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A possible solution to the gallium anomaly moving beyond the leptonic wave function factorization

Luca Ferro

luca.ferro@ca.infn.it

University of Cagliari & INFN Cagliari

Presentation based on

PHYSICAL REVIEW D **113,** 033006 (2026)

Reassessing the gallium anomaly using self-consistent electron wave functions

M. Cadeddu,[1,*] N. Cargioli,[1,†] G. Carotenuto,[2,‡] F. Dordei,[1,§] L. Ferro,[2,1,∥] and C. Giunti[3,¶]

[1] *Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Cagliari, Complesso Universitario di Monserrato—Strada Provinciale per Sestu Km 0.700, 09042 Monserrato (Cagliari), Italy*

[2] *Dipartimento di Fisica, Università degli Studi di Cagliari, Complesso Universitario di Monserrato—Strada Provinciale per Sestu Km 0.700, 09042 Monserrato (Cagliari), Italy*

[3] *Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Torino, Via P. Giuria 1, I–10125 Torino, Italy*

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The gallium anomaly, a persistent discrepancy exceeding 4σ in the ^{71}Ga neutrino capture rates from ^{51}Cr and ^{37}Ar radioactive sources by the GALLEX, SAGE, and recently BEST experiments, has challenged particle physics and nuclear theory for over three decades. We present a new calculation of the neutrino capture cross section, abandoning the conventional leading-order approximation for electronic wave functions by numerically solving the Dirac-Coulomb equation for both bound and continuum electron states. Finally, we reevaluate the gallium anomaly, updating its global significance and presenting the most up-to-date status of its interpretation in terms of sterile neutrinos.

M. Cadeddu, **L. Ferro** et al. PHYSICAL REVIEW D 113, 033006 (2026)

A possible solution to the gallium anomaly moving beyond the leptonic wave function factorization

M. Cadeddu,[1, a] N. Cargioli,[1, b] F. Dordei,[1, c] L. Ferro,[2, 1, d] C. Giunti,[3, e] and M. Pitzalis[2, 1, f]

[1] *Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Cagliari, Complesso Universitario di Monserrato - S.P. per Sestu Km 0.700, 09042 Monserrato (Cagliari), Italy*

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(Dated: 21/01/26)

For over thirty years, a $\sim 20\%$ deficit, now exceeding 5σ, has persisted between measured and predicted neutrino capture rates on ^{71}Ga, as observed in radioactive source experiments (namely GALLEX, SAGE, and more recently BEST) using ^{51}Cr and ^{37}Ar. This long-standing discrepancy, referred to as the gallium anomaly, has posed a significant challenge to our understanding of both experimental methods and theoretical predictions. In this work, we revisit the theoretical calculation of the neutrino capture cross-section by moving beyond the standard treatment of the leptonic wave functions, revealing limitations in the commonly used factorization approach based on the detailed balance principle. Incorporating phenomenologically constrained Gamow-Teller transition densities, able to correctly reproduce the precisely measured half-life of ^{71}Ge, we find that the revised cross-section can be significantly reduced, potentially resolving the gallium anomaly without invoking new physics.

M. Cadeddu, **L. Ferro** et al. arXiv.2512.20560

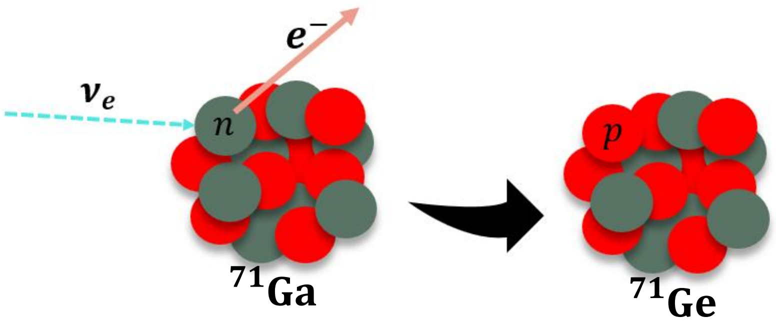
Short-baseline anomaly

Gallium	$\nu_e \to \nu_e$ (disappearance)	Very Significant ($> 5\sigma$)	Unknown (many tests failed)

Gallium Source Experiments

SAGE and **GALLEX** experiments have been studying solar neutrinos through the neutrino capture reaction on a gallium target:

$$\nu_e + {}^{71}_{31}\mathrm{Ga} \rightarrow e^- + {}^{71}_{32}\mathrm{Ge}$$



Artificial neutrinos source used for calibration :

$$e^- + {}^{51}\mathrm{Cr} \rightarrow \nu_e + {}^{51}\mathrm{V}$$

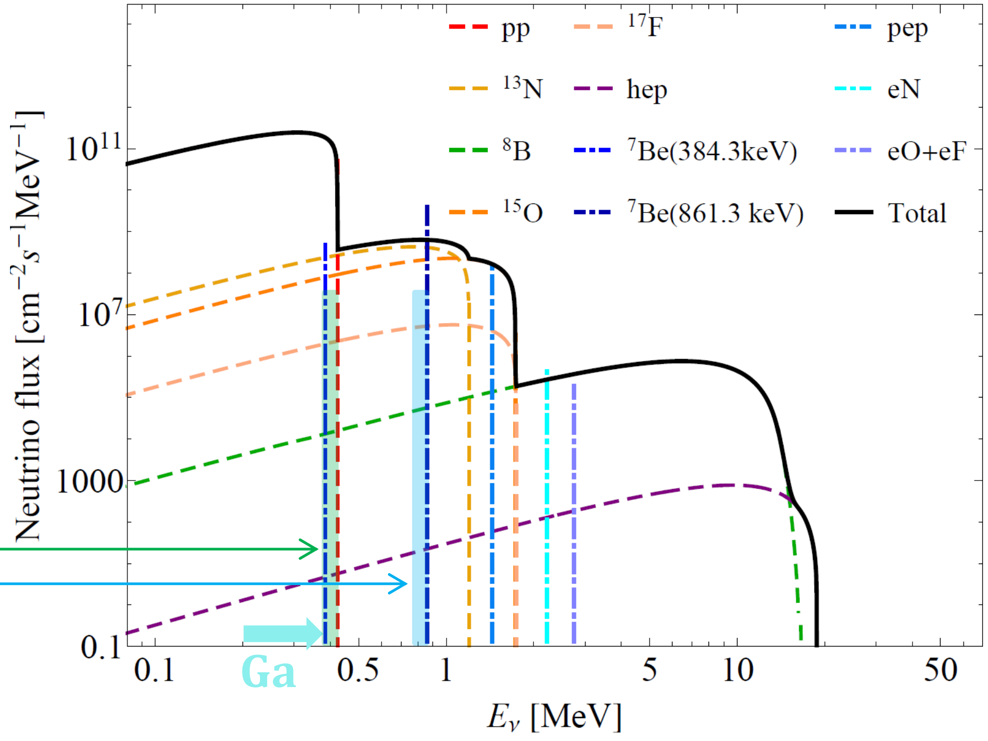
$$e^- + {}^{37}\mathrm{Ar} \rightarrow \nu_e + {}^{37}\mathrm{Cl}$$

$$Q_{EC} = 232.47(9)\ \mathrm{keV}$$

Nuclear Data Sheets 188, 1 (2023).

Gallium can also absorb neutrinos from the solar pp chain.

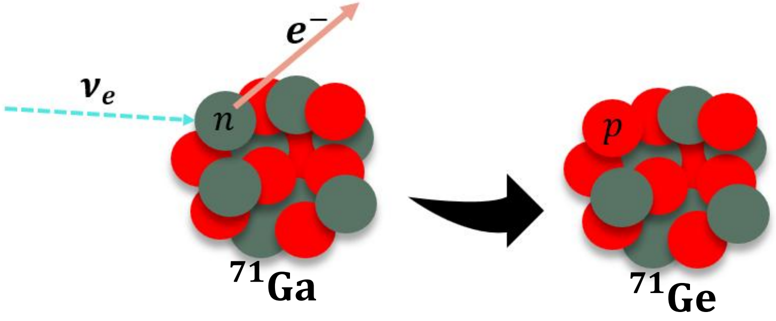
Source	E_ν (MeV)
^{51}Cr	0.7524
	0.7518
	0.7465
	0.4323
	0.4317
	0.4264
^{37}Ar	0.8138
	0.8135
	0.8107



Evidence for the Anomaly

SAGE and **GALLEX** experiments have been studying solar neutrinos through the neutrino capture reaction on a gallium target:

$$\nu_e + {}^{71}_{31}\mathrm{Ga} \rightarrow e^- + {}^{71}_{32}\mathrm{Ge}$$



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$$e^- + {}^{51}\mathrm{Cr} \rightarrow \nu_e + {}^{51}\mathrm{V}$$

$$e^- + {}^{37}\mathrm{Ar} \rightarrow \nu_e + {}^{37}\mathrm{Cl}$$

After the results from **BEST** the anomaly increases its significance to approximately 5.5σ

M. Cadeddu, **L. Ferro** et al. PHYSICAL REVIEW D 113, 033006 (2026)

Source	$\sigma^{\mathrm{db}}_{\mathrm{gs}}$	$\bar{\sigma}^{\mathrm{db}}_{\mathrm{gs}}$	$\bar{\sigma}^{\mathrm{db}}_{(p,n)}$ +6%	$\bar{\sigma}^{\mathrm{db}}_{(^3\mathrm{He},^3\mathrm{H})}$ +9%
^{51}Cr	5.42 ± 0.06	5.28 ± 0.06	$5.57^{+0.28}_{-0.07}$	5.73 ± 0.17
^{37}Ar	6.50 ± 0.07	6.32 ± 0.07	$6.71^{+0.35}_{-0.09}$	6.93 ± 0.22

Considering only the ground-state cross section: $\bar{R}_{gs} = 0.881 \pm 0.031$ → Anomaly at 3.8σ

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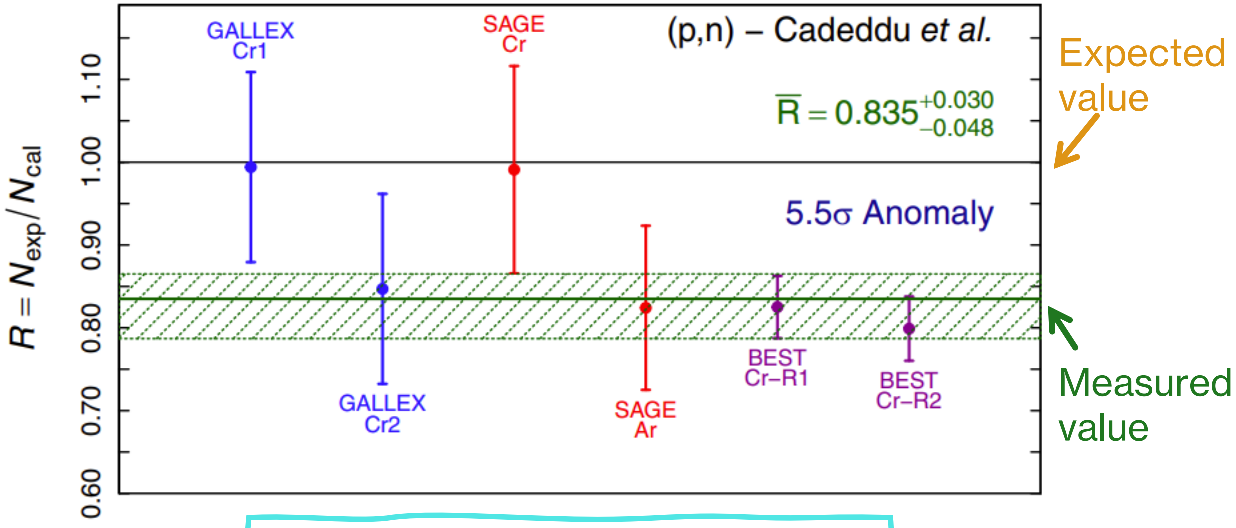
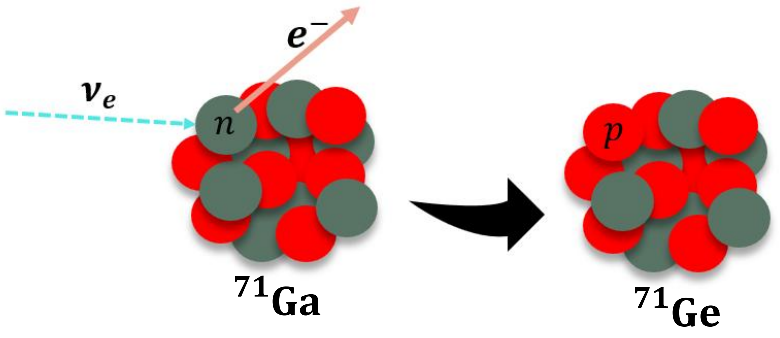


Calibration data require a ~ 20% cross-section reduction.

Neutrino Capture Cross Section

$$\nu_e + {}^{71}_{31}\mathrm{Ga} \rightarrow \mathrm{e}^- + {}^{71}_{32}\mathrm{Ge}$$



Axial-vector coupling const.

CKM Matrix el.

Energy e

Matrix el.

$$\sigma_{\rm gs} = \frac{G_F^2\,|V_{ud}|^2\,g_A^2}{\pi(2J_{\rm Ga}+1)} \sum_j p_e^j E_e^j \left|\mathcal{H}_j^{\rm IBD}\right|^2 \mathcal{B}(E_e^j)$$

Fermi constant

Momentum e

Branching ratio

g_k and f_k : large and small Dirac electron wave-function radial components for a given eigenvalue k, for allowed transition $(\Delta J^\pi = 1^+) \rightarrow k = \pm 1$

j_0 and j_1 : neutrino wave functions, spherical Bessel functions $j_l(qr)$ of order 0 and 1

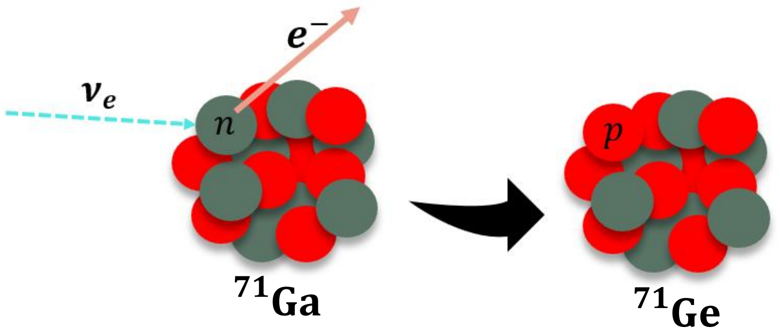
$$|\mathcal{H}^{\rm IBD}|^2 = (4\pi)^2 \left[\left| \int dr\, r^2 \rho_{\rm TD}(r) \left[g_{-1}(r) j_0(qr) + \tfrac{1}{3} f_{-1}(r) j_1(qr) \right] \right|^2 + \left| \int dr\, r^2 \rho_{\rm TD}(r) \left[f_1(r) j_0(qr) - \tfrac{1}{3} g_1(r) j_1(qr) \right] \right|^2 \right]$$

Gamow-Teller transition density represents the overlap of the initial and final nuclear wave functions through the weak interaction operator.

$$\rho_{\rm TD}(\mathbf{r}) = \Psi^*_{^{71}\mathrm{Ge}}(\mathbf{r})\,\hat{H}_{\rm GT}\,\Psi_{^{71}\mathrm{Ga}}(\mathbf{r})$$

Neutrino Capture Cross Section

$$\nu_e + {}^{71}_{31}\mathrm{Ga} \rightarrow e^- + {}^{71}_{32}\mathrm{Ge}$$



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Fermi constant

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Gamow-Teller transition density represents the overlap of the initial and final nuclear wave functions through the weak interaction operator.

In principle, we must **calculate the exact overlap** between the leptonic wave functions and the nuclear transition density.

Required ingredients:

Leptonic wave functions 👍

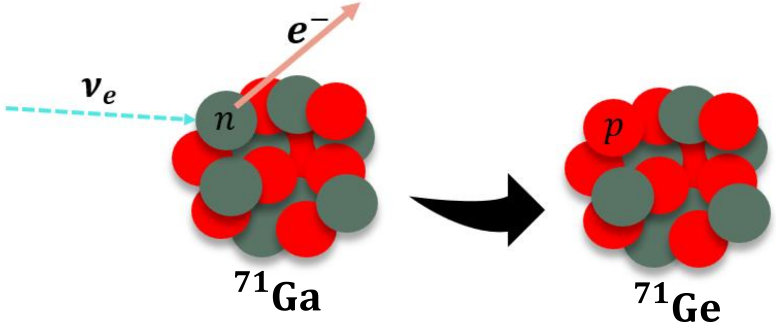
Nuclear transition density 👎

The Factorization Approximation

In the classical treatment, one invokes the **factorization of the leptonic wave functions**: $\nu_e + {}^{71}_{31}\mathrm{Ga} \rightarrow e^- + {}^{71}_{32}\mathrm{Ge}$

Both leptonic wave functions (electron and neutrino) are evaluated at $r_0 \simeq 0$.

$$j_l(qr_0) \simeq \frac{(qr_0)^l}{(2l+1)!!}$$



Factorization scheme:

$$|\mathcal{H}^{\mathrm{IBD}}|^2 = (4\pi)^2 \left[\left| \int dr\, r^2 \rho_{\mathrm{TD}}(r) \left[g_{-1}(r) j_0(qr) + \tfrac{1}{3} f_{-1}(r) j_1(qr) \right] \right|^2 + \left| \int dr\, r^2 \rho_{\mathrm{TD}}(r) \left[f_1(r) j_0(qr) - \tfrac{1}{3} g_1(r) j_1(qr) \right] \right|^2 \right]$$

$$\simeq (4\pi)^2 \left[g_{-1}(r_0)^2 + f_1(r_0)^2 \right] \underbrace{\left| \int dr\, r^2 \rho_{\mathrm{TD}}(r) \right|^2}_{|\mathcal{M}^{\mathrm{IBD}}_{\mathrm{nuc}}|^2}$$

Nuclear matrix element

The Factorization Approximation

In the classical treatment, one invokes the **factorization of the leptonic wave functions**: $\nu_e + {}^{71}_{31}\mathrm{Ga} \to \mathrm{e}^- + {}^{71}_{32}\mathrm{Ge}$

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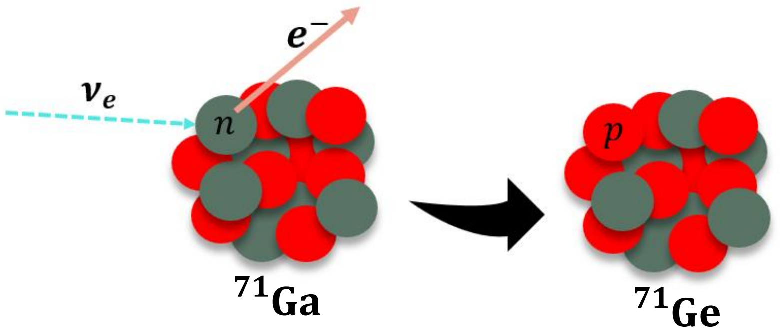


Factorization scheme:

$$|\mathcal{H}^{\mathrm{IBD}}|^2 = (4\pi)^2\left[\left|\int dr\, r^2 \rho_{\mathrm{TD}}(r)\left[g_{-1}(r)j_0(qr) + \tfrac{1}{3}f_{-1}(r)j_1(qr)\right]\right|^2 + \left|\int dr\, r^2\rho_{\mathrm{TD}}(r)\left[f_1(r)j_0(qr) - \tfrac{1}{3}g_1(r)j_1(qr)\right]\right|^2\right]$$

$$\simeq (4\pi)^2\left[g_{-1}(r_0)^2 + f_1(r_0)^2\right]\left|\int dr\, r^2\rho_{\mathrm{TD}}(r)\right|^2$$

$$|\mathcal{M}^{\mathrm{IBD}}_{\mathrm{nuc}}|^2$$

Nuclear matrix element

Challenge:
Direct *ab initio* calculations of the nuclear transition density is challenging

Solution: Detailed Balance!

The **inverse electron-capture** reaction $^{71}\mathrm{Ge} \to {}^{71}\mathrm{Ga}$ exists, and from its lifetime a value for the IBD matrix element can be extracted.

Using Detailed Balance

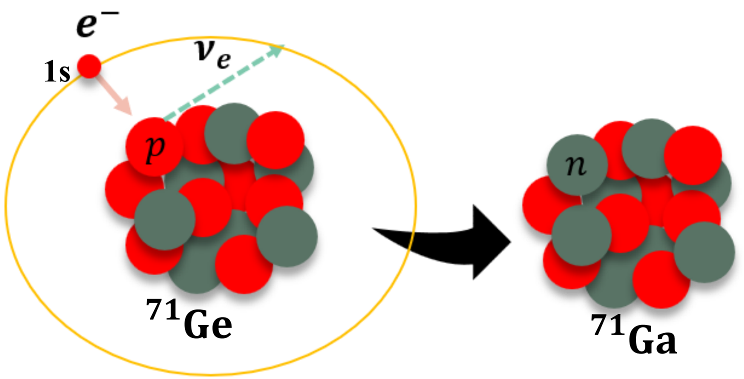
$$e^-_{\rm bound} + {}^{71}_{32}{\rm Ge} \rightarrow {}^{71}_{31}{\rm Ga} + \nu_e$$

$$\frac{1}{t_{1/2}} = \frac{G_F^2\,|V_{ud}|^2\,g_A^2}{\pi\ln 2(2J_{\rm Ge}+1)}(E_\nu^{1s})^2\,|\mathcal{H}^{\rm EC}_{1s}|^2\left[1+\frac{P_{\rm L}+P_{\rm M}}{P_{\rm K}}\right]$$

^{71}Ge half life : 11.465 ± 0.003 d

Exp. measured EC prob. for L, M and K shells.

$$\frac{P_{\rm L}}{P_{\rm K}} = 0.1174 \pm 0.0009$$
$$\frac{P_{\rm M}}{P_{\rm K}} = 0.0188 \pm 0.0004$$



Without factorization scheme: $|H^{\rm IBD}|^2 \neq |H^{\rm EC}|^2$
since the leptonic wave functions are different

$$|\mathcal{H}^{\rm EC}_{1s}|^2 = \left|\int dr\,4\pi\,r^2\rho_{\rm TD}(r)\left[g_{1s}(r)j_0(q\,r) + \tfrac{1}{3}f_{1s}(r)j_1(q\,r)\right]\right|^2$$

g_{1s} and f_{1s} : large and small Dirac electron **bound** wave-function radial components

The **detailed balance** is valid only if the leptonic wave function can be factorized from the nuclear one

$$|\mathcal{M}^{\rm IBD}_{\rm nuc}|^2 \overset{\rm db}{=} |\mathcal{M}^{\rm EC}_{\rm nuc}|^2$$

Using Detailed Balance

$$e^{-}_{\text{bound}} + {}^{71}_{32}\text{Ge} \rightarrow {}^{71}_{31}\text{Ga} + \nu_e$$

$$\frac{1}{t_{1/2}} = \frac{G_F^2\,|V_{ud}|^2\,g_A^2}{\pi \ln 2 (2J_{\text{Ge}}+1)} (E_\nu^{1s})^2\, |\mathcal{H}^{\text{EC}}_{1s}|^2 \left[1 + \frac{P_{\text{L}} + P_{\text{M}}}{P_{\text{K}}}\right]$$

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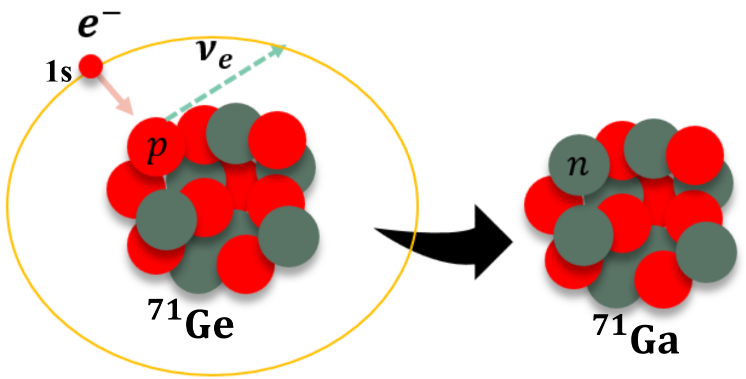


Without factorization scheme: $|H^{\text{IBD}}|^2 \neq |H^{\text{EC}}|^2$
since the leptonic wave functions are different

$$|\mathcal{H}^{\text{EC}}_{1\text{s}}|^2 = \left| \int dr\, 4\pi\, r^2 \rho_{\text{TD}}(r) \left[g_{1\text{s}}(r) j_0(q\,r) + \tfrac{1}{3} f_{1\text{s}}(r) j_1(q\,r) \right] \right|^2$$

g_{1s} and f_{1s} : large and small Dirac electron **bound** wave-function radial components

The **detailed balance** is valid only if the leptonic wave function can be factorized from the nuclear one

$$|\mathcal{M}^{\text{IBD}}_{\text{nuc}}|^2 \overset{\text{db}}{=} |\mathcal{M}^{\text{EC}}_{\text{nuc}}|^2$$

Without factorization, does EC still constrain neutrino capture?

Yes – via transition densities (TDs).

$$\rho^{\text{IBD}}_{\text{TD}}(r) = \rho^{\text{EC}}_{\text{TD}}(r)$$

Constraining the Transition Density

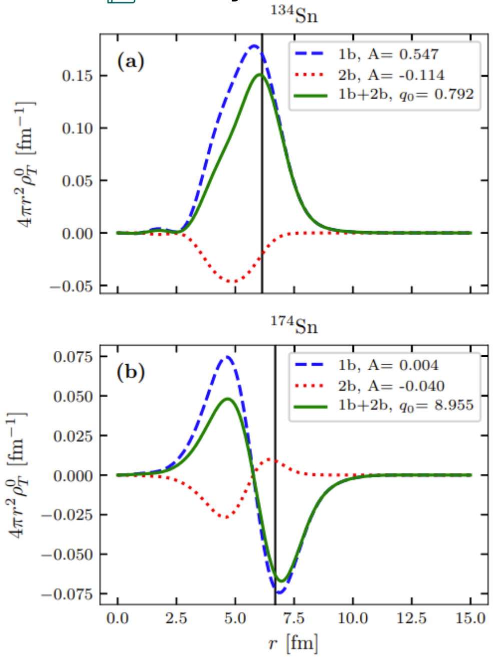
We use phenomenological parameterizations to describe the Transition Density

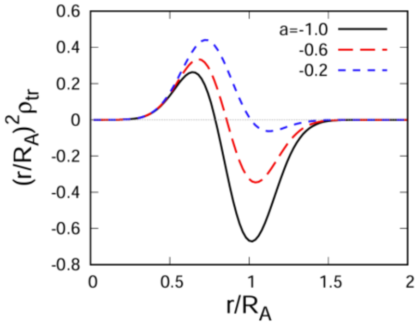
$$\rho_{\rm TD}^{\rm DG}(r,\Theta^{\rm DG}) = A\,e^{-(r-r_a)^2/2a^2} - B\,e^{-(r-r_b)^2/2b^2}$$

$$\rho_{\rm TD}^{\rm mDG}(r,\Theta^{\rm mDG}) = Ar\,e^{-(r-r_a)^2/2a^2} + B\,r^2\,e^{-(r-r_b)^2/2b^2}$$

This choice of parametrizations is guided by other studies [G. B. King, PRC 102, 025501 (2020)] showing that **weak transition densities are generally smooth, extending beyond the nuclear radius**.

E. Ney et al. PRC 105.034349





Horiuchi et al. Arxiv:2103.16815

Results: Solving the Gallium Anomaly

We use phenomenological parameterizations to describe the Transition Density

$$\rho_{\rm TD}^{\rm DG}(r,\Theta^{\rm DG}) = A\,e^{-(r-r_a)^2/2a^2} - B\,e^{-(r-r_b)^2/2b^2}$$

$$\rho_{\rm TD}^{\rm mDG}(r,\Theta^{\rm mDG}) = A r\,e^{-(r-r_a)^2/2a^2} + B\,r^2\,e^{-(r-r_b)^2/2b^2}$$

M. Cadeddu, **L. Ferro** et al. arXiv.2512.20560

Model	$\sigma_{{\rm gs},{}^{51}{\rm Cr}}(\Theta)[10^{-45}\ {\rm cm}^2]$	$\sigma_{{\rm gs},{}^{37}{\rm Ar}}(\Theta)[10^{-45}\ {\rm cm}^2]$	$t_{1/2}(\Theta)$ [d]
DG ✓	4.42	5.13	11.462
mDG ✓	4.50	5.24	11.465

Each transition density parametrization will depend upon a certain set of parameters **Θ**

We investigate which values of **Θ** lead to transition densities that yield a theoretical ground-state (gs) cross section capable of explaining the gallium anomaly.

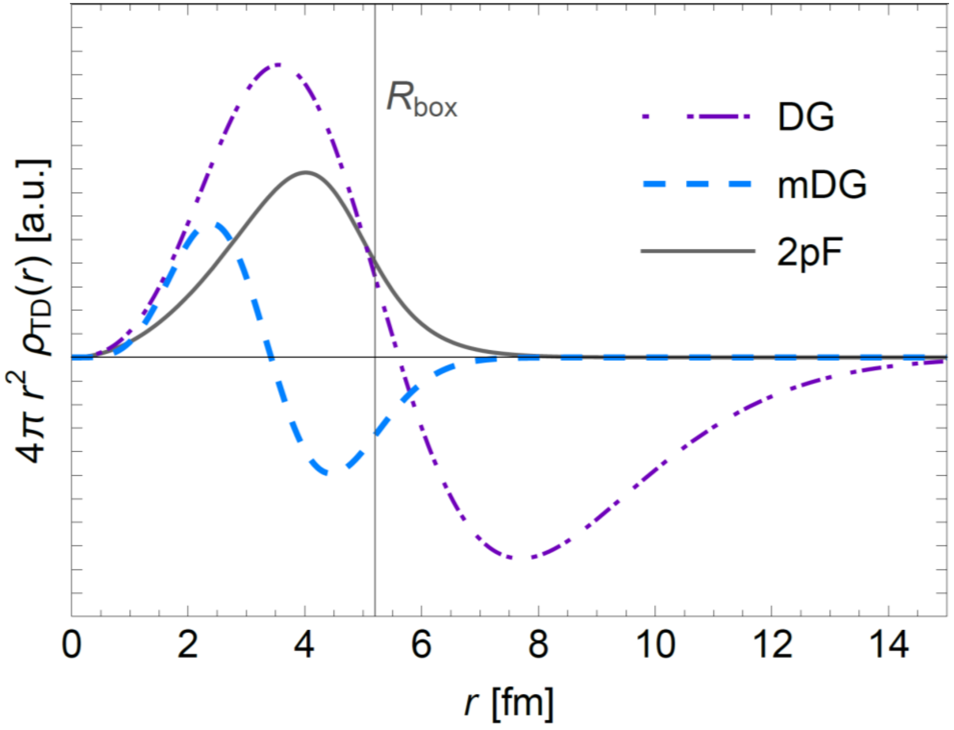
$$\sigma^{\rm exp}_{{\rm gs},{}^{51}{\rm Cr}} = (4.44 \pm 0.33)\times 10^{-45}\ {\rm cm}^2$$

$$\sigma^{\rm exp}_{{\rm gs},{}^{37}{\rm Ar}} = (5.21 \pm 0.72)\times 10^{-45}\ {\rm cm}^2$$

We constrain the Gamow-Teller transition density so that **it reproduces the measured 71Ge half-life** $t^{\rm exp}_{1/2} = 11.465 \pm 0.003$ d

Ground-state cross-section which solves the Gallium Anomaly if a ~5% excited states contributions is present.

M. Cadeddu, **L. Ferro** et al. arXiv.2512.20560



Conclusions

- The standard approach of factorizing leptonic wave functions fundamentally overestimates the theoretical neutrino capture cross-section.

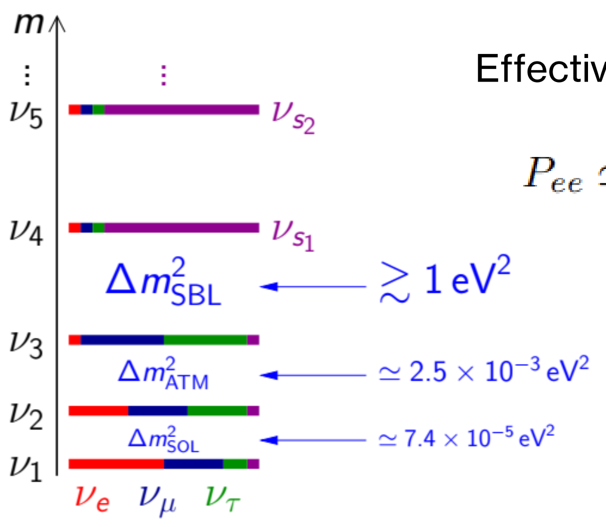
- By incorporating phenomenologically constrained Gamow-Teller transition densities, the revised theoretical cross-section is significantly reduced.

- A **node or sign change** in $\rho_{\mathrm{TD}}(r)$ induces radial cancellations that reduce the ground-state cross section by ~ 20%. This mechanism accurately reproduces the $^{71}\mathbf{Ge}$ half-life, offering a Standard Model resolution to the gallium anomaly without invoking new physics.

Backup

Sterile neutrino interpretation

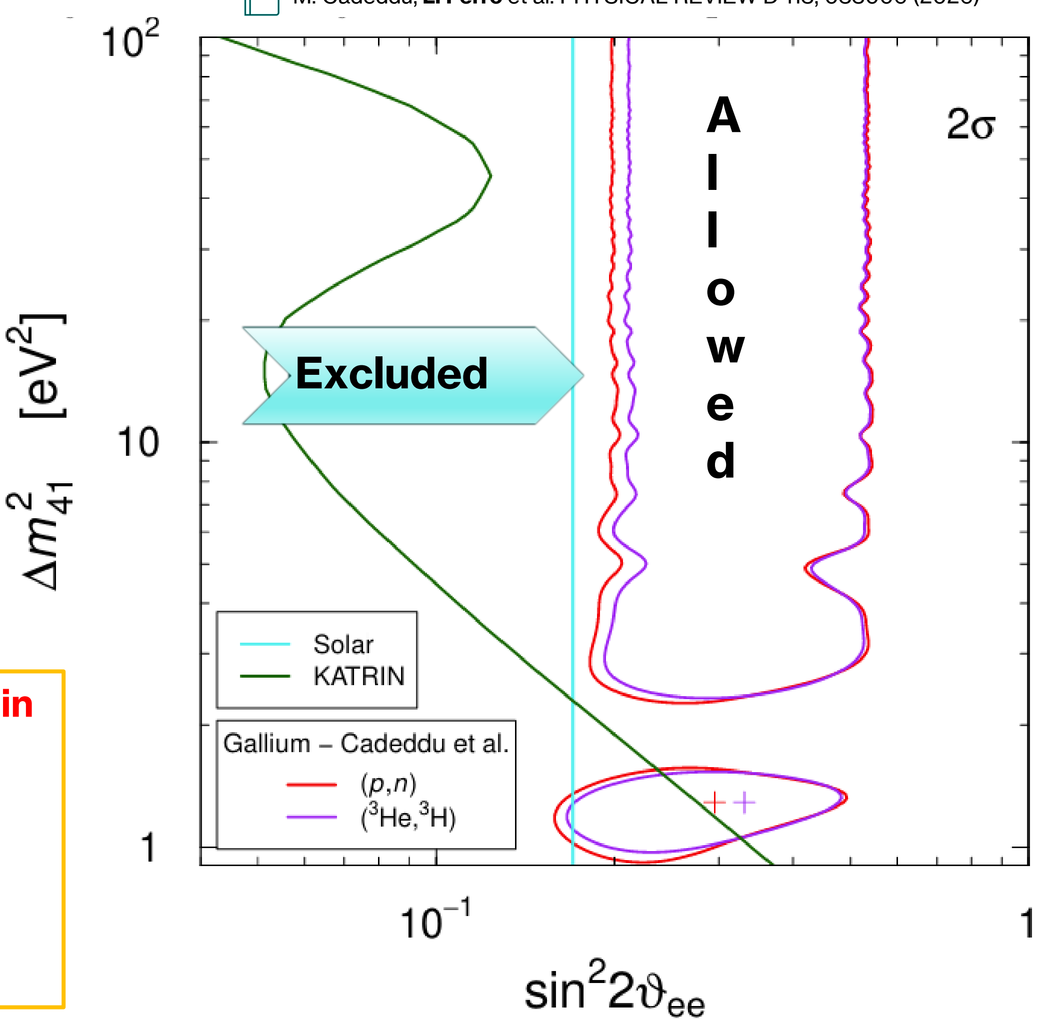
The simplest and most considered explanation of the gallium anomaly is the hypothesis of **short-baseline active-sterile neutrino oscillations.**



Effective oscillation probability in the 3+1 scheme

$$P_{ee} \simeq 1 - \sin^2 2\vartheta_{ee} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

M. Cadeddu, **L. Ferro** et al. PHYSICAL REVIEW D 113, 033006 (2026)



Suggested parameter space :

- $\Delta m^2_{41} \sim 1\ \mathrm{eV}^2$.
- $0.2 < \sin^2 2\theta_{ee} < 0.6$

The allowed region from **the gallium data is in tension** with the bounds obtained from the analysis of reactor antineutrino data [STEREO, DANSS, PROSPECT], the solar neutrino bound [M.C. Gonzalez-Garcia et al. 2411.16840], and the KATRIN neutrino mass measurement [Acharya et al. 2503.18667]

Statistical Fit Procedure

To find the best set of parameters to solve or alleviate the gallium anomaly, given a certain parametrization, we perform a fit using the χ^2 function with $X = ({}^{51}\mathrm{Cr}, {}^{37}\mathrm{Ar})$

M. Cadeddu, **L. Ferro** et al. arXiv.2512.20560

$$\chi^2_{\mathrm{IBD}}(\Theta) = \frac{\eta_1^2}{\delta^2_{\eta_1}} + \sum_X \left(\frac{\sigma^{\mathrm{exp}}_{\mathrm{gs},\,X} - (1+\eta_1)\sigma_{\mathrm{gs},\,X}(\Theta)}{\delta\sigma^{\mathrm{exp}}_{\mathrm{gs},\,X}} \right)^2$$

Free electron wave-functions unc. (0.003%)

The parameters are constrained to maintain the ${}^{71}\mathrm{Ge}$ half-life at its experimental value.

Bound electron wave-functions unc. (0.005)

$$\chi^2_{\mathrm{EC}}(\Theta) = \left(\frac{t^{\mathrm{exp}}_{1/2} - (1+\eta_2+\eta_3)\cdot t_{1/2}(\Theta)}{\delta t_{1/2}} \right)^2 + \frac{\eta_2^2}{\delta^2_{\eta_2}} + \frac{\eta_3^2}{\delta^2_{\eta_3}}$$

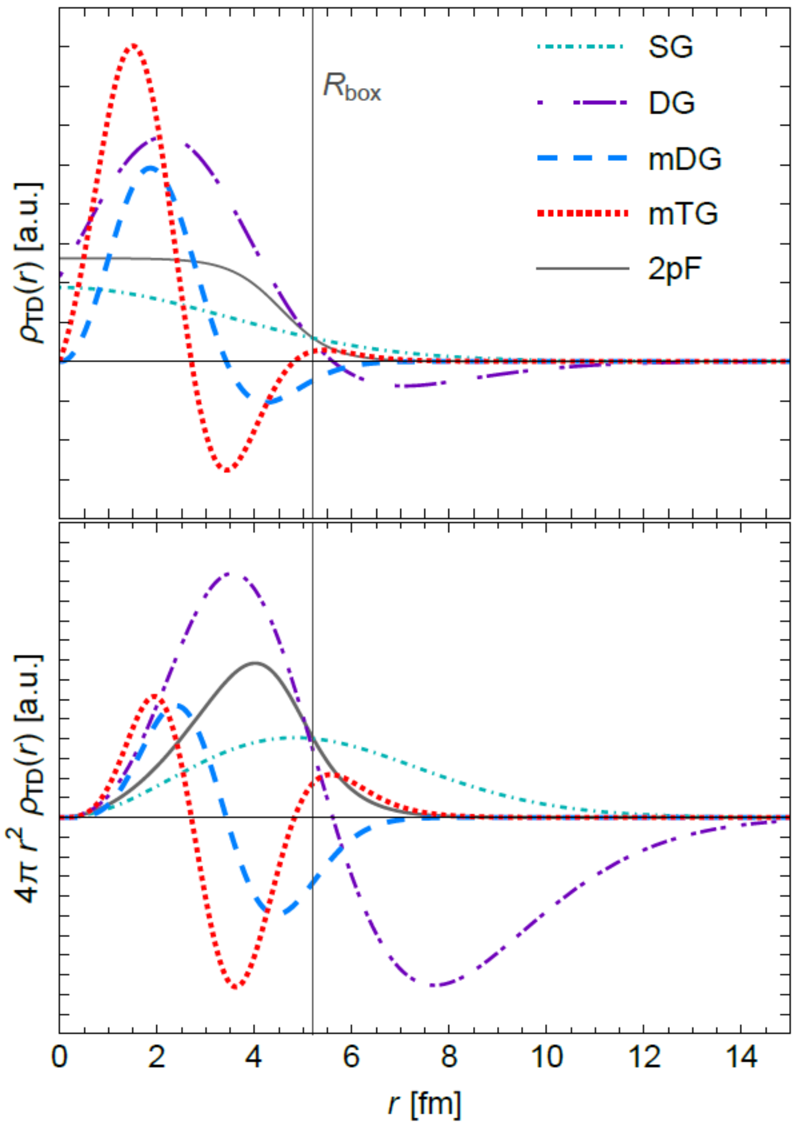
Contribution due to the **exchange** and **overlap** corrections (0.007)

$$\rho^{\mathrm{SG}}_{\mathrm{TD}}(r,\Theta^{\mathrm{SG}}) = A\, e^{-(r-r_a)^2/2a^2},$$

$$\rho^{\mathrm{DG}}_{\mathrm{TD}}(r,\Theta^{\mathrm{DG}}) = A\, e^{-(r-r_a)^2/2a^2} - B\, e^{-(r-r_b)^2/2b^2},$$

$$\rho^{\mathrm{mDG}}_{\mathrm{TD}}(r,\Theta^{\mathrm{mDG}}) = Ar\, e^{-(r-r_a)^2/2a^2} + B\, r^2\, e^{-(r-r_b)^2/2b^2},$$

$$\rho^{\mathrm{mTG}}_{\mathrm{TD}}(r,\Theta^{\mathrm{mTG}}) = Ar\, e^{-(r-r_a)^2/2a^2} + B\, r^2\, e^{-(r-r_b)^2/2b^2} + Cr^3 e^{-(r-r_c)^2/2c^2},$$



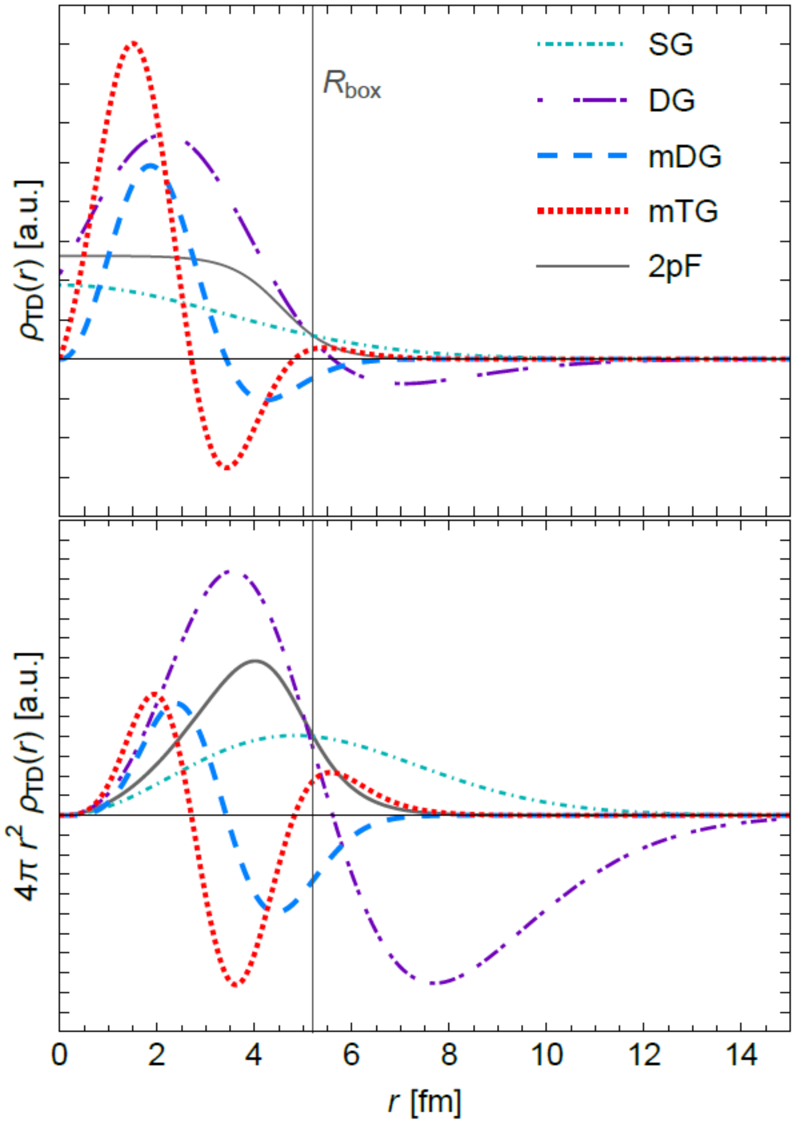
Results

M. Cadeddu, **L. Ferro** et al. arXiv.2512.20560

Model	Best fit parameter values Θ	$\sigma_{\text{gs},^{51}\text{Cr}}(\Theta)$ [10^{-45} cm^2]	$\sigma_{\text{gs},^{37}\text{Ar}}(\Theta)$ [10^{-45} cm^2]	$t_{1/2}(\Theta)$ [d]	χ^2_{IBD}	χ^2_{EC}	GA solved
SG	{0.00094,-0.005,3.42}	5.27	6.30	11.515	8.68	0.24	no
DG	{-0.0231,-0.0802,3.5119,1.855,2.16,2.16}	4.42	5.13	11.462	0.014	0.0005	yes
mDG	{-0.1498,0.3462,1.192,1.3793,2.6192,0.466405}	4.50	5.24	11.465	0.037	0.00003	yes
mTG	{3.514009,4.53567,-1.5007,1.9457,1.3809, 1.14085,0.0029,0.8595,1.513}	4.39	5.10	11.463	0.044	0.0002	yes

$$\rho_{\text{TD}}^{\text{SG}}(r,\Theta^{\text{SG}}) = A\, e^{-(r-r_a)^2/2a^2}\,,$$
$$\rho_{\text{TD}}^{\text{DG}}(r,\Theta^{\text{DG}}) = A\, e^{-(r-r_a)^2/2a^2} - B\, e^{-(r-r_b)^2/2b^2}\,,$$
$$\rho_{\text{TD}}^{\text{mDG}}(r,\Theta^{\text{mDG}}) = Ar\, e^{-(r-r_a)^2/2a^2} + B\, r^2\, e^{-(r-r_b)^2/2b^2}\,,$$
$$\rho_{\text{TD}}^{\text{mTG}}(r,\Theta^{\text{mTG}}) = Ar\, e^{-(r-r_a)^2/2a^2} + B\, r^2\, e^{-(r-r_b)^2/2b^2} + Cr^3 e^{-(r-r_c)^2/2c^2}\,,$$

The mDG and mTG were introduced as they mimic, in a minimal analytic way, the typical polynomial-Gaussian structure of products of single particle wave functions, as for the case of harmonic oscillator radial wave functions



Electron wave function and electron capture rate

Electron wave function: PHYSICAL REVIEW C **111**, 064323 (2025)

$$\Psi_{e^-}(\boldsymbol{r}) = \begin{pmatrix} G_{\kappa_x}(\boldsymbol{r})\chi_{\kappa_x\mu_x} \\ iF_{\kappa_x}(\boldsymbol{r})\chi_{-\kappa_x\mu_x} \end{pmatrix}$$

$$\left(\frac{d}{dr} + \frac{\kappa+1}{r}\right) G_\kappa - (E_e - V(r) + m_e) F_\kappa = 0,$$
$$\left(\frac{d}{dr} - \frac{\kappa-1}{r}\right) F_\kappa + (E_e - V(r) - m_e) G_\kappa = 0,$$

$$V_{\rm DHFS}(r) = V_{\rm nuc}(r) + V_{\rm el}(r) + V_{\rm ex}(r),$$

$$V_{\rm ex}(r) = -C_{\rm ex} e^2 \left(\frac{3}{\pi}\right)^{1/3} [\rho(r)]^{1/3}$$

Cex = 3/2 corresponds to the Slater approximation while Cex = 1 to the Kohn-Sham one

Electron capture rate from an x orbital : A. Ravlic et al. PHYSICAL REVIEW C **111**, 064323 (2025)

$$\lambda_x = \frac{G_F^2 V_{ud}^2}{2\pi} \sum_{i,f} \left(E_0^{(i,f)} + E_x\right)^2 \times \sum_{\kappa_\nu} \sum_{JL} \frac{1}{2J_i+1} |\langle f || \Xi_{JL}(\kappa_x, \kappa_\nu) || i\rangle|^2,$$

$$\begin{aligned}
\langle i || \Xi_{JL}(\kappa_x, \kappa_\nu) || f\rangle = \int dr \Bigg\{ & -g_V \sqrt{\frac{2J+1}{4\pi}} \delta\rho_{C_J}^{(i,f)}(\boldsymbol{r}) [g_{\kappa_\nu} G_{\kappa_x} S_{0JJ}(\kappa_\nu, \kappa_x) - f_{\kappa_\nu} F_{\kappa_x} S_{0JJ}(-\kappa_\nu, -\kappa_x)] \\
& - g_A \sqrt{\frac{2J+1}{4\pi}} \delta\rho_{C_J\gamma_5}^{(i,f)}(\boldsymbol{r}) [g_{\kappa_\nu} F_{\kappa_x} S_{0JJ}(\kappa_\nu, -\kappa_x) + f_{\kappa_\nu} G_{\kappa_x} S_{0JJ}(-\kappa_\nu, \kappa_x)] \\
& + g_V \sqrt{\frac{2L+1}{4\pi}} \delta\rho_{[C_L\otimes\alpha]_J}^{(i,f)}(\boldsymbol{r}) [g_{\kappa_\nu} F_{\kappa_x} S_{1LJ}(\kappa_\nu, -\kappa_x) + f_{\kappa_\nu} G_{\kappa_x} S_{1LJ}(-\kappa_\nu, \kappa_x)] \\
& + g_A \sqrt{\frac{2L+1}{4\pi}} \delta\rho_{[C_L\otimes\Sigma]_J}^{(i,f)}(\boldsymbol{r}) [g_{\kappa_\nu} G_{\kappa_x} S_{1LJ}(\kappa_\nu, \kappa_x) - f_{\kappa_\nu} F_{\kappa_x} S_{1LJ}(-\kappa_\nu, -\kappa_x)] \Bigg\},
\end{aligned}$$