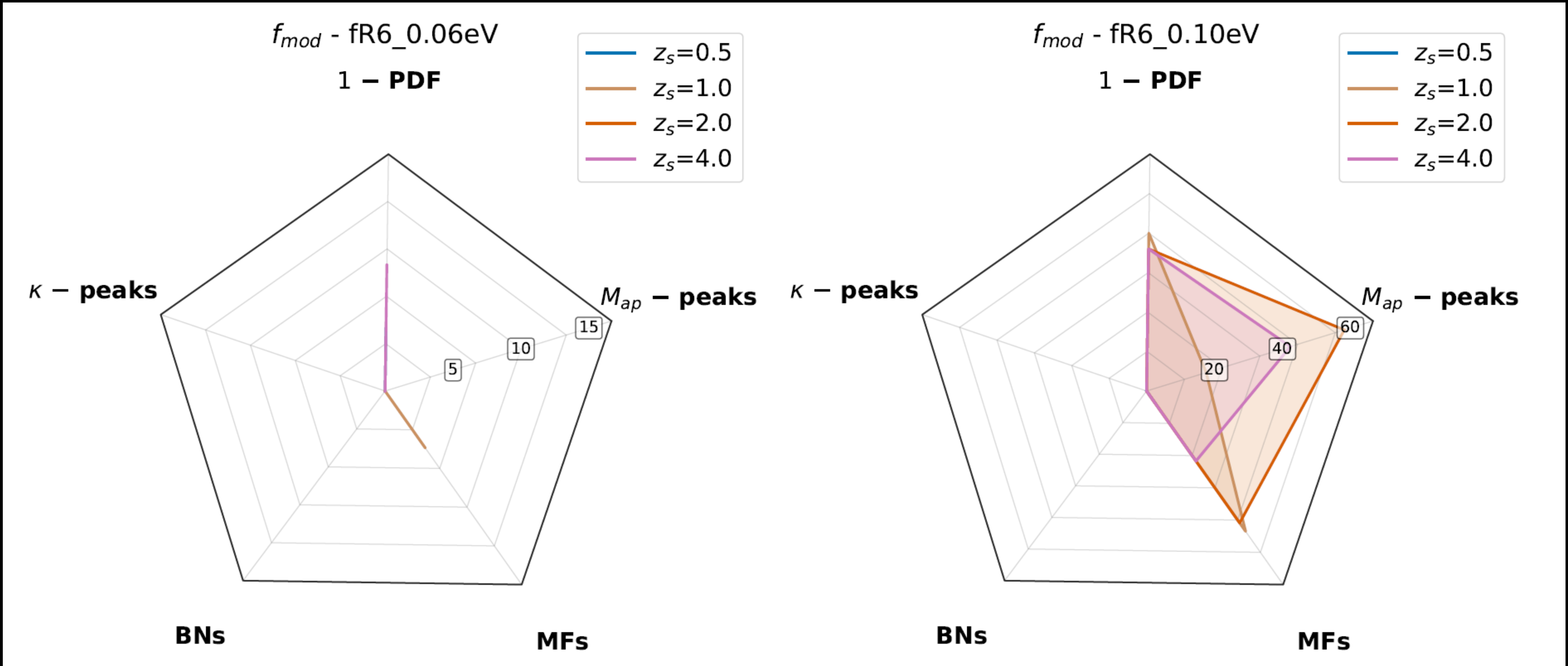
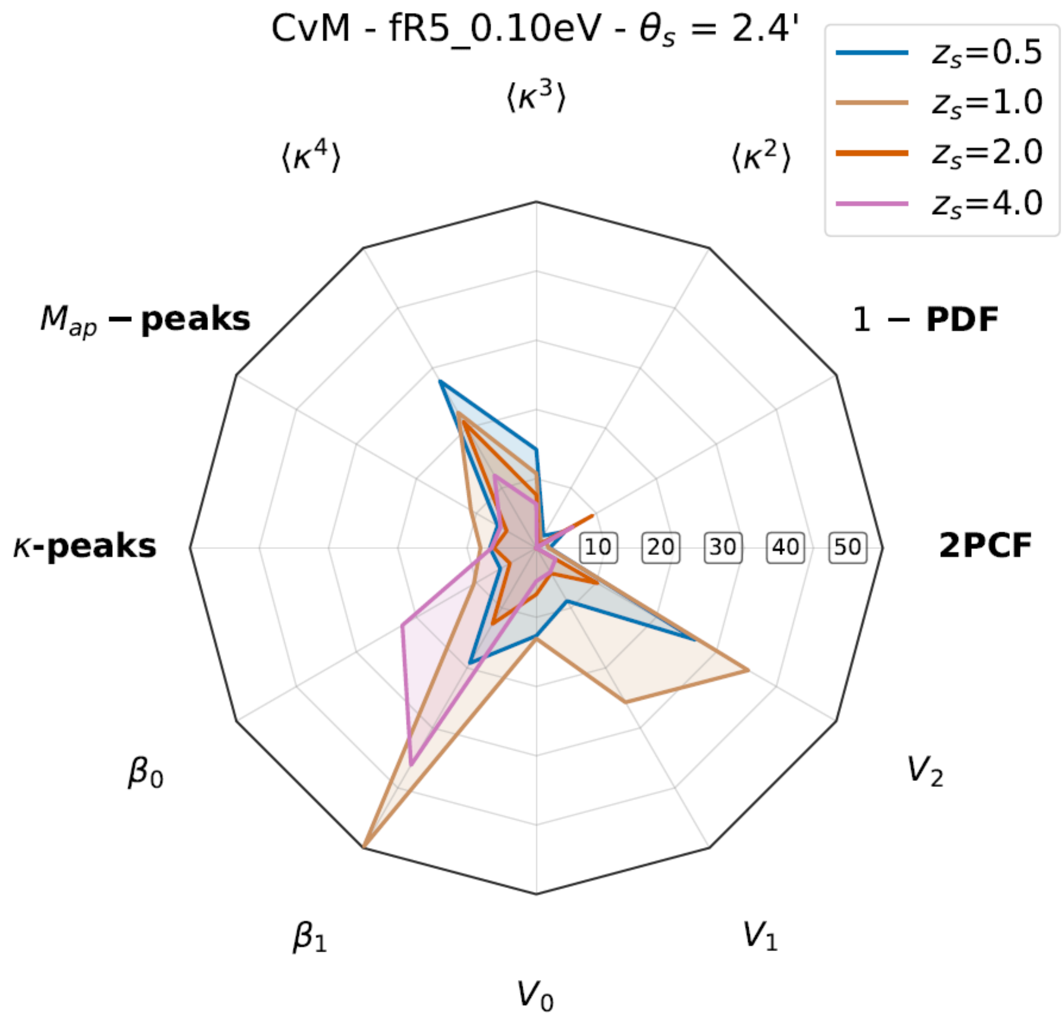
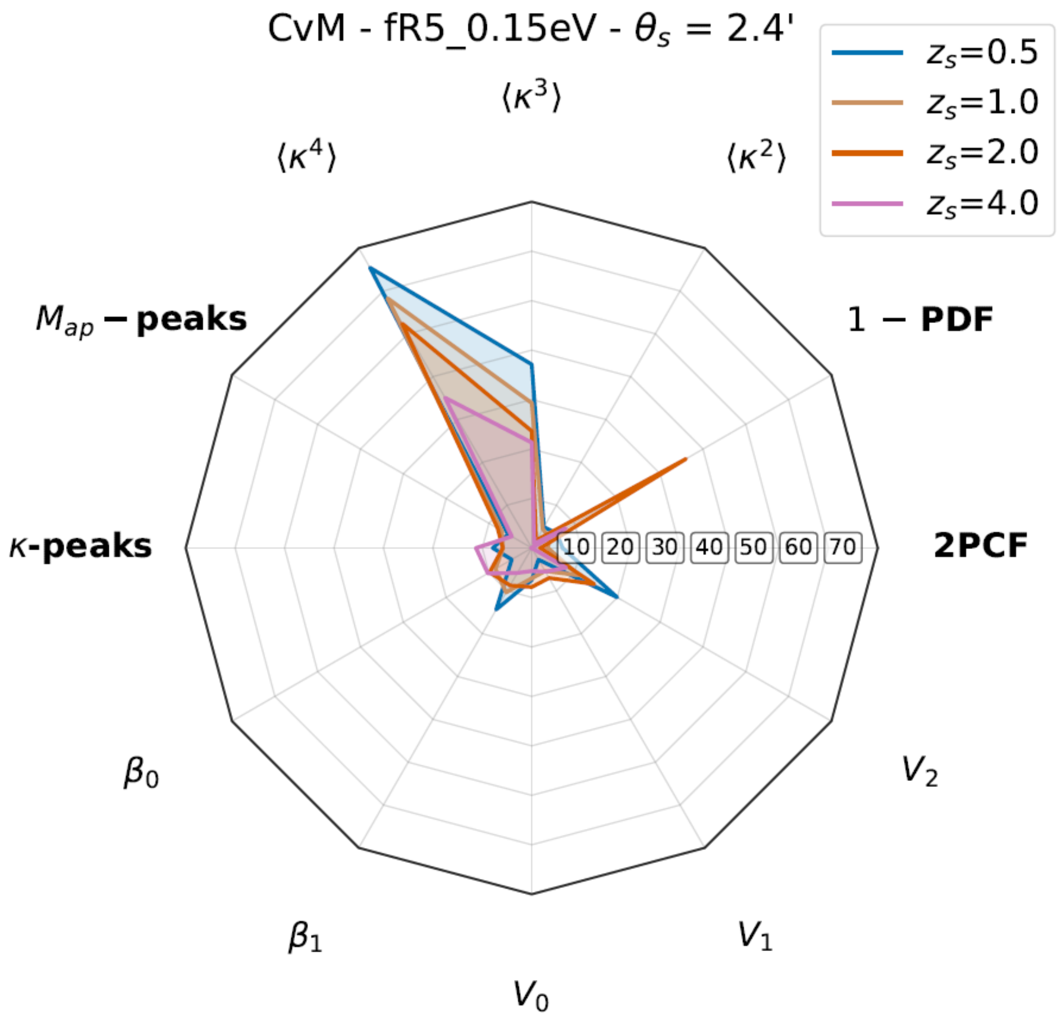


Hypothesis-rejection method - Results





Conclusions

- 2PCF systematically outperformed
- No clear winner! Different configuration → different optimal probe
- For future analyses → risky choosing single HOS on average performance
- Correct strategy → full toolkit, let data tell where to head
- 2PCF still backbone of cosmological analysis → maybe insufficient for beyond GR
- Realistic survey → tomography, systematics: IA, source clustering, baryonic feedback

- Excellent timing: Euclid, LSST, Nancy Roman Space Telescope

  → high-precision data from LSS

Extra Slides

Higher-order statistics

1pt **κ** – PDF

κ - higher-order moments

\+

κ - peak counts
(M_{ap} -)

κ – Minkowski functionals

κ - Betti numbers

Vs.

κ – 2PCF

Higher-order statistics – one-point

1pt **κ** - PDF

$$\mathcal{P}(\kappa) = \int_{-i\infty}^{+i\infty} \frac{dy}{2\pi i} \exp\left[-y\kappa + \phi_{\kappa,\Theta}(y)\right]$$

κ - higher-order moments

$$\langle \kappa^2 \rangle_{ij} = \int_0^\infty dz \frac{\mathcal{W}_i(z)\mathcal{W}_j(z)}{H(z)r^2(z)} \ldots P_{mm}(k_\ell, z)$$

$$\langle \kappa^3 \rangle_{ijk} = \int_0^\infty dz \frac{\mathcal{W}_i(z)\mathcal{W}_j(z)\mathcal{W}_k(z)}{H(z)r^6(z)} \ldots B_{mm}(k_{\ell_1}, k_{\ell_2}, k_{\ell_3}, z)$$

$$\langle \kappa^4 \rangle_{ijkl} = \int_0^\infty dz \frac{\mathcal{W}_i(z)\mathcal{W}_j(z)\mathcal{W}_k(z)\mathcal{W}_l(z)}{H(z)r^8(z)} \ldots T_{mm}(k_{\ell_1}, k_{\ell_2}, k_{\ell_3}, k_{\ell_4} z)$$

κ - ℓ1 norm

$$\ell_1^{\theta_j,i} = \sum_{k=1}^{\#\,\mathrm{coef}\left(S_{\theta_j,i}\right)} \left|S_{\theta_j,i}[k]\right| / N / \Delta B$$

Higher-order statistics – topological

κ - peak counts
(Map -)

$$M_{ap}(\boldsymbol{\theta};\vartheta) = \int d^2\theta' \, U_\vartheta(|\boldsymbol{\theta} - \boldsymbol{\theta'}|) \, \kappa(\boldsymbol{\theta'})$$

κ – Minkowski functionals

$$V_0(\nu) = \frac{1}{A}\int_{Q_\nu} da$$

$$V_1(\nu) = \frac{1}{4A}\int_{\partial Q_\nu} dl$$

$$V_2(\nu) = \frac{1}{2\pi A}\int_{\partial Q_\nu}^{\partial} dl \, \mathcal{K}$$

κ - Betti numbers

$$\beta_0(\nu) = \int_\nu^\infty d\nu' \, \left\{\mathcal{N}_2(\nu') - [1 - g(\nu')]\mathcal{N}_1(\nu')\right\}$$

$$\beta_1(\nu) = \int_\nu^\infty d\nu' \, [g(\nu')\mathcal{N}_1(\nu') - \mathcal{N}_0(\nu')]$$

Gaussianity test

Assuming Gaussian distributed HOS

SMAPE-test (Symmetrized Mean Absolute Percentage Error)

$$y_i = \left(\mathbf{D_i} - \langle \mathbf{D} \rangle\right)^{\mathrm{T}} \mathsf{COV}^{-1} \left(\mathbf{D_i} - \langle \mathbf{D} \rangle\right)$$

If N_d ~ N_f, y_i no more follows χ^2 distribution

500 realizations from multivariate Gaussian, with mean and variance as SLICS

→mean $P_{gauss}(y_i)$ as new target realization

$$\mathcal{S}_{\mathrm{lim}} = \left\langle \mathcal{S}_{\mathrm{obs}}^{\mathrm{gauss}} \right\rangle + 2\sigma \left(\mathcal{S}_{\mathrm{obs}}^{\mathrm{gauss}} \right)$$

S_{mean} and S_{sd} from mean from 500 realizations

Metrics-based method

$$|\Delta\mathcal{M}| > 3\sigma(\Delta\mathcal{M}) \simeq 3\left[\sigma^2\left(\mathcal{M}_{GR}\right) + \sigma^2\left(\mathcal{M}_{MG}\right)\right]^{1/2}$$

i. For metric M and observable O, compute fiducial DV in GR/MG from 256 DUSTGRAIN maps.
ii. Generate 256 DVs (D_{rnd}) for GR/MG from Gaussian centered on fiducial values and with covariance from GR.
iii. For each realization, compute M for either D_{rnd} GR or D_{rnd} MG → M_{rnd} GR, M_{rnd} MG
iv. Repeat steps i. - iii. 500 times. From set (M_{rnd} GR, M_{rnd} MG) estimate (M_{GR}, M_{MG}) and errors: median and symmetrized 68% confidence ranges of distributions.

Hypothesis-rejection method - Results