Modular quintessence and de-Sitter in heterotic orbifolds

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In collaboration with M. Hernandez-Segura, I. Portillo-Castillo, S. Ramos-Sanchez and I. Zavala: arXiv:2509.22781

The Dark Energy puzzle

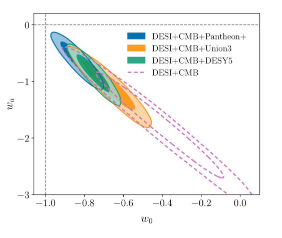
- Traditional ΛCDM:
$$w \equiv \frac{p_{\rm DE}}{\rho_{\rm DE}} = -1\,,$$
but recent observations hint at a time dependent equation of state $w(z)$.[1]

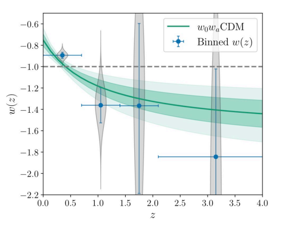
- Dynamical dark energy is often parametrized as
$$w(a) = w_0 + w_a(1-a),$$
ΛCDM recovered for $(w_0, w_a) = (-1, 0)$.

- What is more natural
$$\Lambda = \text{constant} \qquad \text{or} \qquad \rho_{\rm DE} = \rho_{\rm DE}(t)$$
from a UV-complete perspective?

- A stable de Sitter vacuum is in tension with the **de Sitter Swampland Conjecture**.

- Additional problem: Crossing of the phantom divide, $w < -1$





[1] DESI: arXiv:2503.14739, arXiv:2503.14738

Moduli as dynamical fields

- Moduli are complex scalar fields describing the geometry of the compact extra dimensions.
- Their vacuum expectation values control physical parameters of the four-dimensional EFT, including
  - gauge couplings,
  - Yukawa couplings,
  - compactification scales.
- In this work we focus on

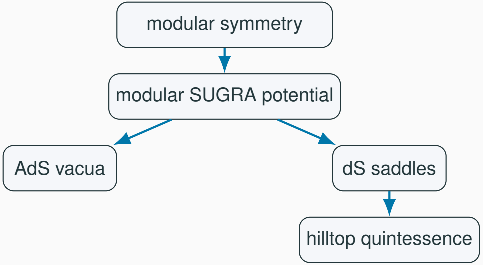
$$S \quad \text{(dilaton)}, \qquad \tau \quad \text{(Kähler modulus)}.$$

- If displaced from their minima, moduli evolve cosmologically and may drive dark energy.

$$\text{Geometry} \Longrightarrow \text{Moduli} \Longrightarrow \text{4D EFT} \Longrightarrow \text{Cosmology}$$

Can target-space modular symmetry in heterotic orbifolds generate controlled late-time quintessence dynamics while respecting swampland constraints?

- Build a modular-invariant $\mathcal{N} = 1$ SUGRA potential from heterotic orbifold ingredients.
- Use a two-moduli truncation: dilaton S and Kähler modulus τ.
- Scan the critical-point structure of the scalar potential.



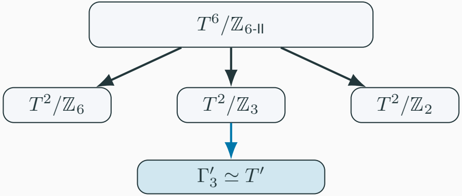
No metastable dS minima are found, but modular symmetry produces many unstable dS saddles suitable for multifield hilltop quintessence.

Geometric origin

- Starting point: the $E_8 \times E_8$ heterotic string compactified on a toroidal orbifold.
- Benchmark compactification: $T^6/\mathbb{Z}_{6\text{-II}}$.
- Orbifold factorization:

$$T^6/\mathbb{Z}_{6\text{-II}} \sim T^2/\mathbb{Z}_6 \otimes T^2/\mathbb{Z}_3 \otimes T^2/\mathbb{Z}_2.$$

- Focus on the $T^2/\mathbb{Z}_3$ sector, where the finite modular group acts on twisted states.



The compactification geometry determines the surviving modular symmetry and the transformation properties of the matter fields.

Target-space modular symmetry

The modular symmetries of the torus are inherited by the low-energy effective field theory.[2]

The modular group $\Gamma := \mathrm{SL}(2,\mathbb{Z})$ is defined as

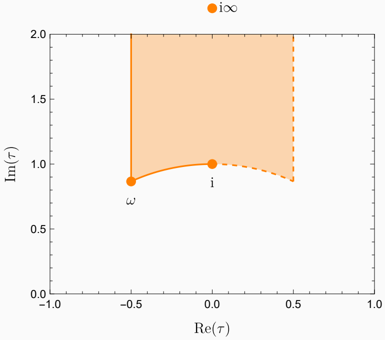
$$\Gamma = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a,b,c,d \in \mathbb{Z}, \quad ad - bc = 1 \right\}$$

Γ acts over the modulus τ as

$$\gamma : \tau \to \frac{a\tau + b}{c\tau + d}\,, \qquad \gamma \in \Gamma$$

- τ lives in the upper half-plane.
- Points related by modular transformations are physically equivalent.

- The fundamental domain removes redundant descriptions.
- Fixed points preserve residual modular symmetries and often correspond to special vacua.



[2] Ferrara, Lust, Theisen: Phys.Lett.B 233 (1989) 147-152
Parameswaran, Ramos-Sanchez, Zavala: arXiv:1009.3931

Benchmark model

The $T^2/\mathbb{Z}_3$ orbifold exhibits an eclectic flavor symmetry[3]: a traditional flavor symmetry $\Delta(54)$ and a finite modular symmetry $\Gamma_3' \cong T'$.

Yukawa couplings are given by modular forms, which are holomorphic functions that transform as

$$Y(\tau) \to Y(\gamma\tau) := (c\tau + d)^{n_Y} \rho_Y(\gamma) Y(\tau), \qquad \gamma \in \Gamma$$

We know the modular forms for T'

$$\hat{Y}^{(1)}(\tau) = \begin{pmatrix} \hat{Y}_1(\tau) \\ \hat{Y}_2(\tau) \end{pmatrix} := \begin{pmatrix} -3\sqrt{2}\frac{\eta^3(3\tau)}{\eta(\tau)} \\ 3\frac{\eta^3(3\tau)}{\eta(\tau)} + \frac{\eta^3(\tau/3)}{\eta(\tau)} \end{pmatrix}$$

[3] Baur, Nilles, Ramos-Sanchez, Trautner, Vaudrevange: arXiv:2405.2037

Low energy effective field theory

At low energies we have a 4D supergravity theory with $\mathcal{N}=1$. The most important functions are given by[4]

The superpotential

$$W = -\frac{1}{\sqrt{2}}\tilde{\lambda}_1 \hat{Y}_1(\tau) + \tilde{\lambda}_2 \hat{Y}_2(\tau) + \frac{\Omega_1(S)H_1(\tau)}{\eta^2(\tau)} + \frac{\Omega_2(S)H_2(\tau)}{\eta^2(\tau)}$$

The Kähler potential

$$K = -\log\left[S + \overline{S} - \frac{1}{8\pi^2}\delta_{\mathsf{GS}}\log\left(\mathrm{i}\overline{\tau} - \mathrm{i}\tau\right)\right] - \log\left(\mathrm{i}\overline{\tau} - \mathrm{i}\tau\right)$$

Some definitions:

$$\begin{aligned} H_i(\tau) &= \left(\frac{E_4(\tau)}{\eta^8(\tau)}\right)^{n_i}\left(\frac{E_6(\tau)}{\eta^{12}(\tau)}\right)^{m_i} P(j(\tau)) \\ j(\tau) &= 1728\frac{E_4(\tau)^3}{E_4(\tau)^3 - E_6(\tau)^2}\,, \qquad \Omega_i(S) = A_i e^{-\alpha_i S} \end{aligned}$$

[4] Cvetic, Font, Ibañez, Lust, Quevedo 1991
Nilles, Ramos-Sanchez, Vaudrevange: arXiv:2004.05200

Scalar potential

The scalar potential is given by[5]

$$V = e^{K}\left(K^{A\overline{B}} D_A W D_{\overline{B}}\overline{W} - 3|W|^2\right) ,$$

where $D_A W = \partial_A W + \partial_A K W$.

For our particular model:

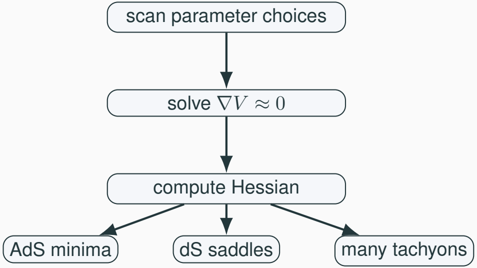
$$V = \mathrm{e}^{K}\left[|YW_S - W|^2 + \frac{Y}{Y - \frac{\delta_{\mathsf{GS}}}{8\pi^2}}\left|\frac{\delta_{\mathsf{GS}}}{8\pi^2}\left(\frac{2W}{Y} - W_S\right) + \mathrm{i}(\mathrm{i}\overline{\tau} - \mathrm{i}\tau)W_\tau - W\right|^2 - 3|W|^2\right]$$

with $Y := S + \overline{S} - \frac{1}{8\pi^2}\delta_{\mathsf{GS}} \log\left(\mathrm{i}\overline{\tau} - \mathrm{i}\tau\right)$.

[5] F. Quevedo: arXiv:1011.1491

Numerical landscape strategy

- Scan over (m_i, n_i) and initial seeds in field-space.
- Identify critical points satisfying $\nabla V \simeq 0$.
- Classify by the sign of V and Hessian eigenvalues.
- Track whether unstable directions are saxionic, axionic, or mixed.



The cosmologically useful points are unstable dS saddles with one light tachyonic direction, especially axionic combinations.

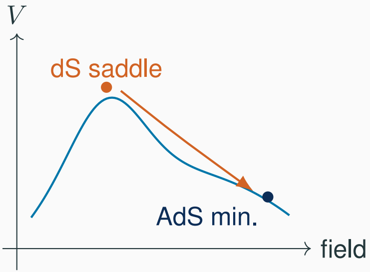
Scan results

- Stable vacua are always anti-de Sitter.
- No metastable de Sitter minima are found.
- Numerous unstable de Sitter saddle points emerge.
- The most interesting class exhibits a single tachyonic direction dominated by axionic components.

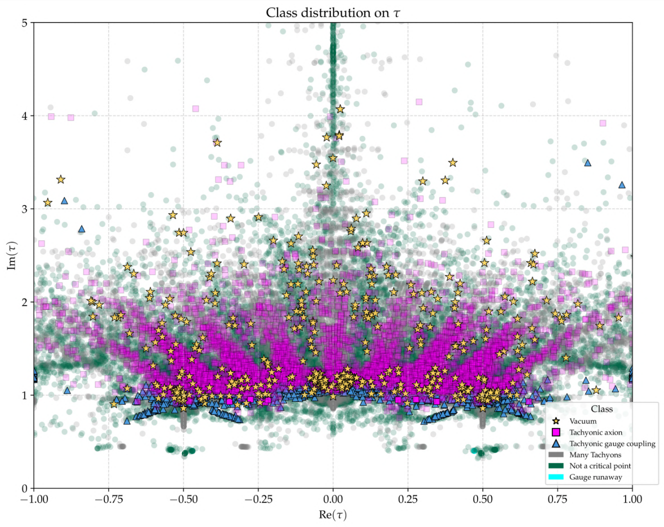
This is precisely the structure required for hilltop quintessence.

Role of modular invariance

- Modular-invariant potential: abundant axionic tachyonic saddles.
- Broken modular symmetry: these saddles become strongly suppressed.



Results for τ



From dS saddles to multifield quintessence

- Evolve the full four-field system:

$$\varphi^i = (\mathrm{Re}S, \mathrm{Im}S, \mathrm{Re}\tau, \mathrm{Im}\tau)$$

- Coupled Einstein-scalar equations with curved field-space metric
- Representative unstable direction:

$$\mathrm{Im}(\psi_S) \sim 0.9\,\mathrm{Im}(S) - 0.4\,\mathrm{Re}(\tau)$$

Thawing behavior[6]

1. Fields frozen by Hubble friction
2. Recent rolling from the saddle
3. Current acceleration
4. Future descent toward

EOM for modular fields:

$$\begin{aligned} 3\left(\frac{\dot{\mathrm{a}}}{\mathrm{a}}\right)^2 &= \frac{1}{2}g_{ij}\dot{\varphi}^i\dot{\varphi}^j + V + 3H_0^2\Omega_{m,0}\mathrm{a}(t)^{-3} + 3H_0^2\Omega_{r,0}\mathrm{a}(t)^{-4}\,, \\ 0 &= \ddot{\varphi}^i + 3\frac{\dot{\mathrm{a}}}{\mathrm{a}}\dot{\varphi}^i + \Gamma^i_{\ jk}\dot{\varphi}^j\dot{\varphi}^k + g^{ij}\frac{\partial V}{\partial \varphi^j}\,, \end{aligned}$$

taking $\Omega_{m,0} = 0.3111$ and $\Omega_{r,0} = 0.0001$. The system is solved in terms of the e-fold number N, with $N = 0$ today.

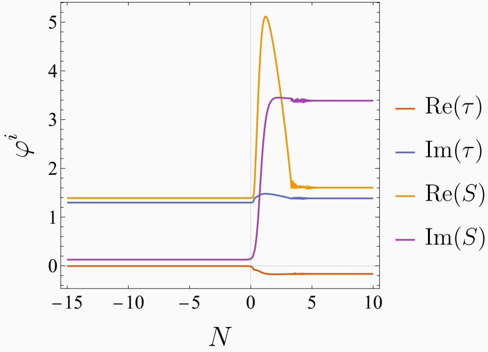
[6] Related model: Olguín-Trejo, Parameswaran, Tasinato, Zavala: arXiv:1810.08634

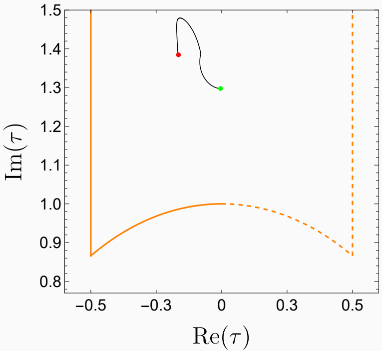
Modular fields evolution

- Fields at the SP $\Rightarrow$ remain static

Modular fields evolution

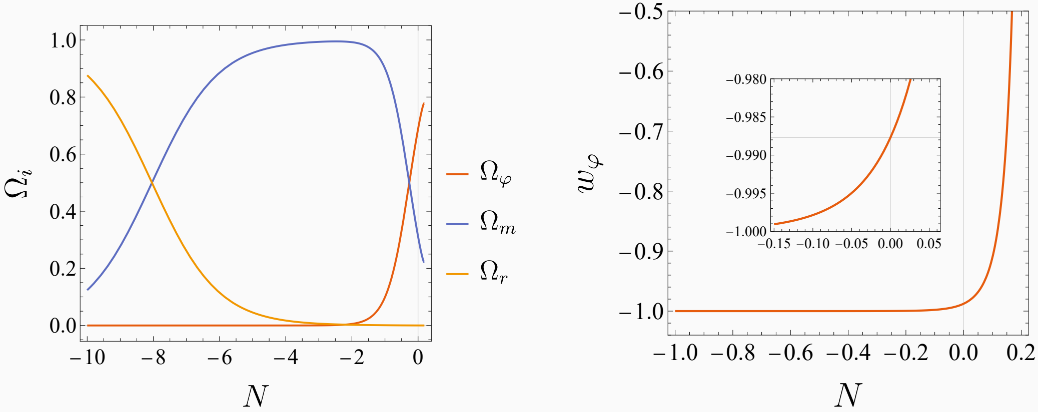
- Fields at the SP $\Rightarrow$ remain static
- Slightly displaced from SP $\Rightarrow$ evolve towards an AdS minimum





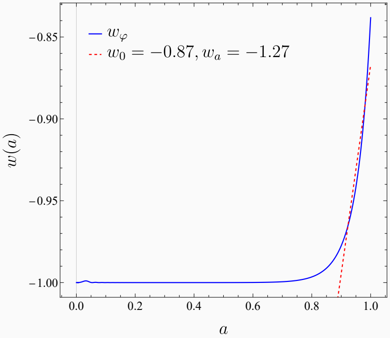
Equation of state and density parameter

$$\Omega_\varphi = \frac{\rho_\varphi}{3H^2}\,, \quad \rho_\varphi = \frac{1}{2} g_{ij}\dot{\varphi}^i\dot{\varphi}^j + V\,, \quad w_\varphi = \frac{\frac{1}{2} g_{ij}\dot{\varphi}^i\dot{\varphi}^j - V}{\rho_\varphi}$$



DESI comparison

CPL parameterization[7]: $w_{\mathrm{DE}}(a) = w_0 + (1-a)\,w_a$



Comparing with DESI+CMB+Union3[8] $w_0 = -0.752 \pm 0.057$ and $w_a = -0.86^{+0.23}_{-0.20}$ we get $\chi^2 \simeq 5.6$.

[7] Chevallier, Polarski: arXiv:gr-qc/0009008, Linder: arXiv:astro-ph/0208512

[8] DESI: arXiv:2503.14738

Summary

- Modular-invariant heterotic compactifications provide a natural framework for dynamical dark energy.
- No metastable de Sitter vacua are found, in agreement with the de Sitter Swampland Conjecture.
- Modular symmetry favors axionic tachyonic saddle points, providing suitable initial conditions for hilltop quintessence.
- Axion-dominated dS saddle points naturally give rise to multifield hilltop quintessence.
- The model reproduces the observed dark-energy equation of state while predicting a future evolution toward AdS.
- Future work: extend the analysis to more realistic compactifications with additional moduli, understand the full *penacho* structure, and explore the role of modular symmetries in other Swampland conjectures.

Thank you

Backup

Explicit model

We take the heterotic string $E8 \times E8$ compactified in an orbifold $\mathbb{Z}_6 -$ II. The shift vector and Wilson lines are:

$$
\begin{aligned}
v &= \frac{1}{6}(0,1,2,-3), \\
V_1 &= \left(-\frac{5}{12},-\frac{5}{12},-\frac{5}{12},-\frac{5}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12}\right),\left(-\frac{5}{12},-\frac{1}{4},-\frac{1}{4},-\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12}\right), \\
A_5 &= \left(-\frac{5}{4},\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{3}{4},\frac{1}{4},\frac{3}{4},\frac{5}{4}\right),\left(-\frac{1}{2},1,1,-\frac{3}{2},-1,0,1,1\right), \\
A_6 &= \left(-\frac{3}{2},-\frac{1}{2},1,1,\frac{1}{2},0,-\frac{1}{2},1\right),\left(-\frac{1}{4},\frac{3}{4},\frac{3}{4},\frac{5}{4},-\frac{1}{4},\frac{7}{4},\frac{1}{4},\frac{7}{4}\right).
\end{aligned}
$$

The 4-dimensional gauge group is:

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \left[\mathrm{SU}(3)^3 \times \mathrm{U}(1)^6\right].$$

Yukawa couplings

$$W_{\mathrm{Yuk}} \supset \left(\hat{Y}^{(1)}_{\mathbf{2}''}(\tau) \otimes \Phi^{(-2/3)}_{\alpha} \otimes \Phi^{(-2/3)}_{\beta} \otimes \Phi^{(-2/3)}_{\gamma}\right)_{\mathbf{1}}$$

$$\begin{aligned} W_{\mathrm{Yuk}} \supset \lambda\, \chi_1\, \chi_2 \Big[-\frac{\hat{Y}_1(\tau)}{\sqrt{2}} & \left(\Phi_{1,3}\Phi_{2,2}\Phi_{3,1} + \Phi_{1,3}\Phi_{2,1}\Phi_{3,2} + \Phi_{1,2}\Phi_{2,3}\Phi_{3,1}\right. \\ & + \left. \Phi_{1,1}\Phi_{2,3}\Phi_{3,2} + \Phi_{1,2}\Phi_{2,1}\Phi_{3,3} + \Phi_{1,1}\Phi_{2,2}\Phi_{3,3}\right) \\ & + \hat{Y}_2(\tau)\left(\Phi_{1,1}\Phi_{2,1}\Phi_{3,1} + \Phi_{1,2}\Phi_{2,2}\Phi_{3,2} + \Phi_{1,3}\Phi_{2,3}\Phi_{3,3}\right)\Big] \end{aligned}$$

The scalar potential

$$
\begin{aligned}
V = {} & \frac{1}{\mathrm{Sr}\,\left(i\,(-i\,\mathrm{T2i}+\mathrm{T2r})-i\,(i\,\mathrm{T2i}+\mathrm{T2r})\right)} \\
& 9.1612\times 10^{-71}\Bigg(-3\Bigg(\frac{1}{100\,000\,000}11\Bigg(\frac{9\,e^{-\frac{2}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\left(1-e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-30i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-42i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)^3}{500\,000\,000\left(1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)}+ \\
& \frac{1}{100\,000\,000\,000}3\Bigg(\frac{3\,e^{-\frac{2}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\left(1-e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-30i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-42i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)^3}{1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}}+ \\
& \frac{\left(1+e^{-\frac{2}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\left(-1-e^{-\frac{2}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-\frac{8}{3}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)\right)^3}{1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}}\Bigg)\Bigg)\Bigg)-\Big(3\,e^{-1-\frac{8}{3}\pi^2(-i\,\mathrm{Si}+\mathrm{Sr})+\frac{5}{6}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})} \\
& \Big(1+240\Big(e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+9\,e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+28\,e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+73\,e^{-8i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+126\,e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+252 \\
& e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+344\,e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+585\,e^{-16i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+757\,e^{-18i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+1134\,e^{-20i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\Big)\Big)\Big)\Big/ \\
& \left(3200\left(1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)^{10}\pi^2\right)-\Big(3\,e^{-1-52.6379(-i\,\mathrm{Si}+\mathrm{Sr})+\frac{65}{6}i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})} \\
& \Big(1+240\Big(e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+9\,e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+28\,e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+73\,e^{-8i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+126\,e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+ \\
& 252\,e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+344\,e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+585\,e^{-16i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+757\,e^{-18i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+1134\,e^{-20i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\Big)\Big)^{10} \\
& \Big(1-504\Big(e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+33\,e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+244\,e^{-6i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+1057\,e^{-8i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+3126\,e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+8052 \\
& e^{-12i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+16\,808\,e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+33\,825\,e^{-16i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+59\,293\,e^{-18i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+103\,158\,e^{-20i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\Big)\Big)^4\Big)\Big/ \\
& \left(320\,000\left(1-e^{-2i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}-e^{-4i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-10i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}+e^{-14i\pi(-i\,\mathrm{T2i}+\mathrm{T2r})}\right)^{130}\pi^2\right)\Bigg)
\end{aligned}
$$

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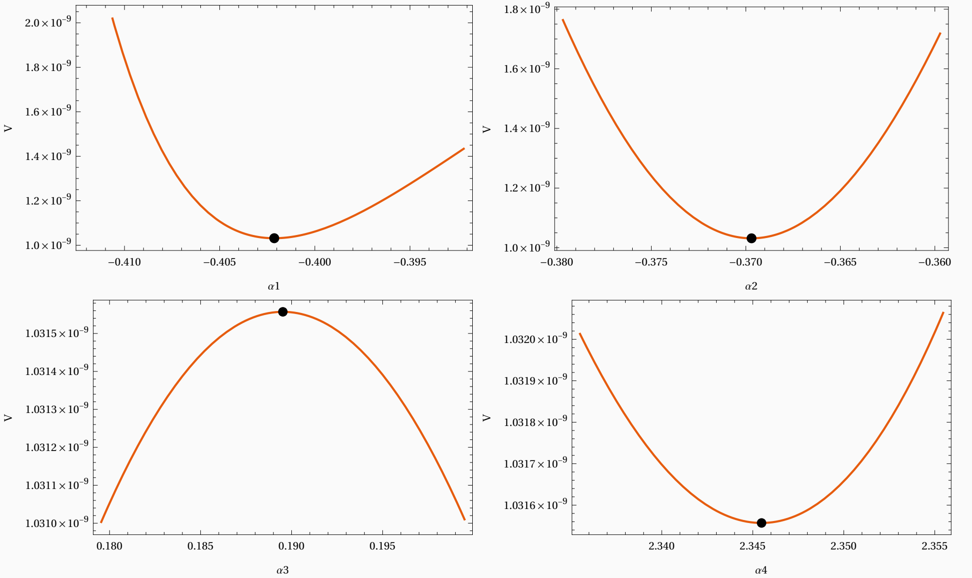
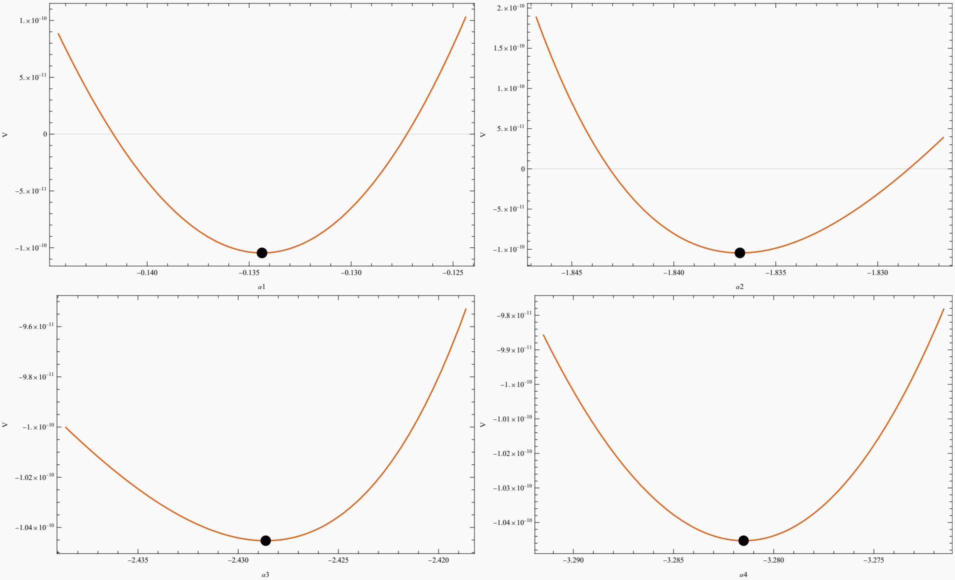
De-Sitter saddle point



AdS minimum



Refined de-Sitter conjecture

The scalar potential V in any EFT consistent with quantum gravity must satisfy[9]

$$|\nabla V| \geq \frac{c}{M_p} V\,, \quad \textbf{or,} \quad \textbf{min}(\nabla_i \nabla_j V) \leq -\frac{c'}{M_p^2} V\,.$$

where c and c' are positive constants of $\mathcal{O}(1)$, and $\min(\nabla^i \nabla_j V)$ denotes the smallest eigenvalue of the Hessian in an orthonormal frame.

According to this conjecture, our theory does not admit any de-Sitter minimum.

[9] Ooguri, Palti, Shiu, Vafa: arXiv:1810.05506