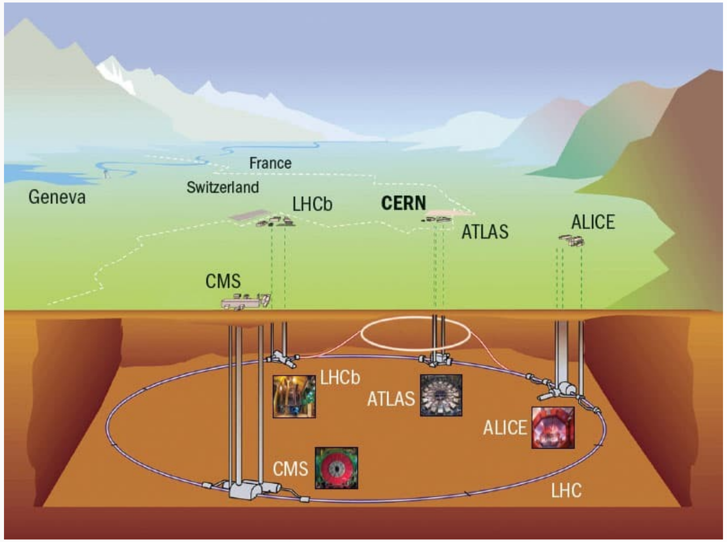
QCD scattering amplitudes at the precision frontier

Heribertus Bayu Hartanto
Asia Pacific Center for Theoretical Physics (APCTP) Pohang

2nd APCTP-INPP Demokritos Meeting
August 26th, 2025

The Large Hadron Collider

Test the Standard Model and search for new physics

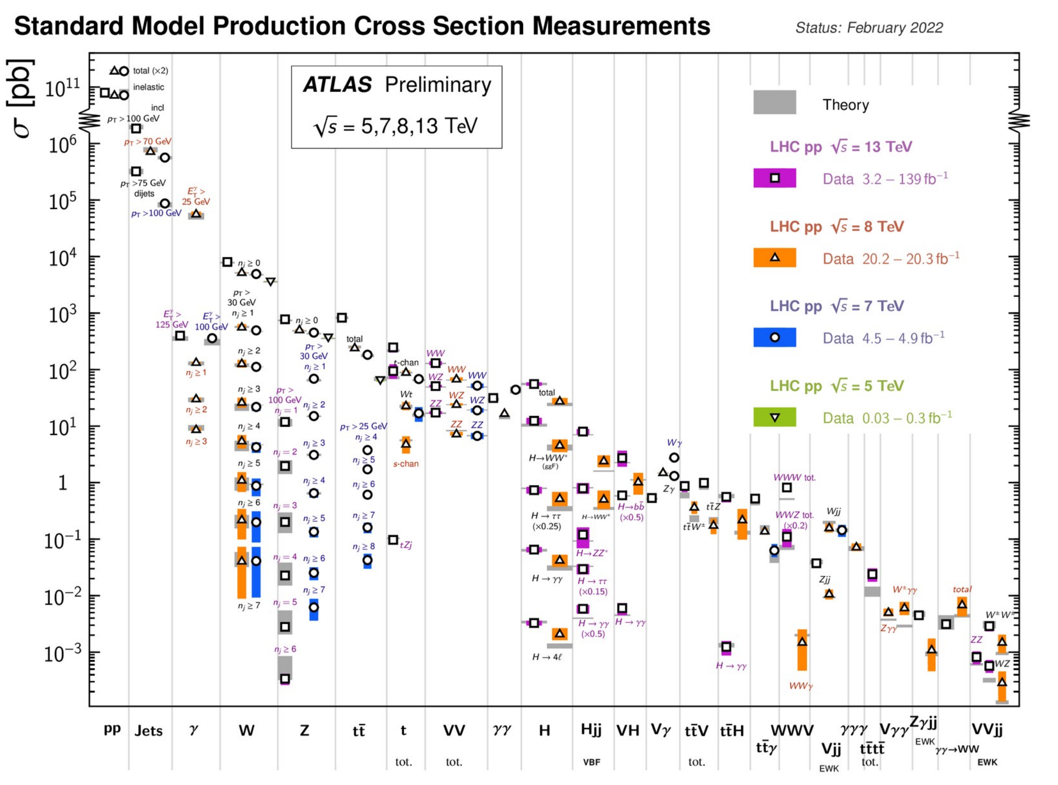


(https://physicsworld.com/a/on-the-road-to-discovery)

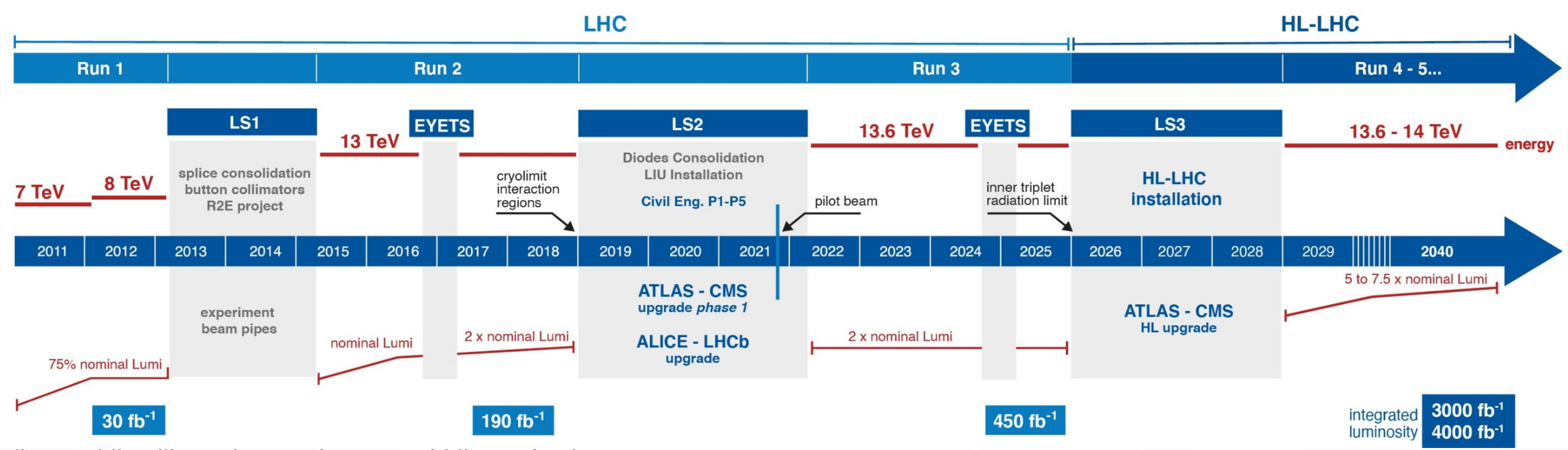
How do we interpret experimental measurements?

Compare with theory predictions!

Measurements on wide range of final states



LHC as a precision machine



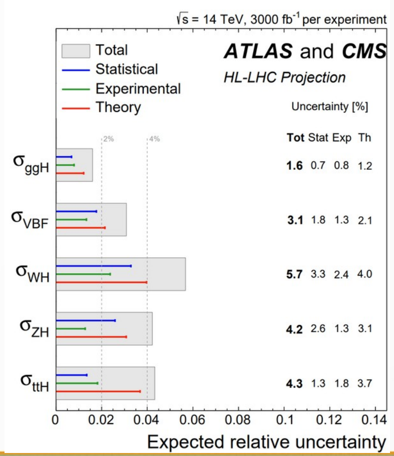
(https://hilumilhc.web.cern.ch/content/hl-lhc-project)

Run 1+2+3 and HL-LHC ⇒ huge amount of data

→ Small uncertainties on experimental measurements

(some may reach % level accuracy)

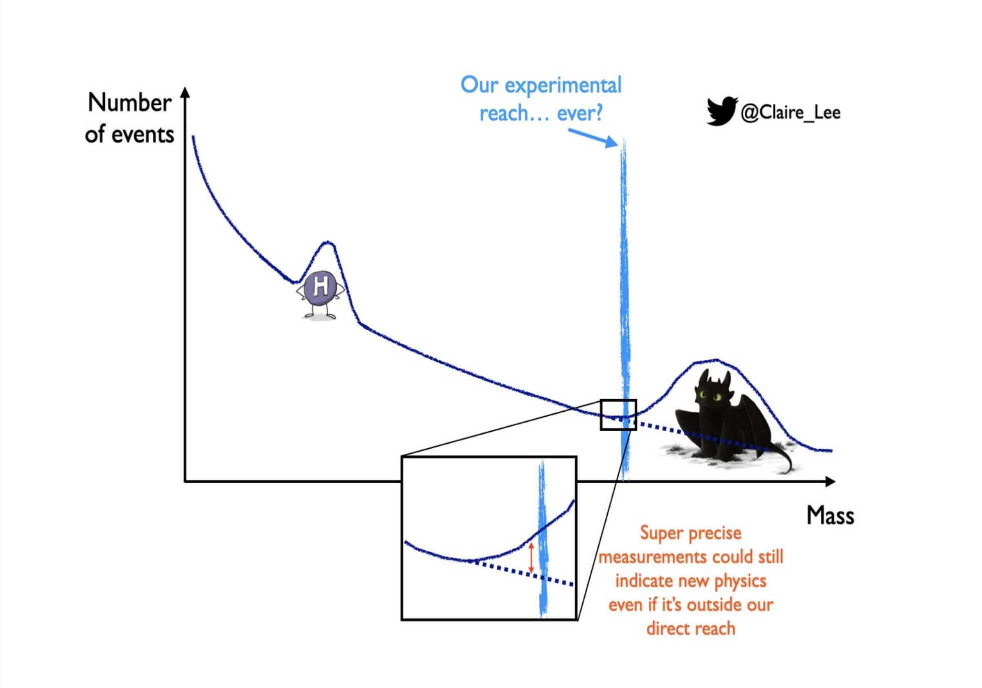
→ Observe rare processes



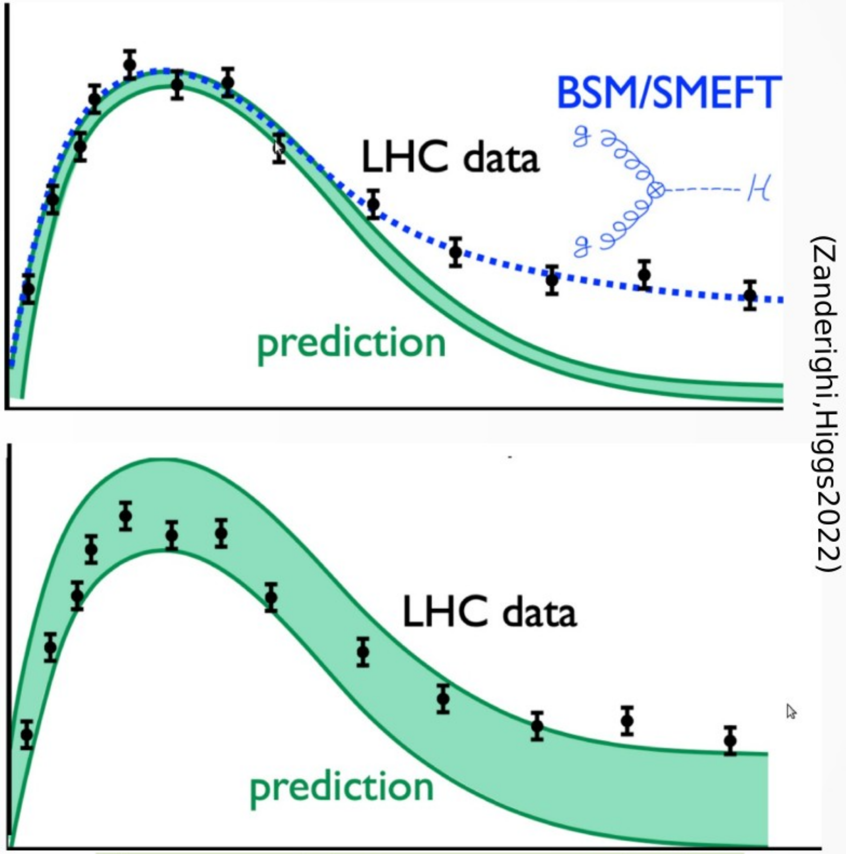
(arXiv:1902.00134)

Searching for New Physics at the LHC

So far: no clear new physics signal at the current energy reach of LHC



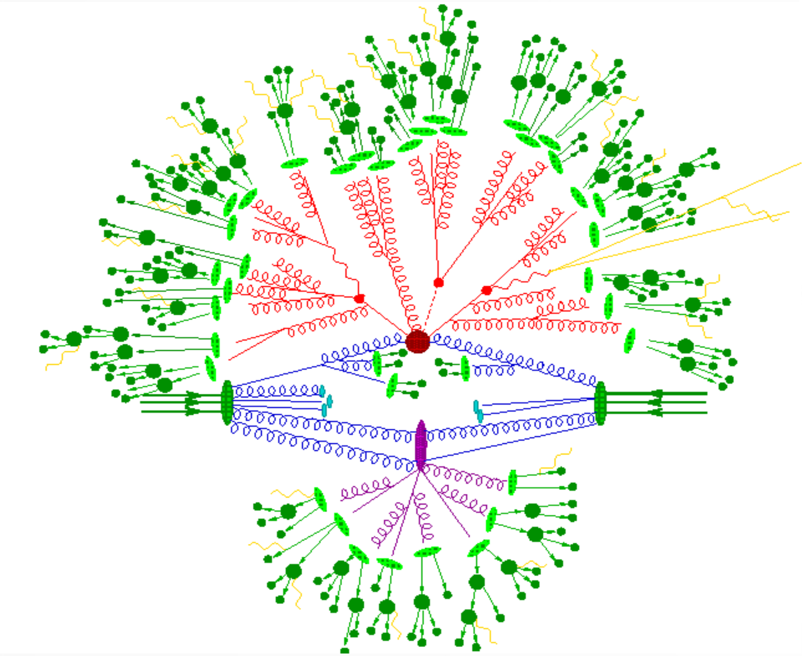
looking for deviations from SM predictions through precision measurements

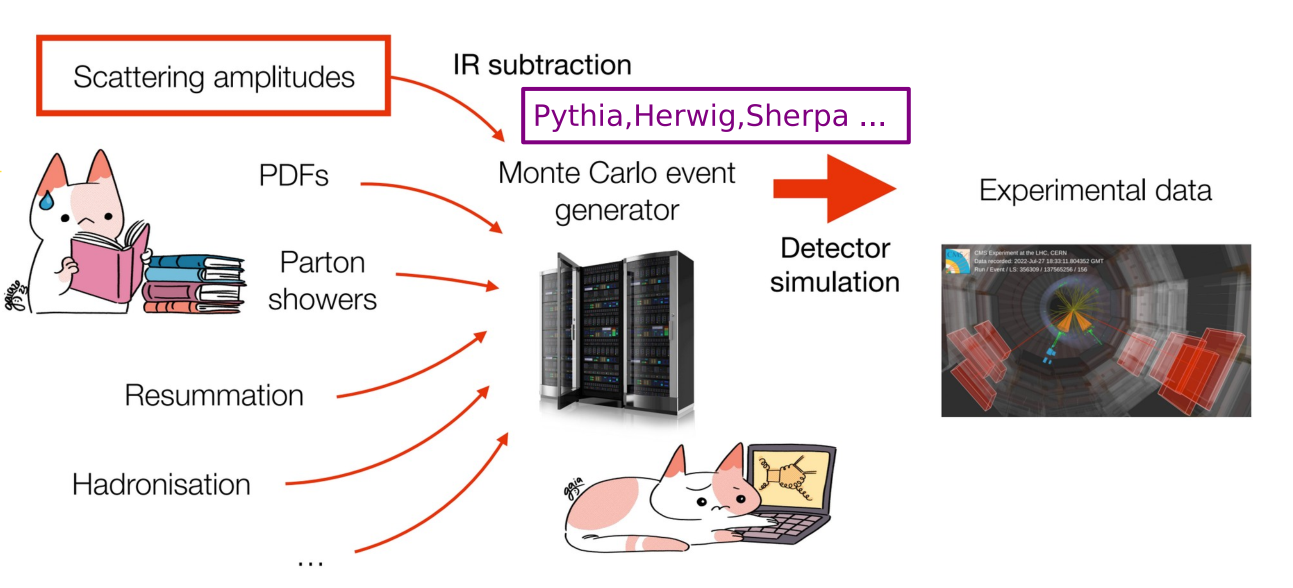


theory with 5x uncertainties

(Zanderighi,Higgs2022)

Theoretical predictions for the LHC





It takes a village to make a prediction!

Taken from Simone Zoia's talk at University of Zurich, December 10, 2024
https://www.physik.uzh.ch/dam/jcr:89cf407b-5202-463a-8985-e1947084478e/zoia_UZH_2024.pdf

Master formula for *pp* cross section:

$$d\sigma(pp \to X) = \sum_{ij} \int dx_1 dx_2 \, f_{i/p}(x_1) f_{j/p}(x_2) \, d\hat{\sigma}(ij \to X)$$

$f_{i/p}(x_1) f_{j/p}(x_2)$ → parton distribution functions (PDF)

$d\hat{\sigma}(ij \to X)$ → partonic cross section → perturbative computation

Perturbative QFT framework

Perturbative calculation of partonic cross sections

$$d\hat{\sigma} = \underbrace{d\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\alpha\, d\hat{\sigma}^{(1)}}_{\delta\text{NLO}} + \underbrace{\alpha^2\, d\hat{\sigma}^{(2)}}_{\delta\text{NNLO}} + \ \ldots$$

LO: Leading Order
NLO: Next-to-Leading Order
NNLO: Next-to-Next-to-Leading Order

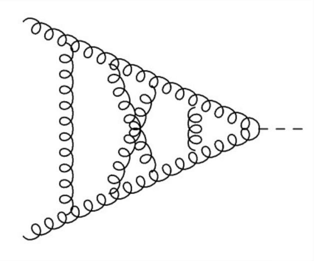
Large quantum corrections come from strong interaction (QCD) $\alpha_s\left(M_Z^2\right) \sim 0.1$

Main source of theoretical error ⇒ truncation of perturbative series (scale dependence)

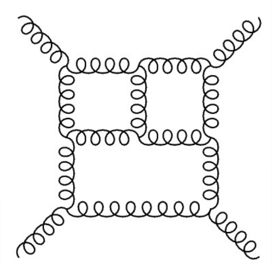
Typical theoretical error on :

LO > 50% NLO QCD ~20-30% NNLO QCD ~1-10%

Loop Frontiers



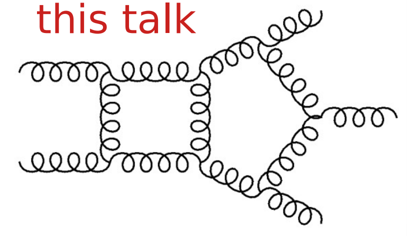
N4LO 2→1



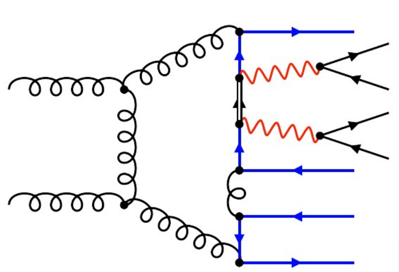
N3LO 2→2

Multiplicity Frontiers

this talk

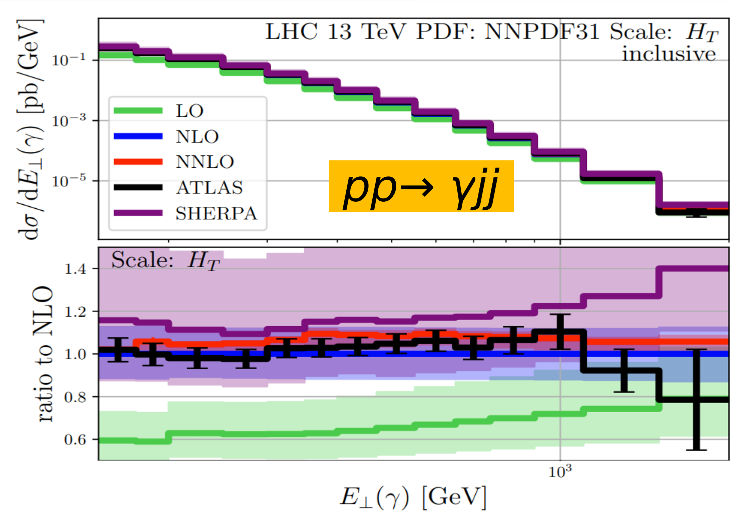


NNLO 2→3

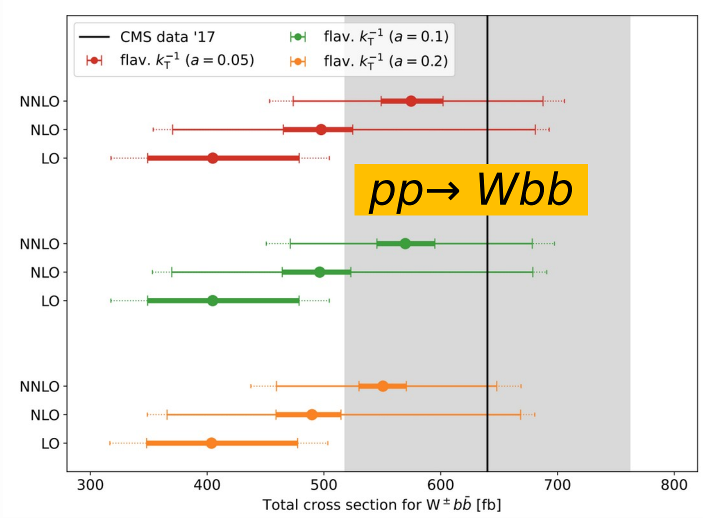


NLO 2→6

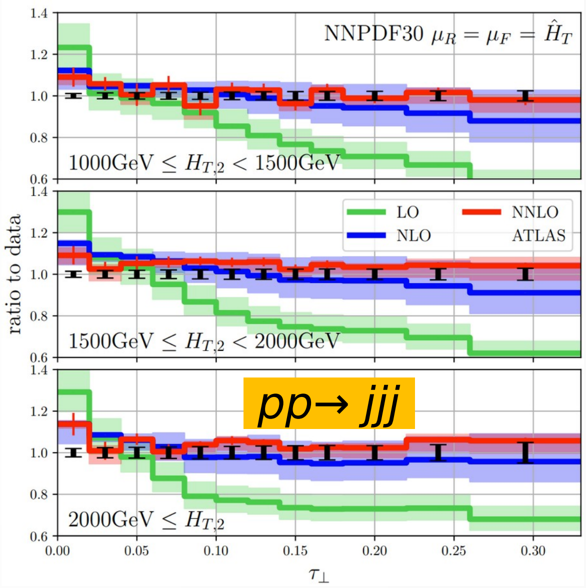
NNLO QCD phenomenology with 2→3 scattering process



[Badger,Czakon,**HBH**,Moodie,Peraro, Poncelet,Zoia(2023)]



[**HBH**,Poncelet,Popescu,Zoia(2022)]



[Czakon,Mitov,Poncelet(2021)][Chen,Gehrmann, Glover,Huss,Marcoli(2022)]Alvarez,Cantero, Czakon,Llorente,Mitov,Poncelet(2023)]

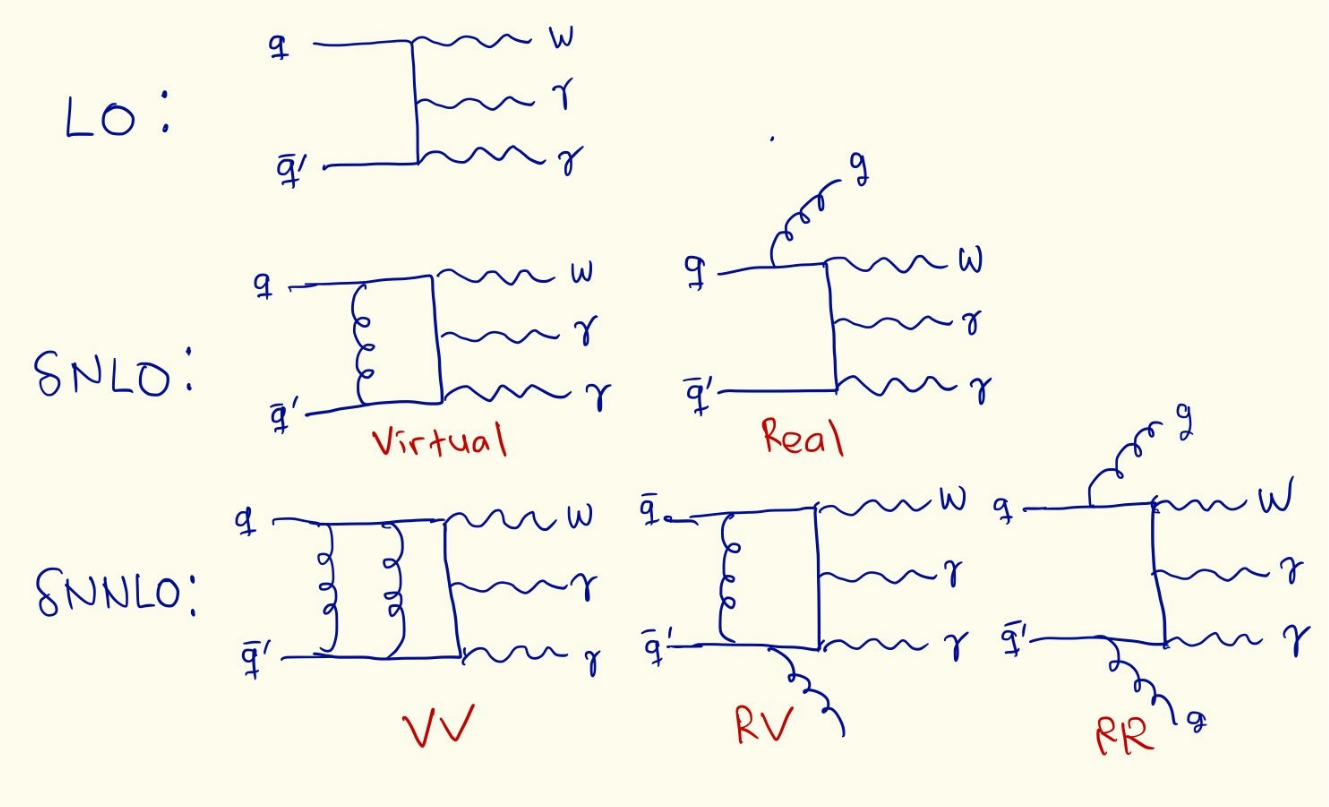
pp→γγγ, pp→γγj, pp→Wbb, pp→Zbb

[Chawdhry,Czakon,Mitov,Poncelet(2019)]
[Kallweit,Sotnikov,Wiesemann(2020)]
[Czakon,Mitov,Poncelet(2020)]
[Buonocore,Devoto,Kallweit,Mazzitelli,Rottoli, Savoini(2022)]
[Mazzitelli,Sotnikov,Wiesemann(2024)]
[Buccioni,Chen,Feng,Gehrmann,Huss,Marcolli(2025)]

pp→ttW, pp→ttH (using approximated two-loop amplitudes)

[Buonocore,Devoto,Grazzini,Kallweit,Mazzitelli,Rottoli,Savoini(2022)]
[Catani,Devoto,Grazzini,Kallweit,Mazzitelli,Savoini(2020)]

NNLO QCD predictions: under the hood



IR singularities:

- Soft: $E_g \rightarrow 0$
- Collinear: $p_g = \lambda\, p_q$

QCD corrections: virtual and real parton (quark & gluon) emissions

Combining different (separately IR divergent) contributions → subtraction scheme

Two-Loop Scattering Amplitudes

$$d\hat{\sigma}^{VV} = \int d\mathrm{PS} \left[2\mathrm{Re}\, \mathcal{A}^{(0)*} \mathcal{A}^{(2)} + |\mathcal{A}^{(1)}|^2 \right]$$

Multi-loop amplitude → integrand from Feynman diagrams/recursion relations/trees

→ linear combination of Master Integrals via IBP reduction

$$\mathcal{A}^{(2)} = \int d^d k_1 d^d k_2 \sum_{i=1}^{N_{\mathrm{diag}}} \frac{\mathcal{N}_i(\{k\},\{p\})}{\mathcal{D}_i(\{k\},\{p\})} = \sum_i c_i(\{p\},\epsilon)\, I_i(\{p\},\epsilon) = \sum_i d_i(\{p\},\epsilon)\, \mathrm{MI}_i(\{p\},\epsilon)$$

Dimensional regularization: d=4-2ϵ

Computing master integrals → differential equation method [Gehrmann,Remiddi(1999)][Henn(2013)]

$$\frac{d\mathrm{MI}_i}{dx} = B_{ij}(x,\epsilon)\mathrm{MI}_j \quad \Rightarrow \quad \frac{d\mathrm{MI}'_i}{dx} = \epsilon \sum_\alpha \frac{\tilde{B}^\alpha_{ij}(x,\epsilon)}{x - x_\alpha} \mathrm{MI}'_j \quad \Rightarrow \quad \mathrm{MI}'_i(x,\epsilon) = \sum_{j=0}^{4} \sum_k d_{jk,i}\, \epsilon^j\, \mathrm{mon}_{jk}(f)$$

“canonical” basis

basis of special functions

Two-Loop Scattering Amplitudes

Finite remainder → subtract universal UV/IR singularities [Catani][Becher,Neubert][Gardi,Magnea]

$$\mathcal{F}^{(2)} = \mathcal{A}^{(2)} - \mathcal{P}^{(2)}_{\mathrm{UV+IR}} = \sum_i r_i(\{p\})\, \mathrm{mon}_i(f) + \mathcal{O}(\epsilon)$$

⇒ analytic pole cancellation, simplest representation of the amplitude

<u>Challenges</u>:

- **Analytic** → obtaining canonical basis
  → determination of special functions: f_i = ln, Li_n, Elliptic
  → intricate branch cuts
  → efficient evaluation for physical kinematics

- **Algebraic** → expression swell in the intermediate stage
  → high-degree rational functions in r_i for multiscale process

Two-Loop Scattering Amplitudes

Finite remainder → subtract universal UV/IR singularities [Catani][Becher,Neubert][Gardi,Magnea]

$$\mathcal{F}^{(2)} = \mathcal{A}^{(2)} - \mathcal{P}^{(2)}_{\mathrm{UV+IR}} = \sum_i r_i(\{p\})\, \mathrm{mon}_i(f) + \mathcal{O}(\epsilon)$$

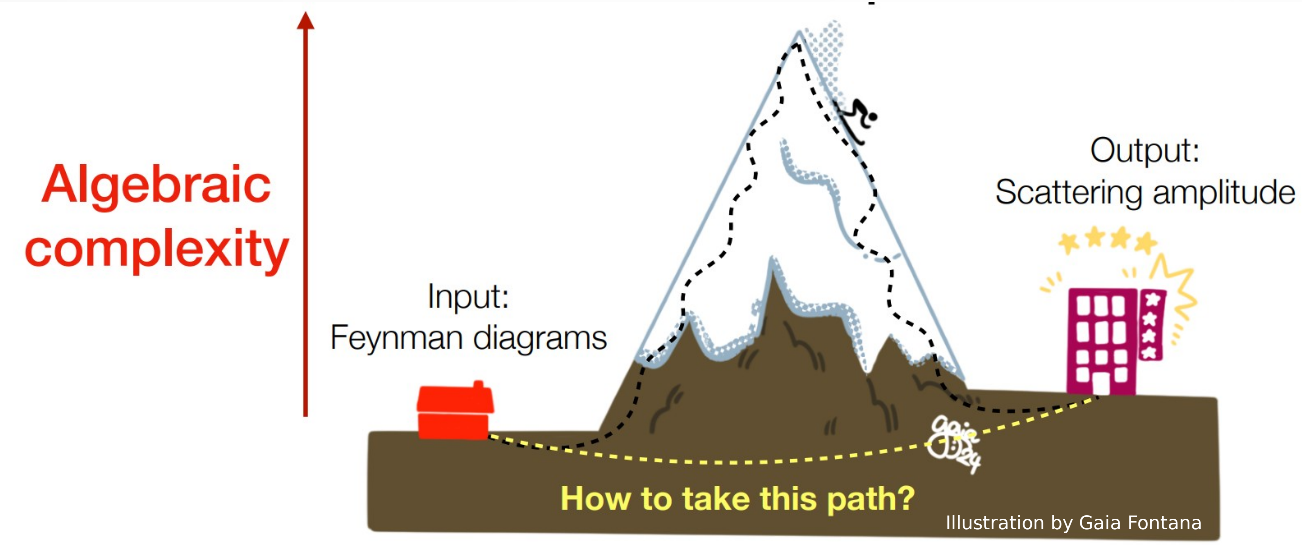
⇒ analytic pole cancellation, simplest representation of the amplitude

<u>Challenges</u>:

- **Analytic** → obtaining canonical basis
  → determination of special functions: f_i = ln, Li_n, Elliptic
  → intricate branch cuts
  → efficient evaluation for physical kinematics

- **Algebraic** → expression swell in the intermediate stage
  → high-degree rational functions in r_i for multiscale process

Taming algebraic complexity: analytics from numerics



$$A^{(2)}(\{p\},\epsilon) = \sum_i (\text{Feynman diagram})_i$$

↓ map loop numerators to $\mathcal{I}$

$$A^{(2),h}(\{p\},\epsilon) = \sum_i c_i^h(\{p\},\epsilon)\, \mathcal{I}_i(\{p\},\epsilon)$$

↓ IBP reduction

$$A^{(2),h}(\{p\},\epsilon) = \sum_i d_i^h(\{p\},\epsilon)\, \mathrm{MI}_i(\{p\},\epsilon)$$

↓ map to special functions

↓ subtract UV/IR poles

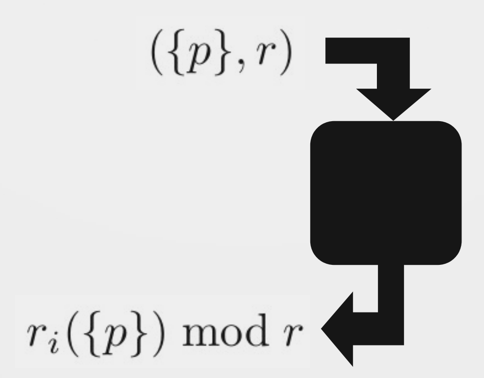
↓ ϵ expansion

$$F^{(2),h}(\{p\}) = \sum_i r_i^h(\{p\})\, \mathrm{mon}_i(f) + \mathcal{O}(\epsilon)$$

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\})\, \mathrm{mon}_i(f)$$

Replace algebraic operations with numerical evaluation over finite fields

Reconstruct analytic expressions from numerical samples



Reconstructed in $\dfrac{\#\text{ points} \times \text{eval. time}}{\#\text{ cores}}$

Taming algebraic complexity: analytics from numerics

```
In[1]:= << FiniteFlow`

In[4]:= prime = 9 223 372 036 854 775 783;
        a = -3 / 2;
        b = 5 / 3;

In[9]:= aFF = FFRatMod[a, prime]

Out[9]= 4 611 686 018 427 387 890

In[10]:= bFF = FFRatMod[b, prime]

Out[10]= 3 074 457 345 618 258 596
```

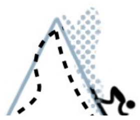


Illustration by Gaia Fontana

```
In[14]:= abFF = FFRatMod[aFF * bFF, prime]

Out[14]= 4 611 686 018 427 387 889

In[15]:= FFRatRec[abFF, prime]

Out[15]= -5/2
```

$$A^{(2)}(\{p\},\epsilon) = \sum_i (\text{Feynman diagram})_i$$

↓ map loop numerators to $\mathcal{I}$

$$A^{(2),h}(\{p\},\epsilon) = \sum_i c_i^h(\{p\},\epsilon)\, \mathcal{I}_i(\{p\},\epsilon)$$

↓ IBP reduction

$$A^{(2),h}(\{p\},\epsilon) = \sum_i d_i^h(\{p\},\epsilon)\, \mathrm{MI}_i(\{p\},\epsilon)$$

↓ map to special functions

↓ subtract UV/IR poles

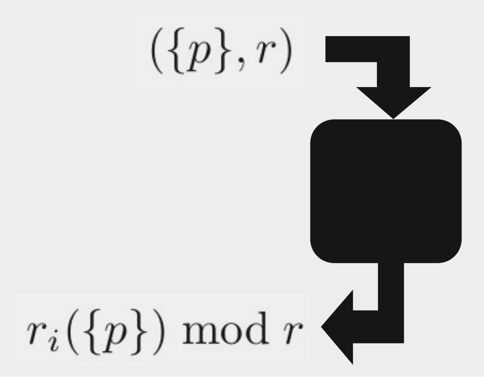
↓ ϵ expansion

$$F^{(2),h}(\{p\}) = \sum_i r_i^h(\{p\})\, \mathrm{mon}_i(f) + \mathcal{O}(\epsilon)$$

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\})\, \mathrm{mon}_i(f)$$

Replace algebraic operations with numerical evaluation over finite fields

Reconstruct analytic expressions from numerical samples



Reconstructed in $\dfrac{\#\text{ points} \times \text{eval. time}}{\#\text{ cores}}$

Reconstructing the finite remainders (1)

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\})\, \mathrm{mon}_i(f)$$

→ helicity amplitudes: $\mathcal{A}(\langle ij\rangle, [ij], \langle i|k_\ell|j])$

need rational parameterization of external kinematics → momentum twistor variables

Ex: massless 5-point parameterization

$$|1\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \quad |2\rangle = \begin{pmatrix}0\\1\end{pmatrix}, \quad |3\rangle = \begin{pmatrix}\frac{1}{x_1}\\1\end{pmatrix}, \quad |4\rangle = \begin{pmatrix}\frac{1}{x_1}+\frac{1}{x_1 x_2}\\1\end{pmatrix}, \quad |5\rangle = \begin{pmatrix}\frac{1}{x_1}+\frac{1}{x_1 x_2}+\frac{1}{x_1 x_2 x_3}\\1\end{pmatrix},$$

$$|1] = \begin{pmatrix}1\\\frac{x_4-x_5}{x_4}\end{pmatrix}, \quad |2] = \begin{pmatrix}0\\x_1\end{pmatrix}, \quad |3] = \begin{pmatrix}x_1 x_4\\-x_1\end{pmatrix}, \quad |4] = \begin{pmatrix}x_1(x_2 x_3 - x_3 x_4 - x_4)\\-\frac{x_1 x_2 x_3 x_5}{x_4}\end{pmatrix}, \quad |5] = \begin{pmatrix}x_1 x_3(x_4 - x_2)\\\frac{x_1 x_2 x_3 x_5}{x_4}\end{pmatrix}.$$

Momentum conservation:

$$\langle 1|2|5] + \langle 1|3|5] + \langle 1|4|5] = 0$$

$$-x_1^2 x_3(x_2 - x_4) - x_1^2 x_3(-x_2 + x_4 + x_2 x_5) + x_1^2 x_3 x_2 x_5 = 0$$

Schouten identity:

$$\langle 12\rangle\langle 34\rangle + \langle 13\rangle\langle 42\rangle + \langle 14\rangle\langle 23\rangle = 0$$

$$-\frac{1}{x_1 x_2} + \frac{1+x_2}{x_1 x_2} - \frac{1}{x_1} = 0$$

$$\mathrm{tr}_5(ijkl) = 4\,\mathrm{i}\,\varepsilon_{\mu\nu\rho\sigma}\, p_i^\mu p_j^\nu p_k^\rho p_l^\sigma$$

can also use Mandelstam invariants: $A_{\text{phase-free}} = c_{\text{even}}(s_{ij}) + \mathrm{tr}_5(1234)\, c_{\text{odd}}(s_{ij})$

Reconstructing the finite remainders (2)

$$\mathcal{F}^{(2)} = \sum_i r_i(\{p\})\ \mathrm{mon}_i(f)$$

need to reduce the complexity of the rational coefficients!!

- set one of the kinematic variables to one ($s_{12} = 1$ or $x_1 = 1$)
- find linear relations among coefficients and reconstruct the simpler ones

$$\sum_i y_i r_i = 0, \qquad \text{we can supply ansatz} \rightarrow \qquad \sum_i y_i r_i + \sum_j \tilde{y}_j \tilde{r}_j = 0,$$

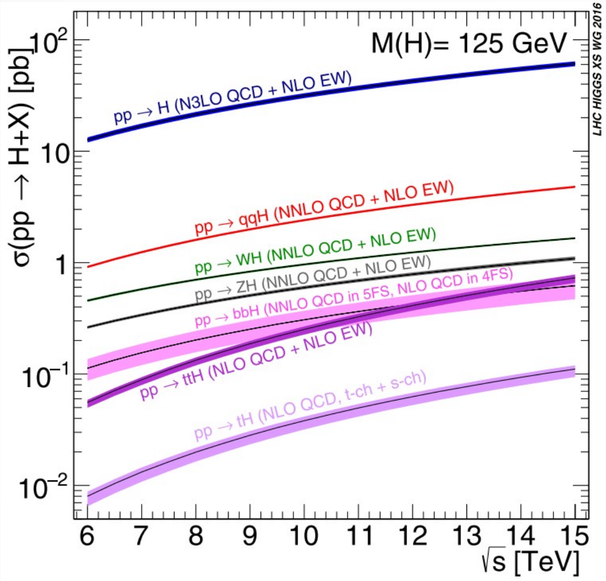
- guess the denominators → from letters of the alphabet
- on-the-fly univariate partial fraction → one fewer variables, lower degrees

$$f(x,y) = \frac{y^4 + 13xy^2 + x^2}{(y-x)(y+x)^2}, \qquad \text{ansatz} \Rightarrow f(x,y) = \frac{u_{110}(x)}{y-x} + \frac{u_{210}(x)}{y+x} + \frac{u_{220}(x)}{(y+x)^2} + r(x) + v_1(x)y$$

- factor matching: letters, spinor products/strings
- reconstructed expressions can be further simplifed: *MultivariateApart, pfd-parallel*

Application: two-loop amplitudes for pp→bbH

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]



bbH: one of the main Higgs production channel, yet to be observed

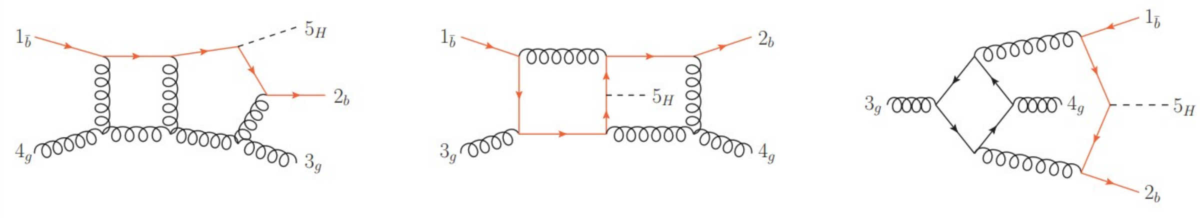
→ study bottom-Yukawa coupling, background to pp→HH

→ work in 5-flavour scheme: massless b, finite Yukawa coupling

Amplitude with massive b quark → massification

$$\mathcal{M}_2^m = \mathcal{M}_2^{m=0} + Z^1_{[q]}\mathcal{M}_1^{m=0} + Z^2_{[q]}\mathcal{M}_0^{m=0}$$

$$Z^l_{[q]} = f\left(\epsilon, \log m_b^2/Q^2\right)$$



$A_{34}^{(2),N_c^2}, A_{34}^{(2),1}, A_{43}^{(2),1}$ $\quad$ $A_{34}^{(2),1}$ $\quad$ $A_{34}^{(2),n_f/N_c}, A_{43}^{(2),n_f/N_c}, A_{\delta}^{(2),n_f/N_c^2}$

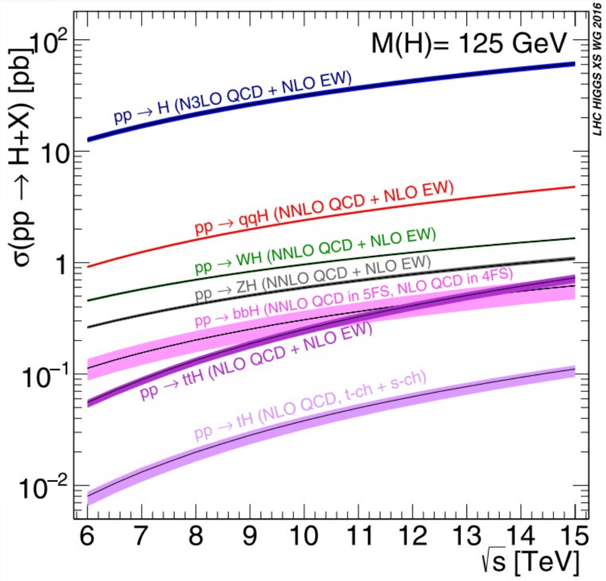
$$A_{34}^{(2)} = N_c^2 A_{34}^{(2),N_c^2} + A_{34}^{(2),1} + \frac{1}{N_c^2}A_{34}^{(2),1/N_c^2} + N_c n_f A_{34}^{(2),N_c n_f} + \frac{n_f}{N_c}A_{34}^{(2),n_f/N_c} + n_f^2 A_{34}^{(2),n_f^2},$$

$$A_{\delta}^{(2)} = N_c A_{\delta}^{(2),N_c} + \frac{1}{N_c}A_{\delta}^{(2),1/N_c} + n_f A_{\delta}^{(2),n_f} + \frac{n_f}{N_c^2}A_{\delta}^{(2),n_f/N_c^2}.$$

First analytic result for full-colour two-loop five-point amplitude with an external mass

Application: two-loop amplitudes for pp→bbH

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]



bbH: one of the main Higgs production channel, yet to be observed

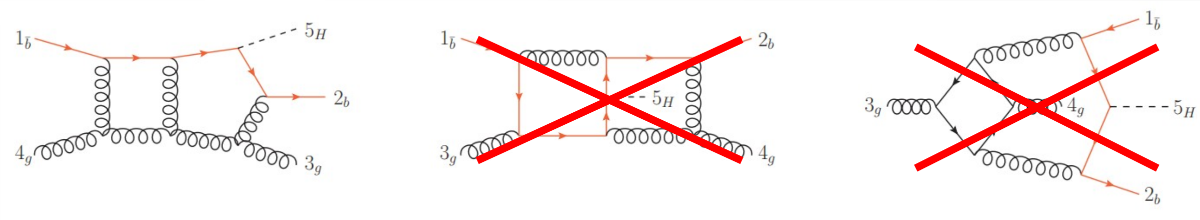
→ study bottom-Yukawa coupling, background to pp→HH

→ work in 5-flavour scheme: massless b, finite Yukawa coupling

Amplitude with massive b quark → massification

$$\mathcal{M}_2^m = \mathcal{M}_2^{m=0} + Z^1_{[q]}\mathcal{M}_1^{m=0} + Z^2_{[q]}\mathcal{M}_0^{m=0}$$

$$Z^l_{[q]} = f\left(\epsilon, \log m_b^2/Q^2\right)$$



$A_{34}^{(2),N_c^2}, A_{34}^{(2),1}, A_{43}^{(2),1}$ $\quad$ $A_{34}^{(2),1}$ $\quad$ $A_{34}^{(2),n_f/N_c}, A_{43}^{(2),n_f/N_c}, A_{\delta}^{(2),n_f/N_c^2}$

Leading-colour approximation

$$\mathcal{A}_{34}^{(2)} = N_c^2 A_{34}^{(2),N_c^2} + A_{34}^{(2),1} + \frac{1}{N_c^2}A_{34}^{(2),1/N_c^2} + N_c n_f A_{34}^{(2),N_c n_f} + \frac{n_f}{N_c}A_{34}^{(2),n_f/N_c} + n_f^2 A_{34}^{(2),n_f^2},$$

$$\mathcal{A}_{\delta}^{(2)} = N_c A_{\delta}^{(2),N_c} + \frac{1}{N_c}A_{\delta}^{(2),1/N_c} + n_f A_{\delta}^{(2),n_f} + \frac{n_f}{N_c^2}A_{\delta}^{(2),n_f/N_c^2}.$$

Application: two-loop amplitudes for pp→bbH

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

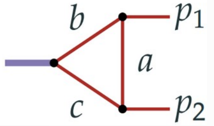
IBP reduction to master integrals: *usually* the main botteleneck

full set of integrals | master integrals

$$I_i(\epsilon,\{p\}) = \sum_j B_{ij}(\epsilon,\{p\})\,\mathrm{MI}_j(\epsilon,\{p\})$$

One-loop triangle example

$I_{a,b,c} \equiv$



IBP identity [Chetyrkin,Tkachov(1981)]

$$0 = \int \frac{d^D l_1}{i\pi^{D/2}} \cdots \frac{d^D l_L}{i\pi^{D/2}} \sum_{i=1}^{L} \frac{\partial}{\partial l_i^\mu} \frac{v_i^\mu}{D_1 \ldots D_k}$$

Syzygy relations [Gluza,Kajda,Kosower(2011)]

$$\sum_{i=1}^{L} v_i^\mu \frac{\partial D_j}{\partial l_i^\mu} = f_j D_j$$

remove doubled propagators

$$\begin{gathered}
(d-6)\,I_{2,1,1} - I_{1,2,1} - I_{1,1,2} = 0 \\
s\,I_{1,2,1} - s\,I_{1,1,2} = 0 \\
s\,I_{1,1,2} + I_{0,2,1} + I_{0,1,2} = 0 \\
(d-4)\,I_{1,1,1} - I_{0,2,1} - I_{0,1,2} = 0 \\
-2\,I_{-1,3,1} - I_{-1,2,2} + (d-3)\,I_{0,2,1} = 0 \\
-2\,I_{-1,1,1} - s\,I_{0,1,1} = 0 \\
-I_{0,3,1} + I_{0,1,3} = 0 \\
s\,I_{-1,2,1} - s\,I_{-1,1,2} = 0 \\
-I_{-1,2,1} - I_{-1,1,2} + (d-2)\,I_{0,1,1} = 0 \\
-I_{-1,2,1} - I_{-1,1,2} - s\,I_{0,1,2} + I_{0,1,1} = 0 \\
-I_{-1,2,2} - 2I_{-1,1,3} + (d-3)\,I_{0,1,2} = 0 \\
s\,I_{-1,2,2} - 2s\,I_{-1,1,3} + s\,I_{0,1,2} = 0 \\
-2\,I_{-1,2,2} - s\,I_{0,2,2} + I_{0,2,1} + I_{0,1,2} = 0 \\
-I_{-1,2,2} - 2\,I_{-1,1,3} - 2s\,I_{0,1,3} + I_{0,1,2} = 0 \\
-I_{0,2,1} + I_{0,1,2} = 0
\end{gathered}
\quad\Longleftrightarrow\quad
\begin{pmatrix}
d-6 & -1 & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & s & -s & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -s & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & -1 & \cdot \\
\cdot & \cdot & \cdot & d-4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & -1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & -2 & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d-3 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot & \cdot & -s \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & s & -s & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d-2 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & -1 & \cdot & \cdot & \cdot & \cdot & -s & \cdot & 1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & -2 & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d-3 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -2s & s & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & s & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2 & \cdot & \cdot & \cdot & \cdot & \cdot & -s & 1 & 1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -2 & -1 & \cdot & \cdot & \cdot & \cdot & -2s & \cdot & 1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & -1 & \cdot
\end{pmatrix}
\begin{pmatrix}
I_{2,1,1} \\ I_{1,2,1} \\ I_{1,1,2} \\ I_{1,1,1} \\ I_{-1,3,1} \\ I_{-1,1,3} \\ I_{-1,2,2} \\ I_{-1,2,1} \\ I_{-1,1,2} \\ I_{-1,1,1} \\ I_{0,3,1} \\ I_{0,1,3} \\ I_{0,2,2} \\ I_{0,1,2} \\ I_{0,2,1} \\ I_{0,1,1}
\end{pmatrix} = 0$$

After Gaussian elimination (108 operations, $\sim N^2_{\text{integrals}}$):

$$\begin{pmatrix}
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -4(d-3)/((d-6)\,s^2) \\
\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2(d-3)/s^2 \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -2(d-3)/s^2 \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2(d-3)/((d-4)\,s) \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & (d-3)(d-2)/(4s) \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & (d-3)(d-2)/(4s) \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & (d-4)(d-3)/(2s) \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & (2-d)/2 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & (2-d)/2 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & s/2 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & -(d-4)(d-3)/(2s^2) \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & -(d-4)(d-3)/(2s^2) \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & -(d-6)(d-3)/s^2 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & (d-3)/s \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & (d-3)/s
\end{pmatrix}
\begin{pmatrix}
I_{2,1,1} \\ I_{1,2,1} \\ I_{1,1,2} \\ I_{1,1,1} \\ I_{-1,3,1} \\ I_{-1,1,3} \\ I_{-1,2,2} \\ I_{-1,2,1} \\ I_{-1,1,2} \\ I_{-1,1,1} \\ I_{0,3,1} \\ I_{0,1,3} \\ I_{0,2,2} \\ I_{0,1,2} \\ I_{0,2,1} \\ I_{0,1,1}
\end{pmatrix} = 0
\quad\Longleftrightarrow\quad
\begin{pmatrix}
I_{2,1,1} \\ I_{1,2,1} \\ I_{1,1,2} \\ I_{1,1,1} \\ I_{-1,3,1} \\ I_{-1,1,3} \\ I_{-1,2,2} \\ I_{-1,2,1} \\ I_{-1,1,2} \\ I_{-1,1,1} \\ I_{0,3,1} \\ I_{0,1,3} \\ I_{0,2,2} \\ I_{0,1,2} \\ I_{0,2,1}
\end{pmatrix} =
\begin{pmatrix}
4(d-3)/((d-6)\,s^2) \\ 2(d-3)/s^2 \\ 2(d-3)/s^2 \\ -2(d-3)/((d-4)\,s) \\ -(d-3)(d-2)/(4s) \\ -(d-3)(d-2)/(4s) \\ -(d-4)(d-3)/(2s) \\ -(2-d)/2 \\ -(2-d)/2 \\ -s/2 \\ (d-4)(d-3)/(2s^2) \\ (d-4)(d-3)/(2s^2) \\ (d-6)(d-3)/s^2 \\ -(d-3)/s \\ -(d-3)/s
\end{pmatrix} I_{0,1,1}$$

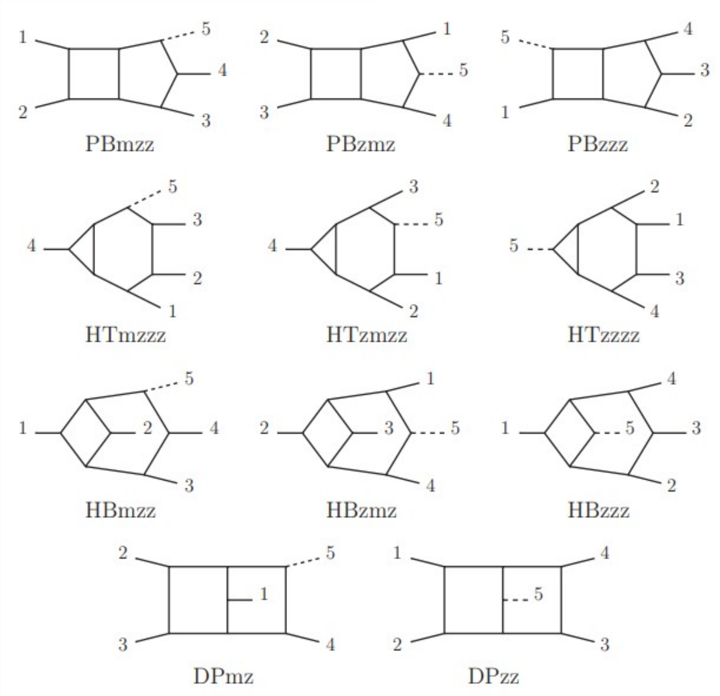
Vitaly Margerya, Mathematical Structure in Feynman Integrals (2023)

Application: two-loop amplitudes for pp→bbH

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

IBP reduction to master integrals: solving linear system of eqs for all integral families

Solve IBP systems numerically over FF, permute the solutions, chain to the next stage of computation



	$A_{34}^{(2),N_c^2}$	$A_{34}^{(2),1}$	$A_{34}^{(2),1/N_c^2}$	$A_{\delta}^{(2),N_c}$	$A_{\delta}^{(2),1/N_c}$	NEATIBP file size
# of scalar integrals	20905	147235	83731	215394	204375	
N_{perm}(PBmzz)	2	16	7	8	8	8MB
N_{perm}(PBzmz)	1	7	4	4	3	10MB
N_{perm}(PBzzz)	2	4	7	12	24	5.9MB
N_{perm}(HTmzzz)	2	8	12	24	24	1.9MB
N_{perm}(HTzmzz)	2	14	8	24	20	3.8MB
N_{perm}(HTzzzz)	1	4	4	12	12	4MB
N_{perm}(HBmzz)	0	10	6	10	12	17MB
N_{perm}(HBzmz)	0	5	3	5	5	13MB
N_{perm}(HBzzz)	0	4	2	6	6	38MB
N_{perm}(DPmz)	0	18	8	16	24	71MB
N_{perm}(DPzz)	0	4	1	2	2	376MB

Application: two-loop amplitudes for pp→bbH

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

Analytic reconstruction data

Subleading colour terms:

- more complicated rational coeffs
- slower evaluation time

exploit relations of rational coefficients among different partial amplitudes

can we use this data to construct primitive amplitudes?

	$A_{34}^{(2),N_c^2}$	$A_{34}^{(2),1}$		$A_{\delta}^{(2),N_c}$		$A_{\delta}^{(2),1/N_c}$	
helicity set	$\vec{h}_g$	$\vec{h}'_g$		$\vec{h}_g$		$\vec{h}_g$	
$x_1 = 1$	133/132	176/175	176/175	265/269	265/269	214/207	214/207
ansatz for linear relations	-	[illegible]	[illegible]	[illegible]	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ [illegible]	[illegible]	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ $A_{34}^{(2),1/N_c^2}$ $A_{43}^{(2),1/N_c^2}$
linear relations	133/132	17[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	135/130
number of independent coefficients	162	572	216	582	151	581	258
denominator matching	133/0	176/0	110/0	233/0	119/0	141/0	135/0
univariate partial fraction	32/25	38/33	25/22	36/33	36/33	32/28	32/28
factor matching	28/0	38/0	24/0	36/0	36/0	32/0	32/0
number of sample points	16711	47608	10382	38221	37002	21150	20337
evaluation time per point	t_0	$100t_0$	$67t_0$	$106t_0$	$57t_0$	$74t_0$	$62t_0$

N/D: max polynomial degrees in the numerator/denominator

Application: two-loop amplitudes for pp→bbH

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

Analytic reconstruction data

Subleading colour terms:

- more complicated rational coeffs
- slower evaluation time

exploit relations of rational coefficients among different partial amplitudes

can we use this data to construct primitive amplitudes?

	$A_{34}^{(2),N_c^2}$	$A_{34}^{(2),1}$		$A_{\delta}^{(2),N_c}$		$A_{\delta}^{(2),1/N_c}$	
helicity set	$\vec{h}_g$	$\vec{h}'_g$		$\vec{h}_g$		$\vec{h}_g$	
$x_1 = 1$	133/132	176/175	176/175	265/269	265/269	214/207	214/207
ansatz for linear relations	-	-	$A_{34}^{(2),1/N_c^2}$	-	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ $A_{34}^{(2),1/N_c^2}$ $A_{43}^{(2),1/N_c^2}$ $A_{\delta}^{(2),1/N_c}$	-	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ $A_{34}^{(2),1/N_c^2}$ $A_{43}^{(2),1/N_c^2}$
linear relations	133/132	176/175	110/109	233/233	121/115	141/140	135/130
number of independent coefficients	162	572	216	582	151	581	258
denominator matching	133/0	176/0	110/0	233/0	119/0	141/0	135/0
univariate partial fraction	32/25	38/33	25/22	36/33	36/33	32/28	32/28
factor matching	28/0	38/0	24/0	36/0	36/0	32/0	32/0
number of sample points	16711	47608	10382	38221	37002	21150	20337
evaluation time per point	t_0	$100t_0$	$67t_0$	$106t_0$	$57t_0$	$74t_0$	$62t_0$

Application: two-loop amplitudes for pp→bbH

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

Analytic reconstruction data

Subleading colour terms:

- more complicated rational coeffs
- slower evaluation time

exploit relations of rational coefficients among different partial amplitudes

can we use this data to construct primitive amplitudes?

	$A_{34}^{(2),N_c^2}$	$A_{34}^{(2),1}$		$A_{\delta}^{(2),N_c}$		$A_{\delta}^{(2),1/N_c}$	
helicity set	$\vec{h}_g$	$\vec{h}'_g$		$\vec{h}_g$		$\vec{h}_g$	
$x_1 = 1$	133/132	176/175	176/175	265/269	265/269	214/207	214/207
ansatz for linear relations	-	-	$A_{34}^{(2),1/N_c^2}$	-	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ $A_{34}^{(2),1/N_c^2}$ $A_{43}^{(2),1/N_c^2}$ $A_{\delta}^{(2),1/N_c}$	-	$A_{34}^{(2),1}$ $A_{43}^{(2),1}$ $A_{34}^{(2),1/N_c^2}$ $A_{43}^{(2),1/N_c^2}$
linear relations	133/132	176/175	110/109	233/233	121/115	141/140	135/130
number of independent coefficients	162	572	216	582	151	581	258
denominator matching	133/0	176/0	110/0	233/0	119/0	141/0	135/0
univariate partial fraction	32/25	38/33	25/22	36/33	36/33	32/28	32/28
factor matching	28/0	38/0	24/0	36/0	36/0	32/0	32/0
number of sample points	16711	47608	10382	38221	37002	21150	20337
evaluation time per point	t_0	$100t_0$	$67t_0$	$106t_0$	$57t_0$	$74t_0$	$62t_0$

Application: two-loop amplitudes for pp→bbH

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

Analytic structure of the full-colour amplitude

Planar scattering

V. Sotnikov, QCD meets Gravity 2023

Only linear or quadratic letters vanish in $\mathcal{P}_{\text{phys}}$, poles always canceled, i.e. $h^{(0)} = 0$

New feature of nonplanar scattering

Square roots of quartic polynomials $\sqrt{\Sigma_5}$ can vanish in $\mathcal{P}_{\text{phys}}$ $\implies$ new types of divergences

1. Integrable square root: $\quad \mathrm{d}\log \frac{a+\sqrt{\Sigma_5}}{a-\sqrt{\Sigma_5}} \xrightarrow{\Sigma_5 \to 0} \frac{\mathrm{d}\Sigma_5}{a\sqrt{\Sigma_5}} \xrightarrow{t \to t^\star} \frac{C}{\sqrt{t-t^\star}} + \ldots$
2. Uncompensated poles: $\quad \mathrm{d}\log\sqrt{\Sigma_5} \xrightarrow{\Sigma_5 \to 0} \frac{\mathrm{d}\Sigma_5}{2\Sigma_5} \xrightarrow{t \to t^\star} \frac{C}{t-t^\star} + \ldots \implies$ log divergence!

- Choose basis functions to localize non-analytic behavior
- Functions with type 2 divergence cancel out in physical results? → YES!!
  (at the level of bare amplitude)
  !!! need to check for other processes !!!
- Numerical evaluation more challenging

Application: two-loop amplitudes for pp→bbH

[Badger,**HBH**,Poncelet,Wu,Zhang,Zoia;arXiv:2412.06519]

zenodo | Search records... | Communities | My dashboard | Log in | Sign up

Published **Analytic expressions & C++ implementation** Open

Ancillary files for "Full-colour double-virtual amplitudes for associated production of a Higgs boson with a bottom-quark pair at the LHC"

Badger, Simon[1]; Hartanto, Heribertus Bayu[2,3]; Poncelet, Rene[4]; Wu, Zihao[5]; Zhang, Yang[6]; Zoia, Simone[7]

Show affiliations

Ancillary files for the article "*Full-colour double-virtual ampl*... *at the LHC*" by Simon Badger, Heribertus Bayu Hartanto, R... https://arxiv.org/abs/2412.06519 and https://doi.org/10.100... anc_cpp.tar.gz for a description of the files.

Versions

Version v1 Dec 9, 2024

...cite all versions by ...io.14328502. This ...and will always ...ad more.

Cluster algebras for Feynman integrals

Dmitry Chicherin* and Johannes M. Henn†
Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, 80805 München, Germany

Georgios Papathanasiou‡
DESY Theory Group, DESY Hamburg, Notkestrasse 85, 22607 Hamburg, Germany

"data" to study cluster algebras in Feynman integrals & scattering amplitudes

Files

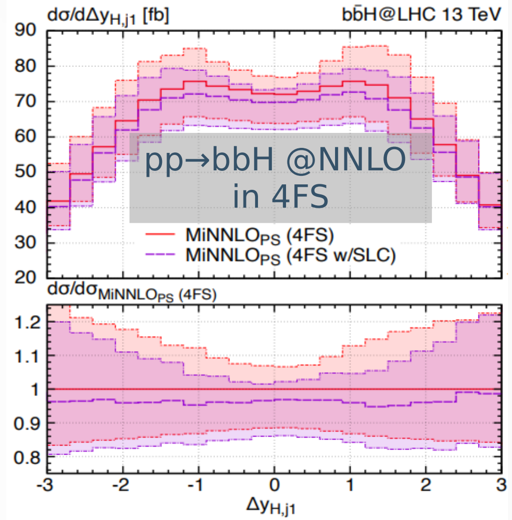
Files (540.7 MB)		
Name	**Size**	Download all
anc_cpp.tar.gz md5:0a01076854b45b18379a6e8b60fa2565	309.4 MB	Download
anc_math.tar.gz md5:504fbcd23f0d9e239bb240ef7dfec6db	231.4 MB	Download

External resources

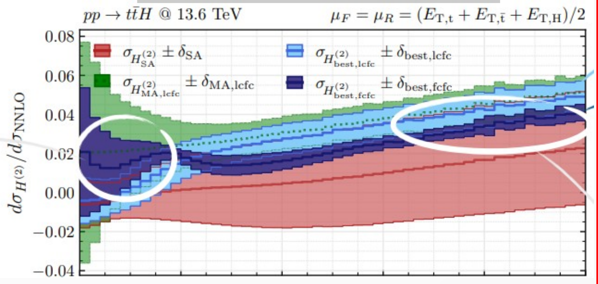
Indexed in

OpenAIRE

Keywords and subjects

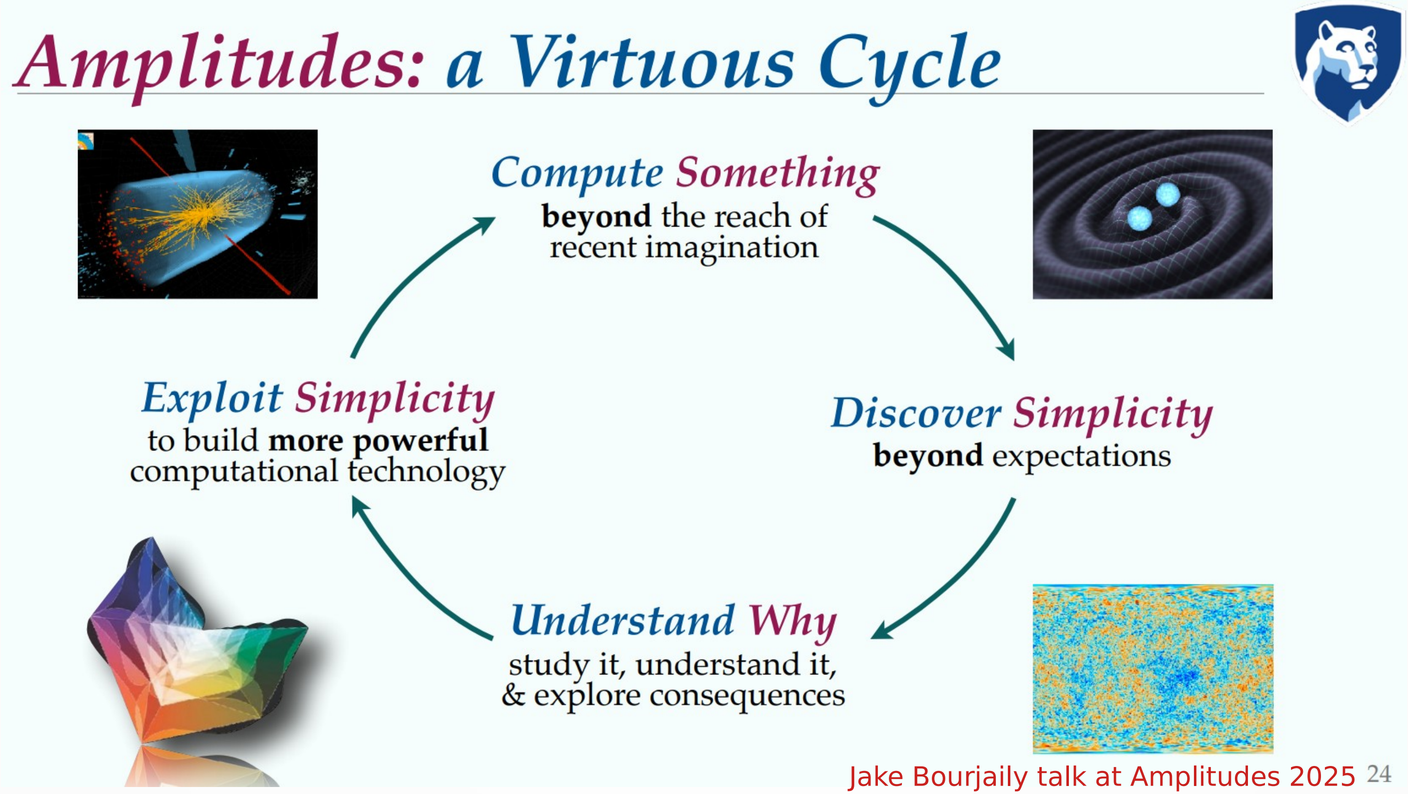


[Biello,etal;arXiv:2412.09510)



Chiara Savoini, SM@LHC2025

Summary



Jake Bourjaily talk at Amplitudes 2025

- Can we do better? (reconstruction algorithm, better kinematic variables & ansatz, …)
- What’s the limit of analytic calculation?

Back-up Slides

Momentum Twistor Variables

$$p_i \cdot \sigma_{a\dot{a}} = \lambda_{ia}\tilde{\lambda}_{i\dot{a}} \qquad p_i^\mu = x_i^\mu - x_{i-1}^\mu \qquad \mu_i^{\dot{a}} = x_i \cdot \tilde{\sigma}^{\dot{a}a}\lambda_{ia}$$

Momentum twistor variables $Z_i(\lambda_i, \mu_i)$ for each momentum

$\tilde{\lambda}_i$ are obtained via

$$\tilde{\lambda}_i = \frac{\langle i, i+1\rangle \mu_{i-1} + \langle i+1, i-1\rangle \mu_i + \langle i-1, i\rangle \mu_{i+1}}{\langle i, i+1\rangle\langle i-1, i\rangle}$$

5-point parameterization:

$$Z = \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 \\ \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{1}{x_1} & \frac{1}{x_1} + \frac{1}{x_1 x_2} & \frac{1}{x_1} + \frac{1}{x_1 x_2} + \frac{1}{x_1 x_2 x_3} \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \frac{x_4}{x_2} & 1 \\ 0 & 0 & 1 & 1 & 1 - \frac{x_5}{x_4} \end{pmatrix}$$

$$s_{12} = x_1, \; s_{23} = x_1 x_4, \; s_{45} = x_1 x_5$$

$$\langle 12\rangle = 1, \; [12] = -x_1, \; \langle 23\rangle = -\frac{1}{x_1}, \; [23] = x_1^2 x_4, \; \langle 45\rangle = -\frac{1}{x_1 x_2 x_3}, \; [45] = x_1^2 x_2 x_3 x_5$$