Testing Einstein's equivalence principle from black hole image observation

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Einstein's equivalence principle

1. WEP: Weak equivalence principle states, geodesic of a freely falling particle (not influenced by any non-gravity force and small enough not to be influenced by tidal force) is independent of its internal structure or composition.

2. LLI: Local Lorentz invariance states, result of any local non-gravitational experiment is independent of the velocity of the freely falling frame of reference.

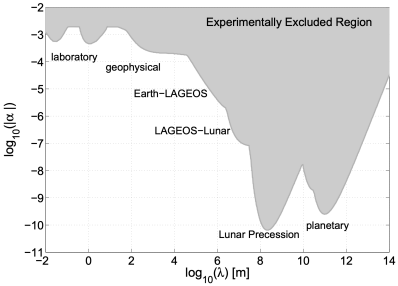
3. LPI: Local position invariance tells, outcomes of any local non-gravitational experiment are independent of the location and time of occurrence of the experiment.

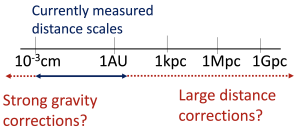
GR & equivalence principle

- Einstein's equivalence principle(EEP) — All laws of physics are the same w.r.t all freely falling frames or observers.
- Unlike other gauge invariant theories, EEP is not a fundamental symmetry but an experimental fact.
- Testing EEP $\longleftrightarrow$ test of the underlying principle of GR.
- It has been tested in the weak field limit(solar system scale).
- LIGO-VIRGO-KAGRA & black-hole image by Event Horizon Telescope(EHT) probe GR in strong field regime.

Modification to GR

2018 J. BERGÈ





- Any modifications to GR ↔ Yukawa like potential: $V(r) = -\frac{GMm}{r}\left(1 + \alpha \exp^{-r/\lambda}\right)$. α is dimensionless & λ causes deviation in Newtonian potential.
- The constraint on α and λ suggests that the gravitational potential $\sim 1/r$ predicted from GR is accurate in solar scale and in weak field limit.
- With better observational set-up obtaining constraints on α and λ requires modification to GR—1)Strong field, 2) large scale.

2022 SS & J. P Johnson

Modification in strong field regime: Non-minimal coupling

- Non-minimal coupling(NMC) of any field or matter to gravity is one way to modify GR—matter(fields, or both) couples to curvature of space-time.

- Electromagnetic(EM) waves are key ingredients in astrophysical observations → NMC of EM fields.

- Introduced as quantum corrections to gravity. For Ricci flat backgrounds (Schwarzchild, Kerr), [Drummond & Hathrell, PRD 1980]

$$S_{NC} = \int d^4x\sqrt{-g}\left[\frac{R}{\kappa} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{2}R^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\right]$$

- Invariant under general coordinate transformation as well as CPT. Violates Einstein's equivalence principle.

Signatures of NMC

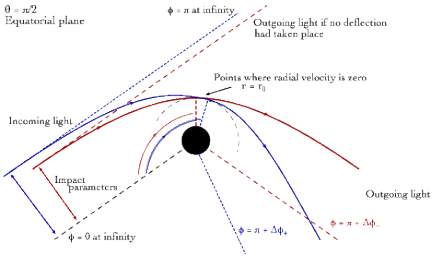


Figure: Polarization dependent deflection of photon (2018 Plascencia.et al)

- Modified dispersion relations → Modified photon trajectory? Modified frequency shift? Or, time delay?
- Can the non-minimal coupling be detected from the black hole observations?
- We aim to find signatures of NMC of electromagnetic fields—deviation in EEP from BH shadows.

Black hole shadow?

- Recently EHT has detected of M87 & SgrA images in 2019 & 2022.
- The shadow is formed due bending of light by the BH.
- The shape depends on the accretion(glowing gas circling the BH at high speed) flow around it. However,
  - The shadow is independent of the spin and orientation of the BH. (Flacke. et al, 2000).
  - Presence of accreting plasma and gravitational lensing makes the brightness of the accretion flow highly non-uniform.

Our aim & route to be followed

1. NMC will lead to modified dispersion relations leading to different propagation for the two polarization modes.

2. We analyze the black hole shadow for the two modes for Schwarzschild(and slowly rotating Kerr) BH.

   - For $(+)$ mode shadow radius $< \sqrt{27}GM/c^2$ and $(-)$ mode shadow radius $> \sqrt{27}GM/c^2$ for $\lambda < 0$. These trends reverse for $\lambda > 0$.

   - The shadow size independent of a for Kerr.

3. Finally, we discuss the possibility of constraining the NMC constant using EHT and ngEHT observations.

Propagation of non-minimally coupled EM field

- Equation of motion

$$\nabla_\nu F^{\mu\nu} = 2\lambda \left(R^{\mu\nu\rho\sigma} \left(\nabla_\nu F_{\rho\sigma} \right) + \left(\nabla_\rho R^{\mu}_{\sigma} \right) F^{\rho\sigma} \right)$$

- Schwarzchild/Kerr space-time $\rightarrow R_{\mu\nu} = 0$

- Obtain the dispersion relation in local inertial frame using the eikonal approximation $\nabla_{(\mu)} F^{(\mu)(\nu)} = p_{(\mu)} F^{(\mu)(\nu)}$

- Modified dispersion relation is obtained using

$$\mathrm{Det}\left[p^{(\mu)} p_{(\mu)} \delta^{(j)}_{(k)} + 4\lambda \left(-\frac{p^{(j)}}{p_{(0)}} \epsilon^{(0)}{}_{(k)} + \epsilon^{(j)}{}_{(k)} \right) \right] = 0, \quad \epsilon^{(j)}{}_{(k)} \equiv R^{(j)(a)(b)}{}_{(k)} p_{(a)} p_{(b)}$$

Photon trajectory in Schwarzchild space-time

- Consider the Schwarzchild space-time in the dimensionless variables $t = t/r_s$, $r = r/r_s$

$$d\tilde{s}^2 = \left[-\left(1 - \frac{1}{r}\right) dt^2 + \left(1 - \frac{1}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \right]$$

- Modified dispersion relation in local inertial frame is given by

$$p^2 = \mathcal{C}_{\pm} p^2_{(\phi)}, \quad \mathcal{C}_- = \frac{6\lambda}{4\lambda - r^3}, \quad \mathcal{C}_+ = \frac{6\lambda}{2\lambda + r^3}, \quad \lambda = \lambda/r_s^2, \quad \mathcal{C}_+ \neq -\mathcal{C}_-$$

- The final dispersion relation in non-inertial frame using constants of motion $\tilde{E} = -\tilde{p}_t$ and $\tilde{L} = \tilde{p}_\phi$

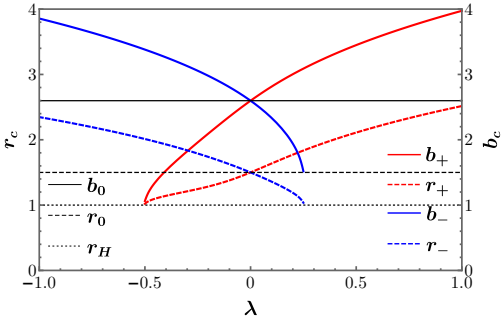
$$\dot{r}^2 = \tilde{L}^2 \left[\frac{1}{b^2} - \left(1 - \frac{1}{r}\right)(1 + \mathcal{C}_{\pm}) \frac{1}{r^2} \right], \quad b = \frac{\tilde{L}}{\tilde{E}}$$

Position of the photon ring

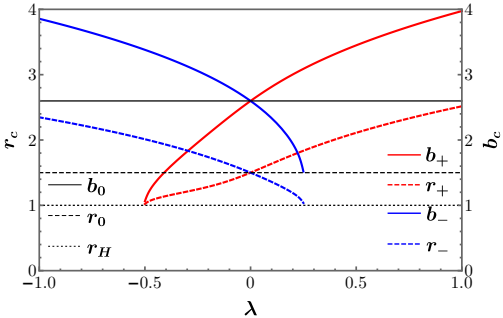
- Unstable circular orbit of the photon satisfies

$$\dot{r} = 0, \quad \ddot{r} = 0$$

- For minimally coupled case, this is satisfied for $r = 1.5$, $b = 1.5\sqrt{3}$.
- For non-minimally coupled case, the above equations can be numerically solved.



Position of the photon ring

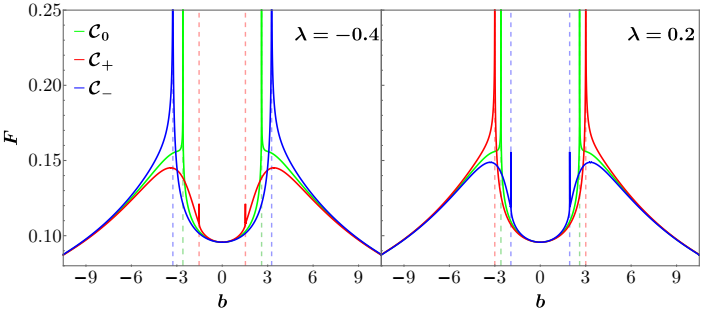


- Photon ring doesn't form for
  - $+$ mode when $\lambda < -0.5$
  - $-$ mode when $\lambda > 0.25$
- $b \rightarrow$ Apparent position of the photon ring in the observer's sky.

Observed image of the black hole

- Schwarzchild black hole enveloped by radially free-falling optically thin light-emitting accretion flow [Bambi, PRD 2013]

$$F(x,y) \propto \int \frac{g^3 p_t dr}{r^2 |p_r|}, \quad g \equiv \frac{p_\alpha u^\alpha_{obs}}{p_\beta u_e^\beta}, \quad u_{obs} = (1,0,0,0), \quad u_e = \left(\left(1-\frac{1}{r}\right)^{-1}, -\frac{1}{\sqrt{r}}, 0, 0\right)$$



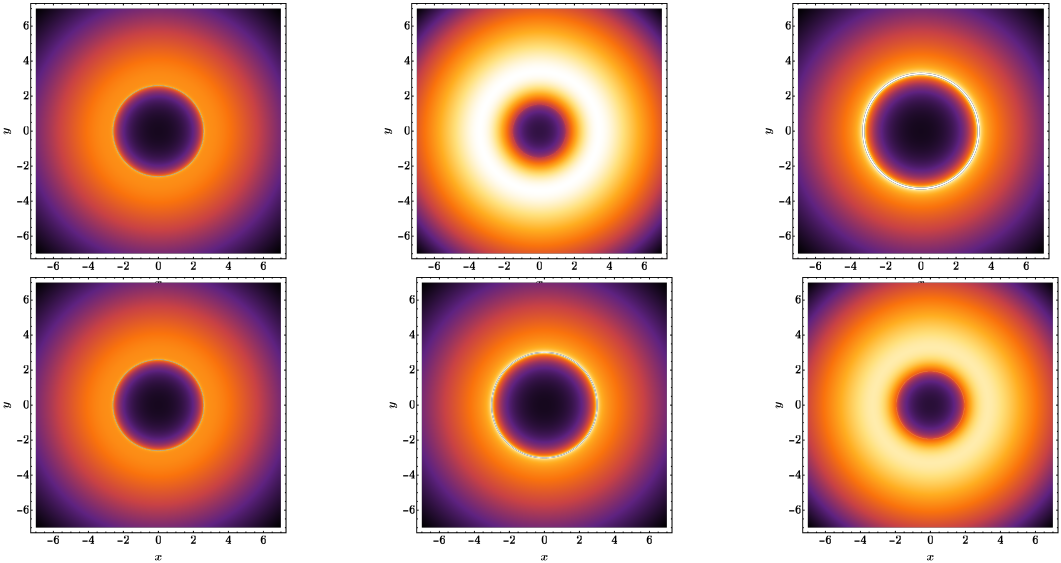


Figure: $\lambda = -0.4$(top), $\lambda = 0.2$(bottom). Left: $\lambda = 0$, Center: '+' mode, Right: '−' mode.

Photon trajectory in Kerr space-time

- $d\tilde{s}^2 = -\left(1 - \frac{r}{\rho^2}\right) dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{ra^2}{\rho^2} \sin^2\theta\right) \sin^2\theta d\phi^2 - \frac{2ra\sin^2\theta}{\rho^2} dt d\phi$

  $a = J/2M^2,\ \rho^2 = r^2 + a^2\cos^2\theta,\ \Delta = r^2 - r + a^2,\ \Xi = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta$

- Dispersion relation of the non-minimally coupled photon

$$p^2{}_{\pm} = \frac{6(p^2_{(\chi)} + p^2_{(\phi)})\lambda\rho^3}{3\lambda\rho^3 \mp \sqrt{\rho^{12} + 9\lambda^2\rho^6 + (3\rho^2 - 4r^2)[2\lambda\rho^2 r - 8\lambda^2 r^2(3\rho^2 - 4r^2)]}}$$

- $$p^2 = p_r^2 \frac{\Delta}{\rho^2} - \frac{(a^2\tilde{E} - a\tilde{L} + \tilde{E}r^2)^2}{\Delta\rho^2} + p_\chi^2 \frac{1 - \chi^2}{\rho^2} + \frac{(\tilde{L} - a\tilde{E}(1 - \chi^2))^2}{(1 - \chi^2)\rho^2}$$

- Analysis for the full Kerr metric is complicated.

- Spin of supermassive blackholes ($SgtA*$) are poorly constrained, can be slowly rotating. [Fragione & Loeb, APJ 2020]

Photon trajectory in the slowly rotating case

- Photon dispersion relation in the linear order in a

$$p_\theta^2 + \frac{b^2\,\tilde{E}^2}{sin^2\theta} + p_r^2\,(r^2 - r)\,\frac{c_\pm}{c_\mp} + \frac{\tilde{E}^2\,r^3}{r-1}\,\frac{c_\pm}{c_\mp} \pm 2\,a\,b\,\tilde{E}^2\,\frac{6\,\lambda\,r \pm c_\mp}{(r-1)\,c_\mp} = 0$$

where $c_+ = r^3 + 2\,\lambda$ and $c_- = r^3 - 4\,\lambda$

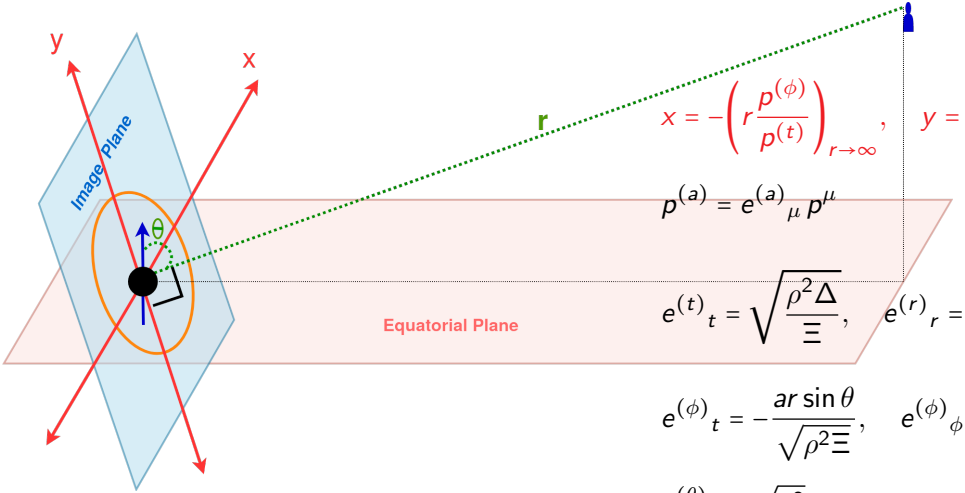
- Separation constant

$$\tilde{k}_\pm \tilde{E}^2 = p_\theta^2 + \frac{b^2\tilde{E}^2}{sin^2\theta} = -p_r^2\,(r^2 - r)\,\frac{c_\pm}{c_\mp} - \frac{\tilde{E}^2 r^3}{r-1}\frac{c_\pm}{c_\mp} \mp 2\,a\,b\,\tilde{E}^2\,\frac{6\,\lambda\,r \pm c_\mp}{(r-1)\,c_\mp}$$

- Circular orbit

$$\dot{r}^2 = \tilde{E}^2\left[1 + \tilde{k}_\pm(1-r)\frac{c_\mp}{c_\pm r^3} - \frac{2\,a\,b\,(c_\mp \pm 6\,\lambda\,r)}{c_\pm r^3}\right], \quad \dot{r} = 0, \quad \ddot{r} = 0$$

Observed image of a rotating blackhole



$$x = -\left(r\frac{p^{(\phi)}}{p^{(t)}}\right)_{r\to\infty}, \quad y = \left(r\frac{p^{(\theta)}}{p^{(t)}}\right)_{r\to\infty}$$

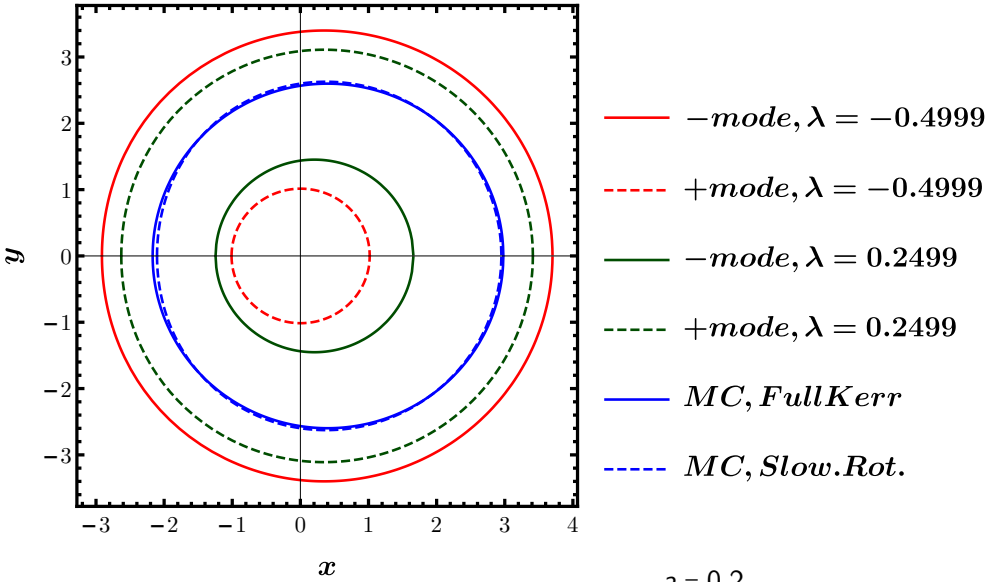
$$p^{(a)} = e^{(a)}{}_{\mu}\, p^{\mu}$$

$$e^{(t)}{}_{t} = \sqrt{\frac{\rho^2\Delta}{\Xi}}, \quad e^{(r)}{}_{r} = \sqrt{\frac{\rho^2}{\Delta}}$$

$$e^{(\phi)}{}_{t} = -\frac{ar\sin\theta}{\sqrt{\rho^2\Xi}}, \quad e^{(\phi)}{}_{\phi} = \sqrt{\frac{\Xi}{\rho^2}}\sin\theta$$

$$e^{(\theta)}{}_{\theta} = \sqrt{\rho^2}$$

[Bardeen etal., APJ 1972]


3
2
1
0
−1
−2
−3
y
−3
−2
−1
0
1
2
3
4
x
$-mode, \lambda = -0.4999$
$+mode, \lambda = -0.4999$
$-mode, \lambda = 0.2499$
$+mode, \lambda = 0.2499$
$MC, FullKerr$
$MC, Slow.Rot.$

$a = 0.2$

Current observational status

- Current resolution of EHT: $24\mu as \rightarrow W = \Delta x = 2.4$ for $SgtA^*$
- Deviation of shadow size for $\lambda = -0.49$
  - $\Delta x_{-,0} = 1.55$: Bigger shadow
  - $\Delta x_{+,0} = -3.03$: Smaller shadow, photon could be superluminal
  - $\Delta x_{+,-} = -4.58$: Need information regarding the polarization
- Deviation of shadow size for $\lambda = 0.249$
  - $\Delta x_{-,0} = -2.16$: Smaller shadow, photon could be superluminal
  - $\Delta x_{+,0} = 0.98$: Bigger shadow.
  - $\Delta x_{+,-} = 3.14$: Need information regarding the polarization
- The deviation is too small to be detected with the current resolution of EHT.

Potential signatures in upcoming ngEHT observations

- Resolution of upcoming ngEHT: $15\mu as \rightarrow W = \Delta x = 1.5$ for $SgrA^*$
- Deviation of shadow size for $\lambda = -0.49$
  - $\Delta x_{-,0} = 1.53$: Bigger shadow, detectable.
  - $\Delta x_{+,0} = -2.27$: Smaller shadow, detectable (Photon could be superluminal).
  - $\Delta x_{+,-} = -4.3$: detectable (Need information regarding the polarization).
- Deviation of shadow size for $\lambda = 0.24$
  - $\Delta x_{-,0} = -1.75$: Smaller shadow, detectable (Photon could be superluminal).
  - $\Delta x_{+,0} = 0.94$: Bigger shadow, deviation too small to be detected.
  - $\Delta x_{+,-} = 2.69$: detectable (Need information regarding the polarization).
- Non-minimal coupling could be detected by ngEHT.

Conclusions

- Non-minimal coupling modifies the photon trajectory → deviation in the blackhole shadow observations.
- Analytical study for a slowly rotating blackhole confirms this.
- Signatures could be detected by the next generation Event Horizon Telescope.
- Analysis for the full Kerr space-time requires numerical computations.
- Study the effect of coupling on the observed polarized image of the black hole.

Thank You

Potential signatures in upcoming ngEHT observations

- Resolution of upcoming ngEHT: $15\mu as \rightarrow W = \Delta x = 1.5$ for $SgtA^*$
- Schwarzchild blackhole

ΔW	W_0	$W(-,0.24)$	$W(+,0.24)$	$W(-,-0.49)$	$W(+,-0.49)$
W_0	0	1.9	-0.95	**-1.54**	2.9
$W(-,0.24)$	-1.9	0	-2.85	-	-
$W(+,0.24)$	0.95	2.85	0	-	-
$W(-,-0.49)$	**1.54**	-	-	0	4.44
$W(+,-0.49)$	-2.9	-	-	-4.44	0

- Slowly Rotating blackhole

ΔW	W_0	$W(-,0.24)$	$W(+,0.24)$	$W(-,-0.49)$	$W(+,-0.49)$
W_0	0	1.75	-0.94	**-1.53**	2.77
$W(-,0.24)$	-1.75	0	-2.69	-	-
$W(+,0.24)$	0.94	2.69	0	-	-
$W(-,-0.49)$	**1.53**	-	-	0	4.3
$W(+,-0.49)$	-2.27	-	-	-4.3	0