

Non-Adiabatic Mixing(Transitions) in the Density Matrix Formalism

Shreya Girishkumar Pandit

Based on a paper with Pasquale Di Bari[†] and Ye-Ling Zhou^{††}

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School of Physics and Astronomy, University of Southampton

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[†] University of Southampton

^{††} HIAS, UCAS, Hangzhou

Why Study Mixing of Quantum States?

In High Energy Physics:

- Neutrino Oscillations
- Active-sterile mixing
- Leptogenesis
- Dark photon kinetic mixing
- Axion-photon conversion
- Early universe phase transitions
- Resonant particle production
-

Usually, one often considers mixing between two quantum states.

Two-state systems across different fields of physics:

- Nuclear Magnetic Resonance → Spin-1/2 in magnetic fields
- Quantum Chemistry → Avoided level crossing
- Quantum Information → Qubit evolution
-

"Different physics, same mathematics!"

Quantum Mechanics Refresher

- In the energy eigenbasis, $|E_1(t)\rangle$ and $|E_2(t)\rangle$, the Hamiltonian matrix is simply given by

$$D_H(t) \equiv \begin{pmatrix} E_1(t) & 0 \\ 0 & E_2(t) \end{pmatrix} = \frac{E_1(t) + E_2(t)}{2} I + \frac{E_1(t) - E_2(t)}{2} \sigma_3$$

so that $D_{\Delta H}(t) = -\frac{1}{2} \Delta E(t) \sigma_3$, $\Delta E(t) \equiv E_2(t) - E_1(t)$.

- Let us now consider the interaction basis, $|\lambda_1\rangle$ and $|\lambda_2\rangle$, described by $|E_i(t)\rangle = \hat{U}(t)|\lambda_i\rangle (i = 1, 2)$ where the unitary matrix elements, $U_{ij}(t) = \langle \lambda_i | \hat{U}(t) | \lambda_j \rangle$, are parametrised as

$$U(t) = \begin{pmatrix} \cos \theta(t) e^{i\alpha(t)} & \sin \theta(t) e^{i[\alpha(t) + \phi(t)]} \\ -\sin \theta(t) & \cos \theta(t) e^{i\phi(t)} \end{pmatrix}, \quad \theta(t) \equiv \text{mixing angle}$$

- In the interaction basis, the Hamiltonian matrix is simply given by

$$H(t) = \begin{pmatrix} H_{11}(t) & H_{12}(t) \\ H_{12}(t)^* & H_{22}(t) \end{pmatrix} = \frac{E_1(t) + E_2(t)}{2} I + \Delta H(t)$$

where $\Delta H(t) = U(t) D_{\Delta H}(t) U^\dagger(t) = -\frac{1}{2} \Delta E(t) \begin{pmatrix} \cos 2\theta(t) & -\sin 2\theta(t) \\ -\sin 2\theta(t) & -\cos 2\theta(t) \end{pmatrix}$

- We will consider the traditional case where $H_{12}(t)$ is real.

- Time evolution of a state vector, $|\psi(t)\rangle = \psi_1(t)|\lambda_1\rangle + \psi_2(t)|\lambda_2\rangle$, in the interaction basis is given by Schrödinger equation

$$i\frac{\partial\psi(t)}{\partial t} = H(t)\psi(t), \quad \psi(t) = \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix}$$

where $\psi_1(t) \equiv \langle\lambda_1|\psi(t)\rangle$ and $\psi_2(t) \equiv \langle\lambda_2|\psi(t)\rangle$.

- Thus, the probability to find the system in state $|\lambda_1\rangle$ and $|\lambda_2\rangle$ at time t are

$$P_{i\rightarrow 1}(t) = |\langle\lambda_1|\psi(t)\rangle|^2 = |\psi_1(t)|^2, \quad P_{i\rightarrow 2}(t) = |\langle\lambda_2|\psi(t)\rangle|^2 = |\psi_2(t)|^2$$

- Traditional way to get an analytical expression: **Hamiltonian Formalism**
- Begin with Schrödinger evolution of states:

$$i\frac{\partial}{\partial t} \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix} = H(t) \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix}$$

Hamiltonian Formalism in Time-independent case

- For a state produced as $|\lambda_1\rangle$ at time $t = 0$,

$$|\psi(0)\rangle = |\lambda_1\rangle$$

- Formal solution is simply

$$|\psi(t)\rangle = e^{-i\hat{H}_0 t} |\psi(0)\rangle = e^{-i\hat{H}_0 t} |\lambda_1\rangle$$

- The time-independent Effective Hamiltonian in the interaction basis is

$$\Delta H_0 = -\frac{1}{2} \Delta E_0 \begin{pmatrix} \cos 2\theta_0 & -\sin 2\theta_0 \\ -\sin 2\theta_0 & -\cos 2\theta_0 \end{pmatrix}, \quad \Delta E_0 \equiv E_2 - E_1.$$

- Probability of detecting state $|\lambda_2\rangle$ is simply

$$P_{1 \rightarrow 2}(t) = |\langle \lambda_2 | \psi(t) \rangle|^2 = |\langle \lambda_2 | e^{-i\hat{H}_0 t} | \lambda_1 \rangle|^2 = |e^{-i\Delta H_{12} t}|^2$$

$$P_{1 \rightarrow 2}(t) = \sin^2 2\theta_0 \sin^2 \left(\frac{\Delta E_0 t}{2} \right)$$

- Solar neutrino oscillations in vacuum: in the ultra-relativistic limit $\Delta E_0 \simeq \Delta m_{12}^2 / 2E$, for neutrinos of given momentum $E \equiv |\vec{p}|$, the appearance probability of $|\nu_{\mu+\tau}\rangle$ for $E \ll 2\text{MeV}$ is

$$P(\nu_e \rightarrow \nu_{\mu+\tau}) = \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{12}^2 l_{\odot}}{4E} \right), \quad l_{\odot} \simeq ct.$$

Hamiltonian Formalism in Time-dependent case

- For a general time-dependent Hamiltonian, $H(t) = H_0 + V(t)$, we can write the Effective Hamiltonian as

$$\Delta H(t) = -\frac{1}{2} \begin{pmatrix} \Delta E_0 \cos 2\theta_0 - \Delta V(t) & -\Delta E_0 \sin 2\theta - 2V_{12}(t) \\ -\Delta E_0 \sin 2\theta_0 - 2V_{12}^*(t) & -\Delta E_0 \cos 2\theta_0 + \Delta V(t) \end{pmatrix}$$

Time-dependent diagonal interaction terms give $\Delta V(t) = V_{11}(t) - V_{22}(t)$.
Non-standard interactions give off-diagonal term $V_{12}(t)$.

- Diagonalize $\Delta H(t)$, and we get the time-dependent energy levels and mixing angle

$$\Delta E(t) = \Delta E_0 \sqrt{|\sin 2\theta_0 - v_{12}(t)|^2 + [\cos 2\theta_0 - \Delta v(t)]^2}; \tan 2\theta(t) = \frac{\sin 2\theta_0 - v_{12}(t)}{\cos 2\theta_0 - \Delta v(t)}$$

where $\Delta v(t) \equiv \Delta V(t)/\Delta E_0$ and $v_{12}(t) \equiv -2V_{12}(t)/\Delta E_0$ are dimensionless quantities.

- Time evolution of the state $|\psi(t)\rangle$ in the interaction basis, projected on the energy eigenstates is

$$i \frac{\partial}{\partial t} \begin{pmatrix} \langle E_1(t) | \psi(t) \rangle \\ \langle E_2(t) | \psi(t) \rangle \end{pmatrix} = \begin{pmatrix} \frac{E_1(t) - E_2(t)}{2} & -i\dot{\theta}(t) \\ i\dot{\theta}(t) & -\frac{E_1(t) - E_2(t)}{2} \end{pmatrix} \begin{pmatrix} \langle E_1(t) | \psi(t) \rangle \\ \langle E_2(t) | \psi(t) \rangle \end{pmatrix}$$

Adiabatic Mixing of Two Quantum States

- Adiabaticity parameter

$$\gamma(t) = \frac{|E_1(t) - E_2(t)|}{2|\dot{\theta}(t)|}, \quad \Delta E(t) \equiv E_2(t) - E_1(t)$$

- In the limit $\gamma(t) \gg 1 \implies |\dot{\theta}(t)| \ll \frac{|\Delta E(t)|}{2}$, adiabatic conversions are favored.
- For initial state $|\psi(0)\rangle = |\lambda_1\rangle$, the appearance probability of state $|\lambda_2\rangle$ is

$$P_{1 \rightarrow 2}(t) = \frac{1}{2} - \frac{1}{2} \cos 2\theta(t) \cos 2\theta(0) - \frac{1}{2} \sin 2\theta(t) \sin 2\theta(0) \cos \left\{ \int_0^t \Delta E(t') dt' \right\}$$

- Solar Neutrino Oscillations: Mikheyev–Smirnov–Wolfenstein (MSW) Effect
 $\Delta V(t) = \sqrt{2} G_F n_e(r)$, where $n_e(r)$ is electron density, and $V_{12}(t) = 0$
For $E \gg 2\text{MeV}$, $\Delta v(0) \gg 1 \implies \theta(0) = -\pi/2$.
During detection on Earth, appearance probability

$$P(\nu_e \rightarrow \nu_{\mu+\tau}) = \rho_{22} = \cos^2 \theta_{12} \simeq 0.69$$

for $\theta_0 = \theta_{12} \simeq 33.7^\circ$ and $r \simeq t$.

Landau-Zener-Stuckelberg-Majorana(LZSM) approximation

- For small mixing angles θ_0 , adiabatic solution is not valid.
- LZSM formula describes the crossing probability between the two energy eigenstates. In the case $v_{12}(t) = 0$ and in the linear approximation $\Delta v \simeq \Delta v(t_{res}) + \dot{\Delta v}(t_{res})(t - t_{res})$, for small mixing angles ($\theta_0 \ll 1$), the conversion probability

$$P_{1 \rightarrow 2}(t_f) = 1 - e^{-\frac{\pi}{2} \gamma_{res}} \quad , \text{ LZSM approximation}$$

where $\gamma_{res} = \gamma(t_{res})$.

- One can easily find adiabaticity parameter at resonance from

$$\gamma(t) = \frac{|\Delta E(t)|}{2|\dot{\theta}(t)|}, \quad \implies \gamma_{res} = \sin^2 2\theta_0 \left| \frac{\Delta E_0}{\Delta \dot{v}(t_{res})} \right|$$

- Resonance is where the time-dependent mixing angle is maximal and energy gap between energy eigestates is minimal.

$$\tan 2\theta(t) = \frac{\sin 2\theta_0 - v_{12}(t)}{\cos 2\theta_0 - \Delta v(t)} \implies \theta(t_{res}) = \frac{\pi}{4}$$

The adiabaticity parameter is minimum ($\cos 2\theta(t) = 0$) at the resonance.

Why Hamiltonian formalism is not enough?

Realistic Environments are time-dependent and interacting which makes analytic control challenging.

- Thermal background
- Scattering
- Decoherence
- Mixed states
- Particle production

More general mathematical structure possible? Yes!

We are interested in getting a general analytical expression for pure state non-adiabatic case.

- **von Neumann equation** corresponding to density operator $\hat{\rho}(t) = |\psi(t)\rangle\langle\psi(t)|$:

$$\frac{\partial \hat{\rho}(t)}{\partial t} = i [\hat{\rho}(t), \hat{H}(t)] \quad \xrightarrow[\text{in a basis } \{|\lambda_\alpha\rangle\}_{\alpha=1,\dots,n}]{\text{Matrix Representation}} \quad \frac{\partial \rho(t)}{\partial t} = i [\rho(t), H(t)].$$

- The Hamiltonian matrix $H(t)$ and the density matrix $\rho(t)$ in the above mentioned basis

$$H(t) = \frac{E_1(t) + E_2(t)}{2} I + \Delta H(t), \quad \rho(t) = \begin{pmatrix} \rho_{11}(t) & \rho_{12}(t) \\ \rho_{21}(t) & \rho_{22}(t) \end{pmatrix}$$

- **Vectorial Notation Parametrization:**

$$\rho(t) = \frac{1}{2}(1 + \vec{P}(t) \cdot \vec{\sigma}), \quad \Delta H(t) = \frac{1}{2}\vec{V}(t) \cdot \vec{\sigma}$$

in terms of the *polarization vector* $\vec{P}(t) = (P_x(t), P_y(t), P_z(t))$, *potential vector* $\vec{V}(t) = (V_x(t), V_y(t), V_z(t))$ and *Pauli matrices* $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ which gives

$$P_x(t) = 2\text{Re}[\rho_{12}(t)], \quad P_y(t) = -2\text{Im}[\rho_{12}(t)], \quad P_z(t) = \rho_{11}(t) - \rho_{22}(t)$$

$$V_x(t) = 2\text{Re}[\Delta H_{12}(t)], \quad V_y(t) = -2\text{Im}[\Delta H_{12}(t)], \quad V_z(t) = 2\Delta H_{11}(t) = -2\Delta H_{22}(t)$$

- **Evolution:**

$$\frac{\partial \vec{P}(t)}{\partial t} = \vec{V}(t) \times \vec{P}(t)$$

a Larmor-like equation describing the precession of the polarization vector around the potential vector, given explicitly in components as

$$\begin{pmatrix} \dot{P}_x(t) \\ \dot{P}_y(t) \\ \dot{P}_z(t) \end{pmatrix} = \begin{pmatrix} 0 & -V_z(t) & V_y(t) \\ V_z(t) & 0 & -V_x(t) \\ -V_y(t) & V_x(t) & 0 \end{pmatrix} \begin{pmatrix} P_x(t) \\ P_y(t) \\ P_z(t) \end{pmatrix}$$

- **Interaction Basis:** $\{|\lambda_1\rangle, |\lambda_2\rangle\}$

Energy Eigenstate Basis: $\{|E_1(t)\rangle, |E_2(t)\rangle\}$

- **Basis transformation:**

$$|E_i(t)\rangle = \hat{U}(t)|\lambda_i\rangle \text{ with the Unitary matrix } U(t) = \begin{pmatrix} \cos \theta(t)e^{i\alpha(t)} & \sin \theta(t)e^{i[\alpha(t)+\phi(t)]} \\ -\sin \theta(t) & \cos \theta(t)e^{i\phi(t)} \end{pmatrix}$$

$U(t)$: Mixing Matrix

and

$\theta(t)$: Mixing Angle

Density Matrix Formalism

- In the interaction basis, Effective Hamiltonian

$$\Delta H(t) = U(t)D_{\Delta H}(t)U^\dagger(t) = -\frac{1}{2}\Delta E(t) \begin{pmatrix} \cos 2\theta(t) & -\sin 2\theta(t)e^{i\alpha(t)} \\ -\sin 2\theta(t)e^{-i\alpha(t)} & -\cos 2\theta(t) \end{pmatrix}$$

- With $\vec{V}(t) = \Delta E(t)(\sin 2\theta(t) \cos \alpha(t), -\sin 2\theta(t) \sin \alpha(t), -\cos 2\theta(t))$, the evolution equation is

$$\begin{pmatrix} \dot{P}_x(t) \\ \dot{P}_y(t) \\ \dot{P}_z(t) \end{pmatrix} = \Delta E(t) \begin{pmatrix} 0 & \cos 2\theta(t) & -\sin 2\theta(t) \sin \alpha(t) \\ -\cos 2\theta(t) & 0 & -\sin 2\theta(t) \cos \alpha(t) \\ \sin 2\theta(t) \sin \alpha(t) & \sin 2\theta(t) \cos \alpha(t) & 0 \end{pmatrix} \begin{pmatrix} P_x(t) \\ P_y(t) \\ P_z(t) \end{pmatrix}$$

- Constant phase $\alpha(t) \equiv$ real effective Hamiltonian
- This specialises the potential vector into

$$\vec{V}(t) = \Delta E(t)(\sin 2\theta(t), 0, \cos 2\theta(t))$$

and the evolution equation for polarisation vector into

$$\begin{pmatrix} \dot{P}_x(t) \\ \dot{P}_y(t) \\ \dot{P}_z(t) \end{pmatrix} = \Delta E(t) \begin{pmatrix} 0 & \cos 2\theta(t) & 0 \\ -\cos 2\theta(t) & 0 & -\sin 2\theta(t) \\ 0 & \sin 2\theta(t) & 0 \end{pmatrix} \begin{pmatrix} P_x(t) \\ P_y(t) \\ P_z(t) \end{pmatrix}$$

General Hamiltonian

- For a general time-dependent Hamiltonian, $H(t) = H_0 + V(t)$, one can write the Effective Hamiltonian as

$$\Delta H(t) = -\frac{1}{2} \begin{pmatrix} \Delta E_0 \cos 2\theta_0 - \Delta V(t) & -\Delta E_0 \sin 2\theta_0 - 2V_{12}(t) \\ -\Delta E_0 \sin 2\theta_0 - 2V_{12}^*(t) & -\Delta E_0 \cos 2\theta_0 + \Delta V(t) \end{pmatrix}$$

where $\Delta V(t) \equiv V_{11}(t) - V_{22}(t)$ and $V_{12}(t)$ is a possible off-diagonal term.

- Note that for $\theta_0 = 0$ or $\Delta E_0 = 0$, mixing is still possible if $V_{12} \neq 0$!
- If $\Delta E_0 \neq 0$, one can write

$$\Delta H(t) = -\frac{1}{2} \Delta E_0 \begin{pmatrix} \cos 2\theta_0 - \Delta v(t) & -\sin 2\theta_0 + v_{12}(t) \\ -\sin 2\theta_0 + v_{12}^*(t) & -\cos 2\theta_0 + \Delta v(t) \end{pmatrix}$$

where $\Delta v(t) \equiv \Delta V(t)/\Delta E_0$ and $v_{12}(t) \equiv -2V_{12}(t)/\Delta E_0$

- We get the potential vector once again in a cleaner form

$$\vec{V}(t) = \Delta E_0 (\sin 2\theta_0 - \text{Re}[v_{12}(t)], \text{Im}[v_{12}(t)], \Delta v(t) - \cos 2\theta_0)$$

General Hamiltonian

- Diagonalizing $\Delta H(t)$, we get the time-dependent energy levels and mixing angle

$$\Delta E(t) = \Delta E_0 \sqrt{|\sin 2\theta_0 - v_{12}(t)|^2 + [\cos 2\theta_0 - \Delta v(t)]^2}$$

$$\tan 2\theta(t) = \frac{\sin 2\theta_0 - v_{12}(t)}{\cos 2\theta_0 - \Delta v(t)}$$

- Compact form of potential vector

$$\vec{V}(t) = \Delta E(t)(\sin 2\theta(t), 0, \cos 2\theta(t))$$

- For $\cos 2\theta_0 = \Delta v(t)$, one observes a resonance where $\theta(t_{res}) = \pm\pi/4$ is maximal and the energy level spacing $\Delta E(t)$ is minimum.

-

$$\sin 2\theta(t) = \frac{\sin 2\theta_0 - v_{12}(t)}{\sqrt{[\sin 2\theta_0 - v_{12}(t)]^2 + [\cos 2\theta_0 - \Delta v(t)]^2}}$$

$$\cos 2\theta(t) = \frac{\cos 2\theta_0 - \Delta v(t)}{\sqrt{[\sin 2\theta_0 - v_{12}(t)]^2 + [\cos 2\theta_0 - \Delta v(t)]^2}}$$

Time-dependent Hamiltonian: Adiabatic Mixing

- **Evolution Equation** with a change of variables $\vec{P}(t) \rightarrow \vec{P}'(t) = R(t)\vec{P}(t)$ and introducing the **adiabaticity parameter** $\gamma(t) \equiv \frac{1}{2} \left| \frac{\Delta E(t)}{\dot{\theta}(t)} \right|$,

$$\begin{pmatrix} \dot{P}'_x(t) \\ \dot{P}'_y(t) \\ \dot{P}'_z(t) \end{pmatrix} = \Delta E(t) \begin{pmatrix} 0 & 1 & \frac{1}{\gamma(t)} \text{sgn}(\Delta E \dot{\theta}) \\ -1 & 0 & 0 \\ \frac{1}{\gamma(t)} \text{sgn}(\Delta E \dot{\theta}) & 0 & 0 \end{pmatrix} \begin{pmatrix} P'_x(t) \\ P'_y(t) \\ P'_z(t) \end{pmatrix}$$

where the rotation matrix $R(t) \equiv R_y(2\theta(t)) = \begin{pmatrix} \cos 2\theta(t) & 0 & \sin 2\theta(t) \\ 0 & 1 & 0 \\ -\sin 2\theta(t) & 0 & \cos 2\theta(t) \end{pmatrix}$.

- **Adiabatic Mixing** ($\gamma \gg 1$):

$$P_{1 \rightarrow 1}(t) = \rho_{11}(t) = \frac{1}{2} + \frac{1}{2} \cos 2\theta(t) \cos 2\theta(0) + \frac{1}{2} \sin 2\theta(t) \sin 2\theta(0) \cos \left(\int_0^t \Delta E(t') dt' \right)$$

$$P_{1 \rightarrow 2}(t) = \rho_{22}(t) = \frac{1}{2} - \frac{1}{2} \cos 2\theta(t) \cos 2\theta(0) - \frac{1}{2} \sin 2\theta(t) \sin 2\theta(0) \cos \left(\int_0^t \Delta E(t') dt' \right)$$

- Probabilities obtained for a more general Hamiltonian with no restrictions on diagonal and off-diagonal interactions.

Mikheyev–Smirnov–Wolfenstein (MSW) Effect

- Important application is MSW effect, describing solar neutrino oscillations
- Time dependence originates in the diagonal term $\Delta V(t) = \sqrt{2}G_F n_e(r)$, where $n_e(r(t))$ is the electron number density inside the sun,

$$n_e(r) = n_e(0)e^{-8.4 \frac{r}{R_\odot}}$$

- Thus, the effective Hamiltonian specialises to

$$\Delta H(t) = -\frac{\Delta m_{12}^2}{4E} \begin{pmatrix} \cos 2\theta_{12} - v_e(t) & -\sin 2\theta_{12} \\ -\sin 2\theta_{12} & -\cos 2\theta_{12} + v_e(t) \end{pmatrix}$$

where $\Delta v(t) = v_e(t) = \frac{2E}{\Delta m_{12}^2} \sqrt{2}G_F n_e(r)$

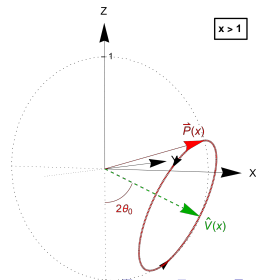
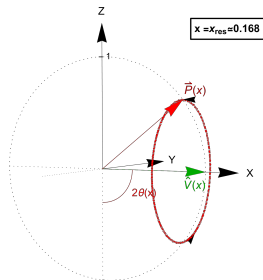
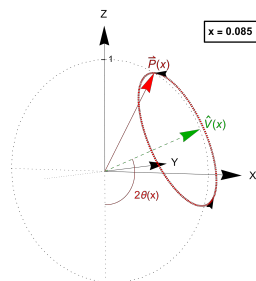
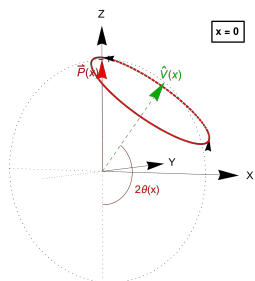
- Survival and appearance probabilities:

$$P(\nu_e \rightarrow \nu_e) = \rho_{11} = \sin^2 \theta_{12} \simeq 0.31$$

$$P(\nu_e \rightarrow \nu_{\mu+\tau}) = \rho_{22} = \cos^2 \theta_{12} \simeq 0.69$$

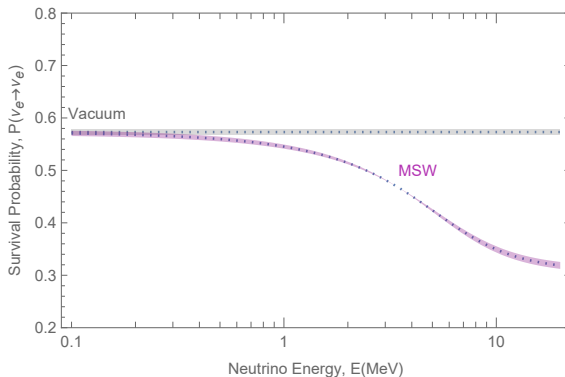
- Let's define a variable $x \equiv r/R_\odot$ and normalised potential vector $\hat{V}(t) \equiv \bar{V}(t)/\Delta E(t)$.

Mikheyev–Smirnov–Wolfenstein (MSW) Effect



Mikheyev–Smirnov–Wolfenstein (MSW) Effect

- Disappearance probability as a function of the neutrino energy E ,



Time-dependent Hamiltonian: Non-adiabatic Mixing

- General evolution of polarisation vector without any assumptions about the system

$$\begin{pmatrix} \dot{P}_x(t) \\ \dot{P}_y(t) \\ \dot{P}_z(t) \end{pmatrix} = \begin{pmatrix} 0 & -V_z(t) & 0 \\ V_z(t) & 0 & -V_x(t) \\ 0 & V_x(t) & 0 \end{pmatrix} \begin{pmatrix} P_x(t) \\ P_y(t) \\ P_z(t) \end{pmatrix}.$$

- In the approximation $P_z \simeq 1$ (small mixing angle),

$$P_{1 \rightarrow 2}(t) = \rho_{22}(t) = \frac{1}{2} \int_0^t dt' V_x(t') \int_0^{t'} dt'' V_x(t'') \cos \left(\int_{t''}^{t'} dt''' V_z(t''') \right)$$

- **Landau-Zener-Stuckelberg-Majorana (LZSM) approximation**

Under proper assumptions as mentioned earlier, when $\gamma_{res} \ll 1$

$$P_{1 \rightarrow 2}^f \simeq \frac{\pi}{2} \gamma_{res}$$

- **Extremely Non-Adiabatic Limit** Considering $\Delta V(t)$ to be non-vanishing in time $\Delta t = [t_{on}, t_{off}]$, if $\sin 2\theta_0 - v_{12}(t)$ is sufficiently small than $\dot{\theta}$ in Δt , then the adiabaticity parameter is so small that the variation in polarisation vector $\Delta \vec{P}$ is negligible. One recovers the time-independent case.

Including Decoherence

- Lindblad equation for realistic quantum systems:

$$\frac{\partial \rho(t)}{\partial t} = -i[H(t), \rho(t)] - \Gamma(t)[\sigma_z, [\sigma_z, \rho(t)]]$$

- In vectorial representation

$$\frac{\partial \vec{P}(t)}{\partial t} = \vec{V}(t) \times \vec{P}(t) - \Gamma(t)[P_x \hat{x} + P_y \hat{y}]$$

- This modifies the conversion probability to

$$P_{1 \rightarrow 2}(t) = \rho_{22}(t) = \frac{1}{2} \int_0^t dt' V_x(t') \int_0^{t'} dt'' V_x(t'') e^{-\Gamma(t'')t''} \cos \left(\int_{t''}^{t'} dt''' V_z(t''') \right)$$

- More general expression including the effect of decoherence.

Example

- Active-sterile neutrino oscillations, Phys. Rev. D 65, 045003 e-Print:[0103192](#) [hep-ph] P. Di Bari, R. Foot

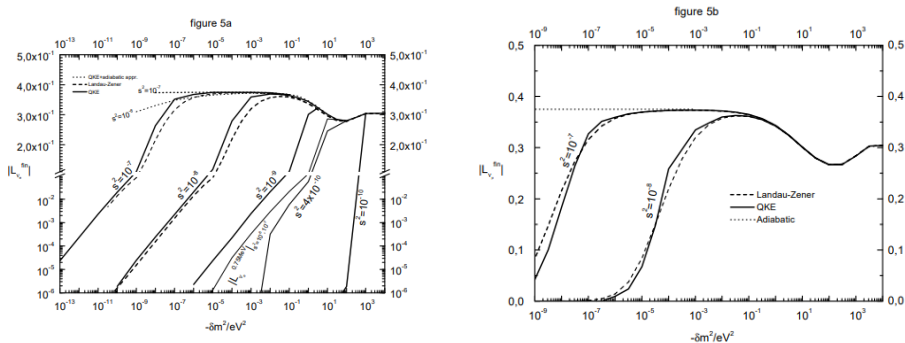


Figure 5. Final values of the neutrino asymmetry in the case $\alpha = \epsilon$ (a) and $\alpha = \mu$ (b). Solid lines are solutions from the QKE's. Dashed lines are the solution with Landau-Zener approximation. Dotted lines are the adiabatic limit. In the figure 5a we also show with a thin solid line the value of neutrino asymmetry at $T = 0.75$ MeV (for mixing angles $\gtrsim 10^{-8}$) that is approximately the freezing temperature of the neutron to proton ratio.

Conclusion

- Density Matrix Formalism works very well for time-independent and adiabatic case, successfully reproducing well known oscillation probabilities.
- Using a general Hamiltonian, we found a more general analytical solution for non-adiabatic mixing of two quantum states in the small mixing angle limit.
- Showed that under proper assumptions one recovers LZSM formula but this work is more general and allows to extend even beyond LZSM approximation.
- Provides a geometric picture to visualize two-state oscillations.
- We plan to apply these very general results to various problems,
 - ▶ solar active-sterile neutrino mixing
 - ▶ active-sterile neutrino mixing in the early universe
 - ▶ very heavy dark matter production from sterile-sterile neutrino mixing
 - ▶ photon-dark photon mixing in the early universe
- We should note that Density Matrix Formalism has wider applications beyond particle physics and serves a general quantum mechanical description for mixing of two quantum states.

Thank you!