From Configurations to Correlators

— Lect. 1 : The Anatomy of a Lattice Measurement
— Lect. 2 : Machine Learning in Measurements

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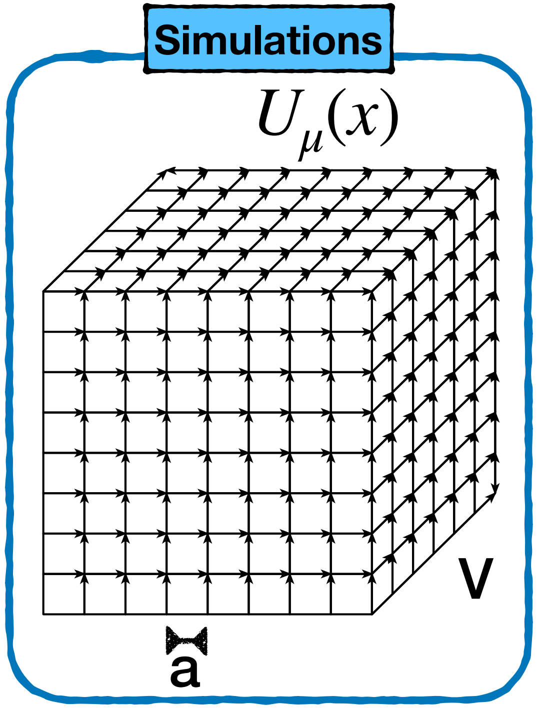
The Anatomy of a Lattice Measurement



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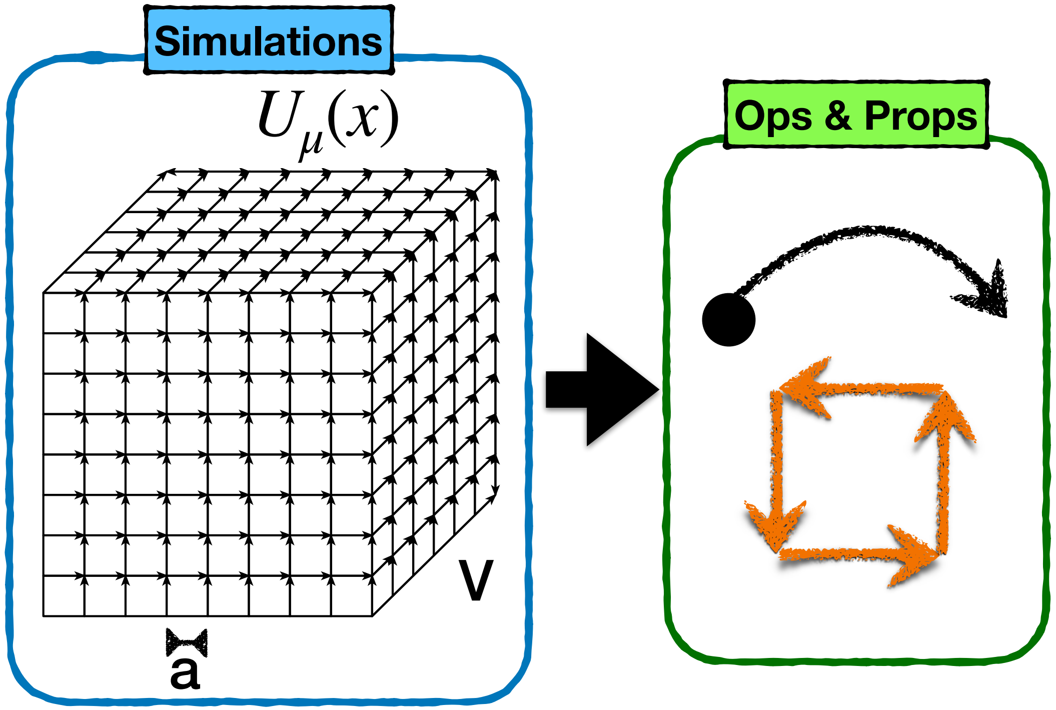

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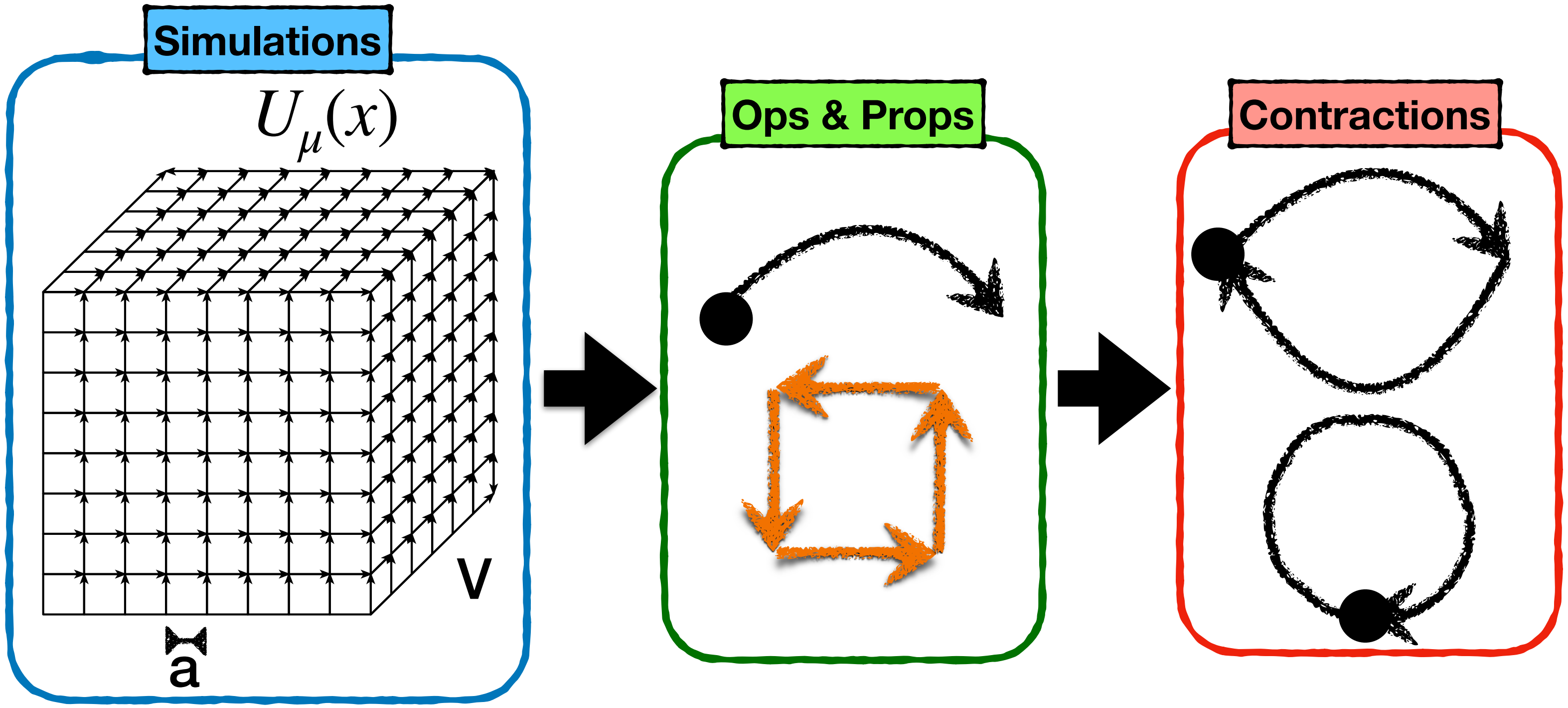

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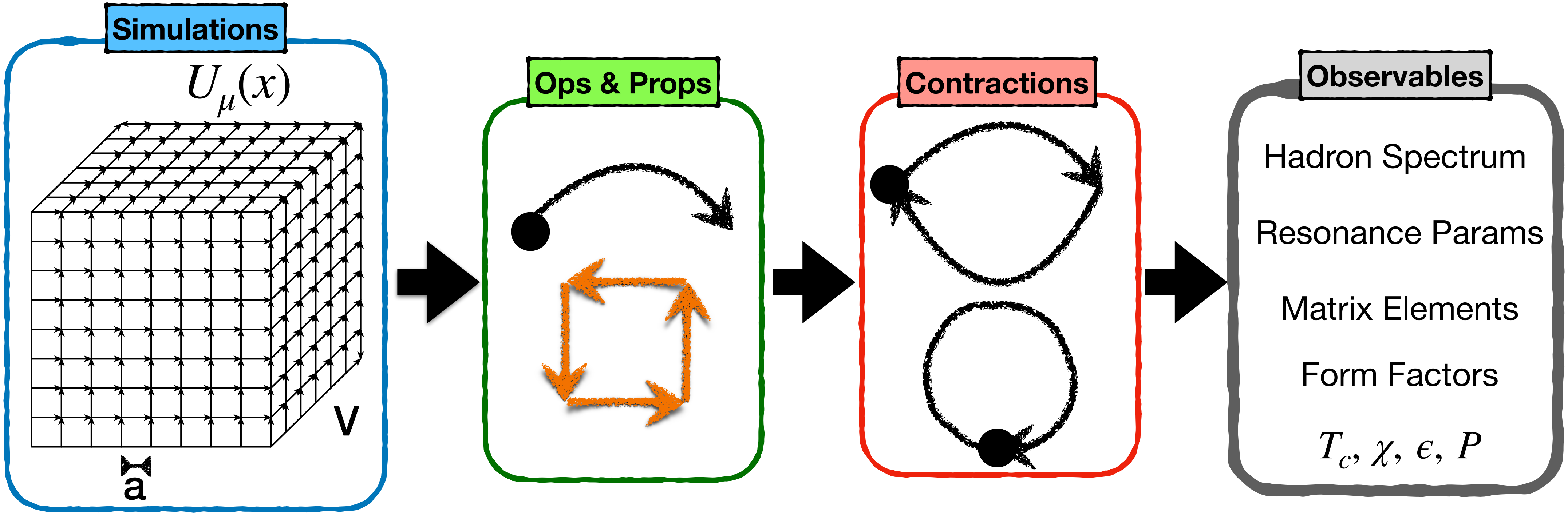

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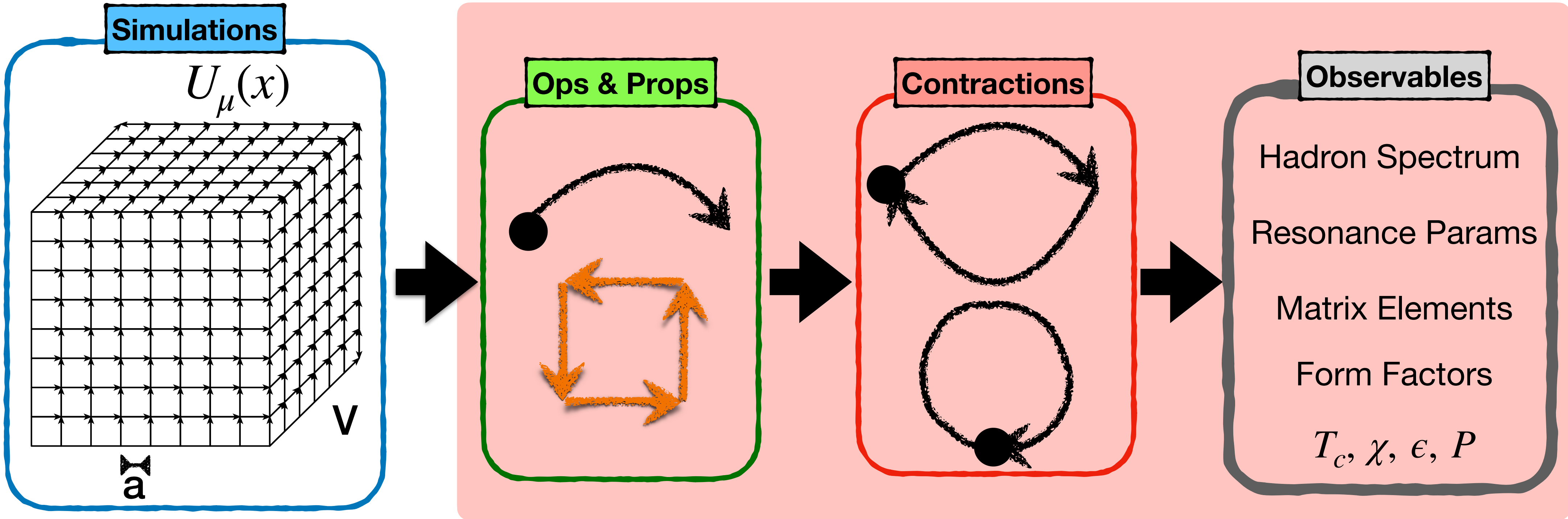

$$\langle \mathcal{O} \rangle_{a,V,n_f} = \frac{1}{\mathcal{Z}} \int \mathbb{D}[U]^V \ \mathbb{D}[\psi\bar{\psi}]^{Vn_f} \ \mathcal{O}(U,\psi,\bar{\psi}) \ e^{-S(a;\,U,\psi,\bar{\psi})}$$

Simulation vs Measurement — Two Different Problems

Every ensemble → $U^{(k+1} = T[U^{(k)}]$, sampling probability $e^{-S[U]}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathbb{D}[U] \;\; \mathcal{O}[U] \;\; e^{-S[U]} \xrightarrow{\text{MCMC}} E[\mathcal{O}] = \frac{\sum_k \mathcal{O}[U^{(k)}]}{N_{\text{cfg}}}$$

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$$\text{Var}[\mathcal{O}] = \tau_{int} \;\; \frac{2\sigma^2}{N}$$

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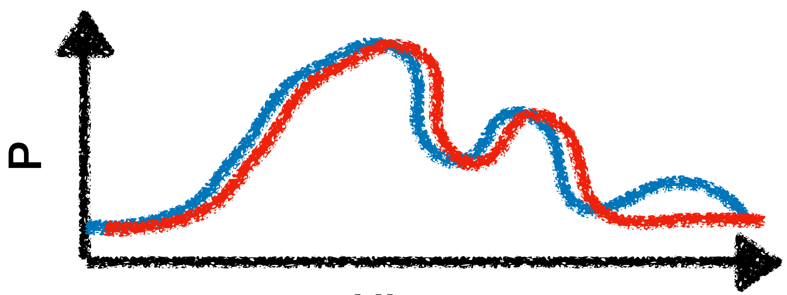


$$\text{Var}[\mathcal{O}] = \tau_{int} \frac{2\sigma^2}{N}$$

The **probability measure** and the **observable definition** need **good overlap**.

Simulation vs Measurement — Two Different Problems

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathbb{D}[U]\, \mathcal{O}[U]\, e^{-S[U]}$$

The **probability measure** and the **observable definition** are **decoupled**.

Reweighting 🙂

$$\langle \mathcal{O} \rangle_{\beta'} = \frac{\langle \mathcal{O}[U] w(U) \rangle_{\beta}}{\langle w(U) \rangle_{\beta}}, \quad w(U) = e^{-\left(S_{\beta'} - S_{\beta}\right)}$$

Scan the neighborhood of parameter space of β.

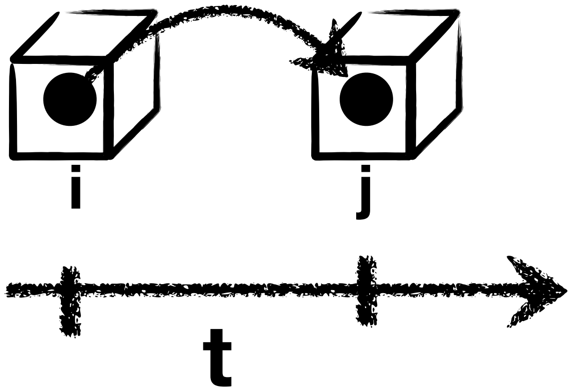
Sign Problem 😫

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O}[U] e^{i\phi[U]} \rangle_P}{\langle e^{i\phi[U]} \rangle_P}$$

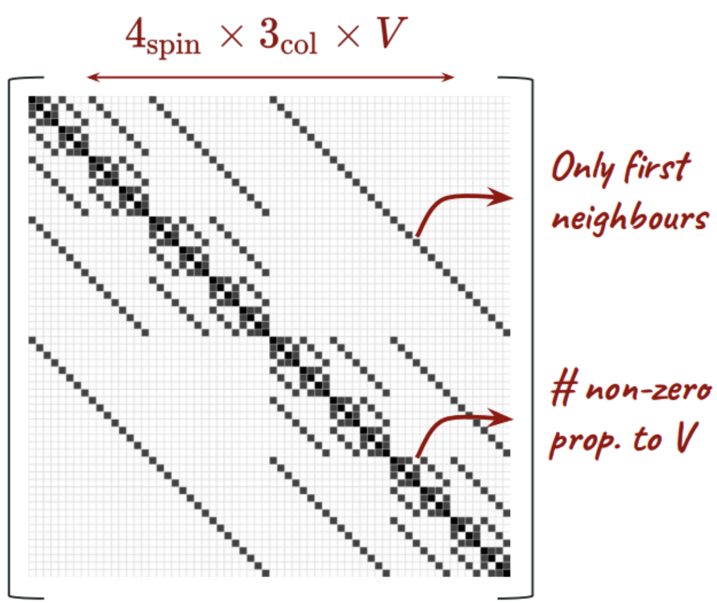
What the Simulation hands off to Measurements ?

$$\langle \mathcal{O}\rangle_{n_f} = \frac{1}{\mathcal{Z}}\int \mathbb{D}[U]\ \mathbb{D}[\psi\bar{\psi}]\ \mathcal{O}(U,\psi,\bar{\psi})\, e^{-S(U,\psi,\bar{\psi})}$$

$$\underbrace{\left\langle \psi_j\bar{\psi}_i\right\rangle}_{n_f=1} = \frac{1}{\mathcal{Z}}\int [\mathbb{D}U]\ \underbrace{D^{-1}_{ji}[U]}_{\text{valence propagator}}\ \underbrace{\det D[U]\, e^{-S_{\mathrm{YM}}[U]}}_{\text{sea weight}}$$



$$\boxed{D \equiv \gamma^{\mu}\mathcal{D}_{\mu}[U] + m_f}$$



What's the issue with D^{-1}?

$$D \equiv \gamma^{\mu}\mathscr{D}_{\mu}[U] + m_f \implies D^{-1} \propto \frac{1}{m_f}$$ **[High Condition number $D^{\dagger}D$]**

- Solve $D\ x = b$

- Most **expensive** part, usually by iterative method (BiCGStab)

  $x_{k+1} = f(D, b, x_k)$ with $x_k \to x$ (true solution)

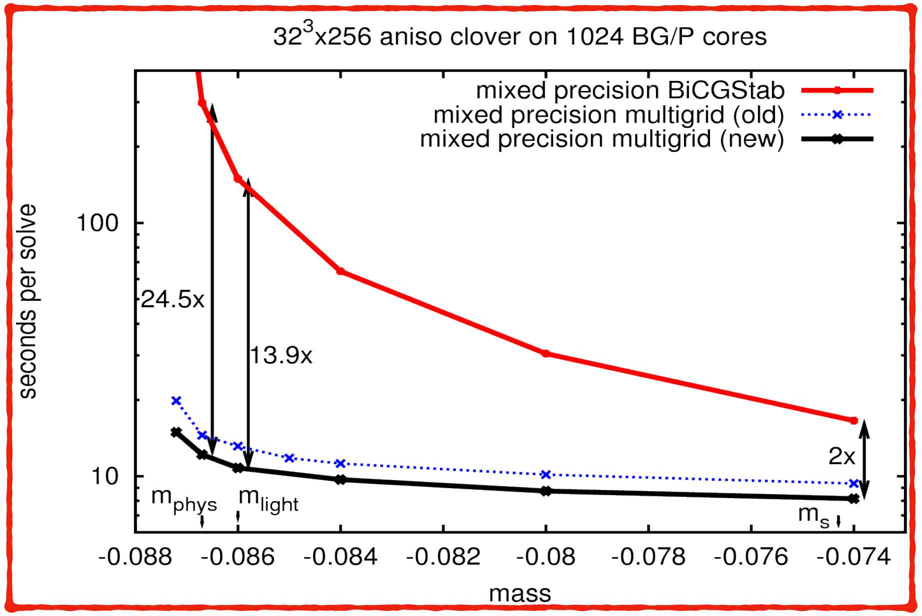
  $r_k = b - D\,x_k$ with $r_k \to 0$

- Find a **preconditioner** M such that $M \approx D^{-1}$ s.t

  $(DM)\ v = b, \quad v = M^{-1}x$ **[Low Condition number]**

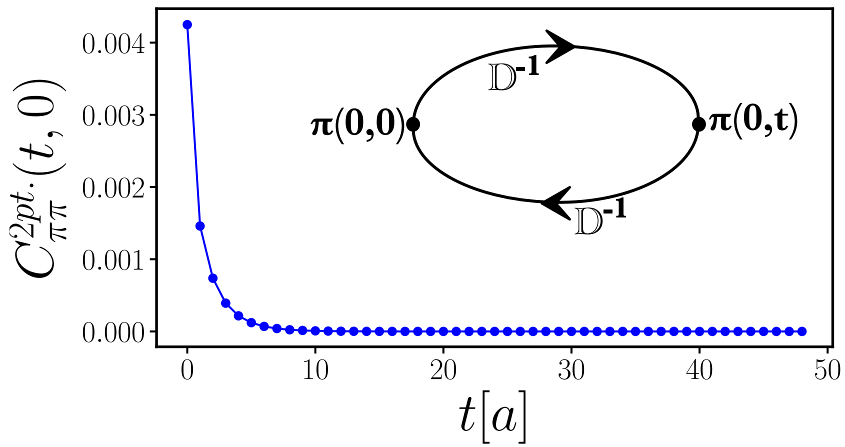
- **Multigrid** approximates **low mode** and **high modes.**

$$D = \sum_n \lambda_n |n\rangle\langle n|$$



[Babich et.al., 1011.2775]

What we measure?



Euclidean 2pt Correlation function

$$C^{2pt.}_{ab}(t) \equiv \left\langle 0 \left| \mathcal{O}_b(t)\ \mathcal{O}^\dagger_a(0) \right| 0 \right\rangle$$

$$= \frac{\text{Tr}[\ e^{-(L_t - t)H}\ \mathcal{O}_b\ e^{-tH}\ \ \mathcal{O}^\dagger_a\ ]}{\text{Tr}[\ e^{-L_t H}\ ]} \qquad t > 0$$

$$= \sum_n \left\langle 0 \left| \mathcal{O}_b(0) \right| n, L \right\rangle \left\langle n, L \left| \mathcal{O}^\dagger_a(0) \right| 0 \right\rangle e^{-E_n t} \qquad L_t \to \infty$$

$$= \sum_n Z_{b,n} Z^\dagger_{a,n} e^{-E_n t}$$

Classification of Lattice Observables

- Quark Propagator based $\rightarrow$ (Mesons, Baryons, Form factors)
- Pure Gauge $\rightarrow$ (Wilson loops, Glueballs, Polyakov loop)
- Mixed/Hybrid $\rightarrow$ (hybrids, extended-light)
- Derived $\rightarrow$ (Scattering, Thermodynamics)

Propagator Based : Mesons

$$\mathcal{O}_{\text{meson}}(x) = \sum_{a,\alpha,\beta} \bar{q}_1(x)^a_\alpha \, \Gamma_{\alpha,\beta} \, q_2(x)^a_\beta$$

- **<u>Flavor Structure</u>**: Flavor quantum numbers by q_1 and q_2
- **<u>Spin & Parity:</u>** Γ matrix choice.
    - $\mathbb{1}$ is **<u>scalar</u>** ($J^P = 0^+$) : σ [Not sure]
    - γ_5 is **<u>pseudoscalar</u>** ($J^P = 0^-$) : $\pi^\pm, \pi^0$
    - γ_μ is **<u>vector</u>** ($J^P = 1^-$) : ρ, ω
    - $\gamma_\mu\gamma_5$ is **<u>axial-vector</u>** ($J^P = 1^+$) : a_1

$f_0(500)$ $I^G(J^{PC}) = 0^+(0^{++})$

also known as σ; was $f_0(600)$, $f_0(400\text{–}1200)$

See the review on "Scalar Mesons below 1 GeV."

Mass (T-Matrix Pole $\sqrt{s}$) = (400–550)$-i$(200–350) MeV
Mass (Breit-Wigner) = 400 to 800 MeV
Full width (Breit-Wigner) = 100 to 800 MeV

$f_0(500)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	seen	–
$\gamma\gamma$	seen	–

Propagator Based : Baryons

$$\mathcal{O}_{\text{baryon}}(x) = \epsilon_{abc}\, P_{\pm}\, \Gamma_A\, q_1(x)^a \, \left(q_2^T(x)^b\, \Gamma_B\, q_3(x)^c \right)$$

- **<u>Spin Selection:</u>**

$$J = \frac{1}{2} \;\rightarrow\; (\Gamma^A, \Gamma^B) = (\mathbb{I}, C\gamma_5), (\gamma_5, C), \text{ or } (\mathbb{I}, i\gamma_4\, C\,\gamma_5)$$

$$J = \frac{3}{2} \;\rightarrow\; (\Gamma^A, \Gamma^B) = \;(\mathbb{I}, C\gamma_i)$$

- **<u>Parity Selection:</u>** $P_{\pm} = \frac{1}{2}\left(\mathbb{I} \pm \gamma_4\right)$

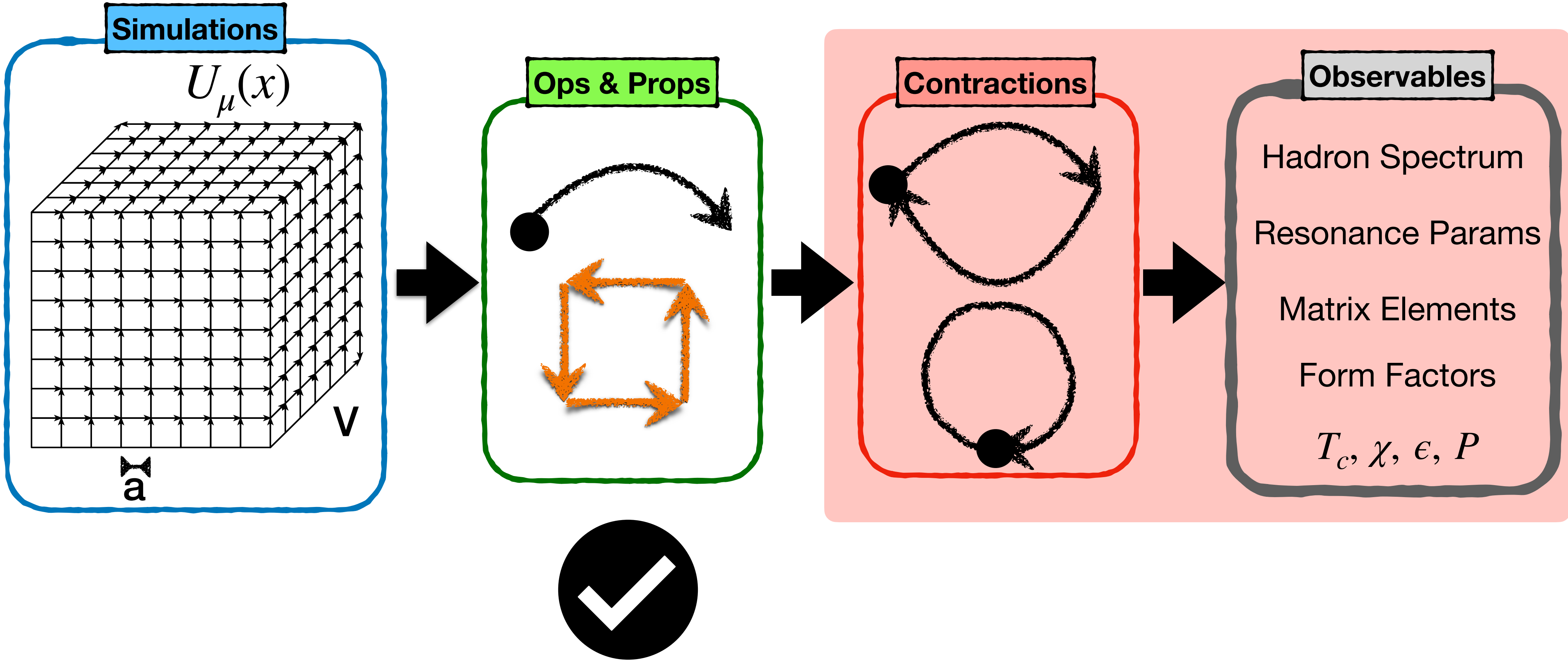
- **<u>Charge Conjugation:</u>** C is Charge Conjugation matrix. $C \equiv i\gamma_2\gamma_4$

- ϵ_{abc} ensures antisymmetry in the color indices.

Propagator Based : Baryons

Baryon	Quark content	Interpolating field	I	I_z
p	uud	$\epsilon_{abc}\left(u_a^T C\gamma_5 d_b\right)u_c$	1/2	+1/2
n	udd	$\epsilon_{abc}\left(d_a^T C\gamma_5 u_b\right)d_c$	1/2	-1/2
Λ	uds	$\frac{1}{\sqrt{6}}\epsilon_{abc}\left[2\left(u_a^T C\gamma_5 d_b\right)s_c+\left(u_a^T C\gamma_5 s_b\right)d_c-\left(d_a^T C\gamma_5 s_b\right)u_c\right]$	0	0
Σ^+	uus	$\epsilon_{abc}\left(u_a^T C\gamma_5 s_b\right)u_c$	1	+1
Σ^0	uds	$\frac{1}{\sqrt{2}}\epsilon_{abc}\left[\left(u_a^T C\gamma_5 s_b\right)d_c+\left(d_a^T C\gamma_5 s_b\right)u_c\right]$	1	0
Σ^-	dds	$\epsilon_{abc}\left(d_a^T C\gamma_5 s_b\right)d_c$	1	-1
Ξ^0	uss	$\epsilon_{abc}\left(s_a^T C\gamma_5 u_b\right)s_c$	1/2	+1/2
Ξ^-	dss	$\epsilon_{abc}\left(s_a^T C\gamma_5 d_b\right)s_c$	1/2	-1/2
Δ^{++}	uuu	$\epsilon_{abc}\left(u_a^T C\gamma_\mu u_b\right)u_c$	3/2	+3/2
Δ^+	uud	$\frac{1}{\sqrt{3}}\epsilon_{abc}\left[2\left(u_a^T C\gamma_\mu d_b\right)u_c+\left(u_a^T C\gamma_\mu u_b\right)d_c\right]$	3/2	+1/2
Δ^0	udd	$\frac{1}{\sqrt{3}}\epsilon_{abc}\left[2\left(d_a^T C\gamma_\mu u_b\right)d_c+\left(d_a^T C\gamma_\mu d_b\right)u_c\right]$	3/2	-1/2
Δ^-	ddd	$\epsilon_{abc}\left(d_a^T C\gamma_\mu d_b\right)d_c$	3/2	-3/2

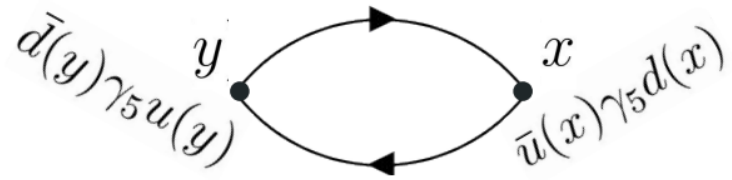
The Anatomy of a Lattice Measurement



Wick Contractions: Structure of Correlators

$$\left\langle \eta_{i_1}\bar{\eta}_{j_1}\ldots\eta_{i_n}\bar{\eta}_{j_n}\right\rangle_F = = (-1)^n \sum_{P(1,2,\ldots,n)} \mathrm{sign}(P)\left(D^{-1}\right)_{i_1 j_{P_1}}\left(D^{-1}\right)_{i_2 j_{P_2}}\ldots\left(D^{-1}\right)_{i_n j_{P_n}}$$

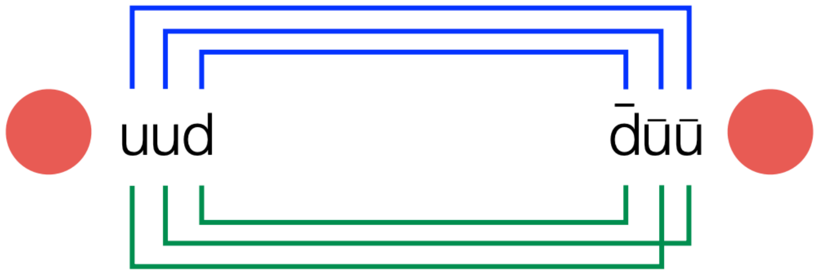
Lets' compute for a pion,



$$\left\langle \mathcal{O}_{\pi^+}(y)\mathcal{O}^{\dagger}_{\pi^+}(x)\right\rangle = \left\langle \bar{d}(y)\gamma_5 u(y)\bar{u}(x)\gamma_5 d(x)\right\rangle = -\,\mathrm{Tr}\left[\gamma_5\, D_d^{-1}(x;y)\gamma_5\, D_u^{-1}(y;x)\right]$$

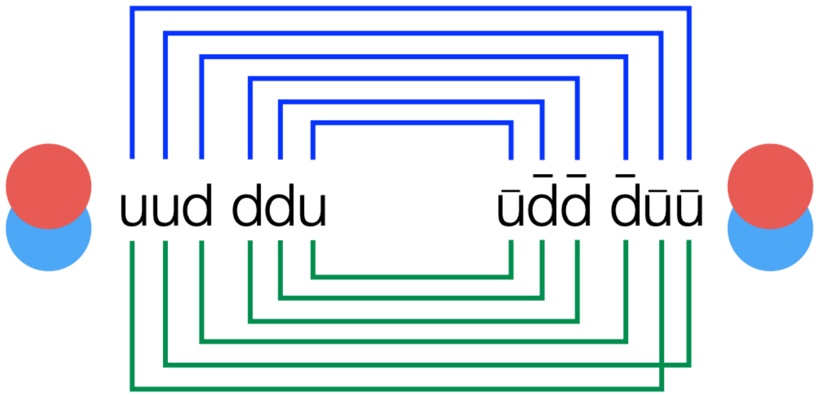
Wick Contractions Explosion: Obstacle

Single Proton



2

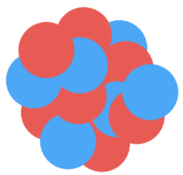
Deuteron



36

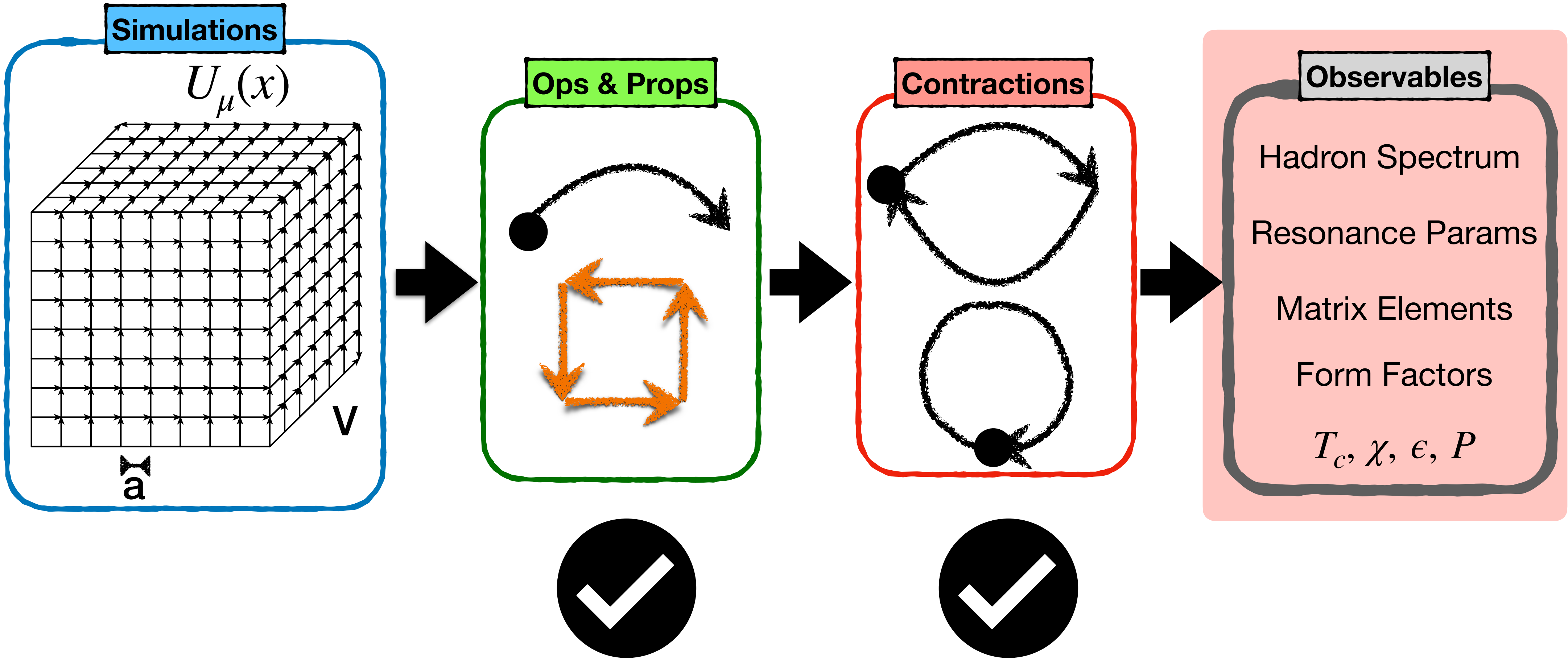
+ many more!

Wick Contractions Explosion: Obstacle

Single Proton		uud	2
Deuteron		uud ddu	36
Dineutron		ddu ddu	48
^{A}Z Nucleus			u! d! (A+Z)! (2A-Z)!

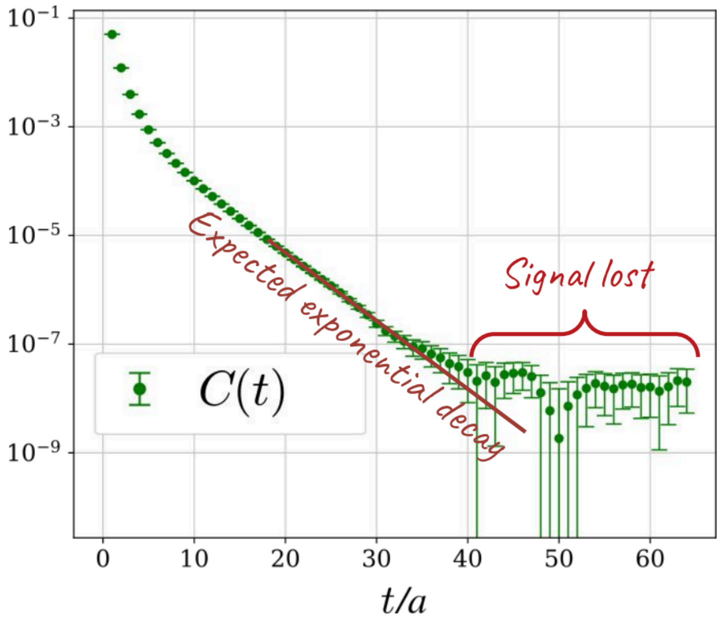
Nontrivial concern: writing correct Wick contractions!

The Anatomy of a Lattice Measurement



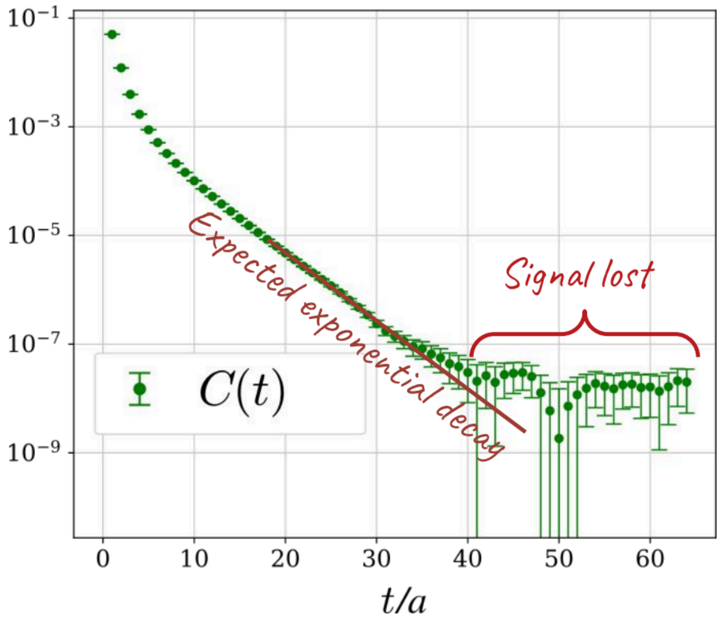
Extract energies from two-point functions

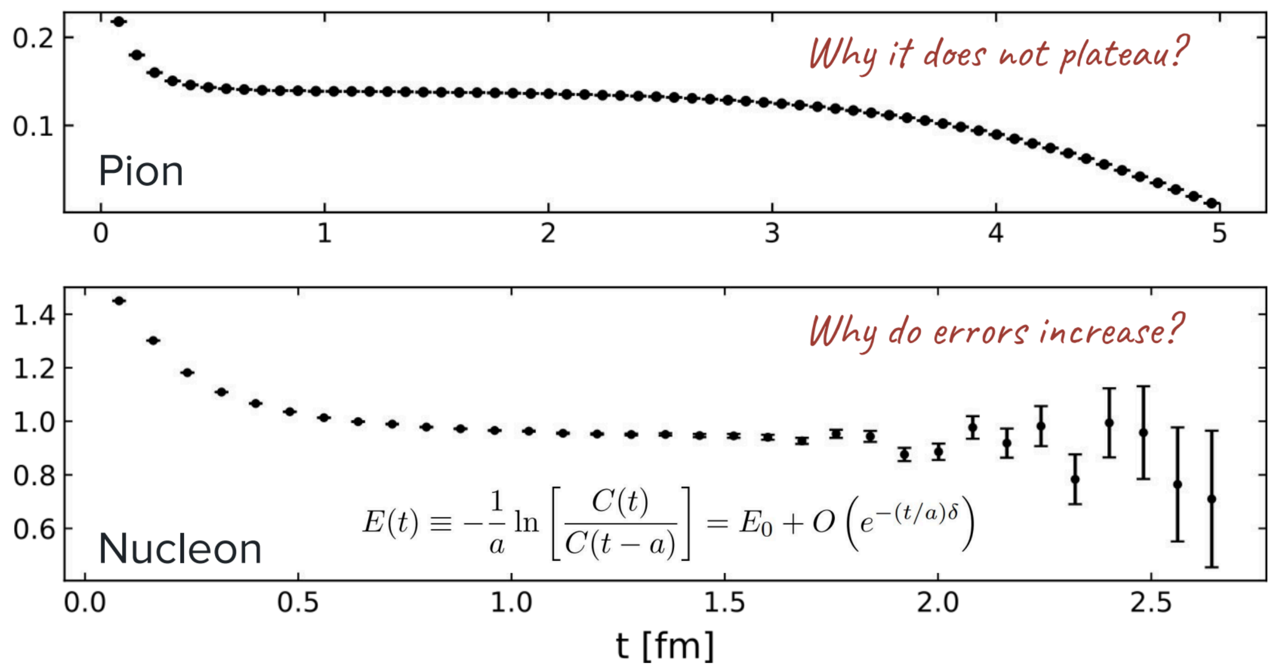
$$C_{\mathcal{O}}(t) = \sum_n |Z_{\mathcal{O},n}|^2 e^{-E_n t}$$



Extract energies from two-point functions

$$C_{\mathcal{O}}(t) = \sum_n |Z_{\mathcal{O},n}|^2 e^{-E_n t} \implies E(t) \equiv \frac{1}{a}\log\left[\frac{C_{\mathcal{O}}(t)}{C_{\mathcal{O}}(t-a)}\right] = E_0 + O\left(e^{-(t/a)\delta}\right)$$





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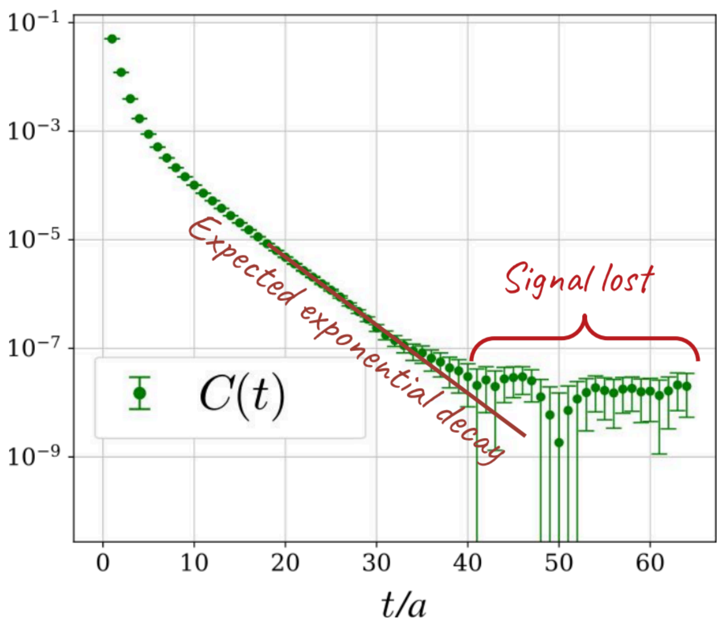
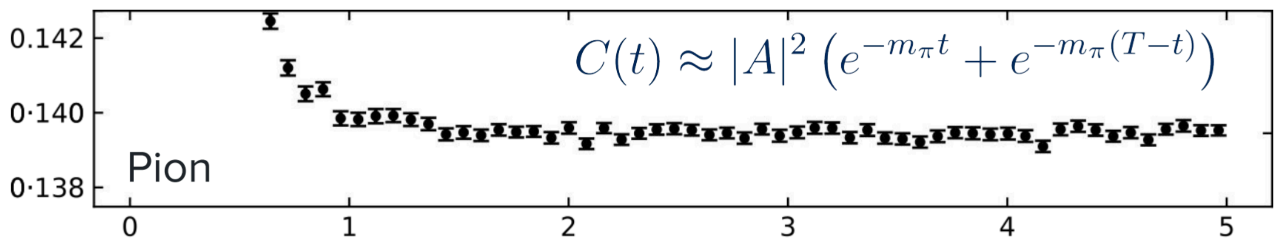
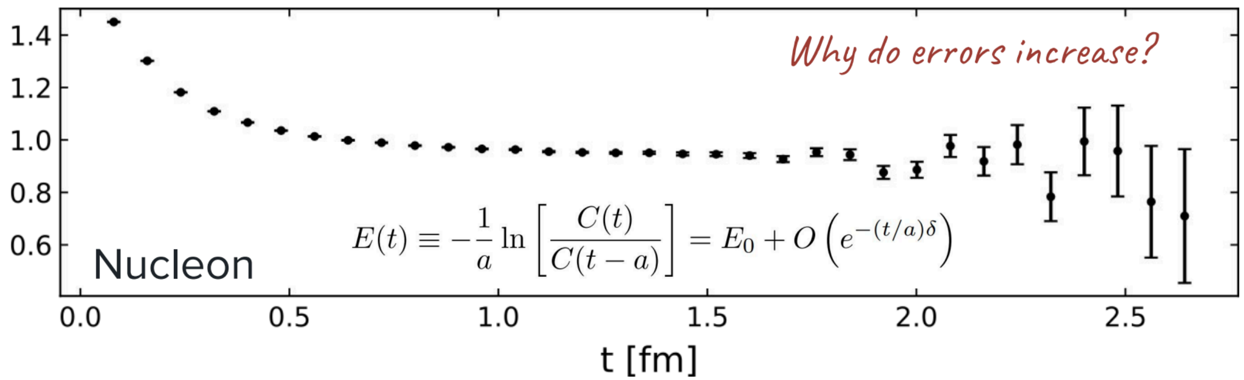






Signal-to-Noise in Euclidean Correlators

$$C(t) \equiv \left\langle 0 \left| \mathcal{O}(t)\ \mathcal{O}^\dagger(0) \right| 0 \right\rangle$$ [Parisi-Lepage]

$$\text{Var}[X] = E[X^2] - E[X]^2$$

$$\sigma_C^2(t) \equiv \left\langle 0 \left| \mathcal{O}(t)\ \mathcal{O}^\dagger(0)\mathcal{O}(t)\ \mathcal{O}^\dagger(0) \right| 0 \right\rangle - \left\langle 0 \left| \mathcal{O}(t)\ \mathcal{O}^\dagger(0) \right| 0 \right\rangle^2$$

$$\frac{\sigma_C(t)}{C(t)} = \sqrt{\frac{\left\langle 0 \left| \mathcal{O}(t)\ \mathcal{O}^\dagger(0)\mathcal{O}(t)\ \mathcal{O}^\dagger(0) \right| 0 \right\rangle}{\left\langle 0 \left| \mathcal{O}(t)\ \mathcal{O}^\dagger(0) \right| 0 \right\rangle^2} - 1}$$

$$\overset{t>>1}{\approx} \sqrt{\frac{|B|^2 e^{-t E_{0,C^2}}}{|A|^4 e^{-2t E_{0,C}}} - 1}$$

$$\frac{\sigma_C(t)}{C(t)} \propto e^{t\left(E_{0,C} - \frac{1}{2}E_{0,C^2}\right)}$$

$$E_{0,C^2} \leq 2\, E_{0,C}$$

Signal-to-Noise in Euclidean Correlators tail

$$C(t) \equiv \left\langle 0 \middle| \mathcal{O}(t)\ \mathcal{O}^\dagger(0) \middle| 0 \right\rangle$$ [Parisi.] [Lepage,]

$$\text{Var}[X] = E[X^2] - E[X]^2$$

$$\sigma_C^2(t) \equiv \left\langle 0 \middle| \mathcal{O}(t)\ \mathcal{O}^\dagger(0)\mathcal{O}(t)\ \mathcal{O}^\dagger(0) \middle| 0 \right\rangle - \left\langle 0 \middle| \mathcal{O}(t)\ \mathcal{O}^\dagger(0) \middle| 0 \right\rangle^2$$

$$\frac{\sigma_C(t)}{C(t)} \propto e^{t\left(E_{0,C} - \frac{1}{2}E_{0,C^2}\right)}$$

$$E_{0,C^2} \leq 2\,E_{0,C}$$

Pion: $\dfrac{\sigma_C(t)}{C(t)} \propto$ const

Nucleon: $\dfrac{\sigma_C(t)}{C(t)} \propto e^{t(m_N - \frac{3}{2}m_\pi)}$

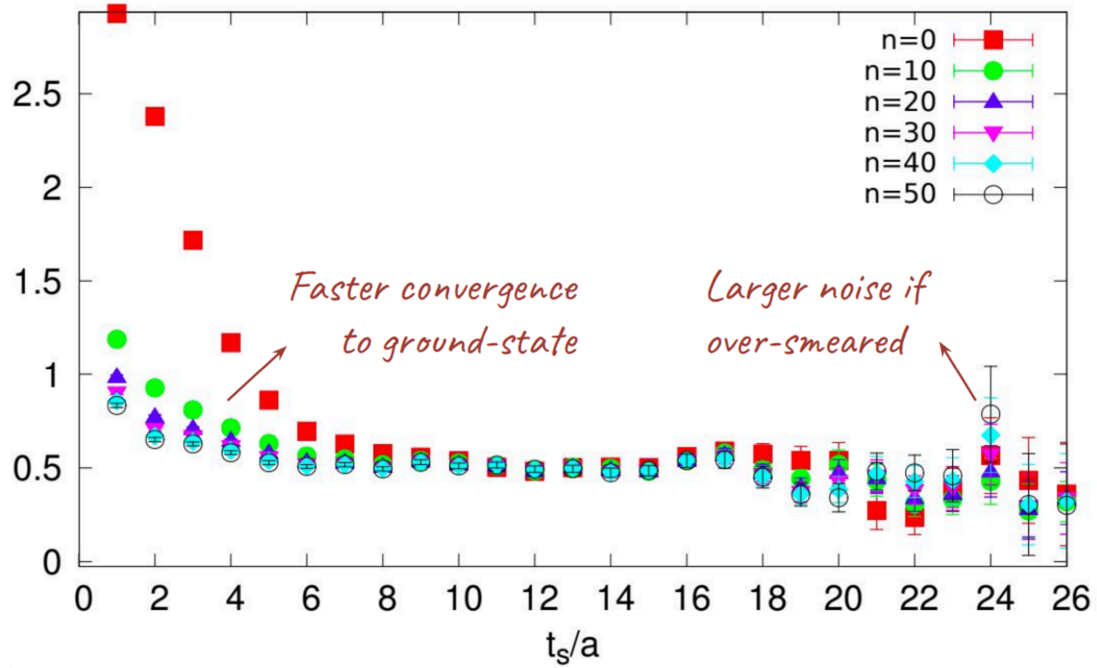
What can be done at early timeslices?

Smearing

$$\tilde{\mathcal{O}}_{a\alpha}(\vec{x},t) = \sum_{y} S_{ab}(\vec{x},\vec{y})\ \mathcal{O}_{b\alpha}(\vec{y},t)$$

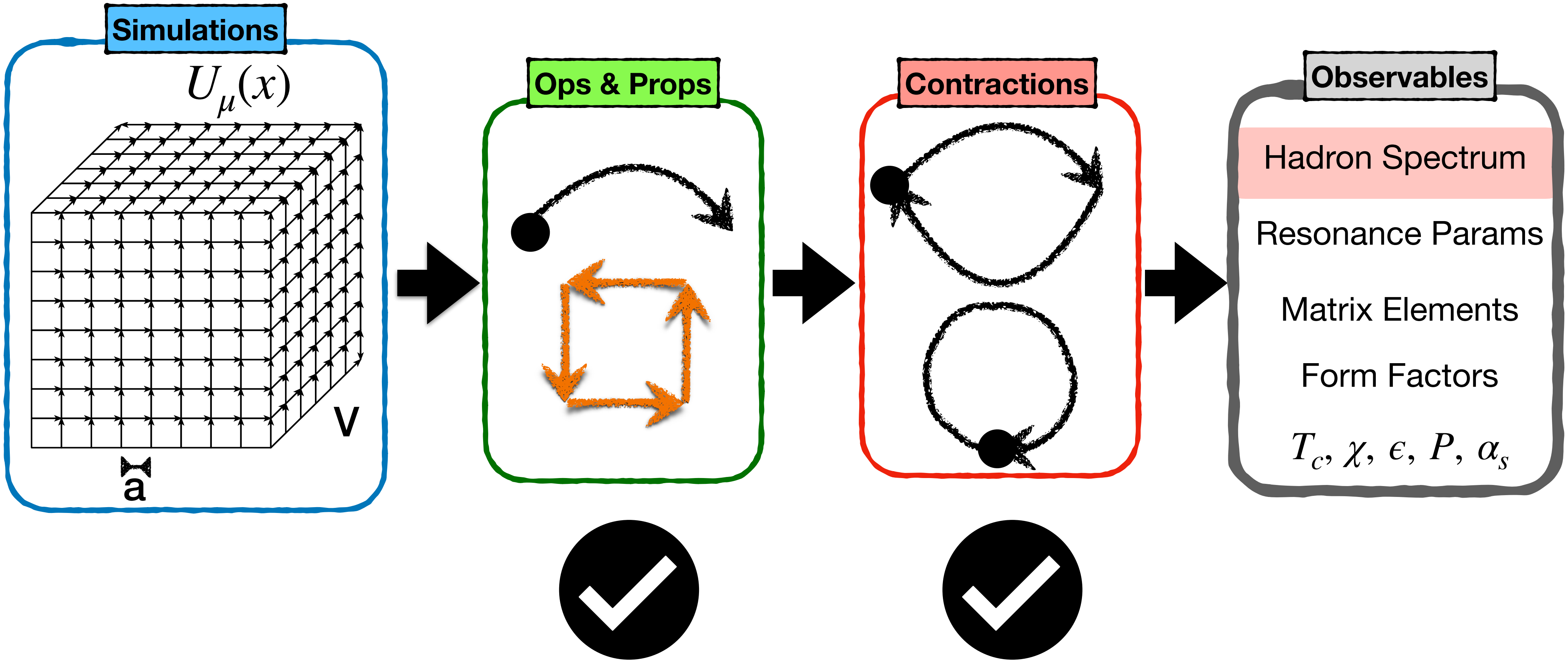
$$= \sum_{y} \left(\mathbb{1} + \alpha H(\vec{x},\vec{y};U(t)\right)^{n} \mathcal{O}(\vec{y},t)$$

Needs tuning

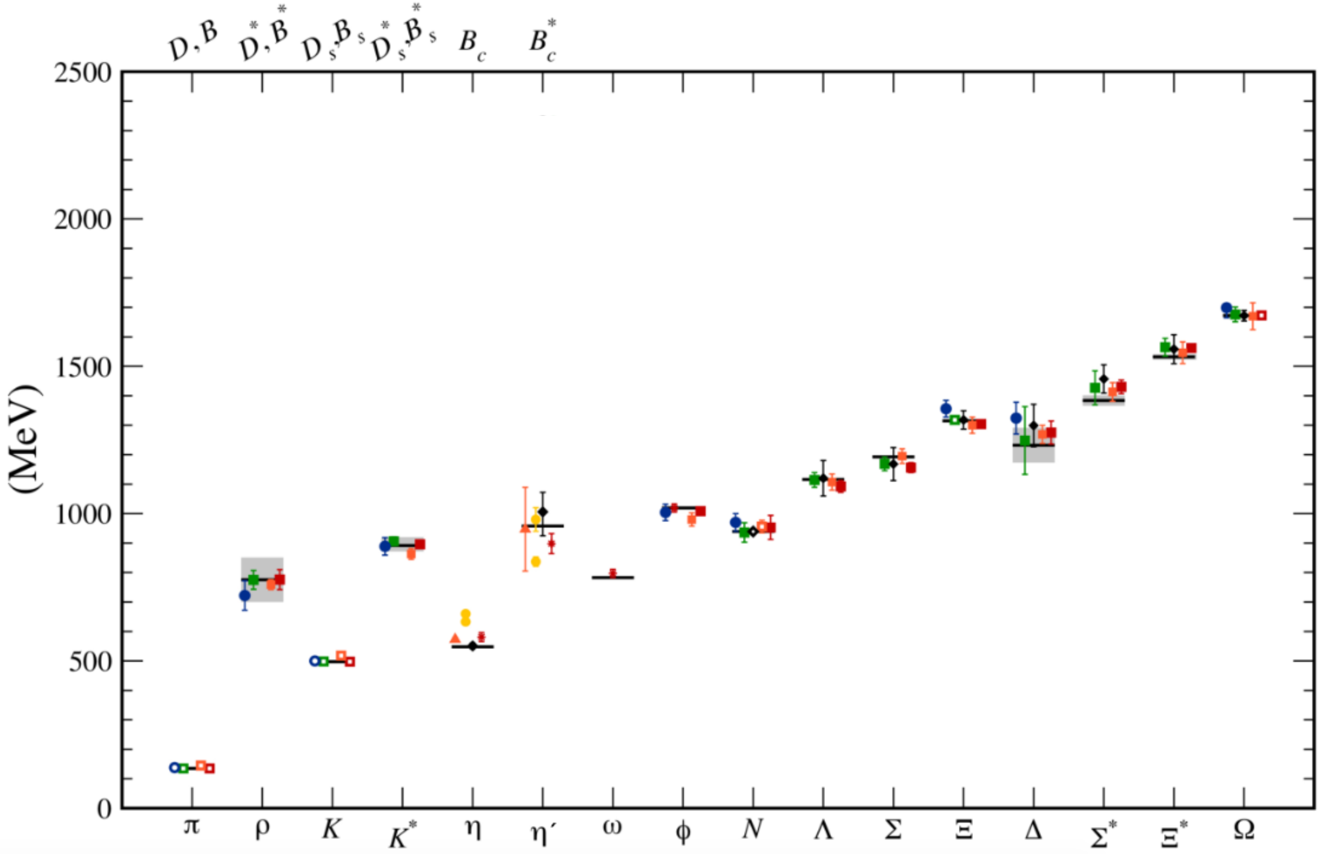


$$H(\vec{x},\vec{y};U(t)) = \sum_{k=1}^{3}\left(U_k(\vec{x},t)\delta_{\vec{x},\vec{y}-\hat{k}} + U_k^{\dagger}(\vec{x}-\hat{k},t)\delta_{\vec{x},\vec{y}+\hat{k}}\right)$$

The Anatomy of a Lattice Measurement

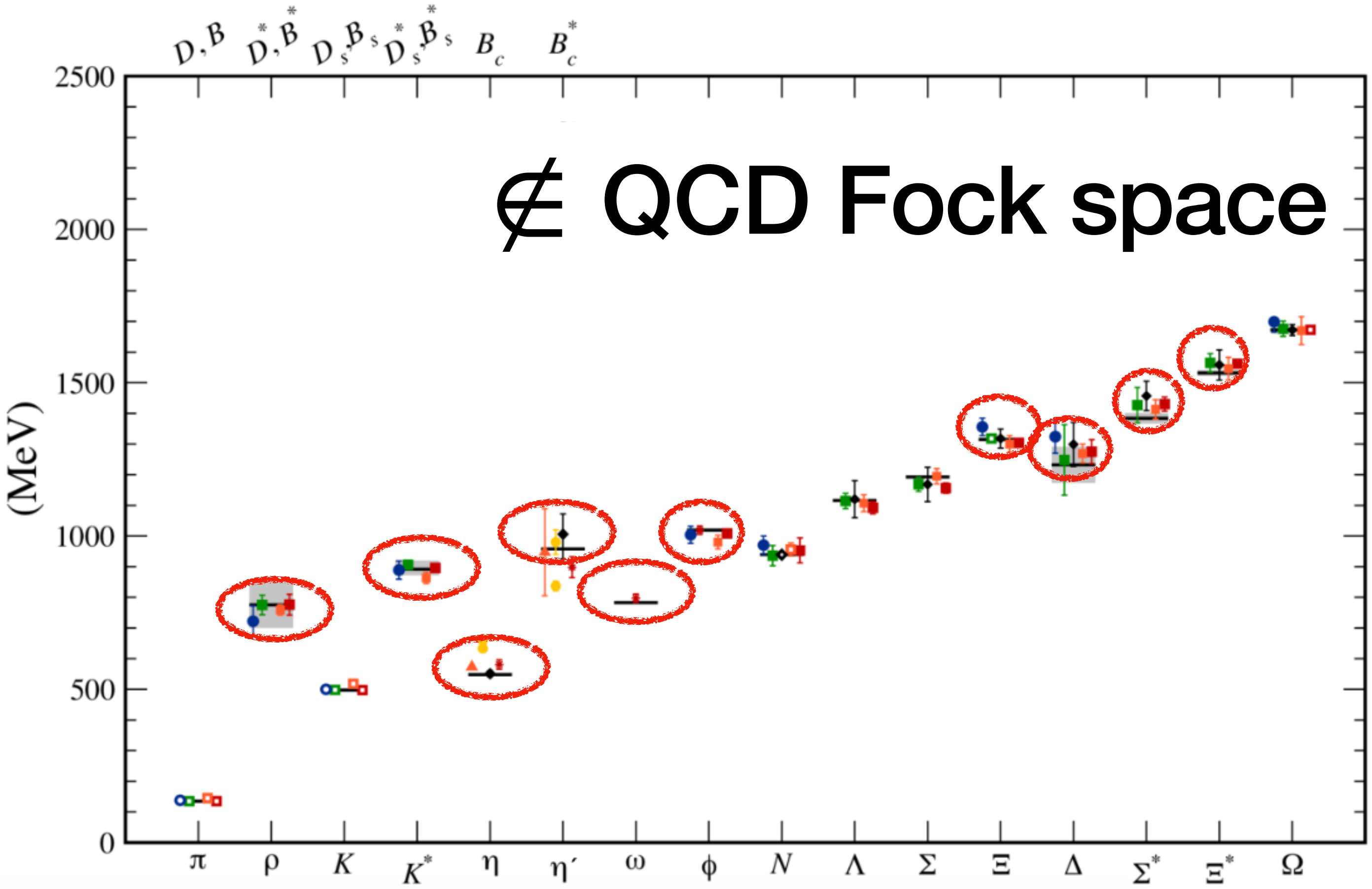


Are we done then?



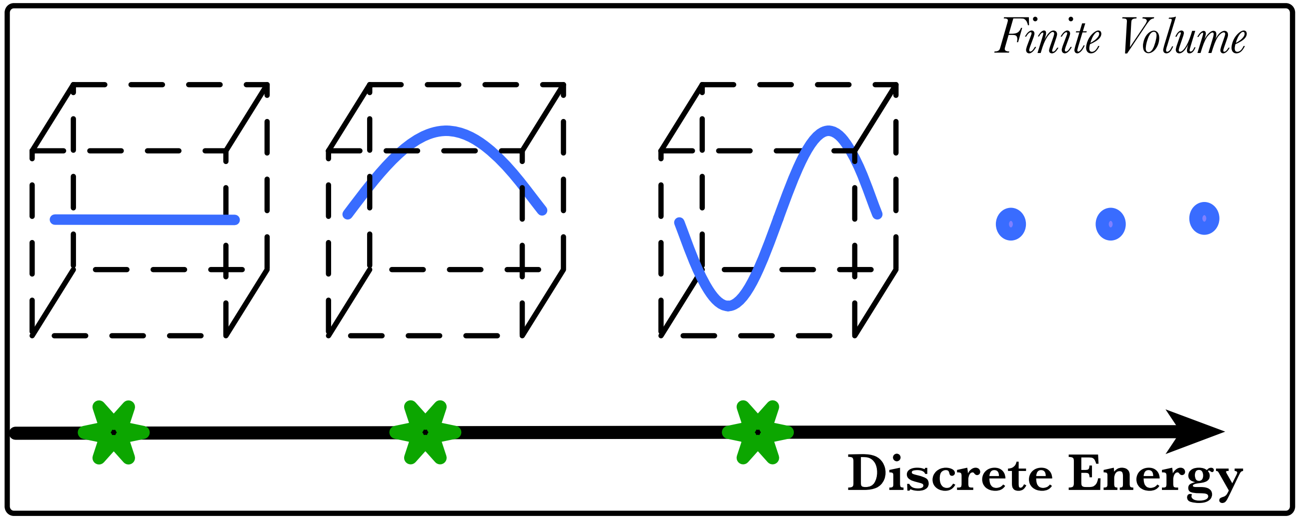
[PDG Review 2024], η,η' from HSC, RBC-UKQCD, Michael-Ottnad-Urbach

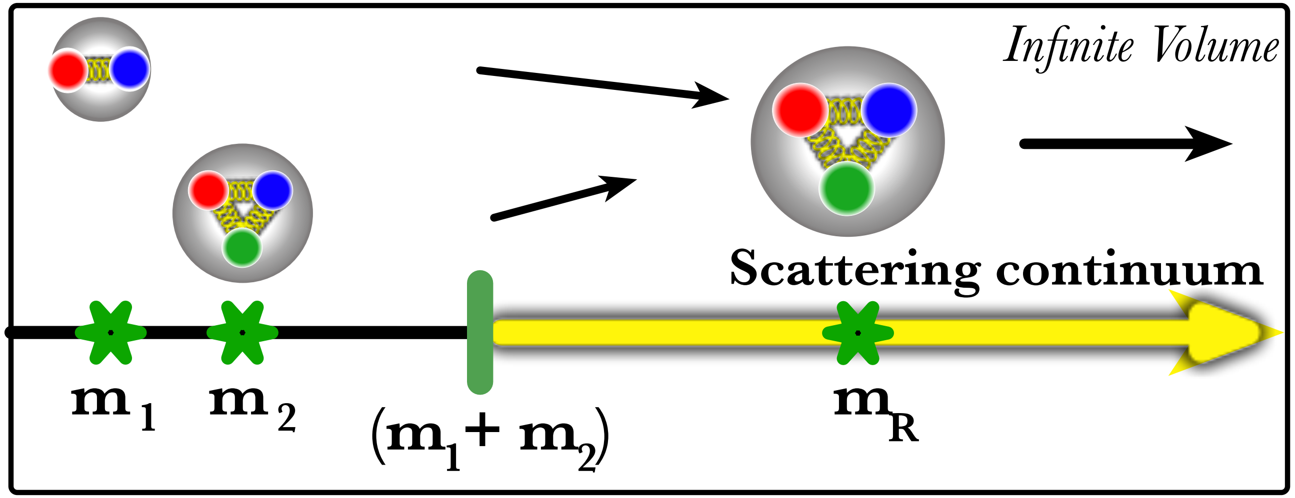
Are we done then? 🙁. NOT YET



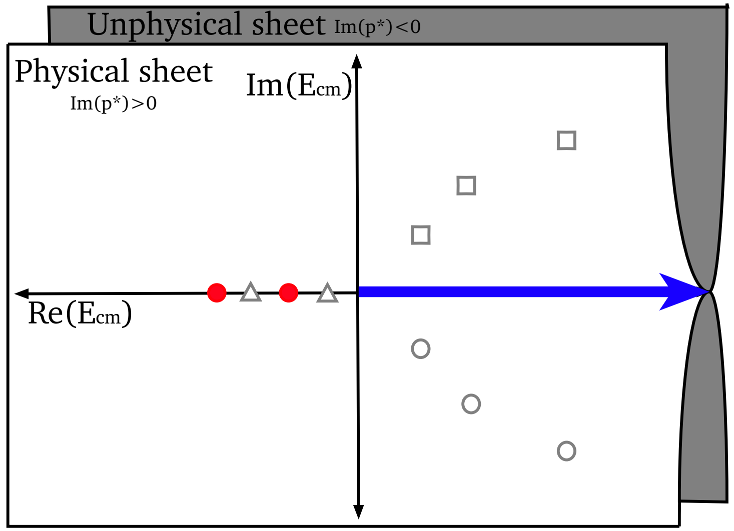
[PDG Review 2024], η,η' from HSC, RBC-UKQCD, Michael-Ottnad-Urbach

Scattering states on the lattice





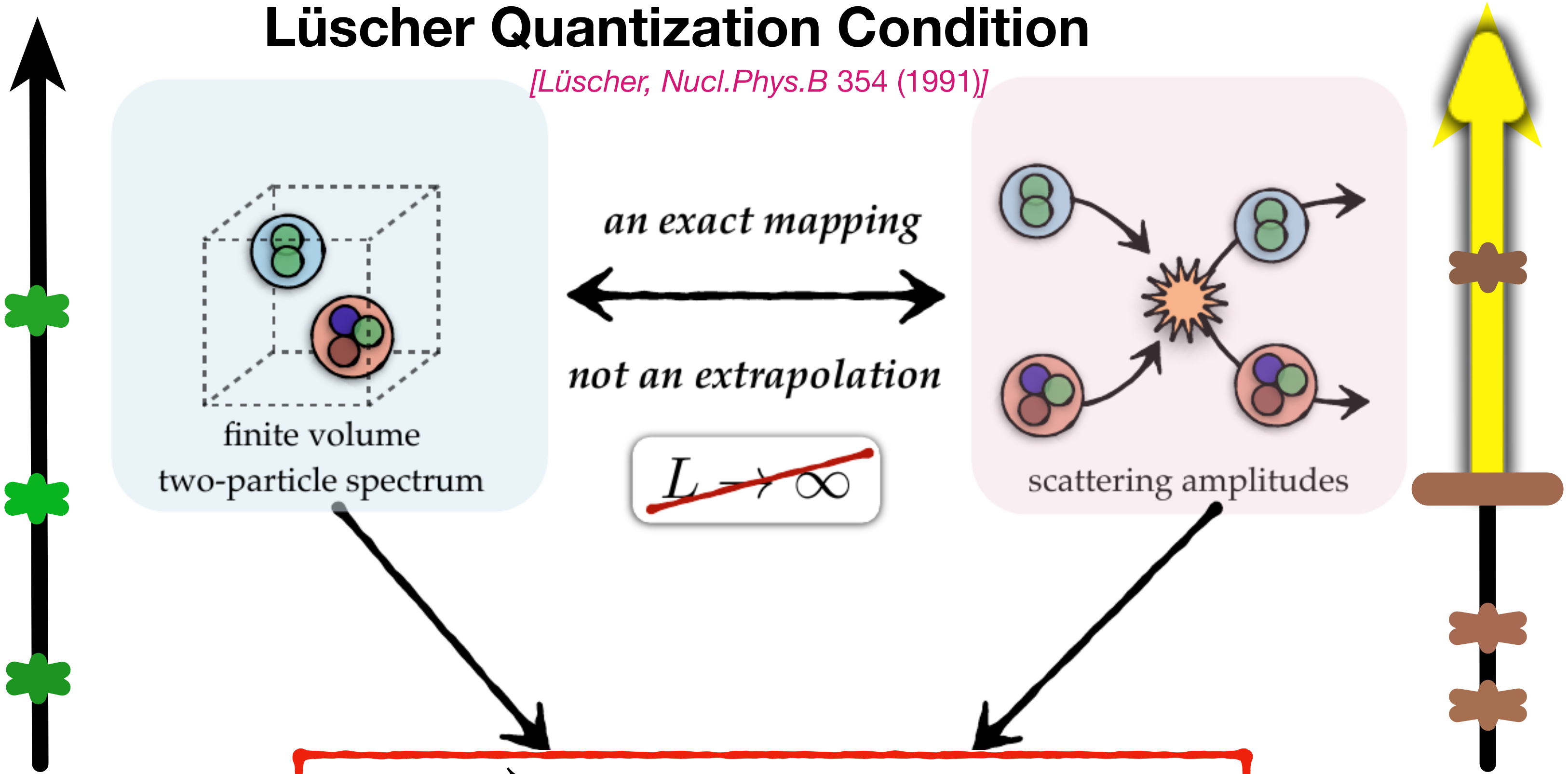
Issues
No Continuum
-No cuts
-No Sheet structure
-No Poles



Elastic (“near” Threshold) Scattering states on the lattice

Lüscher Quantization Condition

[Lüscher, Nucl.Phys.B 354 (1991)]



$$\mathcal{M}_{\ell}^{\vec{P},\Lambda}(E,L) = \cot\delta_{\ell}$$

Elastic (“near” Threshold) Scattering states on the lattice

Lüscher Quantization Condition

[Lüscher, Nucl.Phys.B 354 (1991)*]*

Assumptions

- Interaction range $< L$
- Two spin 0 particles only
- Elastic scattering
- Periodic box
- Upto Exponential corrections

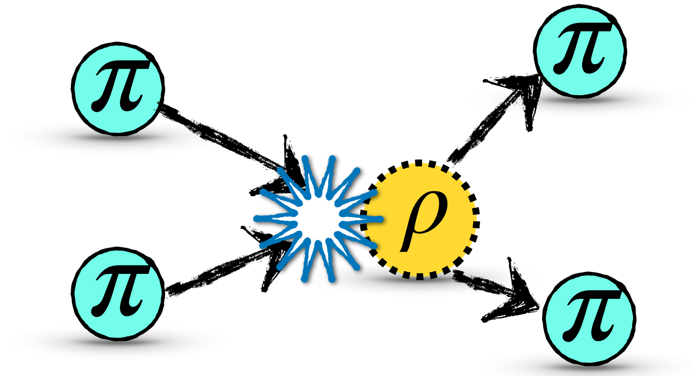
$$\mathcal{M}_{\ell}^{\vec{P},\Lambda}(E,L) = \cot\delta_{\ell}$$

Let's get into a real LQCD calculation..

Lattice Box : $32^3 \times 96$
L : 3.65 fm
a : 0.114 fm
$m_\pi L$: 5.86
Number of configurations: 1000
m_π ~320 MeV

Gauge Action: tree-level Symanzik action $\mathcal{O}(a^2)$
Fermion Action: Wilson clover $\mathcal{O}(a)$
Boundary: Periodic

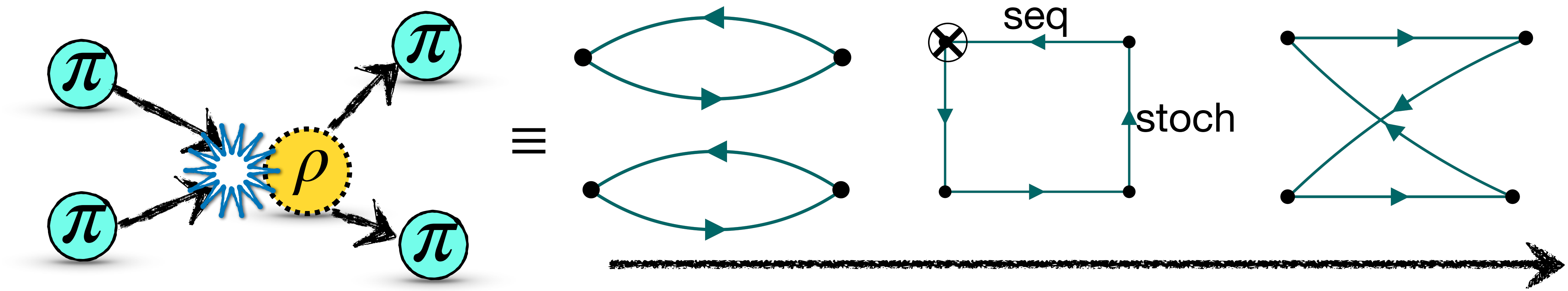
$$I = 1\,, J^P = 1^+$$



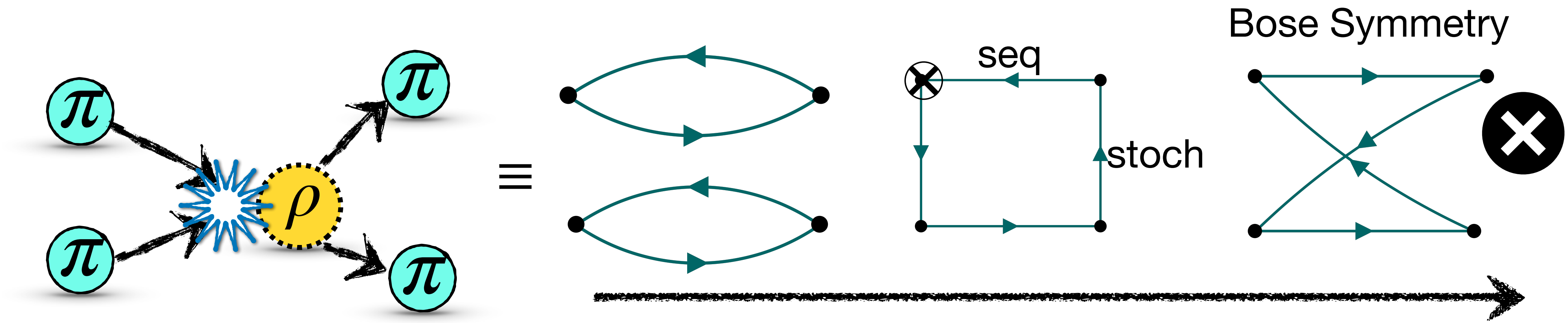
Energy Range to probe
using $\pi\pi$ Ops in $I = 1$

0.64 MeV ⟷ 1.0 GeV

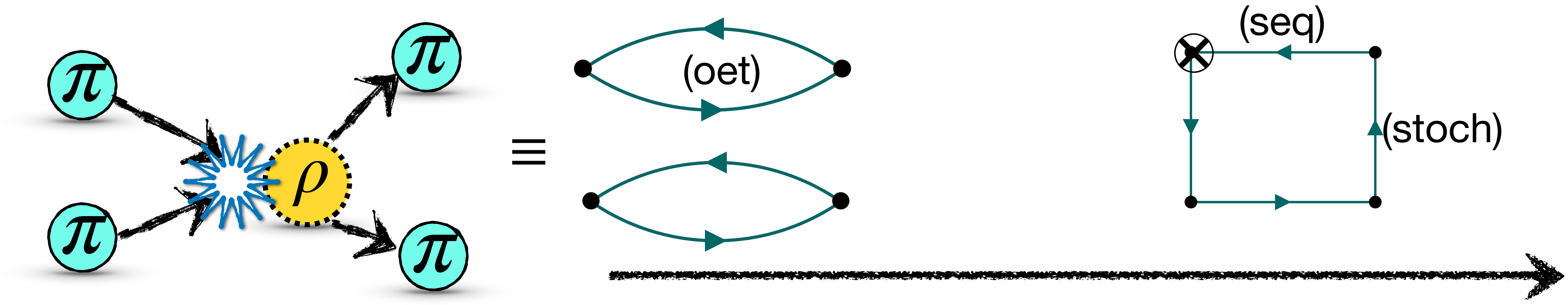
Ingredients for $n > 2$, n-point functions



Ingredients for $n > 2$, n-point functions



Ingredients for $n > 2$, n-point functions



⊗ Sequential Source/Propagators (seq): $\left(D^{-1}\Gamma_{i_2}(\vec{p}_{i_2})\, D^{-1}\right)(x,y)^{a,b}_{\alpha,\beta}$

Stochastic Propagator (stoch): $\phi^r(x)^a_\alpha\, \eta^{r*}(t,\vec{y})^b_\beta = \left(D^{-1}\,\eta^r\,\eta^{r\dagger}\right)(x;t,\vec{y})^{a,b}_{\alpha,\beta}$

One-end trick (oet):

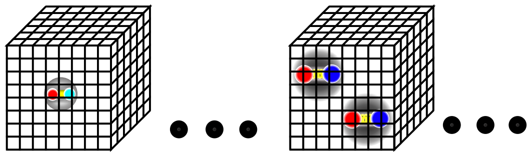
$$D_u^{-1}\,\Gamma_X\,D_d^{-1} \approx \phi\Gamma_X\gamma_5\,\phi^\dagger$$

η^r stochastic timeslice source ("full time dilution"), $r = 1, \ldots, N_{\mathrm{samples}}$,
fixed set of source locations $\{y\}$

Hadron Spectrum with excited states

Motivated from the scattering Experiments

Multihadron Op.

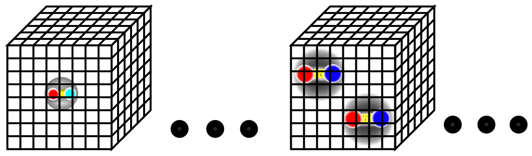


Same Quantum no.?

Hadron Spectrum with excited states

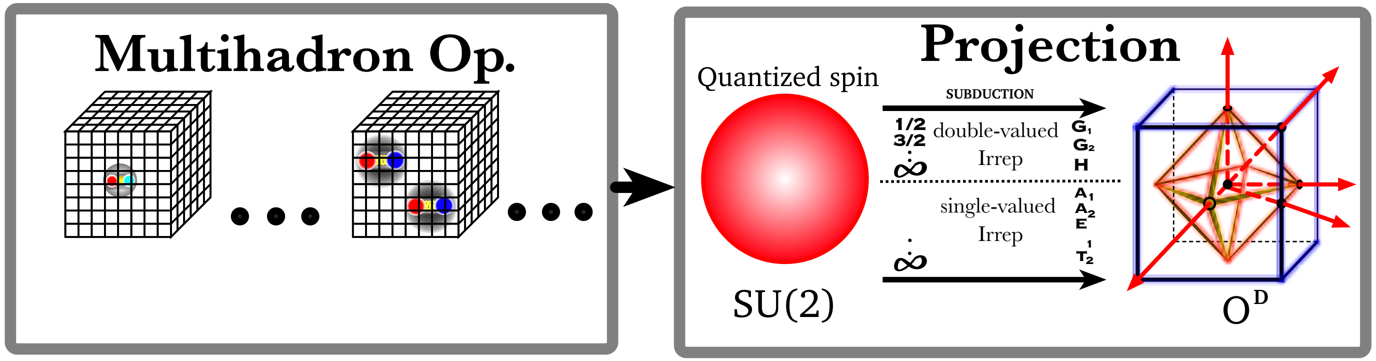
Motivated from the scattering Experiments

Multihadron Op.



Same Quantum no.?
But the two hadrons can’t move freely on the lattice!
Orbital Angular momentum constrained!

Hadron Spectrum with excited states



[Johnson, Phys.Lett.B 114 (1982) 147-151]

$\vec{P}\ [\frac{2\pi}{L}]$	Little Group	Irrep Λ	J
$(0,0,0)$	O_h	T_1^-	$1^-,3^-,\ldots$
$(0,0,1)$	D_{4h} (Dic$_4$)	A_2^- (A_1)	$1^-,3^-,\ldots$
$(0,0,1)$	D_{4h} (Dic$_4$)	E^- (E)	$1^-,3^-,\ldots$
$(0,1,1)$	D_{2h} (Dic$_2$)	B_1^- (A_1)	$1^-,3^-,\ldots$
$(0,1,1)$	D_{2h} (Dic$_2$)	B_2^- (B_1)	$1^-,3^-,\ldots$
$(0,1,1)$	D_{2h} (Dic$_2$)	B_3^- (B_2)	$1^-,3^-,\ldots$
$(1,1,1)$	D_{3d} (Dic$_3$)	A_2^- (A_1)	$1^-,3^-,\ldots$
$(1,1,1)$	D_{3d} (Dic$_3$)	E^- (E)	$1^-,3^-,\ldots$

$$O_{\bar{q}q}^{\Lambda,\vec{P}}(t) = \frac{\dim(\Lambda)}{N_{LG(\vec{P})}} \sum_{\hat{R}\in LG(\vec{P})} \chi_\Lambda(\hat{R})\hat{R}\, O_{\bar{q}q}(t,\vec{P}),$$

$$O_{\pi\pi}^{\Lambda,\vec{P}}(t) = \frac{\dim(\Lambda)}{N_{LG(\vec{P})}} \sum_{\hat{R}\in LG(\vec{P})} \chi_\Lambda(\hat{R}) \Big(\pi^+(t,\vec{P}/2+\hat{R}\vec{p})\,\pi^0(t,\vec{P}/2-\hat{R}\vec{p}) - \pi^0(t,\vec{P}/2+\hat{R}\vec{p})\,\pi^+(t,\vec{P}/2-\hat{R}\vec{p}) \Big),$$

where

$$\vec{p} = \frac{\vec{P}}{2} + \frac{2\pi}{L}\vec{m}, \ \ \vec{m}\in\mathbb{Z}^3.$$

Hadron Spectrum with excited states

Motivated from the scattering Experiments

$$\Omega \equiv \sum_i v_i^* \mathcal{O}_i$$

$$\langle 0 | \Omega(t) \Omega^\dagger(0) | 0 \rangle = \sum \left| \langle n | \Omega^\dagger(0) | 0 \rangle \right|^2 e^{-E_n t}$$

Hadron Spectrum with excited states

Motivated from the scattering Experiments

$$\Omega \equiv \sum_i v_i^* \mathcal{O}_i$$

$$\langle 0 | \Omega(t) \Omega^\dagger(0) | 0 \rangle = \sum \left| \langle n | \Omega^\dagger(0) | 0 \rangle \right|^2 e^{-E_n t}$$

Minimize $\langle 0 | \Omega(t) \Omega^\dagger(0) | 0 \rangle = \sum_{i,j} v_i^* \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle v_j = \sum_{i,j} v_i^* C_{ij}(t) v_j$

$$\sum_{i,j} v_i^* \langle 0 | \mathcal{O}_i \left(t_0\right) \mathcal{O}_j(0) | 0 \rangle v_j = N$$ **Normalization condition**

Hadron Spectrum with excited states

<u>**Motivated from the scattering Experiments**</u>

$$\Omega \equiv \sum_i v_i^* \mathcal{O}_i$$

$$\langle 0 | \Omega(t) \Omega^\dagger(0) | 0 \rangle = \sum \left| \langle n | \Omega^\dagger(0) | 0 \rangle \right|^2 e^{-E_n t}$$

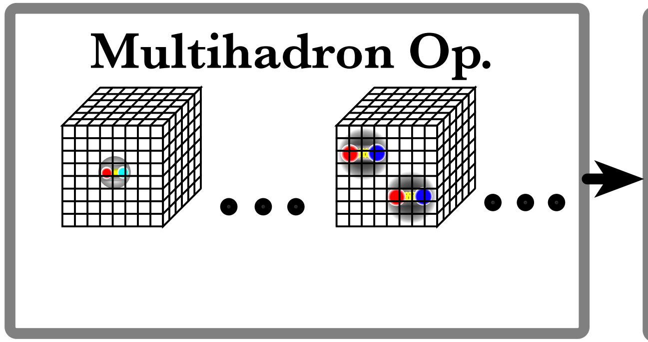
<u>**Minimize**</u> $\langle 0 | \Omega(t) \Omega^\dagger(0) | 0 \rangle = \sum_{i,j} v_i^* \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle v_j = \sum_{i,j} v_i^* C_{ij}(t) v_j$

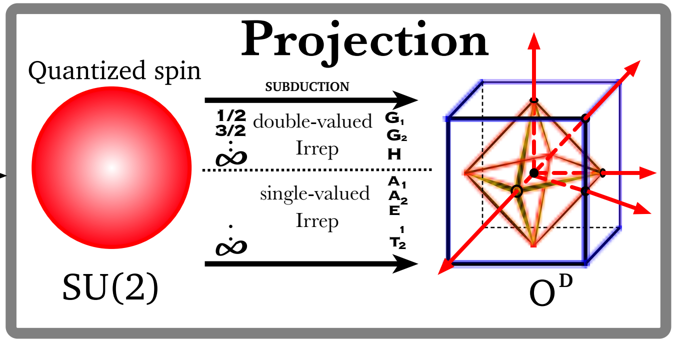
$$\sum_{i,j} v_i^* \langle 0 | \mathcal{O}_i \left(t_0\right) \mathcal{O}_j(0) | 0 \rangle v_j = N$$ <u>**Normalization condition**</u>

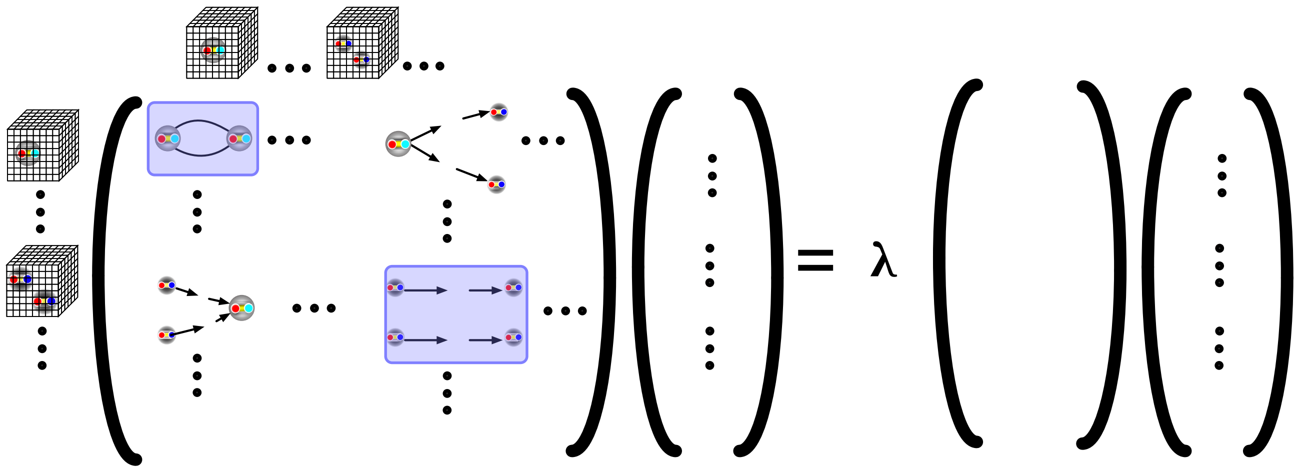
<u>**Using Lagrange Multipliers**</u>

$$\Lambda \left(v_1, \cdots, v_n, \cdots, \lambda\right) = \sum_{i,j} v_i^* C_{ij}(t) v_j - \lambda \left[\sum_{i,j} v_i^* C_{ij} \left(t_0\right) v_j - N \right]$$

Hadron Spectrum with excited states



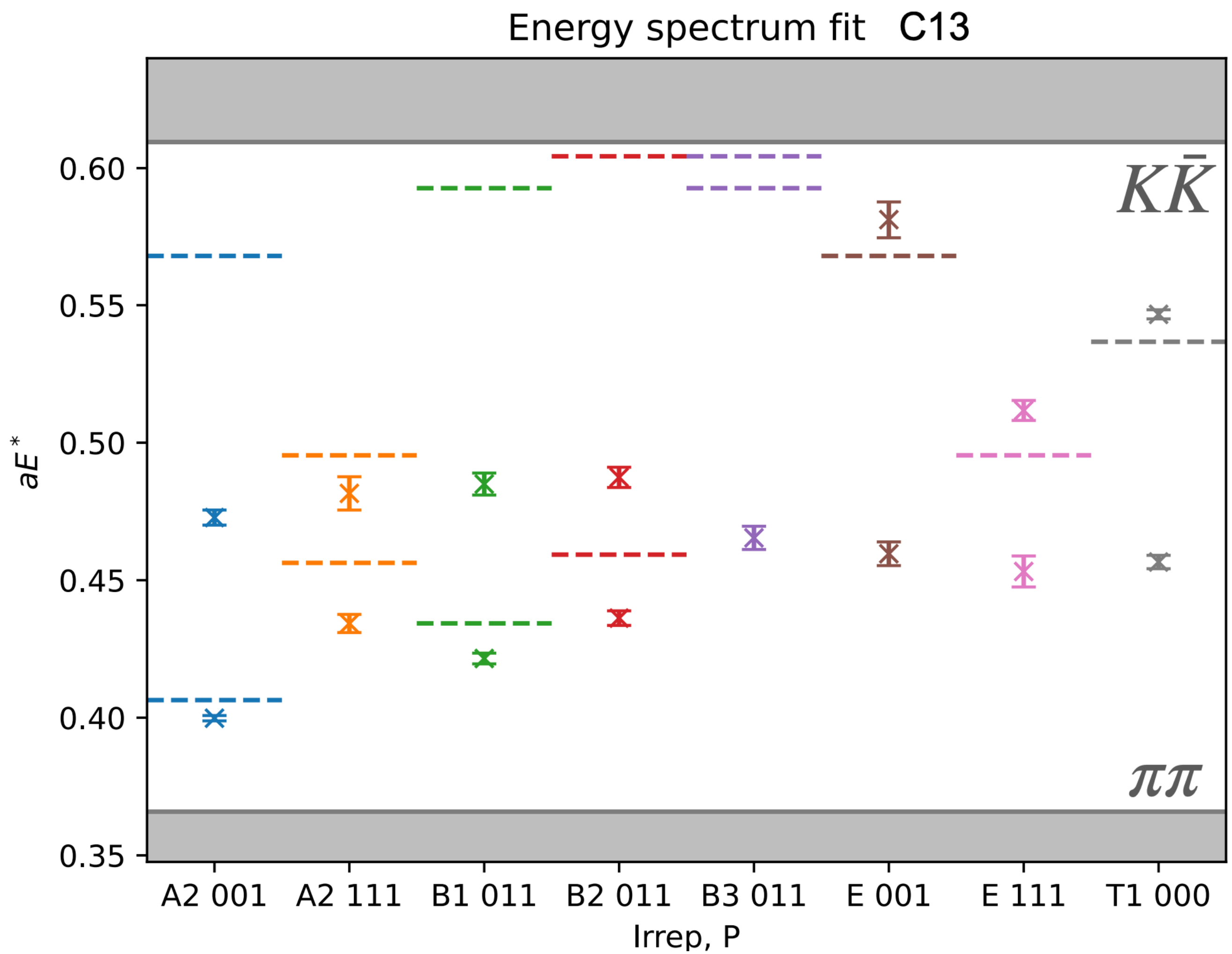


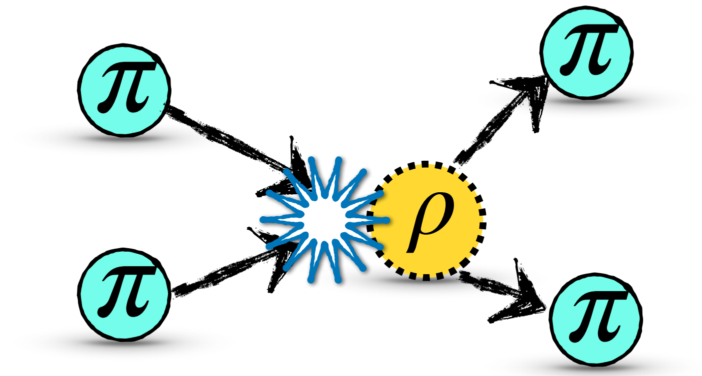


GEVP: $C(t)\ v(t,t_0) = \lambda(t,t_0)\, C(t_0)\ v(t,t_0)$

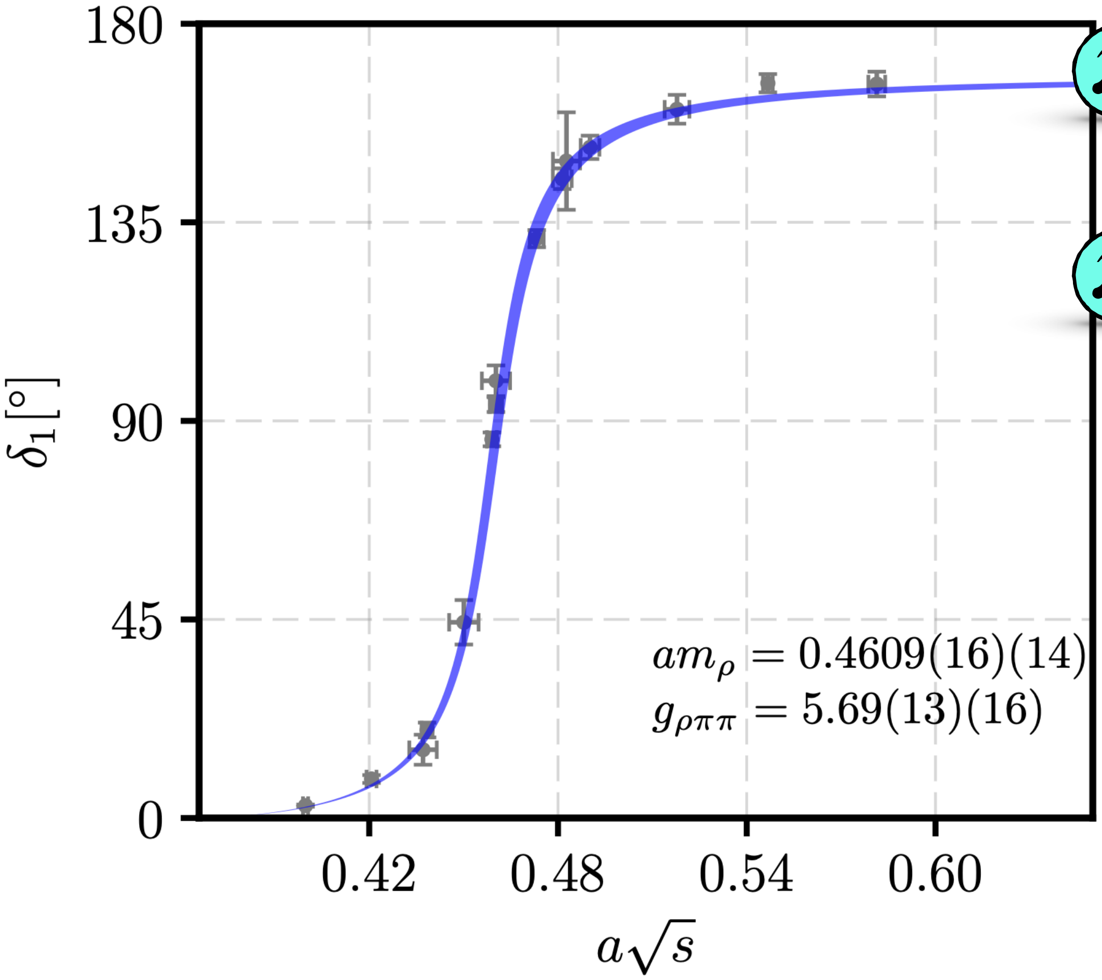
$\lambda(t,t_0) \overset{t\to\infty}{\sim} e^{-Et}$ $\quad E_{eff}(t,t_0) = \log[\lambda(t,t_0)/\lambda(t+1,t_0)]$

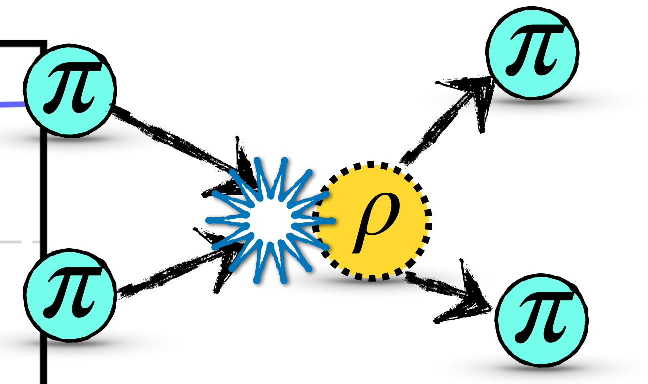
$\pi\pi$ Finite Volume Spectrum





$\pi\pi$ Phase Shift $I = 1$





<u>Our result</u>

$m_\rho = 797.6(6.5)$ MeV

$g_{\rho\pi\pi} = 5.69(21)$

The Anatomy of a Lattice Measurement