Matter coupled Aristotelian gravity

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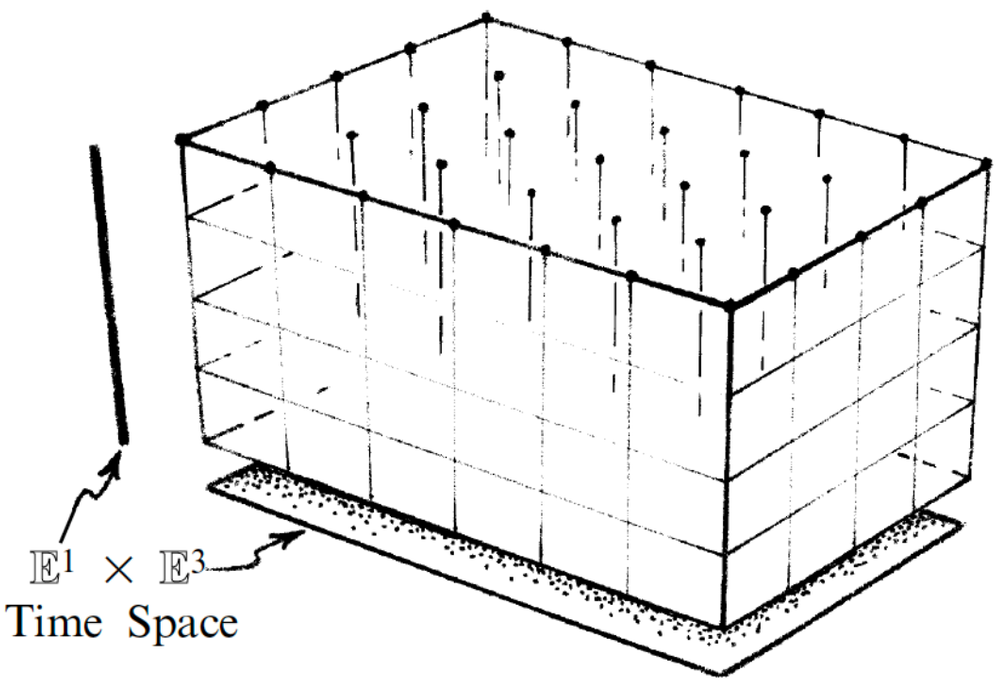
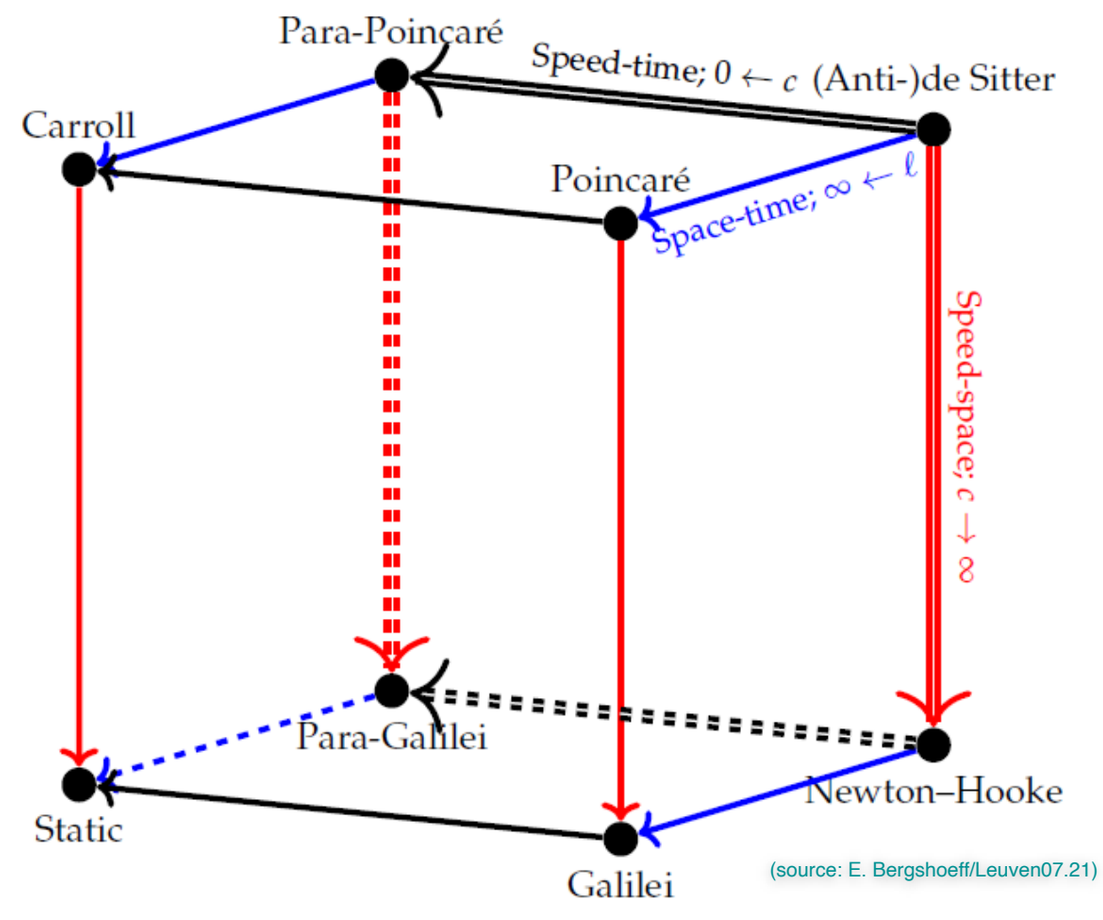
Based on:

- **E. A. Bergshoeff, G. Giorgi, J. Rosseel, PSR - 2507.16943 [hep-th]**
- **J. K. Ghosh, F. Peña-Benítez, PSR - 2504.20361 [cond-mat.str-el]**

Non-Lorentzian Limits of Poincaré

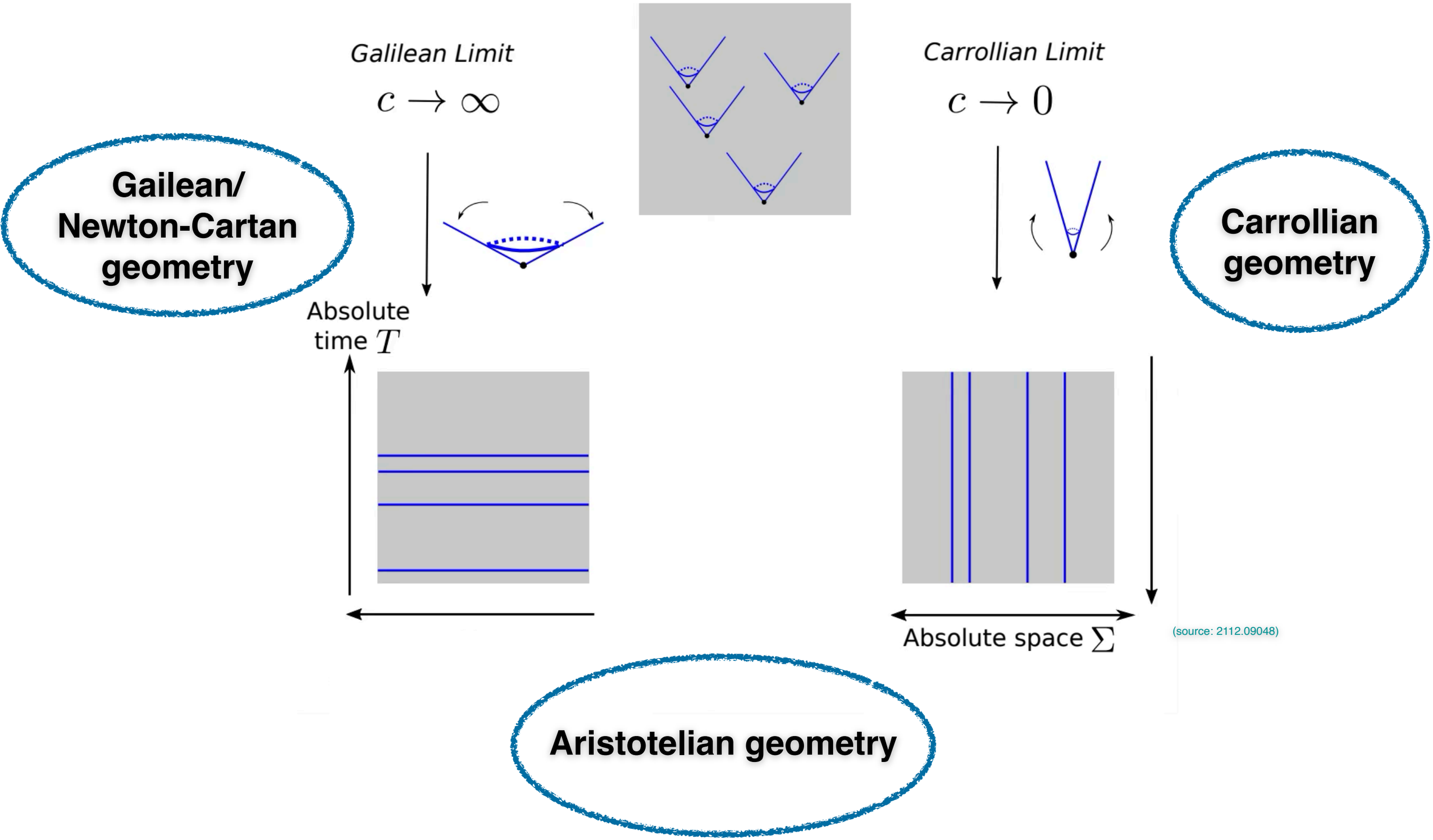
$$x^i \to R^i{}_j x^j + a^i, \quad t \to t + a$$





(source: E. Bergshoeff/Leuven07.21)

Non-Lorentzian Geometry



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Aristotelian Field Theory

(source: Road to Reality. R. Penrose)

- **Naturalness** [Horava (2016)]

- **Lifshitz field theory** [Kachru, Liu, Mulligan (2018)]

$$x^i \to b\, x^i, \quad t \to b^z t$$

- **Bimetric theory for the fractional quantum Hall effect** [Gromov, Son (2017)]

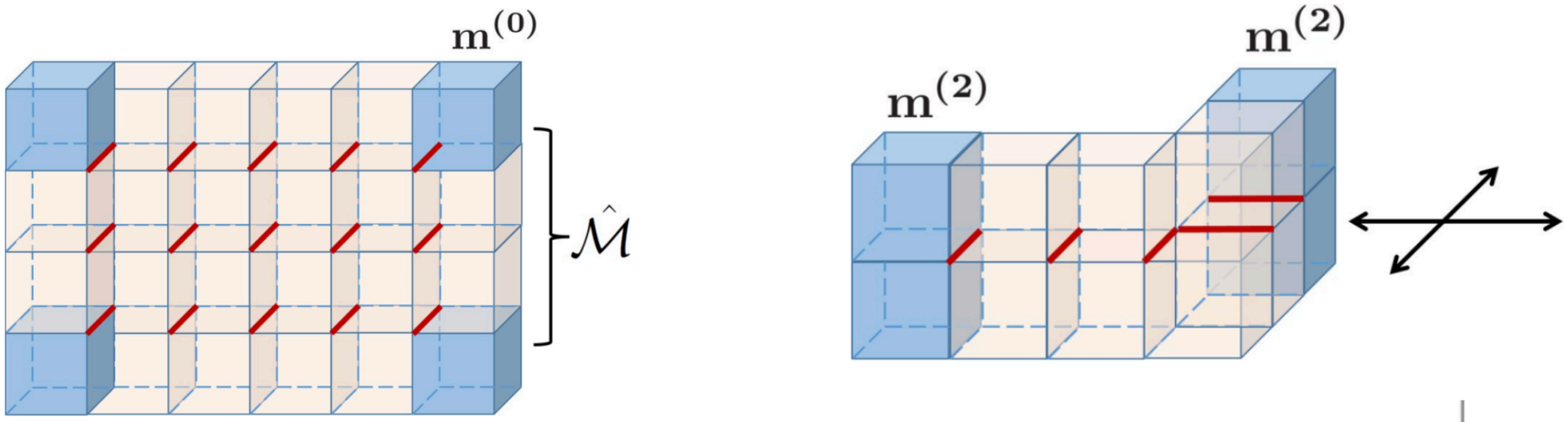
- **Hydrodynamics** [Armas, Jain (2021)], [Pinzani-Fokeeva, Yarom (2022)]

- **Field theories for fractons** [Pretko (2016)], [Bidussi, Hartong, Have, Musaeus, Prohazka (2021)], [Grosvenor, Hoyos, Peña-Benítez, Surówka (2021)]

(source: 2001.01722)

Fractons

- **Quantum error correction** [Haah (2011)], [Vijay, Haah, Fu (2016)], [Shirley, Slagle, Chen (2018)]



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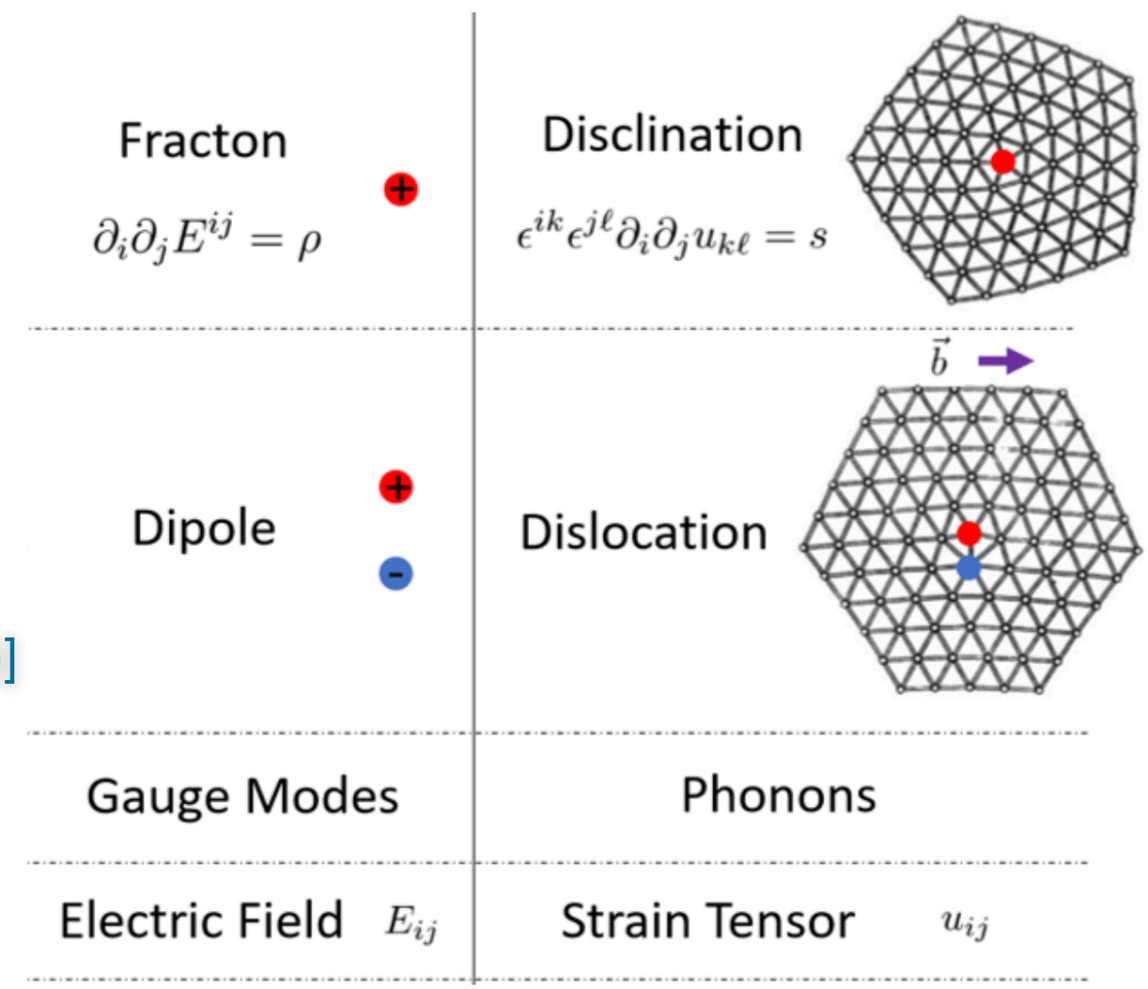
- **Higher-rank gauge theory** [Pretko (2016)]
- **Duality with Carroll symmetry** [Figueroa-O'Farrill, Peréz, Prohazka (2023)], [Marsot, Zhang, Chrenodub, Horvathy (2022)]
- **Fracton-elasticity duality** [Pretko, Radzihovski (2017)]
- **FQHE** [Du, Mehta, Nguyen, Son (2024)]



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Fractons

- **Field theories with fractonic behavior introduce conservation of higher moment charges** [Pretko (2016)]

$$Q = \int d^D x \rho \qquad D^a = \int d^D x x^a \rho$$

$$[J_{ab}, J_{cd}] = -4\delta_{[a[c}J_{d]b]} \qquad [P_a, J_{bc}] = -2\delta_{a[b}P_{c]}$$

$$[D_a, J_{bc}] = -2\delta_{a[b}D_{c]} \qquad [P_a, D_b] = \delta_{ab}Q$$

- **Extension to higher moment charges: multipole algebra** [Gromov (2018)]

- **Continuity equation** $\partial_0 \rho + \partial_a \partial_b J^{ab} = 0$

- **Scalar field theory with dipole symmetry**

$$\mathcal{S} = \frac{1}{2}\int d^D x \left(\partial_0 \Phi \partial_0 \Phi^* - k\, X_{ab} X^{ab*}\right) \qquad X_{ab} = \partial_a \Phi \partial_b \Phi - \Phi \partial_a \partial_b \Phi$$

$$\delta\Phi = \left(i\lambda_{U(1)} + i\lambda^a_{\mathrm{dp}}\, x_a\right)\Phi$$

- **Gauging the U(1) symmetry requires to introduce a higher rank gauge field**

$$\partial_0 \to \partial_0 - iA_0 \qquad \delta A_0 = \partial_0 \lambda_{U(1)}$$

$$X_{ab} \to X_{ab} - iA_{ab}\Phi^2 \qquad \delta A_{ab} = \partial_a \partial_b \lambda_{U(1)}$$

- **Scalar charge theory**

$$\mathcal{S}_{\mathrm{SC}} = \frac{1}{2}\int d^D x \left(F_{0bc}\, F^{0bc} - F_{ab,c}\, F^{ab,c}\right) \qquad F_{0ab} = \partial_0 A_{ab} - \partial_a \partial_b A_0$$

$$F_{ab,c} = \partial_a A_{bc} - \partial_b A_{ac}$$

Outline

- p-brane Aristotelian geometry
- Conformal extension
- Matter coupled Aristotelian gravity
- Aristotelian geometry for multi-Weyl semimetals

p-brane Aristotelian geometry

- **Aristotelian G structure** $$G = \mathrm{SO}(p,1) \times \mathrm{SO}(D-p-1) \qquad 0 \leq p \leq D-2$$

- **Connection** $$\Omega_\mu = \frac{1}{2}\omega_\mu{}^{AB} M_{AB} + \frac{1}{2}\omega_\mu{}^{ab} J_{ab}$$

- **Geometry** $$\tau^A \qquad A = 0, 1, \ldots, p$$ $$e^a \qquad a = p+1, \ldots, D-1$$

- **The veilbeine and the "inverse" veilbeine satisfy the orthogonality and completeness relations**

$$\tau_A{}^\mu e_\mu{}^a = 0 \qquad e_a{}^\mu \tau_\mu{}^A = 0$$

$$\tau_A{}^\mu \tau_\mu{}^B = \delta_A^B \qquad e_\mu{}^a e_b{}^\mu = \delta_b^a \qquad \tau_\mu{}^A \tau_A{}^\nu + e_\mu{}^a e_a{}^\nu = \delta_\mu^\nu$$

- **They define the spacetime metric**

$$g_{\mu\nu} = \eta_{AB}\tau_\mu^A \tau_\nu^B + \delta_{ab} e_\mu^a e_\nu^b$$

p-brane Aristotelian geometry

- **Transformations**

$$\delta\omega_\mu{}^{AB} = \partial_\mu\lambda^{AB} + 2\lambda^{[A|C|}\omega_{\mu C}{}^{B]} \qquad \delta\omega_\mu{}^{ab} = \partial_\mu\lambda^{ab} + 2\lambda^{[a|c|}\omega_{\mu c}{}^{b]}$$

$$\delta\tau_\mu{}^{A} = \lambda^{A}{}_{B}\tau_\mu{}^{B} \qquad \delta e_\mu{}^{a} = \lambda^{a}{}_{b}e_\mu{}^{b}$$

- **Curvature**

$$R_{\mu\nu}{}^{AB}(M) \equiv 2\partial_{[\mu}\omega_{\nu]}{}^{AB} - 2\omega_{[\mu}{}^{AC}\omega_{\nu]C}{}^{B} \qquad R_{\mu\nu}{}^{ab}(J) \equiv 2\partial_{[\mu}\omega_{\nu]}{}^{ab} - 2\omega_{[\mu}{}^{ac}\omega_{\nu]c}{}^{b}$$

- **Torsion**

$$T_{\mu\nu}{}^{A} \equiv 2\partial_{[\mu}\tau_{\nu]}{}^{A} - 2\omega_{[\mu}{}^{AB}\tau_{\nu]B} \qquad E_{\mu\nu}{}^{a} \equiv 2\partial_{[\mu}e_{\nu]}{}^{a} - 2\omega_{[\mu}{}^{ab}e_{\nu]b}$$

- **Vectors and tensors can be decomposed into longitudinal and transverse components**

$$V_\mu = \tau_\mu{}^{A}V_A + e_\mu{}^{a}V_a \qquad V_A = \tau_A{}^{\mu}V_\mu \qquad V_a = e_a{}^{\mu}V_\mu$$

p-brane Aristotelian geometry

- **Intrinsic torsion: no dependence on spin connection components**

$$T_{ab}{}^{A} \qquad T_{a}{}^{\{AB\}} \qquad T_{a}{}^{A}{}_{A} \qquad E_{AB}{}^{a} \qquad E_{A}{}^{\{ab\}} \qquad E_{A}{}^{a}{}_{a}$$

- **Extrinsic torsion**

$$T_{a}{}^{[AB]} \qquad T_{AB}{}^{C} \qquad E_{A}{}^{[ab]} \qquad E_{ab}{}^{c}$$

- **The vanishing of the extrinsic torsion allows to solve the spin connection in terms of the metric variables**

$$\tau_{\mu\nu}{}^{A} \equiv 2\,\partial_{[\mu}\tau_{\nu]}{}^{A} \qquad\qquad e_{\mu\nu}{}^{a} \equiv 2\,\partial_{[\mu}e_{\nu]}{}^{a}$$

- **Solution:**

$$\omega_{A,BC}(\tau,e) = \frac{1}{2}\tau_{BC,A} - \tau_{A[B,C]}$$
$$\omega_{a,AB}(\tau,e) = -\tau_{a[A,B]}$$
$$\omega_{a,bc}(\tau,e) = \frac{1}{2}e_{bc,a} - e_{a[b,c]}$$
$$\omega_{A,ab}(\tau,e) = -e_{A[a,b]}$$

Conformal extension

- **A conformal extension of the Aristotelian structure group is**

$$\mathfrak{so}(2, p+1) \oplus \mathfrak{so}(1, D-p)$$

longitudinal translations P_A longitudinal Lorentz transformations M_{AB}
longitudinal special conformal transformations K_A longitudinal dilatation D_1

$$[M_{AB}, M_{CD}] = -4\eta_{[A[C}M_{D]B]} \qquad [P_A, M_{BC}] = -2\eta_{A[B}P_{C]}$$

$$[K_A, M_{BC}] = -2\eta_{A[B}K_{C]} \qquad [P_A, K_B] = 2\eta_{AB}D_1 + 2M_{AB}$$

$$[D_1, P_A] = -P_A \qquad [D_1, K_A] = K_A$$

transversal translations P_a transversal rotations J_{ab}
transversal special conformal transformations K_a transversal dilatation D_2

$$[J_{ab}, J_{cd}] = -4\delta_{[a[c}J_{d]b]} \qquad [P_a, J_{bc}] = -2\delta_{a[b}P_{c]}$$

$$[K_a, J_{bc}] = -2\delta_{a[b}K_{c]} \qquad [P_a, K_b] = 2\delta_{ab}D_2 + 2J_{ab}$$

$$[D_2, P_a] = -P_a \qquad [D_2, K_a] = K_a$$

Conformal extension

- **We will restrict the analysis to the following anisotropic sub algebra**

$$\{M_{AB}, J_{ab}, P_A, P_a, D \equiv zD_1 + D_2\}$$

$$[M_{AB}, M_{CD}] = -4\eta_{[A[C}M_{D]B]} \qquad [P_A, M_{BC}] = -2\eta_{A[B}P_{C]}$$

$$[J_{ab}, J_{cd}] = -4\delta_{[a[c}J_{d]b]} \qquad [P_a, J_{bc}] = -2\delta_{a[b}P_{c]}$$

$$[D, P_A] = -zP_A \qquad [D, P_a] = -P_a$$

- **Apart from the vielbeine and the spin connections, we introduce dilatation gauge field** b_μ

- **Gauge transformations**

$$\delta\tau_\mu{}^A = \lambda^A{}_B\tau_\mu{}^B + z\lambda_D\tau_\mu{}^A \qquad \delta e_\mu{}^a = \lambda^a{}_b e_\mu{}^b + \lambda_D e_\mu{}^a \qquad \delta b_\mu = \partial_\mu\lambda_D$$

$$\delta\omega_\mu{}^{AB} = \partial_\mu\lambda^{AB} + 2\lambda^{[A|C|}\omega_{\mu C}{}^{B]} \qquad \delta\omega_\mu{}^{ab} = \partial_\mu\lambda^{ab} + 2\lambda^{[a|c|}\omega_{\mu c}{}^{b]}$$

- **The torsion is modified as**

$$\mathcal{T}_{\mu\nu}{}^A = T_{\mu\nu}{}^A - 2z\, b_{[\mu}\tau_{\nu]}{}^A \qquad \mathcal{E}_{\mu\nu}{}^a = E_{\mu\nu}{}^a - 2b_{[\mu}e_{\nu]}{}^a$$

Conformal extension

- **Intrinsic torsion: no dependence on the spin connection or the dilatation gauge field components**

$$\mathcal{T}_{ab}{}^{A} \qquad \mathcal{T}_{a}{}^{\{A,B\}} \qquad \mathcal{E}_{AB}{}^{a} \qquad \tilde{\mathcal{E}}_{A}{}^{\{a,b\}}$$

- **Extrinsic torsion**

$$\mathcal{T}_{a}{}^{A}{}_{A} \qquad \mathcal{T}_{a}{}^{[A,B]} \qquad \mathcal{T}_{AB}{}^{C}$$

$$\mathcal{E}_{A}{}^{a}{}_{a} \qquad \mathcal{E}_{A}{}^{[a,b]} \qquad \mathcal{E}_{ab}{}^{c}$$

- **Solution:**

$$\omega_{A,BC}(\tau,e,b) = \omega_{A,BC}(\tau,e) - 2z\, b_{[B}\eta_{C]A}$$

$$\omega_{a,AB}(\tau,e,b) = \omega_{a,AB}(\tau,e)$$

$$\omega_{a,bc}(\tau,e,b) = \omega_{a,bc}(\tau,e) - 2\, b_{[b}\delta_{c]a}$$

$$\omega_{A,ab}(\tau,e,b) = \omega_{A,ab}(\tau,e)$$

- **Furthermore, from** $\mathcal{E}_{A}{}^{a}{}_{a} = 0 = \mathcal{T}_{a}{}^{A}{}_{A}$ **, we find**

$$b_a = \frac{1}{z(p+1)} T_a{}^{A}{}_{A} \qquad\qquad b_A =, \frac{1}{D-p-1} E_A{}^{a}{}_{a}$$

Matter coupled Aristotelian gravity

- **Example: Real scalar field** $\mathcal{S}_\rho = \frac{1}{2}\int d^D x \left(c_1\, \partial_A \rho \partial^A \rho + c_2\, \partial_a \rho \partial^a \rho\right)$

- **Global invariance under dilatations** $\delta\rho = w\lambda_D \rho \qquad z = 1 \qquad w = \frac{1}{2}(2-D)$

- **Local invariance: coupling to the conformal Aristotelian gauge fields**

$$D_A \rho = \tau_A{}^\mu \partial_\mu \rho - w\, b_A \rho \qquad D_a \rho = e_a{}^\mu \partial_\mu \rho - w\, b_a \rho$$

$$\mathcal{S}^c_\rho = \frac{1}{2}\int d^D x\, \Omega \Big(c_1\, D_A \rho D^A \rho + c_2\, D_a \rho D^a \rho\Big) \qquad \Omega = \det(\tau_\mu{}^A, e_\mu{}^a)$$

- **Gauge fixing** $\rho = 1$ **leads to a gravity action of electric type** [Henneaux, Salgado-Rebolledo (2021)]

$$\mathcal{S}_{\text{electric}-\text{AG}} = \frac{1}{2}\int d^D x\, \Omega \Big(c_1\, w^2\, b_A b^A + c_2\, w^2\, b_a b^a\Big)$$
$$= \int d^D x\, \Omega \Big(\alpha\, T_a{}^A{}_A T^{aB}{}_B + \beta\, E_A{}^a{}_a E^{Ab}{}_b\Big)$$

$$\beta = \frac{c_1 (D-2)^2}{8(D-p-1)^2} \qquad \alpha = \frac{c_2 (D-2)^2}{8(p+1)^2}$$

Matter coupled Aristotelian gravity

- **There are more Aristotelian gravity actions of this type that can be generated in a similar way**

$$\mathcal{S}_{\text{electric}-\text{AG}} = \int d^D x\, \Omega \Big(\alpha\, T_a{}^A{}_A T^{aB}{}_B + \beta\, E_A{}^a{}_a E^{Ab}{}_b \Big)$$

$$\mathcal{S}_{\text{magnetic}-\text{AG}} = \int d^D x\, \Omega \left(\alpha\, R_{AB}{}^{AB}(M) + \beta\, R_{ab}{}^{ab}(J) \right)$$

$$\mathcal{S}_{\text{el.mag.}-\text{AG}} = \int d^D x\, \Omega \left(\alpha\, E^A{}_a{}^a E_{Ab}{}^b + \beta\, R_{ab}{}^{ab}(J) \right)$$

$$\mathcal{S}_{\text{mag.el.}-\text{AG}} = \int d^D x\, \Omega \left(\alpha\, R_{AB}{}^{AB}(M) + \beta\, T^a{}_A{}^A T_{aB}{}^B \right)$$

Matter coupled Aristotelian gravity

- **Complex scalar field**

$$\mathcal{S}_\Phi = \frac{1}{2}\int d^D x \left(c_1\, \partial_A \Phi \partial^A \Phi^* + c_2\, \partial_a \Phi \partial^a \Phi^*\right)$$

$$\delta\Phi = \left(i\lambda_{U(1)} + w\lambda_D\right)\Phi$$

- **Coupling to conformal Aristotelian geometry**

$$\mathcal{S}^c_\Phi = \frac{1}{2}\int d^D x\, \Omega \left(c_1\, D_A \Phi D^A \Phi^* + c_2\, D_a \Phi D^a \Phi^*\right)$$

- **Gauge fixing the dilatations**

$$\mathrm{Re}(\Phi) = 1 \;\Rightarrow\; \Phi = e^{i\theta}$$

- **Aristotelian gravity coupled to the real scalar field** θ

$$\mathcal{S} = \mathcal{S}_{\mathrm{electric-AG}} + \mathcal{S}^c_\theta$$

$$\mathcal{S}^c_\theta = \frac{1}{2}\int d^D x\, \Omega \left(c_1\, \tau^{A\mu}\tau_A{}^\nu \partial_\mu\theta\, \partial_\nu\theta + c_2\, e^{a\mu} e_a{}^\nu \partial_\mu\theta\, \partial_\nu\theta\right)$$

- **Furthermore, we can introduce a U(1) gauge field** $A_\mu = (A_A, A_a)$ **and consider the action**

$$\mathcal{S} = \frac{1}{2}\int d^D x\, \Omega \Big[c_1 \left(D_A - iA_A\right) \Phi \left(D^A + iA^A\right) \Phi^* + c_2 \left(D_a - iA_a\right) \Phi \left(D^a + iA^a\right) \Phi^* \\ - |\Phi\Phi^*|^{\gamma/2} \left(\frac{b_0}{2} F_{AB} F^{AB} + b_1 F_{Aa} F^{Aa} + \frac{b_2}{2} F_{ab} F^{ab} \right) \Big] \qquad \gamma = \frac{4-D}{w}$$

- **Field strength components**

$$F_{AB} = \tau_A{}^\mu \tau_B{}^\nu F_{\mu\nu}, \qquad F_{Aa} = \tau_A{}^\mu e_a{}^\nu F_{\mu\nu} \qquad F_{ab} = e_a{}^\mu e_b{}^\nu F_{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- **Now we gauge fix both the U(1) and the dilatation symmetry**

$$\Phi = 1$$

- **This leads to Aristotelian gravity coupled to a massive vector field**

$$\mathcal{S} = \mathcal{S}_{\text{electric-AG}} + \mathcal{S}^c_{AE} + \frac{1}{2}\int d^D x\, \Omega \left(c_1\, A_A A^A + c_2\, A_a A^a \right)$$

$$\mathcal{S}^c_{AE} = -\frac{1}{2}\int d^D x\, \Omega \left(\frac{b_0}{2} F_{AB}\, F^{AB} + b_1\, F_{Aa}\, F^{Aa} + \frac{b_2}{2} F_{ab}\, F^{ab} \right)$$

Matter coupled Aristotelian gravity

- **Inspired by fracton field theory models we would like to generalize the previous construction to the case of the following type of action**

$$\mathcal{S}_{\Phi}^{HD} = \frac{1}{2}\int d^D x \left(c_1\, \partial_A \Phi \partial^A \Phi^* + c_2\, X_{ab} X^{ab*}\right) \qquad X_{ab} = \partial_a \Phi \partial_b \Phi - \Phi \partial_a \partial_b \Phi$$

- **Global symmetries** $\delta\Phi = \left(i\lambda_{U(1)} + i\lambda^a_{\rm dp}\, x_a + w\lambda_D\right)\Phi$

$$z = \frac{p-3-D}{p-3} \qquad w = \frac{p-3+D}{p-3}$$

- **Local U(1) symmetry requires to introduce two gauge fields,** A_A **and** A_{ab}**, transforming as**

$$\delta A_A = \partial_A \lambda_{U(1)} - z\lambda_D A_A, \qquad \delta A_{ab} = \partial_a \partial_b \lambda_{U(1)} - 2\lambda_D A_{ab}$$

- **Field strength**

$$F_{Aab} = \partial_A A_{ab} - \partial_a \partial_b A_A, \qquad F_{ab,c} = \partial_a A_{bc} - \partial_b A_{ac}$$

- **Generalized "scalar charge" theory**

$$\mathcal{S}_{\rm SC} = \frac{1}{2}\int d^D x \left(b_1\, F_{Abc}\, F^{Abc} + b_2\, F_{ab,c}\, F^{ab,c}\right)$$

Matter coupled Aristotelian gravity

- **Curved space generalization is problematic** [Slagle, Pretko (2016)]

- **It requires the addition of extra field content** [Bidussi, Hartong, Have, Musaeus, Prohazka (2021)],[Peña-Benitez (2021)], [Jain, Jensen (2021)], [Peña-Benitez, Salgado-Rebolledo (2023)]

- **In our case, we can use the imaginary part of** $\Phi = \rho\, e^{i\theta}$ **and consider the gauge invariant curvatures**

$$\tilde{F}_{Aab} \equiv D_A A_{ab} - D_{(a} D_{b)} A_A - D_A D_{(a} \partial_{b)} \theta + D_{(a} D_{b)} \partial_A \theta$$

$$\tilde{F}_{ab,c} \equiv D_a A_{bc} - D_b A_{ac} - D_a D_{(b} \partial_{c)} \theta + D_b D_{(a} \partial_{c)} \theta$$

- **The final action has the form**

$$\mathcal{S} = \frac{1}{2} \int d^D x\, \Omega \Big(c_1 \, \Big((D_A - i A_A)\, \Phi \, \big(D^A + i A^A \big)\, \Phi^* + c_2\, \tilde{X}_{ab} \tilde{X}^{ab*} \Big) + b_1\, |\Phi \Phi^*|^{\gamma_1/2}\, \tilde{F}_{Abc}\, \tilde{F}^{Abc} + b_2\, |\Phi \Phi^*|^{\gamma_2/2}\, \tilde{F}_{ab,c}\, \tilde{F}^{ab,c} \Big)$$

$$\tilde{X}_{ab} = D_a \Phi D_b \Phi - \Phi D_{(a} D_{b)} \Phi + i A_{ab} \Phi^2$$

$$\gamma_1 = 2 \frac{3p - 9 + D}{p - 3 + D} \qquad \gamma_2 = 2 \frac{3p - 9 - 2D}{p - 3 + D}$$

Matter coupled Aristotelian gravity

- **Gauge fixing**

$$\Phi = 1$$

- **Resulting system**

$$\mathcal{S} = \mathcal{S}_{\rm HDAG} + \mathcal{S}_{\rm Kin} + \mathcal{S}_{\rm Mass}$$

- **Higher derivative gravitational action**

$$\mathcal{S}_{\rm HDAG} = \frac{1}{2}\int d^D x\, \Omega\, \left(c_1\, w^2\, b_A b^A + c_2\, Y_{ab} Y^{ab}\right) \qquad Y_{ab} = w\left(\mathcal{D}_{(a} b_{b)} + 2 b_a b_b - \delta_{ab} b_c b^c\right)$$

$$b_a = \frac{1}{z(p+1)} T_a{}^A{}_A \qquad b_A =, \frac{1}{D-p-1} E_A{}^a{}_a$$

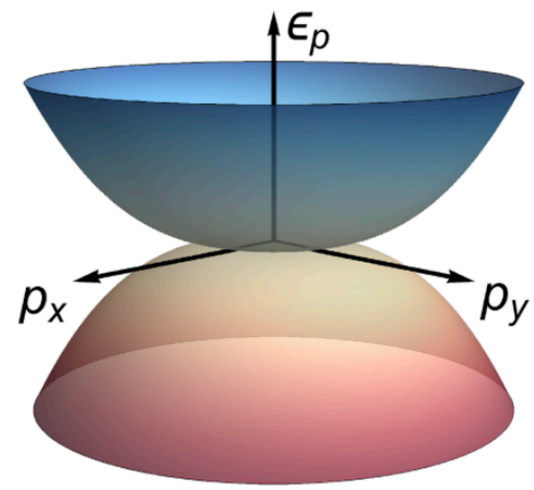
- **Massive gauge fields**

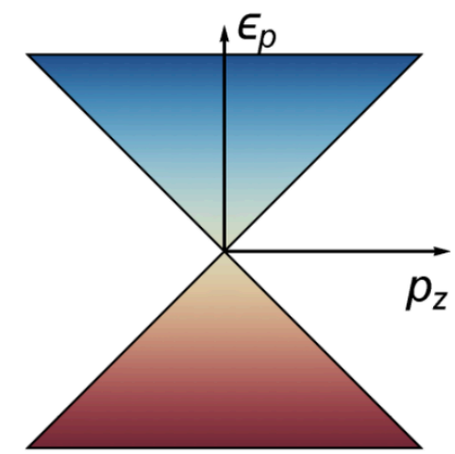
$$\mathcal{S}_{\rm Kin} = \frac{1}{2}\int d^D x\, \Omega\, \left(b_1\, F_{Abc}\, F^{Abc} + b_2\, F_{ab,c}\, F^{ab,c}\right)$$

$$\mathcal{S}_{\rm Mass} = \frac{1}{2}\int d^D x\, \Omega\, \left(c_1\, A_A A^A + c_2\, A_{ab} A^{ab}\right)$$

Aristotelian geometry for multi-Weyl semimetals

- **Multi-well semimetals: topological materials with anisotropic energy dispersion and higher-order topological charge at Weyl nodes**





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- **Higher order dispersion in certain directions**

$$\omega^2 \sim k_{\parallel}^2 + k_{\perp}^{2n}$$

- **Reduced spacetime symmetry**

$$(SO(1,1) \times SO(2)) \ltimes \mathbb{R}^4$$

- **This corresponds to an Aristotelian p-brane structure with** $p = 1$ **(string foliation)**

- **Casimir invariants** $E^2 = -\eta^{AB} p_A p_B$ $\qquad P^2 = \delta^{ab} p_a p_b$

- **Without loss of generality we can choose** $P_\mu = (-E, 0, 0, P)$

- **First law of thermodynamics** $dE = TdS + \alpha dP + \mu dQ - \mathcal{P} dV$

Aristotelian geometry for multi-Weyl semimetals

- **Effective action** [Dantas, Peña-Benitez, Roy, Surówka (2020)]

$$S = \int d^4x \, \mathrm{Re}\, \bar{\phi} \left[i\not{\partial}_{\parallel} + m^{1-n} (i\not{\partial}_{\perp})^n \right] \phi$$

$$\bar{\phi} = \phi^\dagger \gamma^0 \qquad \not{\partial}_{\parallel} = \gamma^0 \partial_0 + \gamma^1 \partial_1 \qquad \not{\partial}_{\perp} = \gamma^2 \partial_2 + \gamma^3 \partial_3 \qquad \gamma^\mu = (\sigma^1, \sigma^0, -i\sigma^3, -i\sigma^2)$$

- **Curved space generalization** $S = \int d^4x \, \Omega \, \mathrm{Re}\, \bar{\phi} \left[i\not{D}_{\parallel} + m^{1-n} (i\not{D}_{\perp})^n \right] \phi$

- **Covariant derivative** $D\phi = \left[d + iA + \frac{1}{4} (\epsilon_{AB} \omega^{AB} + i\, n\, \epsilon_{ab} \omega^{ab}) \sigma^3 \right] \phi$

$$\not{D}_{\parallel} \phi = \tau_A{}^\mu \gamma^A D_\mu \phi \qquad \not{D}_{\perp} = e_a{}^\mu \gamma^a D_\mu \phi$$

- **Energy-momentum, spin, angular, and U(1) currents**

$$\delta S = \int \left(\delta\tau^A \wedge \star \mathcal{T}_A + \delta e^a \wedge \star \mathcal{T}_a + \frac{1}{2} \delta\omega^{AB} \wedge \star \mathcal{S}_{AB} + \frac{1}{2} \delta\omega^{ab} \wedge \star \mathcal{J}_{ab} + \delta A \wedge \star J \right)$$

Aristotelian geometry for multi-Weyl semimetals

- **Generating functional = effective action for the hydrostatic regime**

$$\mathcal{Z}[\tau^A, e^a, \Omega^{AB}, \Omega^{ab}, A] = -\log \int D\bar{\phi} D\phi^{-S_E[\bar{\phi},\phi,\tau^A,e^a,\Omega^{AB},\Omega^{ab},A]}$$

- **Diffeomorphism and gauge invariance constrain the Ward identities and the gauge fields entering the effective action**

$$\delta_\chi \mathcal{Z} = -\int_{\mathcal{M}} \left(\xi^A \frac{\delta \mathcal{Z}}{\delta \xi^A} + \frac{1}{2}\lambda^{AB} \frac{\delta \mathcal{Z}}{\delta \lambda^{AB}} + \frac{1}{2}\lambda^{ab} \frac{\delta \mathcal{Z}}{\delta \lambda^{ab}} + \lambda^{(Q)} \frac{\delta \mathcal{Z}}{\delta \lambda^{(Q)}} \right) + \int_{\mathcal{M}} d \star \tilde{N} \,,$$

$$\tilde{N} = \mathcal{T}_A \xi^A + \frac{1}{2}\mathcal{S}_{AB}\lambda^{AB} + \frac{1}{2}\mathcal{J}_{ab}\lambda^{ab} + J\lambda^{(Q)}$$

- **Hydrostatic equilibrium: Killing parameters associated to steady regime**

$$\chi_K = (\xi^A_K, \xi^a_K, \lambda^{AB}_K, \lambda^{ab}_K, \lambda^{(Q)}_K) \qquad \mathcal{Z} = \int \mathcal{P}\left(T, \alpha, \mu^{AB}, \mu^{ab}, \mu^{(Q)}\right) dV$$

- **Inverse temperature and chemical potentials**

$$\beta^2 = -(\tau^A_\mu \tau_{A\nu})\, \xi^\mu_K, \xi^\nu_K \qquad \frac{\alpha^2}{T^2} = (e^a_\mu e_{a\nu})\, \xi^\mu_K, \xi^\nu_K$$

$$\mu^{AB} \equiv \lambda^{AB}_K \qquad \mu^{ab} \equiv \lambda^{ab}_K \qquad \mu^{(Q)} \equiv \lambda^{(Q)}_K$$

Outlook

- Conformal extensions of p-brane Aristotelian symmetries allow to construct Aristotelian gravity models
- By gauging the conformal Aristotelian symmetries in matter field theories leads to matter coupled Aristotelian gravity
- The method can be generalized to include higher-derivative models of fractonic type
- p-brane Aristotelian geometry for $p = 1$ is suitable for defining an effective description of multi-Weyl semimetals
- Future direction: to consider linearized version of Aristotelian gravity actions and higher-spin extensions in relation to the FQHE
- Future direction: study gravitational anomalies in multi-Weyl semimetals

Thank You!