

Logarithmic corrections to near-extremal entropy of de Sitter black holes

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Based on :

Logarithmic corrections to near-extremal entropy of charged de Sitter black holes;

arXiv: 2503.08617 [hep-th]

Quantum corrections to the path integral of near extremal de Sitter black holes by

Blacker, Castro, Sybesma, Toldo ; 2503.14623 [hep-th]



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Spacetime from quantum systems

- About 50 years ago (early-mid 70s) physicists stumbled upon certain interesting relations for black holes which were very analogous to standard thermodynamics.
- Laws of thermodynamics are *emergent*. Follows from more complex and ergodic fundamental laws i.e. more coarse-grained.
- Black hole entropy, which is geometric also exhibit microscopic, quantum mechanical origin. Standard belief: BH thermodynamics is some kind of *coarse-graining* of underlying complicated quantum system. However, in standard GR, it is simple a spacetime curvature.
- Understanding how BH thermodynamics *emerges* from statistical averaging of some quantum system will give us a better understanding of how spacetime *emerges* from quantum mechanics.

- How robust is the analogy?
- The Laws of Black hole mechanics

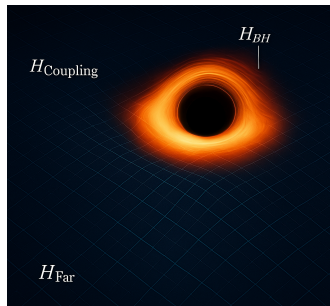
$$\delta M = \frac{\kappa}{8\pi} \delta A + \Omega_H \delta J + \mu \delta Q$$
$$\Delta A > 0$$

Not the absence of \hbar . $\kappa \rightarrow$ surface gravity. Analogous to temperature (?). The relations result purely from diffeomorphism invariance of the theory.

- Temp. is a phenomenological parameter which tells when two systems are in equilibrium. For black holes, cannot connect a heat bath. It will fall into the black hole!
- Also, scaling ambiguity $\kappa \rightarrow \frac{\kappa}{\lambda}$ and $A \rightarrow \lambda A$.
- How powerful really is this analogy?

Black holes: Information paradox and near-horizon

- Consider quantum fields on fixed black hole background [Hawking '74-'75]: $T = \frac{\kappa\hbar}{2\pi}$ and $S = \frac{A}{4G\hbar}$. Can take a classical limit. Entropy scales as area.
- However, when gravity is dynamical, BH evaporates eventually disappearing completely leaving behind thermal radiation. It is unknown how information gets out to purify this radiation.
- Solving the general problem is difficult. But, one can be a less ambitious and look at specific limits.



Split up: $H = H_{BH} + H_{Coupling} + H_{Far}$. We would first like to understand H_{BH} and then treat the remaining as perturbation. Unfortunately there is no diffeo. invariant way to split up the Hamiltonian like this. However, in a particular limit i.e. **extremal limit** there is a geometrical realization at the level of the metric of this split.

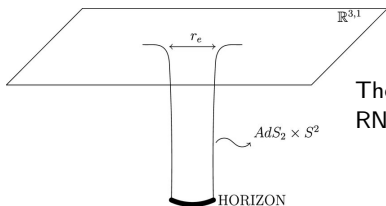
Black Holes at Extremality

- When there are multiple charges, existence of horizon imposes certain bounds:

$$J \leq M^2 \quad , \quad Q \leq M .$$

Black holes (nearly) saturating these bounds are called (nearly) extremal.

- As a black hole 'approaches' extremality a 'throat'-like region develops. In general, symmetry enhances. Observer far away does not notice this but one closer to black hole experiences this 'separate universe'.



The throat region for asymp. flat extremal RN black holes look like $AdS_2 \times S^2$.

Ground state degeneracy from black holes

- If dual quantum system is known, it implies $e^{S_{BH}} \sim$ number of ground states. This is because the black holes are extremal and hence zero temperature.
- Quite strange thermodynamically since most systems do not have high ground state degeneracy unless there is some kind of symmetry.
- In the examples that were known, there was supersymmetry which protects ground state degeneracy. But, what about non-supersymmetric cases. They also seem to predict high degeneracy. Does quantum correction lift this degeneracy?

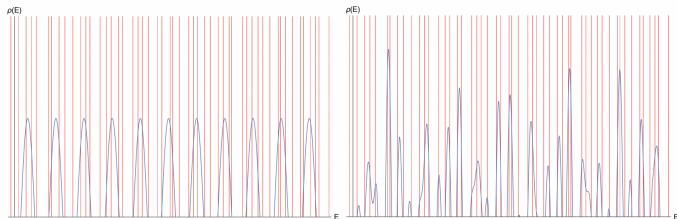


Figure: Including quantum corr. will presumably change the BH density of state which is otherwise coarse-grained. [From Kapec, Law, Toldo '24]

What we did

- Recent investigations have been performed for asymptotically BTZ and flat black holes.
- We decided to focus on dS black holes ($\Lambda > 0$) which are relevant for our current universe.
- We computed the leading quantum correction to 2 classes of near-extremal dS black holes (Cold and Nariai) and it seems the ground state degeneracy is indeed lifted for dS black hole. At some future point if we have some handle on quantum mechanical duals of de Sitter gravity, these quantum corrections can potentially act as checks for density of state for low-lying states.
- Specifically, we focussed on corrections arising from tensor zero modes which gives an entropy correction:

$$\delta \log \mathcal{Z} = \frac{3}{2} \log T .$$

Reissner-Nordström de Sitter (RN-dS₄) Black Holes

- The action of the theory we are concerned with:

$$I^{(4D)} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} (R - 2\Lambda_4 - F_{\mu\nu} F^{\mu\nu}) .$$

- More specifically, we will focus on *electric solution* which is characterized by the three constants M , Q and ℓ_4 .

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2) ,$$

$$A = -\frac{Q}{r_+} \left(1 - \frac{r_+}{r}\right) dt \quad , \quad f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{\ell_4^2} .$$

- The blackening factor $f(r)$ is crucial and decides the location of *horizons*. Even when all roots are real, only three are positive. They correspond to:

Inner horizon r_- ; **Outer horizon** r_+ ; **Cosmological horizon** r_c

Hierarchy of scales: $r_- < r_+ < r_c$.

RN-dS₄ Black Holes: Parameter space

- The presence of the cosmological horizon makes the physics and the extremal limit more interesting.
- Demanding the absence of a naked singularity puts a constraint on the (M, Q) parameter space. Constraint is determined by the positivity of the discriminant of $f(r)$. The valid parameter regime for RN-dS₄ black hole in the (M, Q) space is:

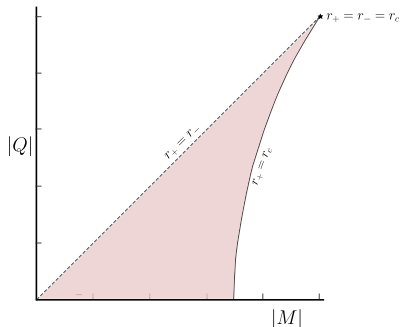


Figure: (M, Q) parameter space for RN-dS₄ metric

RN-dS₄ Black Holes: Thermodynamics

- Each of the three horizons has an associated temperature and entropy. Temperatures, entropies and chemical potentials associated with the outer and cosmological horizon r_+ and r_c are

$$T_+ = \frac{(r_c - r_+)(r_+ - r_-)(r_c + 2r_+ + r_-)}{4\pi\ell_4^2 r_+^2} \quad T_c = -\frac{(r_c - r_+)(r_c - r_-)(2r_c + r_+ + r_-)}{4\pi\ell_4^2 r_+^2}$$
$$S_h = \pi r_h^2 \quad \mu_h = \frac{Q}{r_h} .$$

- These lead to the first law of black hole thermodynamics

$$dM = \pm T_h dS_h + \mu_h dQ .$$

For inner and cosmological horizon, one gets the negative sign.

RN-dS₄ Black Holes: Extremality

- Extremality occurs when two or more horizons *coalesce*, causing the Hawking temperature associated with both horizons to vanish.
- In this case, three different extremal scenarios are possible:
 - 1 **Cold black hole:** The outer horizon and inner horizon coalesce, i.e. $r_- = r_+ \equiv r_0$.
 - 2 **Nariai black hole:** The outer horizon coalesces with the cosmological horizon, i.e. $r_+ = r_c \equiv r_n$.
 - 3 **Ultracold black hole:** When all three horizons coalesce, i.e. $r_- = r_+ = r_c \equiv r_{uc}$.

These are the boundaries of the *shark-fin diagram*.

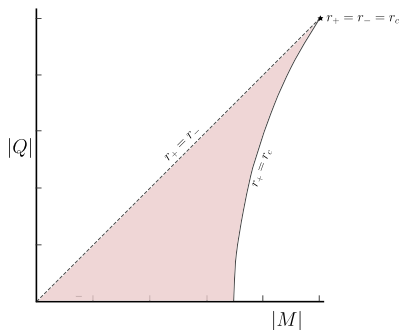


Figure: Dashed line: Cold BH; Solid line: Nariai BH; Tip of shark-fin: Ultra-cold BH

RN-dS₄ Black Holes: The three extremal geometries

The near-extremal geometry naturally gets divided into three classes and gives distinct extremal geometries:

- **Cold BH:** $AdS_2 \times S^2$

$$ds_{CBH}^2 = \frac{r_0^2}{1 - \frac{6r_0^2}{\ell_4^2}} \left(- (y^2 - 1) d\tau^2 + \frac{dy^2}{y^2 - 1} \right) + r_0^2 d\Omega_2^2 \quad ; \quad \ell_4^2 > 6r_0^2$$

- **Nariai BH:** $dS_2 \times S^2$

$$ds_{NBH}^2 = \frac{r_n^2}{\frac{6r_n^2}{\ell_4^2} - 1} \left((1 - y^2) d\tau^2 - \frac{dy^2}{1 - y^2} \right) + r_n^2 d\Omega_2^2 \quad ; \quad \ell_4^2 < 6r_n^2$$

- **Ultra cold BH:** $Mink_2 \times S^2$

$$ds_{UCBH}^2 = -d\tau^2 + dy^2 + r_{uc}^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

The gravity path integral

- AdS/CFT relates the partition function of dual QM system to the gravitational path integral in the AdS throat:

$$Z_{QM}(\beta) = Z_{\text{grav}}(\beta) .$$

The RHS is mostly studied under saddle-point approximation.

- More explicitly, we want to calculate

$$Z_{\text{grav}}(\beta) = \int Dg e^{-S[g]} \quad \text{with} \quad g \rightarrow g_{\text{bdy}}(\beta)$$

after fixing boundary conditions depending on the ensemble.

- In this case, the saddle point are the three corresponding near-horizon extremal geometries. The path integral computation with saddle point reproduces the leading order Hawking-Bekenstein entropy.
- The first correction comes from integrating fluctuations about the saddle. Like usual QFT it will be UV divergent but still there is useful information in them.

The gravity path integral

- In essence at 1-loop what one evaluates is the Gaussian integral

$$\int_{-\infty}^{+\infty} \prod_i dx_i e^{-\lambda_i x_i^2} = \prod_i \sqrt{\frac{\pi}{\lambda_i}}$$

- In this case, what we do is to write the $O(T)$ corrections: $g = g_{NHNE} + h$ and compute

$$\mathcal{Z} = \mathcal{Z}_0 \int Dh e^{-\int h \mathcal{D} h}$$

where \mathcal{D} is known as the Lichnerowicz operator.

- If one decompose h into eigenvectors, $h = \sum_i c_i h_i$, of \mathcal{D} , we get,

$$\int Dh e^{-\int h \mathcal{D} h} \sim \frac{1}{\sqrt{\det \mathcal{D}}}$$

- However, if there is a zero mode, there is a subtlety

$$\int_{-\infty}^{+\infty} e^{-0x^2} \rightarrow \infty .$$

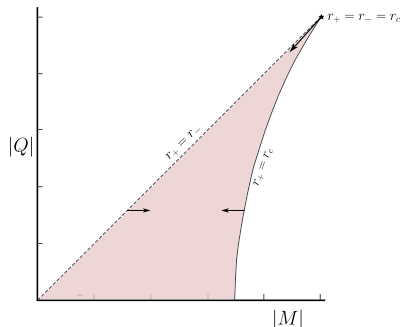
The throat talks to the far-away region!

- The operator \mathcal{D} in fact has infinitely many zero modes! However, they are normalizable.
- This however hints at an IR divergence which is there is AdS_2 throats. Central issue: AdS_2 is a low-dimensional system with long range forces. So, backreaction effect is very strong and lines of force in the bulk does not really have many directions to spread out.
- Essentially the throat really does not decouple from the far away region.
- So, how should we take into account this 'talking' between the throat and far away region. Resolution: Take the limit to keep the near-horizon extremal geometry but keep $O(T)$ subleading terms.

$$g \sim g_{NHE} + Th$$

RN-dS₄ Black Holes: Away from extremality

- We want to move away from extremality giving a *small temperature* to the black hole. Move away from the extremal limit, keeping the electric charge Q fixed i.e. **we describe the black hole in the canonical ensemble** at least for Cold and Nariai B.H.



- The case of ultracold BH is more subtle. Happens to be a single point on shark-fin diagram. Veering away from this limit staying within the space of black hole solutions forces us to correct Q .

RN-dS₄ Black Holes: Near-horizon, Near-Extremal (NHNE) geometry

For **cold BHs**, these corrections are

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu + T \delta g_{\mu\nu} dx^\mu dx^\nu ,$$
$$A = \bar{A} + T \delta A ,$$

where

$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = \frac{r_0^2}{1 - \frac{6r_0^2}{\ell^2_4}} \left((y^2 - 1) d\tau^2 + \frac{dy^2}{y^2 - 1} \right) + r_0^2 d\Omega_2^2 ,$$

$$\delta g_{\mu\nu} dx^\mu dx^\nu = \frac{4\pi r_0^3}{1 - \frac{6r_0^2}{\ell^2_4}} \left(\frac{\left(1 - \frac{4r_0^2}{\ell^2_4}\right)(y+2)}{\left(1 - \frac{6r_0^2}{\ell^2_4}\right)^2} \left(-(y-1)^2 d\tau^2 + \frac{dy^2}{(y+1)^2} \right) + y d\Omega_2^2 \right)$$

$$\bar{A} = \frac{ir_0 \sqrt{1 - \frac{3r_0^2}{\ell^2_4}}}{1 - \frac{6r_0^2}{\ell^2_4}} (y-1) d\tau ,$$

$$\delta A = \frac{2\pi ir_0^2 \sqrt{1 - \frac{3r_0^2}{\ell^2_4}}}{\left(1 - \frac{6r_0^2}{\ell^2_4}\right)^2} (y^2 - 1) d\tau .$$

RN-dS₄ Black Holes: Near-horizon, Near-Extremal (NHNE) geometry

For **Nariai BHs**, these corrections are

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu + T \delta g_{\mu\nu} dx^\mu dx^\nu,$$
$$A = \bar{A} + T \delta A,$$

$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = \frac{r_n^2}{\frac{6r_n^2}{\ell_4^2} - 1} \left((1 - y^2) d\tau^2 + \frac{dy^2}{1 - y^2} \right) + r_n^2 d\Omega_2^2,$$

$$\delta g_{\mu\nu} dx^\mu dx^\nu = \frac{4\pi r_n^3}{\frac{6r_n^2}{\ell_4^2} - 1} \left(\frac{\left(1 - \frac{4r_n^2}{\ell_4^2}\right)}{\left(\frac{6r_n^2}{\ell_4^2} - 1\right)^2} \left(-y (y^2 - 1) d\tau^2 + \frac{y dy^2}{(y^2 - 1)} \right) + y d\Omega_2^2 \right),$$

The gauge field background solution and the first order correction are given by

$$\bar{A} = -i|Q|_E \frac{y}{\frac{6r_n^2}{\ell_4^2} - 1} d\tau = \frac{ir_n \sqrt{1 - \frac{3r_n^2}{\ell_4^2}}}{\frac{6r_n^2}{\ell_4^2} - 1} (1 - y) d\tau,$$
$$\delta A = \frac{2\pi ir_n^2 \sqrt{1 - \frac{3r_n^2}{\ell_4^2}}}{\left(\frac{6r_n^2}{\ell_4^2} - 1\right)^2} (1 - y^2) d\tau.$$

For **Ultra cold BHs**, we obtain similar expressions with a subtlety.

- The metric corrections look like

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu + T^{\frac{1}{4}} \delta g_{\mu\nu} dx^\mu dx^\nu,$$

- We see the gauge field expansion is divergent in the $T \rightarrow 0$ limit

$$A = \frac{i}{2} \left(-\sqrt{\frac{3R_0}{R_0^2 - 6\pi^3}} T^{-\frac{1}{4}} + \frac{\sqrt{2} y}{R_0} - \sqrt{\frac{3R_0^3}{(R_0^2 - 6\pi^3)}} T^{\frac{1}{4}} \right) d\tau + (\sqrt{T}).$$

Since the divergent part in the gauge field is constant, it can be gauged away for any small temperature.

Correction to \mathcal{Z} : Generalities

- Correction to metric induces correction to the Lichnerowicz operator

$$\left(\mathcal{D}^{\alpha\beta\mu\nu} + \delta\mathcal{D}^{\alpha\beta\mu\nu} \right) (h_{\mu\nu} + \delta h_{\mu\nu}) = (\lambda_n + \delta\lambda_n) \left(h^{\alpha\beta} + \delta h^{\alpha\beta} \right),$$

- The first order correction is finally:

$$\delta\lambda_n(T) = \int d^4x \sqrt{\bar{g}} h_{\alpha\beta}^* \delta\mathcal{D}(T)^{\alpha\beta\mu\nu} h_{\mu\nu}.$$

eventually leading to the correction:

$$\log \mathcal{Z} = - \sum_n \log(\delta\lambda_n(T)),$$

Correction to \mathcal{Z} : Cold Black Hole

- The (normalizable) tensor zero modes of the Lichnerowicz operator is given by

$$h_{\mu\nu} dx^\mu dx^\nu = \frac{\sqrt{|n|(n^2-1)}}{2\sqrt{2}\pi\sqrt{1-\frac{6r_0^2}{\ell_4^2}}} \left(\frac{y-1}{y+1}\right)^{\frac{|n|}{2}} e^{in\tau} \left(-d\tau^2 + \frac{2i d\tau dy}{y^2-1} + \frac{dy^2}{(y^2-1)^2}\right),$$

where $|n| \geq 2$.

- The modes $n = 0, \pm 1$ need to be excluded or equivalently one need to take a quotient of the path integral measure by $SL(2, \mathbb{R})$ group since these perturbations correspond to diffeomorphisms generated by the isometries of background metric $\bar{g}_{\mu\nu}$.
- Correction to eigenvalues of Lichnerowicz operator for **cold BH**:

$$\begin{aligned} \delta\lambda_n &= -\frac{16|n|\pi(n^2-1)T}{r_0(\ell_4^2-6r_0^2)} \int_1^\infty dy (y-1)^{|n|-2}(y+1)^{-|n|-4} \\ &\quad \left(\ell_4^2(|n|(-|n|(y+2)+y(y+2)-4)+4y-2) \right. \\ &\quad \left. + r_0^2(4n^2(y+2)-5|n|(y-1)(y+3)+2(y-1)^2(y+4))\right) \\ &= \frac{2\pi n T}{r_n}, \quad n \geq 2 \end{aligned}$$

Correction to \mathcal{Z} : Cold black hole

- Eventually using zeta function regularization

$$\prod_{n \geq 2} \frac{1}{nT} = \frac{1}{\sqrt{2\pi}} T^{3/2} .$$

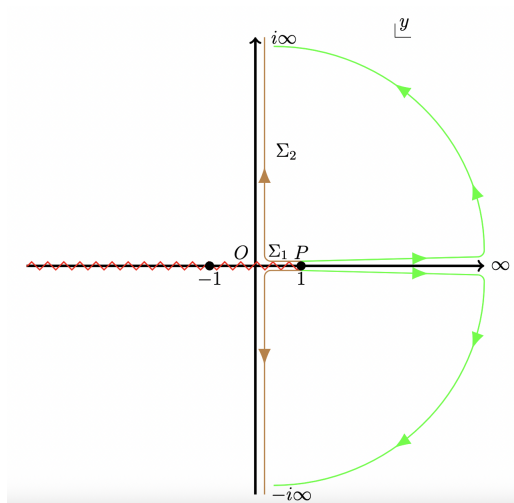
- This leads us to:

$$\delta \log \mathcal{Z} \sim \frac{3}{2} \log T + T - \text{independent constant} .$$

The subtlety of Nariai BH

- There is a subtlety in the case of Nariai BHs. Following the appropriate coordinate transformations which takes one to the NHNE limit, the 'radial' coordinate becomes compact and $y \in (-1, +1)$.
- Naively, it seems such a compact space isomorphic to a 2-sphere does not admit tensorial zero modes resulting from non-normalizable diffeomorphism generators.
- However, such non-normalizable modes do exist in an analytically continued radial patch. Secretly, in this case, one should look at the correction to the Hartle-Hawking wavefunction!
- The path integral in dS_2 scenario involves spacetime metrics with different signatures. Two distinct contours—dubbed the Hartle-Hawking contour and the Maldacena contour have been proposed to calculate the no-boundary wavefunction, each reflecting different choices of spacetime evolution.

The subtlety of Nariai BH



Conventional Hartle-Hawking contour (depicted by brown line), begins with a Euclidean dS space with S^2 topology, from the north pole at $y = 1$ to equator $y = 0$. This Euclidean dS space is then connected at its equator to a Minkowski dS space (obtained following an analytic continuation).

The subtlety of Nariai BH

- The above analytic continuation for the path integral allows one to get normalizable tensor zero modes which result in corrections of the following form

-

$$\begin{aligned}\delta\lambda_n = & \frac{16i\pi|n|(n^2-1)T}{r_n(\ell_4^2-6r_n^2)} \int_0^\infty dy (-iy-1)^{|n|-2}(-iy+1)^{-|n|-4} \\ & \left(\ell_4^2(|n|(-|n|(-iy+2) - iy(-iy+2) - 4) - 4iy - 2) \right. \\ & \left. + r_n^2 \left(4n^2(-iy+2) - 5|n|(-iy-1)(-iy+3) + 2(-iy-1)^2(-iy+4) \right) \right) \\ & + \frac{16i\pi|n|(n^2-1)T}{r_n(\ell_4^2-6r_n^2)} \int_1^0 du (u-1)^{|n|-2}(u+1)^{-|n|-4} \\ & \left(\ell_4^2(|n|(-|n|(u+2) + u(u+2) - 4) + 4u - 2) \right. \\ & \left. + r_n^2 \left(4n^2(u+2) - 5|n|(u-1)(u+3) + 2(u-1)^2(u+4) \right) \right).\end{aligned}$$

- Again, results in the same factor of $\frac{3}{2} \log T$.
- Perhaps the most intricate and non-trivial among all the three extremal limits is the case of ultracold black hole. Deviations from extremality does not exactly heat up the Mink_2 part. heat up the

Conclusions

- Although we focused on the tensor modes, shortly after our work vector and scalar sector corrections were also investigated [\[See Blacker, Castro, Sybesma, Toldo\]](#)
- What happens for *rotating* dS black holes. Has much richer dynamics.
- What happens for fermions but broken SUSY? This will necessitate one to look at path integrals in the first order formalism. Which is more fundamental– first or second order? [\[Work in progress\]](#)