Generalised anomalies, QCD_4, and holography

Mohammad Akhond
INFN Roma Tor Vergata

This talk is based on:

arXiv:2511.05646 [hep-th] with S. Sugimoto

Motivation

The last decade saw rapid developments in generalising the notion of symmetries, and anomalies. This led to many interesting results in field theory, and naturally holographic aspects were explored by many authors.

- New constraints on IR phases of gauge theories ($d < 5$)
- Global features of supergravity theories (backgrounds)
- Refined checks of the holographic correspondence
- Supersymmetry not crucial

I will focus on these developments in the context of massless QCD.
Question: Is holographic QCD a convenient setting for detecting these novel type of anomalies?

Anomalies in QFT

Consider a d-dimensional QFT, coupled to a background $\mathcal{B}$. Under background gauge transformation $\mathcal{B} \to \mathcal{B}^\Lambda$

$$Z_{\mathcal{M}_d}[\mathcal{B}^\Lambda] = \exp\left(-2\pi i \int_{\mathcal{M}_d} \mathcal{A}(\Lambda, \mathcal{B})\right) Z_{\mathcal{M}_d}[\mathcal{B}]\,. \tag{1}$$

Gauge invariance may be restored by attaching a $d+1$-dimensional bulk $\mathcal{N}_{d+1}$, with boundary $\partial\mathcal{N}_{d+1} = \mathcal{M}_d$, supporting degrees of freedom specified by a local Lagrangian $\mathcal{I}_{d+1}$ subject to

$$\delta_\Lambda \mathcal{I}_{d+1}(\mathcal{B}) = \mathrm{d}\,\mathcal{A}(\Lambda, \mathcal{B}) \tag{2}$$

Anomalies from holography

The holographic correspondence

$$\mathrm{QFT}_d \longleftrightarrow \text{String theory on } M_{\mathrm{ext}} \times M_{\mathrm{int}} \tag{3}$$

By the holographic dictionary

$$Z_{\mathrm{QFT}}[\mathcal{B}] \approx \mathrm{e}^{-S_{\mathrm{Sugra}}[\mathcal{B}]} \tag{4}$$

$$S_{\mathrm{Sugra}} = S_{\mathrm{kin}} + S_{\mathrm{top}} \tag{5}$$

- The kinetic terms are manifestly gauge invariant.
- S_{top} is gauge invariant on closed manifolds.
- The anomalous phase must come from boundary gauge variation of S_{top}.

State of the art for QCD

Massless QCD has the symmetry

$$G_{\mathrm{QCD}} = \frac{SU(N_f)_L \times SU(N_f)_R \times U(1)_V}{\mathbb{Z}_N \times (\mathbb{Z}_{N_f})_V} \,. \tag{6}$$

Turning on $SU(N_f)_L \times SU(N_f)_R$ gauge fields leads to

$$2\pi N \int [\mathrm{CS}_5(A_L) - \mathrm{CS}_5(A_R)] \tag{7}$$

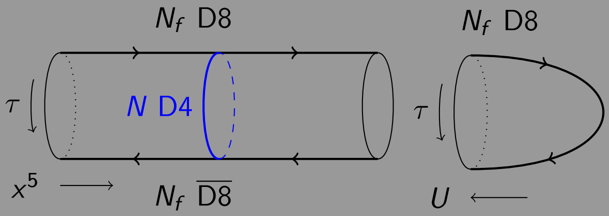
Turning on $(\mathbb{Z}_{N_f})_L \times U(N_f)_V/\mathbb{Z}_N$ [Tanizaki 18]

$$\mathcal{S}_5\left[A_\chi, \tilde{A}, B_2\right] = \pi N \int A_\chi \wedge \mathrm{Tr}\left[\tilde{F}^2\right] + 2\pi N \int A_\chi \wedge B_2 \wedge \mathrm{Tr}\,\tilde{F} \,. \tag{8}$$

The latter is more subtle, depends on the discrete nature of the backgrounds

$$\oint A_\chi \in \frac{1}{N_f}\mathbb{Z} \,, \quad \oint B_2 \in \frac{1}{N}\mathbb{Z} \,.$$

Holographic QCD



- Place N D4-branes on a Euclidean thermal circle.
- The anti-periodic boundary condition gives a tree level mass to the *gaugini*. 1-loop potential for scalars
- Massless quarks from D4-D8, and D4-$\overline{\text{D8}}$ open strings.
- The near horizon limit $\mathbb{R}^{1,3} \times \text{cigar} \times S^4$ [Witten '98].
- In the 't Hooft limit with $N_f \ll N$, treat D8 branes as probes

[Sakai, Sugimoto '04]

An apparent puzzle

In our probe brane background the bulk action has two pieces

$$S_{\mathrm{SuGra}} = S_{\mathrm{bulk}} + S_{\mathrm{D8}} \ . \tag{9}$$

Both the bulk and the brane action contain topological terms

$$S_{\mathrm{bulk}} = S_{\mathrm{kin}} + S_{\mathrm{top}} \ , \qquad S_{\mathrm{D8}} = S_{\mathrm{DBI}} + S_{\mathrm{WZ}} \tag{10}$$

The puzzle: Coupling to D8 branes requires dual fields

$$S^{\mathrm{D8}}_{\mathrm{WZ}} = \int_{M_9} C_9 + \cdots \ , \qquad G_{10} = \mathrm{d}\, C_9 \ , \qquad G_{10} = \star G_0 \tag{11}$$

This suggests a democratic formulation is convenient. **But** the democratic formulation hides topology of the bulk action.

Resolution: Dual formulation of type IIA

Elegant formulation of type IIA [Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen '01]

$$S_{\text{top}} = 2\pi \int \epsilon_0 \left(\Phi + \Omega\right)_< \wedge \left(\mathrm{d}\, V - \Omega\right)$$
$$+ \pi \int \epsilon_0 \left(e^B \Phi\right)_< \wedge \left(e^B \Phi\right)_> + \pi \int \epsilon_0 \Omega_< \wedge \Omega_> \,.$$

- RR fluxes $\Phi = \Phi_0 + \Phi_2 + \Phi_4$
- Lagrange multipliers $V = V_5 + V_7 + V_9$
- D8 gauge field contained in

$$\Omega = \frac{\mathrm{d}\,\tau}{2\pi}\,\mathrm{e}^{-B_2}\Bigg(\sum_{n\geq 0}(-1)^n\,\mathrm{CS}_{2n+1}(A_R - B_1)$$
$$-\sum_{n\geq 0}(-1)^n\,\mathrm{CS}_{2n+1}(A_L - B_1)\Bigg)$$

Anomaly of QCD_4 from holography

Truncating the bulk topological action on the internal S^4, and integrating over the τ direction yields

$$\begin{aligned} S_{\mathrm{top}} = &-2\pi N \int \left(b_2 \, \mathrm{d}\, u_2 + \frac{1}{2} b_2^2 \, \mathrm{d}\, u_0 \right) \\ &+ 2\pi N \int \left[\mathrm{CS}_5(A_L - b_1) - \mathrm{CS}_5(A_R - b_1) \right] \, . \end{aligned} \tag{12}$$

- This action should encode *all* anomalies of QCD_4.
- When restricting the background fields appropriately, it reproduces Tanizaki's anomaly
- It trivially reproduces the continuous anomaly, just set $b = 0$.

Summary & Outlook

- Presented a general expression for the anomaly theory of massless QCD.
- QCD literature only contains partial results, where different subgroups are turned on. All these results are consistent with our expression.
- Would be nice to match this in field theory, the pion effective action, and QCD should reproduce this anomaly.
- Extensions of type IIA formulation. Fermionic sector, backgrounds with H-flux.