Phase transitions and domain walls in holography-inspired hydrodynamics

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[Romuald Janik, MJ, Jacob Sonnenschein 2502.07879, 2106.02642]

[Romuald Janik, MJ, Hesam Soltanpanahi, Jacob Sonnenschein 2205.06274]

Outline

1. Introduction and motivation
2. Extended hydrodynamics: setup
3. Bubble wall velocity
4. Hot remnants in expanding plasma
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Phase transitions at strong coupling

First order transitions and domain walls are fundamental topics in physics

Examples in the context of strongly coupled field theory:

1. QCD
    - (Conjectured) phase transition at finite density probed in heavy-ion collisions
    - Phase transitions in neutron stars and neutron star mergers

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Examples in the context of strongly coupled field theory:

1. QCD
    - (Conjectured) phase transition at finite density probed in heavy-ion collisions
    - Phase transitions in neutron stars and neutron star mergers
2. Strongly coupled extensions of the standard model
    - (First order) phase transitions in the electroweak, hidden sectors, or at higher energies than the standard model
    - Could occur via bubble nucleation in the early universe

Basic setup: Domain walls

Static domain walls at $T \approx T_c$

- At a first order phase transition at $T = T_c$, we can have domains of coexisting phases separated by domain walls
- The pressures on both sides are balanced and the domain wall can be static

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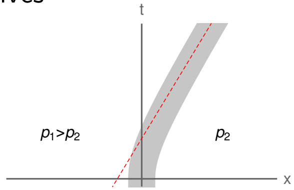
Going away from $T = T_c$, different dynamical situations:

- nucleated bubbles of a stable phase within an supercooled medium ($T < T_c$)
- bubbles of stable phase in overheated medium ($T > T_c$)
- at an interface between phases at different temperatures

Pressure difference over the domain wall drives its motion

- For strong interactions solving the surface tension, wall profile and velocity are hard questions $\Rightarrow$ use gauge/gravity duality?

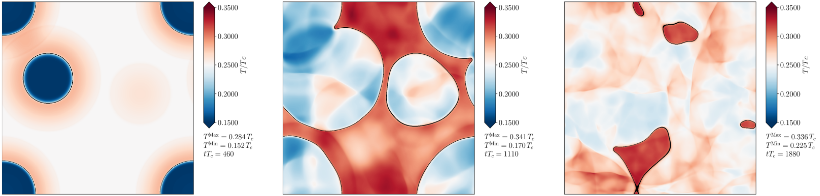


Phase transition through bubble nucleation

Cooling of matter with first order transition (e.g. early universe)

1. The high-T phase becomes supercooled
2. Bubbles of low-T phase form through tunneling

   [Coleman, Linde,. . .]
3. Bubbles expand, collide and coalesce, filling the whole space



[Figure: Cutting, Hindmarsh, Weir 1906.00480]

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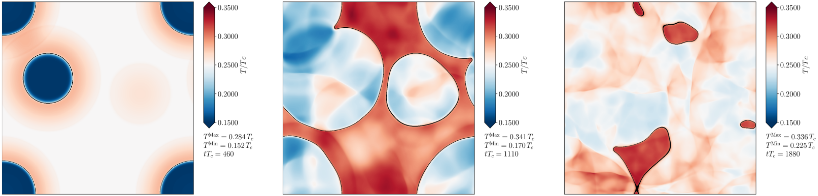
Gravitational waves from bubble collisions, sound waves, turbulence

- May be detectable by observatories (LISA, pulsar arrays, . . .)
- Studied using hydrodynamic simulations – additional insights from gauge/gravity duality?



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Holographic phase transitions

Gauge/gravity duality: study phase transitions and domain walls though numerical analysis in classical higher dimensional gravity

- Finding classical solutions in principle straightforward
- E.g., domain walls: interpolating solutions between two geometries (dual to different phases)

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- Holographic methods add one coordinate
- Solving Einstein equations numerically somewhat challenging
- Typically would like to study confinement-deconfinement transitions – it turns out to be particularly tricky
    - Confining and deconfining geometries have different structure
    - Leads to a "discontinuity" or topology change in the geometry

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Solutions

1. Try to solve the geometry nevertheless (hard)
   [Aharony, Minwalla, Wiseman hep-th/0507219
   Bantilan, Figueras, Mateos 2001.05476]
2. Study simpler (deconfining-deconfining) transitions in gravity
   [See e.g. Bellantuono et al. 1906.00061; Bea et al. 2202.10503]
3. Use effective theory methods, hydrodynamics (this talk)

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Generalized hydrodynamic model

Ingredients:

1. Perfect fluid hydrodynamics of (deconfined) plasma
2. Order parameter of the phase transition, γ

[Janik, MJ, Sonnenschein 2106.02642]

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Description of two phases – following the large N picture

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Key properties

- Hydrodynamics and γ coupled in a specific manner
- Directly defined in space-time coordinates (not holographic) – mostly 2+1 dimensions but also 3+1 possible
- However obtained by comparing to results for a holographic, i.e. higher-dimensional gravity model

[Janik, MJ, Sonnenschein 2106.02642]

Lagrangian for the hydrodynamic model

One can define the model through a Lagrangian

$$\mathcal{L} = \underbrace{(1-\Gamma(\gamma))p(T)}_{\text{deconfined}} + \underbrace{\Gamma(\gamma)}_{\text{confined}} \underbrace{-\frac{1}{2}a(\gamma)\left((\partial\gamma)^2 + V(\gamma, T)\right)}_{\text{domain wall}}$$

- $\Gamma = \gamma = 0$ in deconfined and $\Gamma = \gamma = 1$ in confined phase
- Equations of motion are

  $$\partial_\mu T^{\mu\nu} = 0\ , \qquad \partial^\mu(a\,\partial_\mu\gamma) = -\frac{\partial\mathcal{L}}{\partial\gamma}$$

  with $T_{\mu\nu}$ computed from $\mathcal{L}$

  [Haehl, Loganayagam, Rangamani 1502.00636]
- Coupling between γ and hydro from $(1-\Gamma(\gamma))p(T)$ (and $V(\gamma, T)$)
- Use the conformal equation of state, $p(T) = T^4$

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- Coupling between γ and hydro from $(1-\Gamma(\gamma))p(T)$ (and $V(\gamma, T)$)
- Use the conformal equation of state, $p(T) = T^4$
- $\Gamma(\gamma)$, $V(\gamma, T)$, and $a(\gamma)$ to be determined by comparing to full gravity solutions in a holographic model

Deconfinement transition in Witten's model

- An example of a holographic theory with a first order confinement/deconfinement phase transition: (a lower dimensional variant of) the Witten model [Witten hep-th/9803131; Brandhuber, Itzhaki, Sonnenschein, Yankielowicz hep-th/9803263]
- One compactifies a spatial coordinate (ϕ) on a circle

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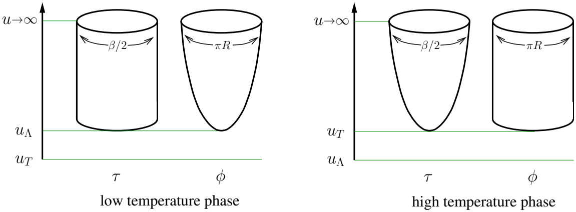
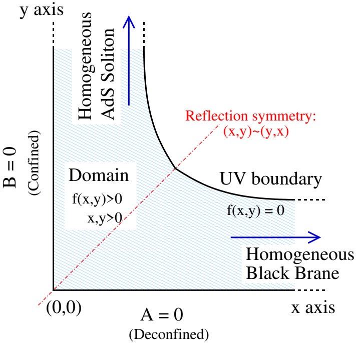
- One compactifies a spatial coordinate (ϕ) on a circle
- At low temperatures the bulk geometry of the ϕ circle closes off into a cigar → confinement (soliton geometry)
- At high temperatures, the geometry of the Euclidean time (τ) circle closes off into a cigar → deconfined phase (black brane)
- In between, there is a first order phase transition [Aharony, Sonnenschein, Yankielowicz hep-th/0604161]



The AMW solution

- AMW constructed numerically a static planar domain wall solution interpolating between confined and deconfined phases [Aharony, Minwalla, Wiseman hep-th/0507219]
- The numerical relativity setup is highly nontrivial due to the different topologies of the geometries in the different phases



- We use the solution with 4+1 bulk dimensions, dual to a 3+1 d boundary theory with one compactified coordinate

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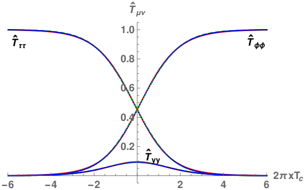
- Extract $T_{\mu\nu}$ from AMW solution using holographic distionary
- Fit it by using the hydrodynamic model

$$\mathcal{L} = (1 - \Gamma(\gamma))p(T) + \Gamma(\gamma) - \frac{1}{2}a(\gamma)\left((\partial\gamma)^2 + T^\beta q_*^2 \gamma^2 (1-\gamma)^2\right)$$

- We set $V(\gamma) \propto \gamma^2(1-\gamma)^2$ as required by the tanh profile of $T_{\tau\tau}$

$$\Gamma(\gamma) = \gamma^2(3 - 2\gamma)\,, \quad a(\gamma) = c$$



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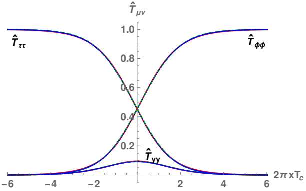

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$$\Gamma(\gamma) = \gamma\,, \quad a(\gamma) = \frac{c}{\gamma(1-\gamma)}$$

however leads to problematic singularities at $\gamma = 0$ and $\gamma = 1$

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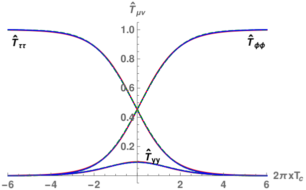

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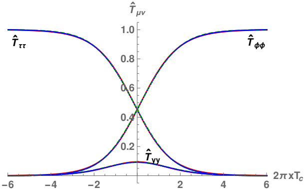

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Coupling of γ to hydrodynamics natural for $\Gamma = \gamma^2(3 - 2\gamma)$, and leads to asymmetric effective potential for γ when $T \neq T_c$

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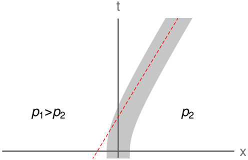
Moving wall at $T > T_c$

Evolve a domain wall at $T > T_c$: expect that the wall first accelerates and later propagates at constant velocity



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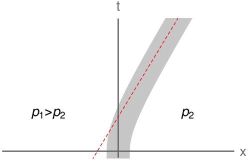

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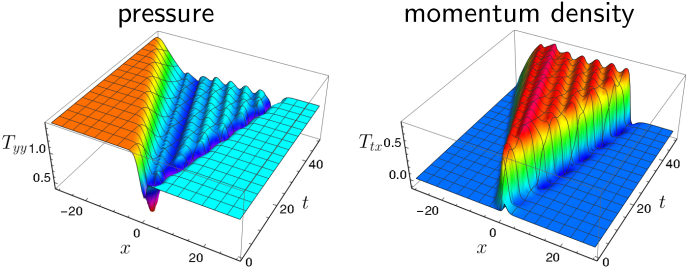
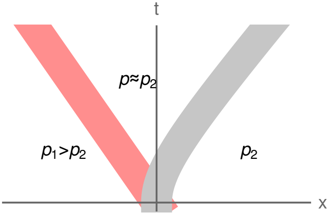
Simulation results in Witten's model:



- Looks rather different, what's going on?

Analysis of the results



- The domain wall moves to the right as expected, with constant speed
- A hydrodynamic wave moving to the left
- Pressure difference only over the hydro wave
- In the middle, plasma moving to the right following the wall

[Janik, MJ, Soltanpanahi, Sonnenschein 2205.06274]

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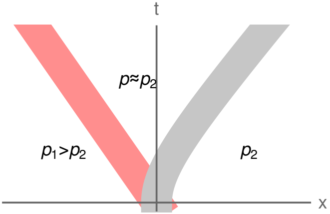

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- A hydrodynamic wave moving to the left
- Pressure difference only over the hydro wave
- In the middle, plasma moving to the right following the wall
- Wall velocity determined by integrating the EOS over the hydro wave $\Rightarrow$ a surprisingly simple formula:

$$\boxed{v_{\text{domain wall}} = \tanh \int_{p_2}^{p_1} \frac{dp}{(\epsilon + p)c_s} = \tanh \int_{T_2=T_c}^{T_1} \frac{dT}{Tc_s}}$$

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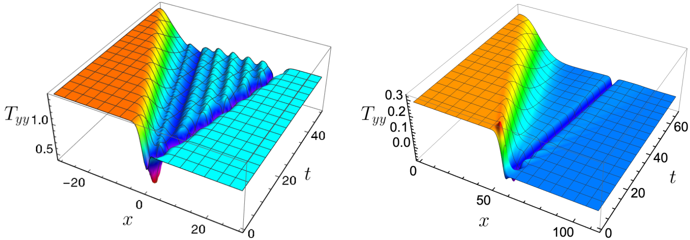
Simulations and the wall speed

Compare to full holographic gravity simulation (for deconfinement-deconfinement transition)

Hydro fitted to Witten's model Gravity simulation



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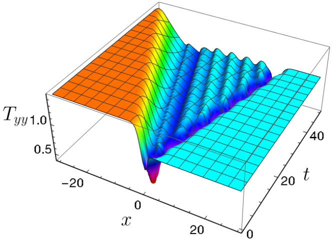
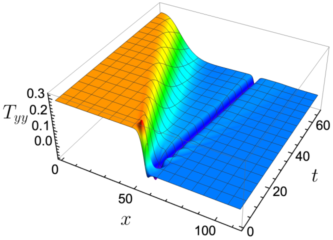
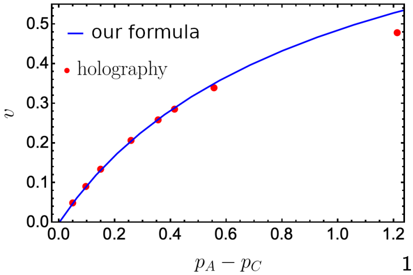


Gravity simulation



Wall velocities from gravity simulations compared to our formula

(N.B. formula also works for bubble nucleation at $T < T_c$)



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Remnants of deconfined matter

Studying expanding plasmas phenomenologically motivated

1. Bjorken expansion after a heavy-ion collisions
2. Matter in expanding (Friedmann–Robertson–Walker) universe

Focus on remnants of deconfined matter

- Last stage of a deconfinement transition in an expanding setup

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Specific large N feature in Yang-Mills: slow decay?

- Deconfined phase has $\epsilon \sim N_c^2$, confined phase $\epsilon \sim N_c^0 \approx 0$
- Consequently, by conservation of energy, decay of plasma balls $1/N_c$ suppressed! [Aharony, Minwalla, Wiseman hep-th/0507219]

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Our study:

- Expanding metric: $ds^2 = -d\tau^2 + \tau^2 dy^2 + dx^2$
- τ = proper time, y = pseudo-rapidity
- Expansion implies cooling — for conformal fluids, $T \propto \tau^{-1/3}$
- Use this in the extended hydro model
- Planar configurations: dependence on τ and x

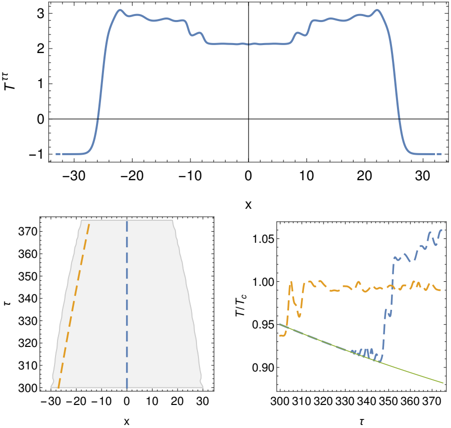
[Janik, MJ, Sonnenschein 2502.07879]

A blob of deconfined matter: early stage

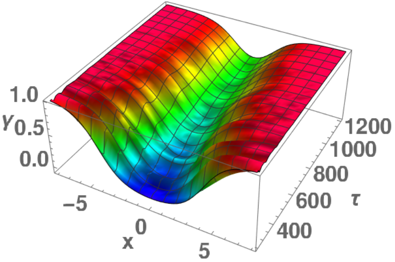
Simulation of a largish blob with $T < T_c$, starting at rest

[Janik, MJ, Sonnenschein 2502.07879]

- Near edges, similar to planar picture
- Hydro waves moving in before the domain wall
- In the middle, T decays due to expansion
- Behind hydro wave $T \approx T_c$ or above



Decay of the remnant



Simulation shows two stages:

1. Initial shrinking with $L \sim 1/\tau$
   - Consistent with energy conservation
   - Temperature $T \approx T_c$
2. Slow increase of γ from zero towards one with unchanged size
   - Starts when size of blob $\sim$ domain wall width

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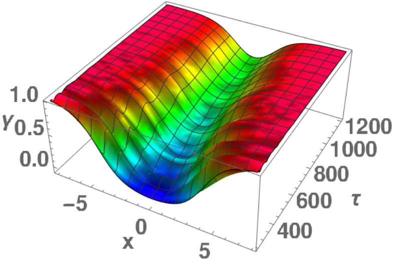

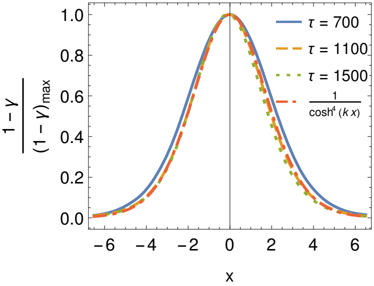

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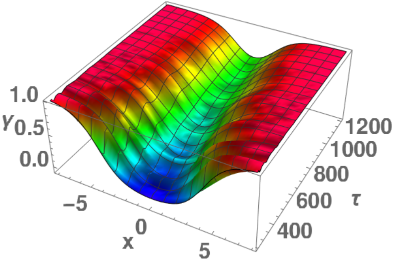
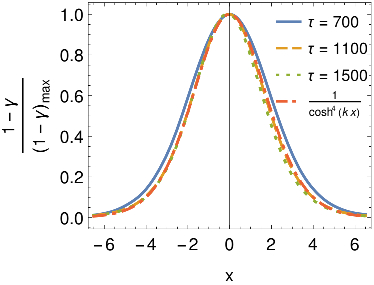
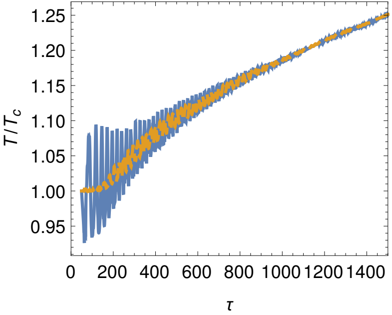


2. Slow increase of γ from zero towards one with unchanged size
    - Starts when size of blob $\sim$ domain wall width
    - Shape of γ profile freezes
    - Temperature increases??



Late time behavior: heating up

In the last stage, T starts to increase above T_c!



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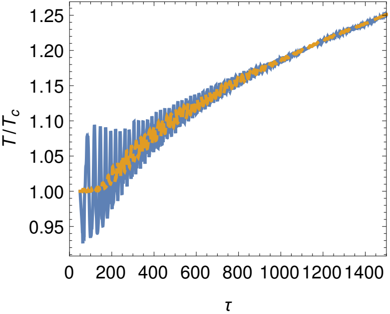
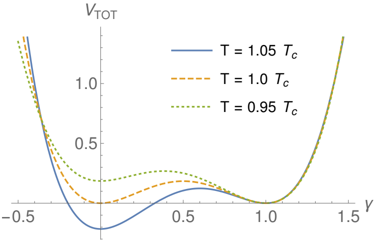
Can be understood from the virial theorem:

$$\frac{c}{2}(\gamma')^2 = V_{\text{TOT}}(\gamma, T)$$
$$\equiv V(\gamma, T) - (p(T) - 1)(1 - \Gamma(\gamma))$$

1. In the middle of the blob, $\gamma' = 0$ and the value $\equiv \gamma_*$ increases towards one
2. Temperature determined from
$$V_{\text{TOT}}(\gamma_*, T) = 0$$





Remnants in expanding universe: setup

We expect that our findings generalize to other expanding setups

A natural geometry to check: the FRW metric (taking $a(t) = \sqrt{t}$, radiation dominated)

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  change $\beta = 2 + 2\Gamma \to 4$
- Consider planar deconfining domains, depending on t and x

Remnants in expanding universe

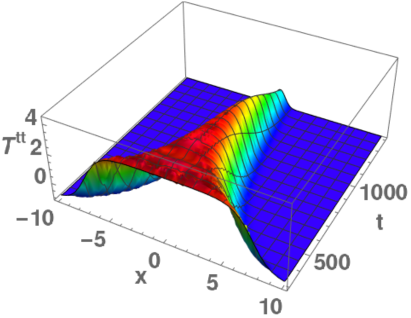
The same two-stage decay observed:

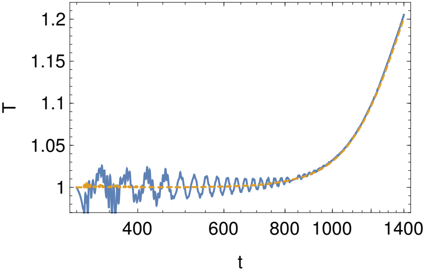
1. In the shrinking stage

   $L \sim 1/a(t)^3$

   as given by energy conservation
2. Consequent decay of gamma, and heating up, described by (modified) virial theorem:

   $$\frac{c}{2}(\gamma')^2 = a(t)^2 V_{\mathrm{TOT}}(\gamma, T)$$





Outline

1. Introduction and motivation
2. Extended hydrodynamics: setup
3. Bubble wall velocity
4. Hot remnants in expanding plasma
5. Conclusion

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An extended hydrodynamic setup: perfect fluid hydro coupled to the order parameter

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- Identified two stages: shrinking and decay
- Bubbles heat up in the last stage!
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Some future directions

- Remnants in medium with faster expansion
- Addition of glueball component – effect on hot remnants?
- More complicated setups, e.g. cylindrical and spherical blobs
- Eventually, bubble collisions and gravitational wave production

Thank you!

Different simplifications

Solving Einstein equations numerically hard, in particular time evolution. Simplifications:

1. Solve the effective potential V for the order parameter from holography – use the effective potential in field theory
   [Ares, Henriksson, Hindmarsh, Hoyos, Jokela 2109.13784, 2110.14442]

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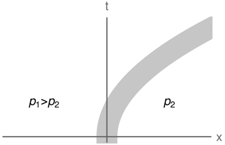
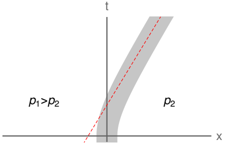
3. Derive a hydrodynamic theory from gravity in an expansion motivated by dynamics high-dimensional black holes
   [MJ, Weissman 2405.17533]

Moving domain walls

- The pressures on both sides are not balanced so one would (naively) expect accelerated motion. . .



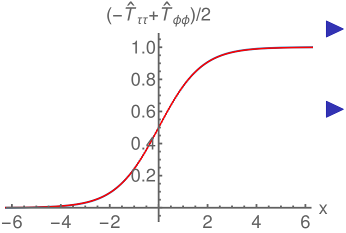
- . . . but analysis shows that eventually the acceleration stops and the domain wall approaches a constant velocity



- Common lore: friction balances the net force – challenging to calculate

Our main result: At strong coupling, the domain wall velocity can be understood in a simpler way using essentially only the equation of state

$T_{\mu\nu}$ of the AMW solution



- $-T_{\tau\tau} + T_{\phi\phi}$ from the numerical solution
- Extremely good fit by

$$-T_{\tau\tau} + T_{\phi\phi} \propto \frac{1}{2}\left(1 + \tanh\frac{q_* x}{2}\right)$$

maps to $1 - \Gamma(\gamma)$ in the model

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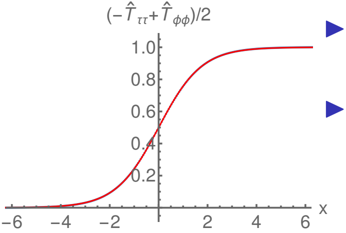

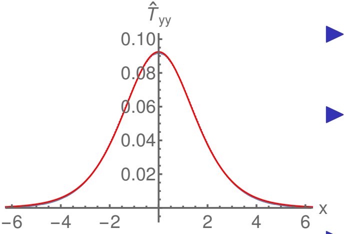

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- This maps to $\sim (\gamma')^2$

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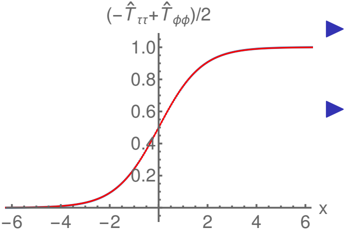

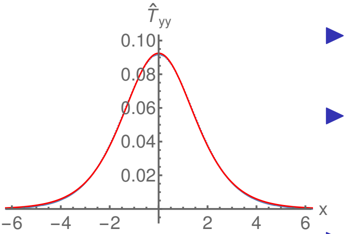

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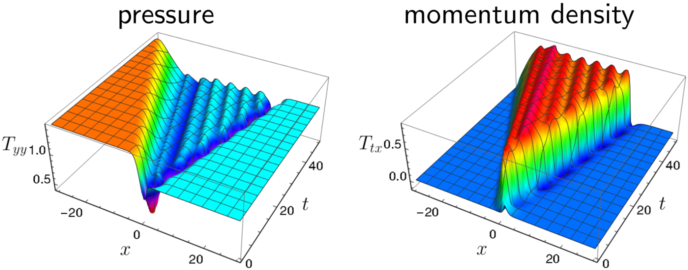
N.B. tanh fit also works well in nonconformal holographic models

[Attems, Bea, Casalderrey-Solana, Mateos, Zilhão 1905.12544]

What's going on?



pressure

momentum density

- The domain wall moves to the right as expected, with constant speed
- Nontrivial dynamics on the deconfined side (left)
- A hydrodynamic wave moving to the left
- Pressure difference over the hydro wave, no difference over the domain wall
- In the middle, plasma moving to the right following the wall

Hydrodynamic wave

- The hydrodynamic wave solution should interpolate between static plasma with $p_A > p_c$ and plasma with $p = p_c$ moving with the domain wall velocity
- Since the wave is deep in the deconfined phase, it is described in terms of perfect fluid hydrodynamics only
- Since $v_{\text{fluid}} = v_{\text{domain wall}}$, we may access $v_{\text{domain wall}}$ through a hydrodynamic computation in the deconfined phase!

Nonlinear hydrodynamic solution: “simple wave”

[Landau, Fluid mechanics]

- One assumes that all hydrodynamic quantities are functions of a single variable (e.g. pressure)
- One gets our main result

$$\boxed{v_{\text{domain wall}} = \tanh \int_{p_c}^{p_A} \frac{dp}{(\epsilon + p) c_s} \equiv \tanh \int_{T_c}^{T_A} \frac{dT}{T c_s}}$$

Hydrodynamic simulations in Witten’s model not precise enough to compare to this formula. We will instead use a different holographic model.

Generating a deconfined blob

Setup with longitudinal Bjorken expansion

$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx^2$

- τ = proper time, y = pseudo-rapidity
- Expansion implies cooling – for conformal fluids, $T \propto \tau^{-1/3}$
- Planar configurations: dependence on τ and x

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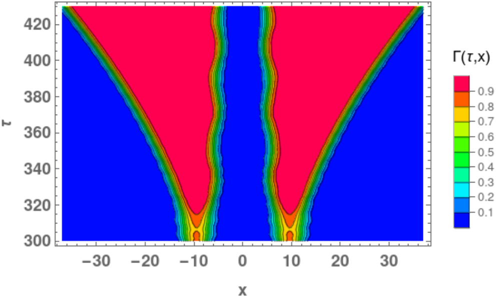
Creating a deconfined blob:



- Start at constant T deconfined phase
- Insert two Gaussian perturbations – model tunneling of bubbles
- Deconfined blob gets trapped between two confined domains
- Energy conservation prevents the decay of the blob

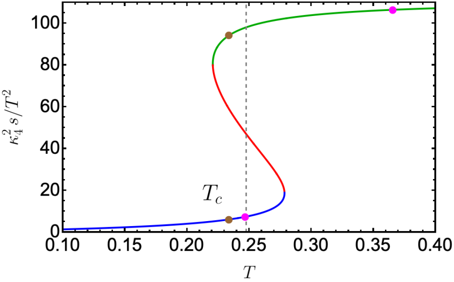
Nonconformal holographic model

- A gravity+scalar field system in D = 3 + 1 bulk dimensions

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\,(\partial\phi)^2 - V(\phi)\right]$$
$$V(\Phi) = -6\cosh\left(\frac{\Phi}{\sqrt{3}}\right) - 0.2\,\Phi^4$$

[Janik, Jankowski, Soltanpanahi 1704.05387;
Bellantuono, Janik, Jankowski, Soltanpanahi 1906.00061]

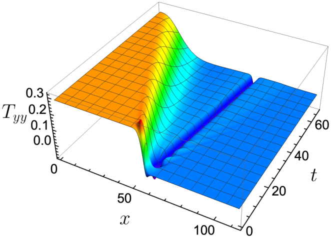
- Two deconfined phases separated by a first order transition

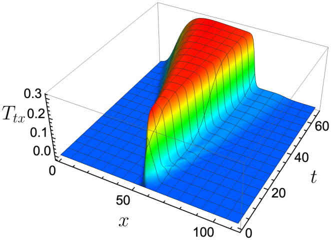


- Horizon in both phases, so much easier to setup a numerical relativity computation

Simulations in the holographic model

Quite similar results as in Witten's model:

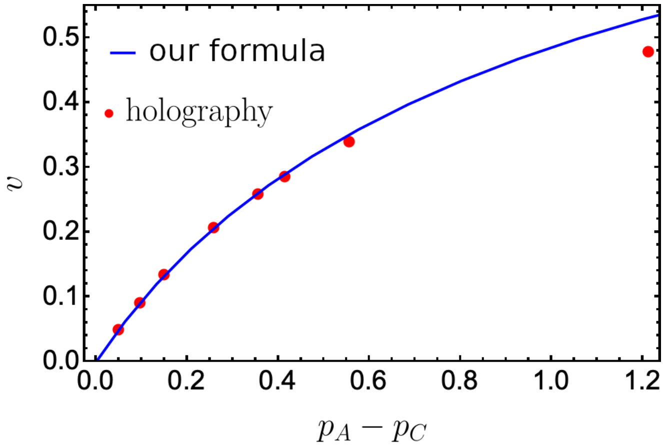




However, some important differences:

- Smooth profiles, due to finite dissipation
- Hydrodynamic evolution also in the low temperature side (instead of empty space)
- Different fluid velocities (not shown) on different sides of the domain wall
- One can show that the low-T phase velocity $v_L \approx v_{\text{domain wall}}$ $\Rightarrow$ apply our formula for the high-T phase

Velocity for the nonconformal model

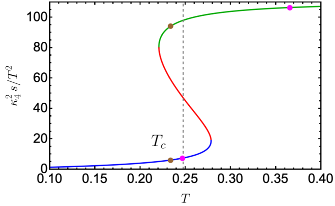


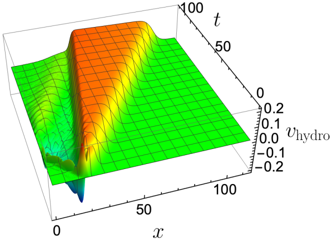
$$v_{\text{domain wall}} = \tanh \int_{p_c}^{p_A} \frac{dp}{(\epsilon + p)c_s}$$

Simulations for bubble nucleation

It is interesting to also study the expansion of small region of the stable low T phase (left) in the supercooled high T phase (right)

- Model for bubble nucleation in the early universe





- Hydrodynamic wave traveling in front of bubble wall (“deflagration”)
- Inside the bubble, fluid eventually at rest
- As earlier, pressure difference over the domain wall (not shown) ≈ 0

Decorating the velocity formula

For the bubble nucleation case, the fluid inside the bubble is at rest

- Use this to improve the velocity formula: we can now compute v_L (instead of just basically ignoring it)
- Skipping details: a correction factor

$$\boxed{v_{\text{domain wall}} = \frac{1}{1 - \frac{\epsilon_L + p_H}{\epsilon_H + p_L}} \tanh \int_{p_c}^{p_A} \frac{dp}{(\epsilon + p) c_s}}$$

Late time behavior: freezing of the profile

The shape of γ frozen at late times?

Use virial theorem with

$$V_{\mathrm{TOT}} \approx q_*^2(1-\gamma)^2\left(1-\sqrt{\frac{1-\gamma}{b(\tau)}}\right)$$

near $\gamma = 1$

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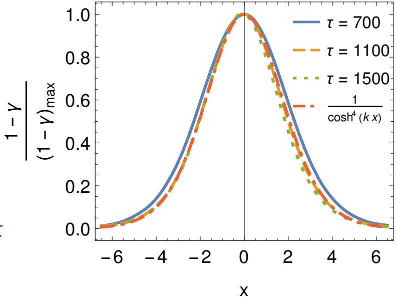
near $\gamma = 1$

- Solution:

$$1 - \gamma = \frac{b(\tau)}{\cosh^4(q_* x/4)}$$

- Fits the profile well!
- We find roughly $b(\tau) \sim 1/\sqrt{\tau}$



Comparison to the velocity formula