Magnetogenesis via plasma processes

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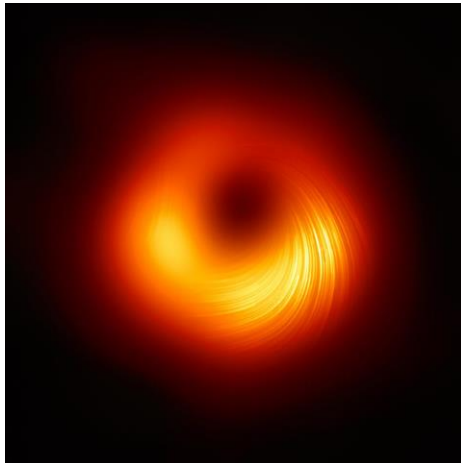
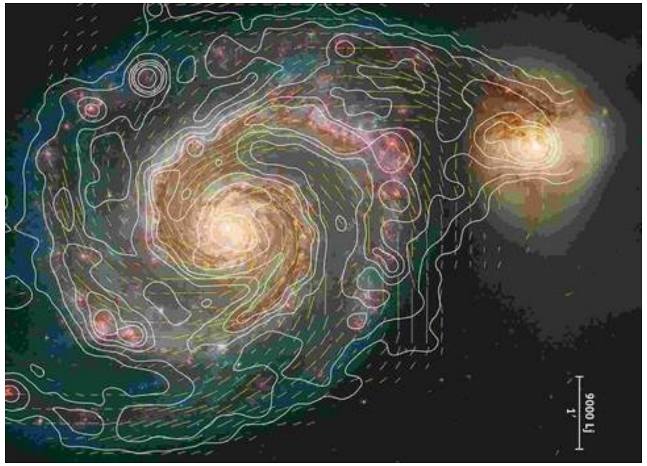
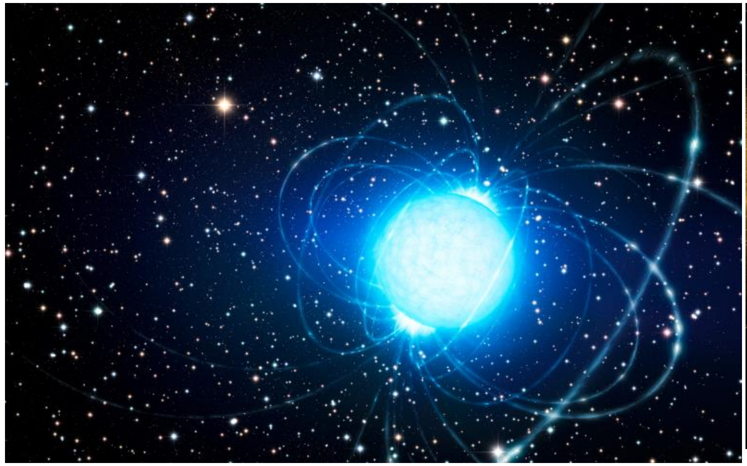
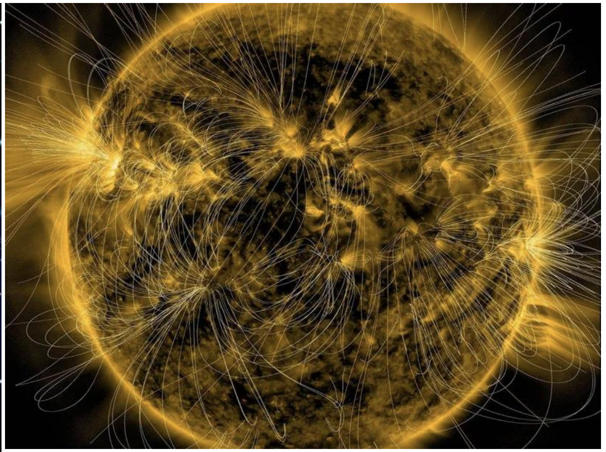
Magnetic fields in the Universe

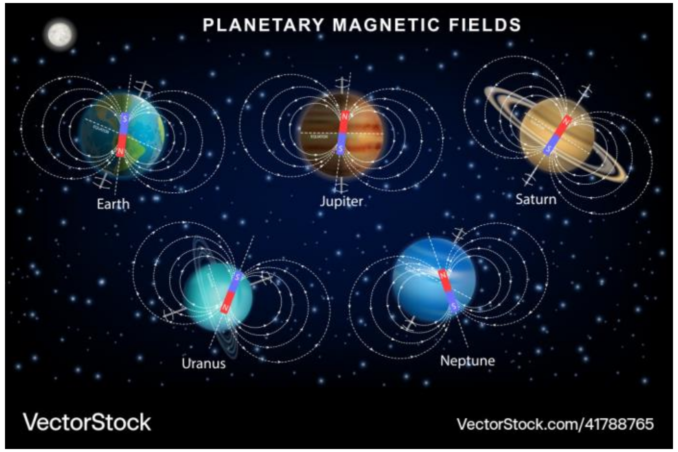
- Magnetic fields are everywhere in the Universe at all scales and strength


Black Holes


Galaxies


Neutron Stars


Stars



Planets

μG --- Interstellar Medium, Galaxies
G --- Black Holes, Stars, Planets
10^{10}G --- Neutron Star (Magnetars)

Standard cosmology: no magnetic fields were created at the big bang

How are cosmic/astrophysical magnetic fields generated?

MHD

- Magnetohydrodynamics (MHD) is the largest-scale description of magnetized plasmas

- One important equation:

$$\boldsymbol{E} + \boldsymbol{U} \times \boldsymbol{B} = 0$$

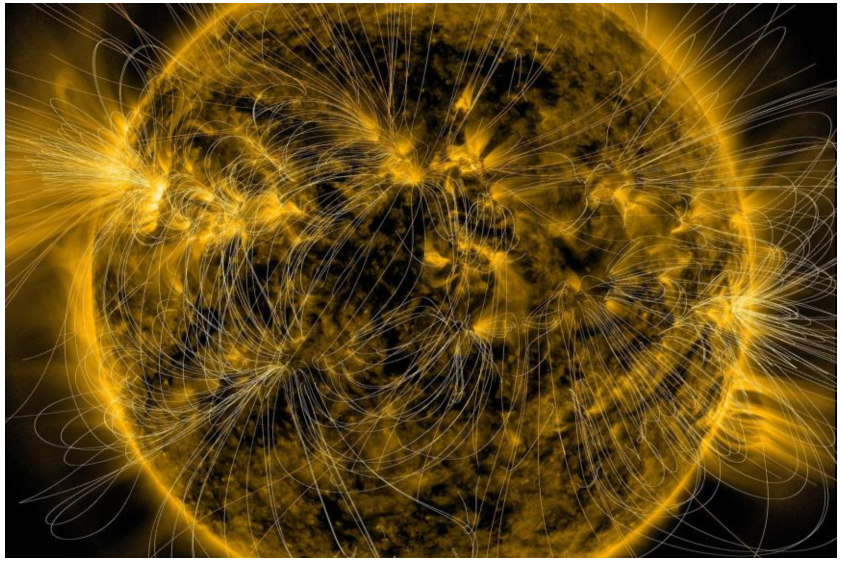
Ideal Ohm's law

- Take curl and use Faraday's law

=

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{U} \times \boldsymbol{B})$$

MHD induction equation

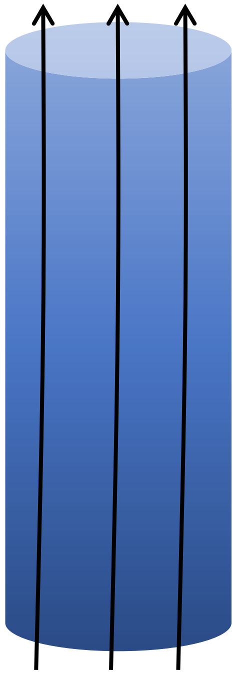


MHD

- So what is the conserved quantity in MHD?
- Magnetic Flux!

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{U} \times \boldsymbol{B}) \;\Rightarrow\; \frac{d}{dt}\int \boldsymbol{B} \cdot d\boldsymbol{S} = 0$$

- A plasma wants to keep its magnetic flux ⇔ Magnetic fields are “frozen into” the plasma



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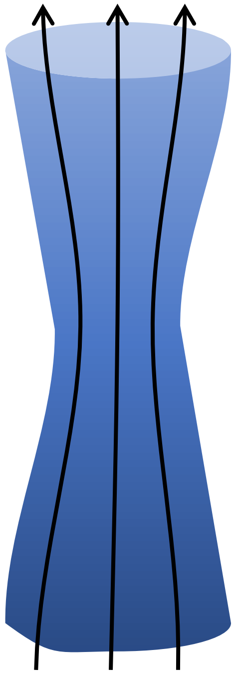

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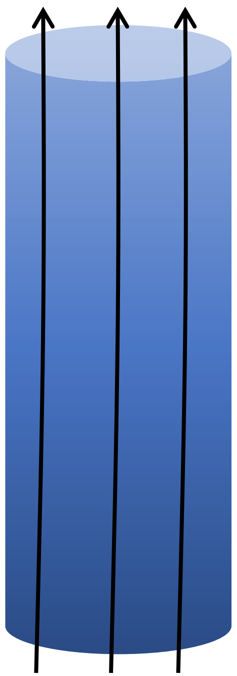

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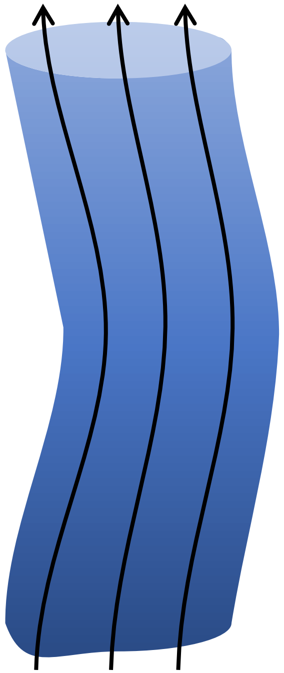


Plasma Dynamo

- Plasma induction equation is the underlying equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{U} \times \boldsymbol{B})$$

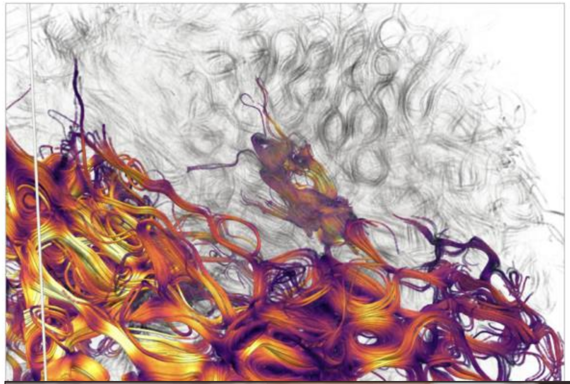
- If $\boldsymbol{B} = 0$ at $t = 0$,

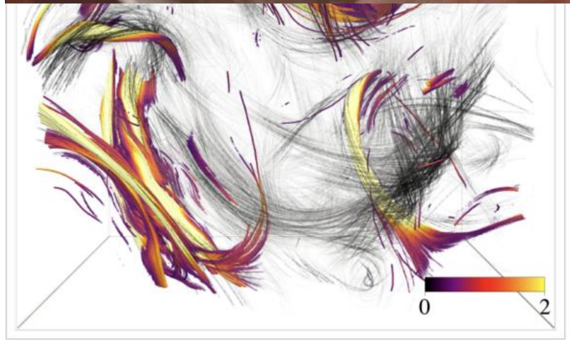
$$\frac{\partial \boldsymbol{B}}{\partial t} = 0$$

- We need a source for the "seed" magnetic field!

- Has to be due to a non-conservative electric field

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$



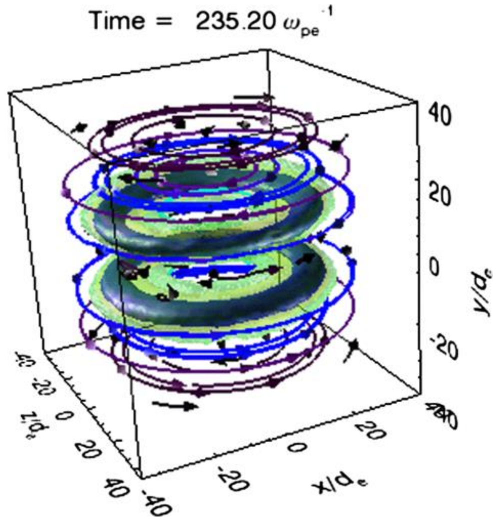


Zhou+, ApJ (2023)

Seed Magnetogenesis

- Two popular mechanisms for seed magnetogenesis
- Biermann battery [Biermann, 1950]
  - Misalignment of density and pressure

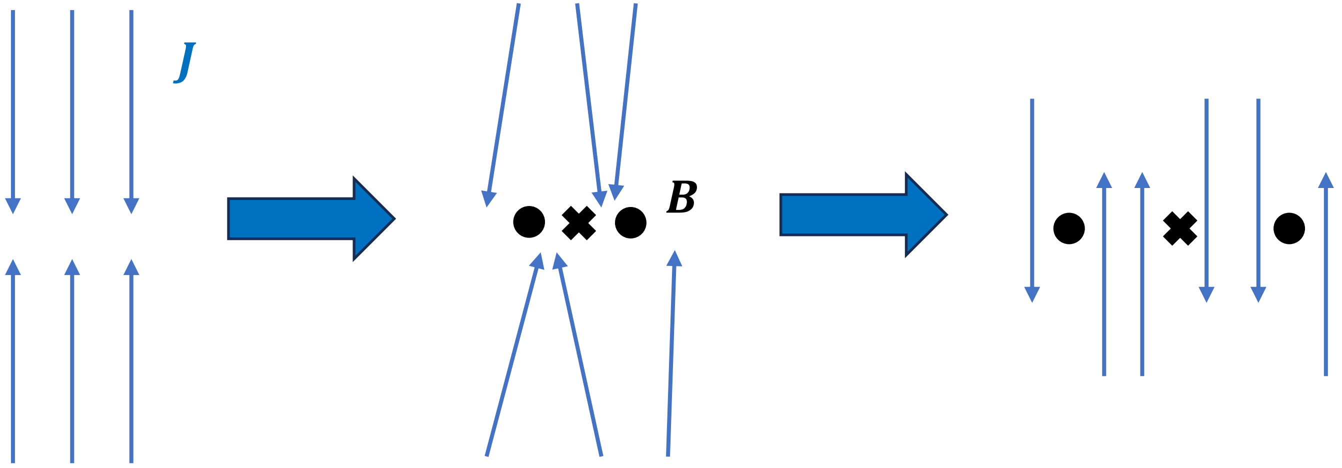
$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{\nabla n \times \nabla p}{en^2}$$

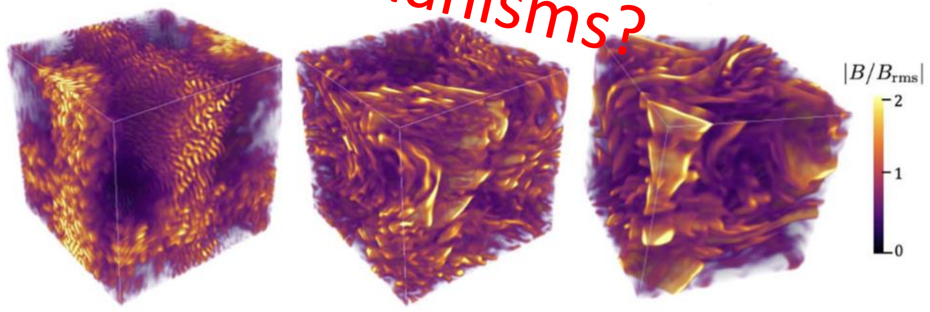


[Schoeffler+, PRL (2014)]

Can we generalize these mechanisms and find new mechanisms?

- Weibel Instability [Weibel, 1959]
  - Anisotropic pressure or, equivalently, counter-propagating beams





[Zhou+, ApJ (2023)]

$$m_\sigma \frac{d\mathbf{u}_\sigma}{dt} = q_\sigma \left(\mathbf{E} + \mathbf{u}_\sigma \times \mathbf{B} \right) - \frac{\nabla \cdot \overleftrightarrow{\mathbf{p}}_\sigma}{n_\sigma}$$

$$m_\sigma \frac{d\mathbf{u}_\sigma}{dt} = q_\sigma \left(\mathbf{E} + \mathbf{u}_\sigma \times \mathbf{B}\right) - \frac{\nabla \cdot \overleftrightarrow{\mathbf{p}}_\sigma}{n_\sigma}$$

Acceleration of plasma species σ

$$m_\sigma \frac{d\mathbf{u}_\sigma}{dt} = \underbrace{q_\sigma \left(\mathbf{E} + \mathbf{u}_\sigma \times \mathbf{B}\right)}_{\text{Lorentz force}} - \frac{\nabla \cdot \overleftrightarrow{\mathbf{p}}_\sigma}{n_\sigma}$$

$$m_\sigma \frac{d\mathbf{u}_\sigma}{dt} = q_\sigma \left(\mathbf{E} + \mathbf{u}_\sigma \times \mathbf{B}\right) - \frac{\nabla \cdot \overleftrightarrow{\mathbf{p}}_\sigma}{n_\sigma}$$

Pressure gradient

$$m\frac{d\mathbf{u}}{dt} = q\left(\mathbf{E} + \mathbf{u} \times \mathbf{B}\right) - \frac{\nabla \cdot \overleftrightarrow{\mathbf{p}}}{n}$$

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Vorticity $\mathbf{\Omega} = \nabla \times \mathbf{u}$

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Vector Calc Identity $\nabla \times [\mathbf{u}\cdot\nabla\mathbf{u}] = -\nabla \times [\mathbf{u}\times\nabla\times\mathbf{u}]$

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Faraday's law $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

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Faraday's law $\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$

$$m\left(\frac{\partial\mathbf{\Omega}}{\partial t}-\nabla\times[\mathbf{u}\times\mathbf{\Omega}]\right)=q\left(-\frac{\partial\mathbf{B}}{\partial t}\right.$$

$$m\nabla \times \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = q\nabla \times (\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \boldsymbol{\nabla} \times \left(\frac{\boldsymbol{\nabla} \cdot \overleftrightarrow{\mathbf{p}}}{n} \right)$$

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$$\frac{\partial}{\partial t}(m\boldsymbol{\Omega} + q\mathbf{B})$$

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$$\frac{\partial}{\partial t}\left(m\boldsymbol{\Omega} + q\mathbf{B}\right)$$

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Canonical Vorticity $\mathbf{Q} = m\boldsymbol{\Omega} + q\mathbf{B}$

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Canonical Vorticity $\mathbf{Q} = m\boldsymbol{\Omega} + q\mathbf{B}$

$$\frac{\partial \mathbf{Q}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{Q}) - \nabla \times \left(\frac{\nabla \cdot \overleftrightarrow{\mathbf{p}}}{n}\right)$$

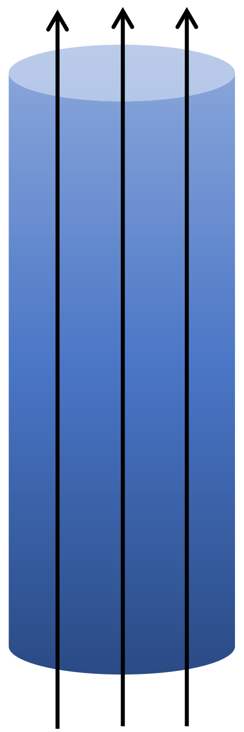
Magnetogenesis

- Canonical induction equation

$$\frac{\partial \boldsymbol{Q}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{Q}) - \nabla \times \left(\frac{\nabla \cdot \overleftrightarrow{\boldsymbol{p}}}{n} \right)$$

$\boldsymbol{Q} = 0$

Sole term responsible for magnetogenesis!



Vorticity-free
Magnetic-field-free

Pressure term

- Does our pressure term generalize both mechanisms?

$$\frac{\partial \boldsymbol{Q}}{\partial t} = -\nabla \times \left(\frac{\nabla \cdot \overleftrightarrow{\boldsymbol{p}}}{n} \right)$$

- Assume isotropic pressure, i.e., $\overleftrightarrow{\boldsymbol{p}} = p\overleftrightarrow{\boldsymbol{I}}$

Biermann Battery!

$$-\nabla \times \frac{\nabla \cdot \overleftrightarrow{\boldsymbol{p}}}{n} = -\nabla \times \frac{\nabla p}{n} = \boxed{\frac{\nabla n \times \nabla p}{n^2}}$$

- So our pressure term generalizes Biermann battery ✓
- Can it generalize Weibel instability as well?
- Assume 1D, i.e., $\nabla = \hat{x}\partial/\partial x$

$$-\left(\nabla \times \frac{\nabla \cdot \overleftrightarrow{\boldsymbol{p}}}{n} \right) \cdot \hat{z} = -\frac{1}{n}\frac{\partial^2 p_{xy}}{\partial x^2}$$

Diagonal　Off-diagonal

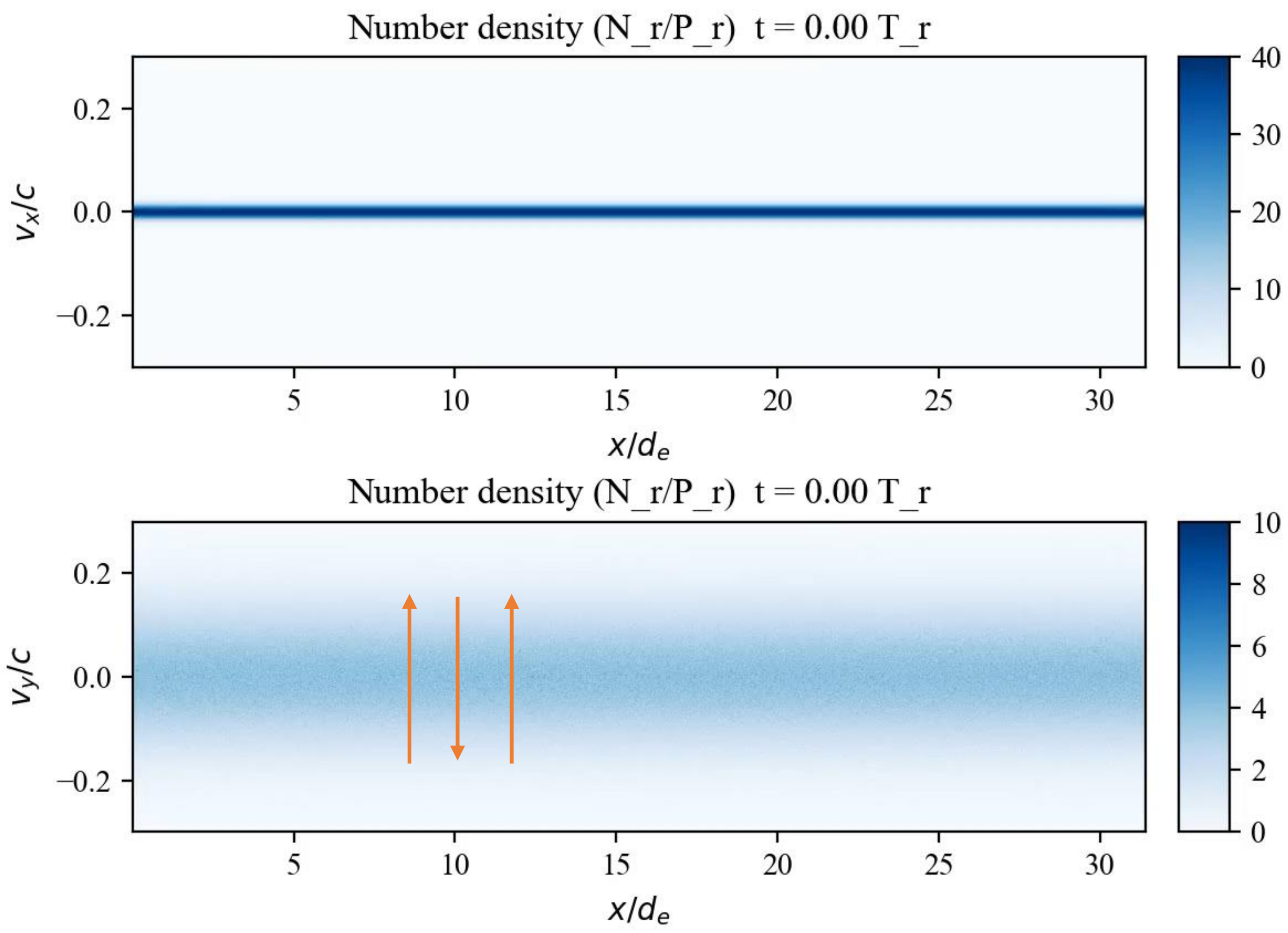
$$\begin{pmatrix} p_{xx} & \boxed{p_{xy}} & p_{xz} \\ p_{xy} & p_{yy} & p_{yz} \\ p_{xz} & p_{yz} & p_{zz} \end{pmatrix}$$

- Mixing between p_{xx} and p_{yy}

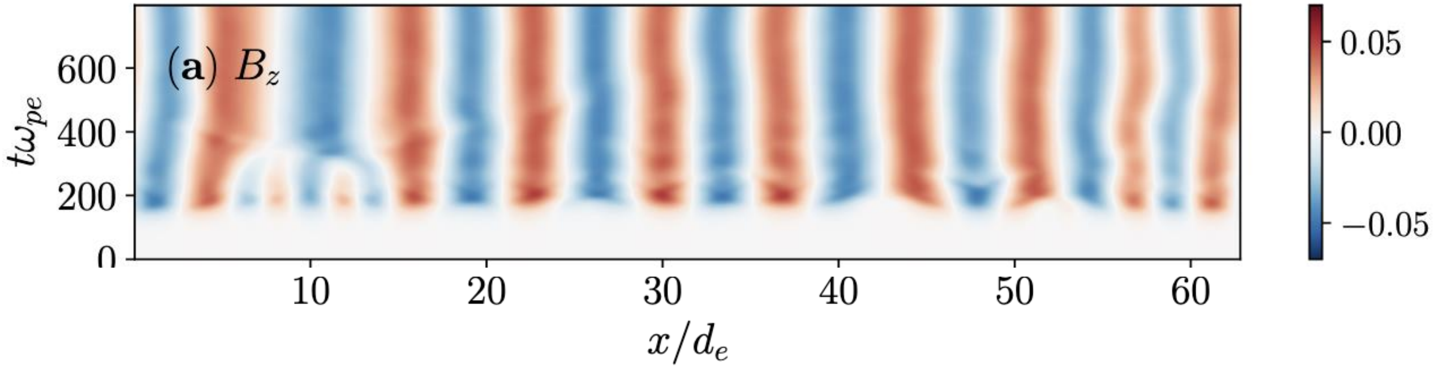
Weibel Instability

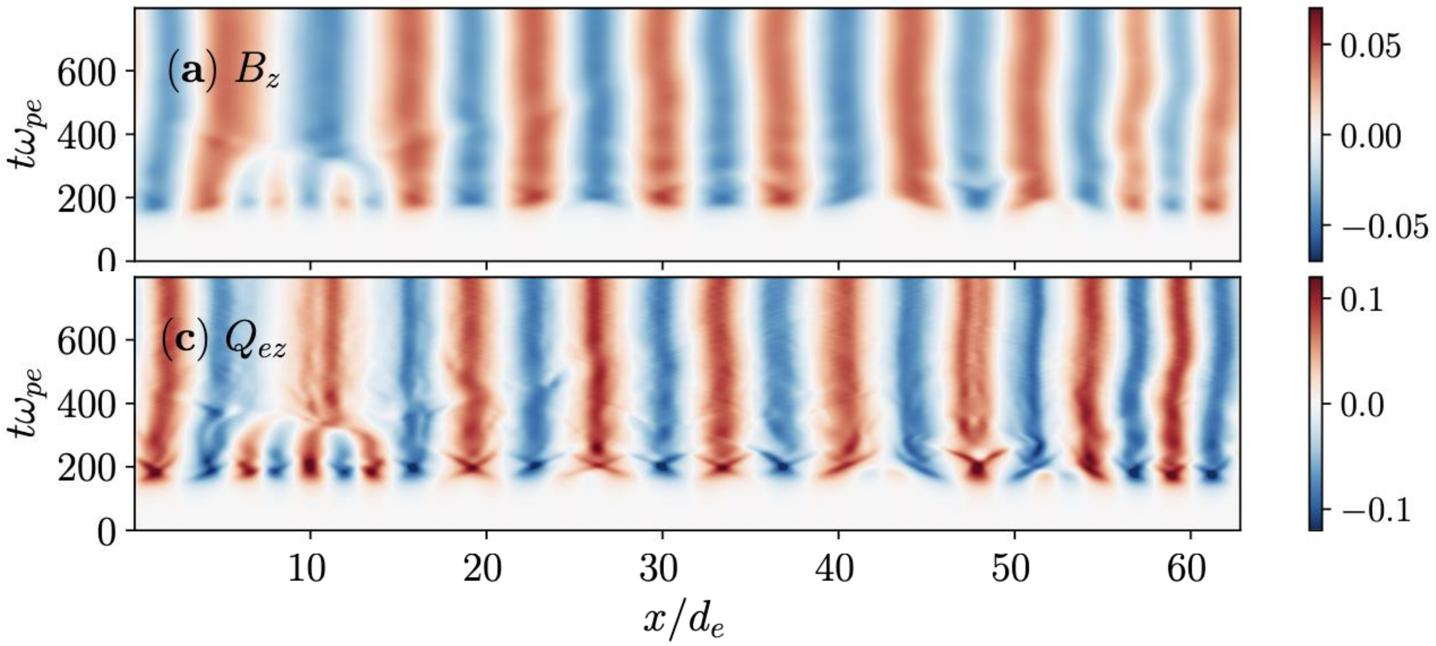
- Particle-in-cell simulations of the Weibel instability

Initially, $p_{yy} \gg p_{xx}$

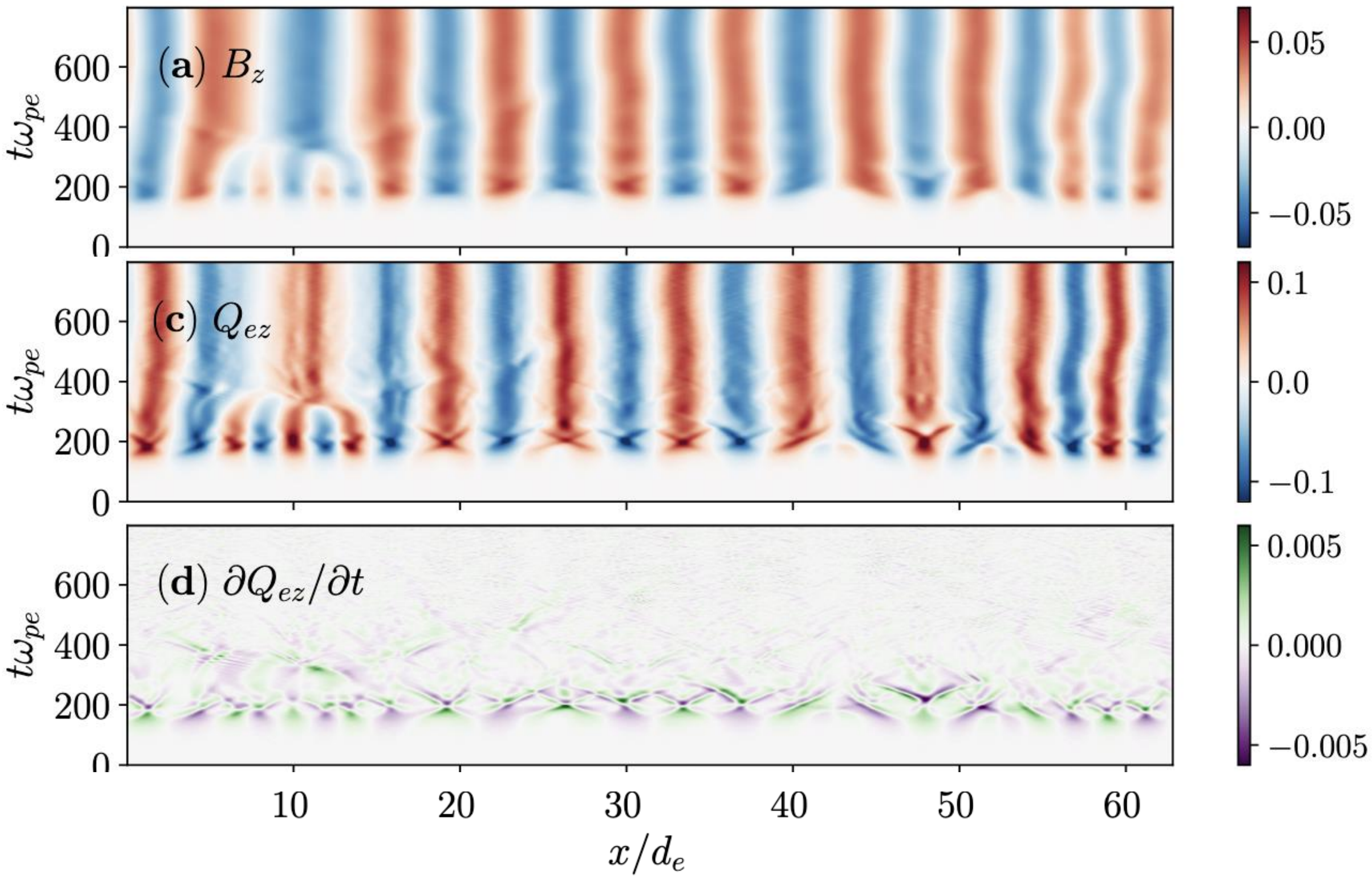


Weibel Instability

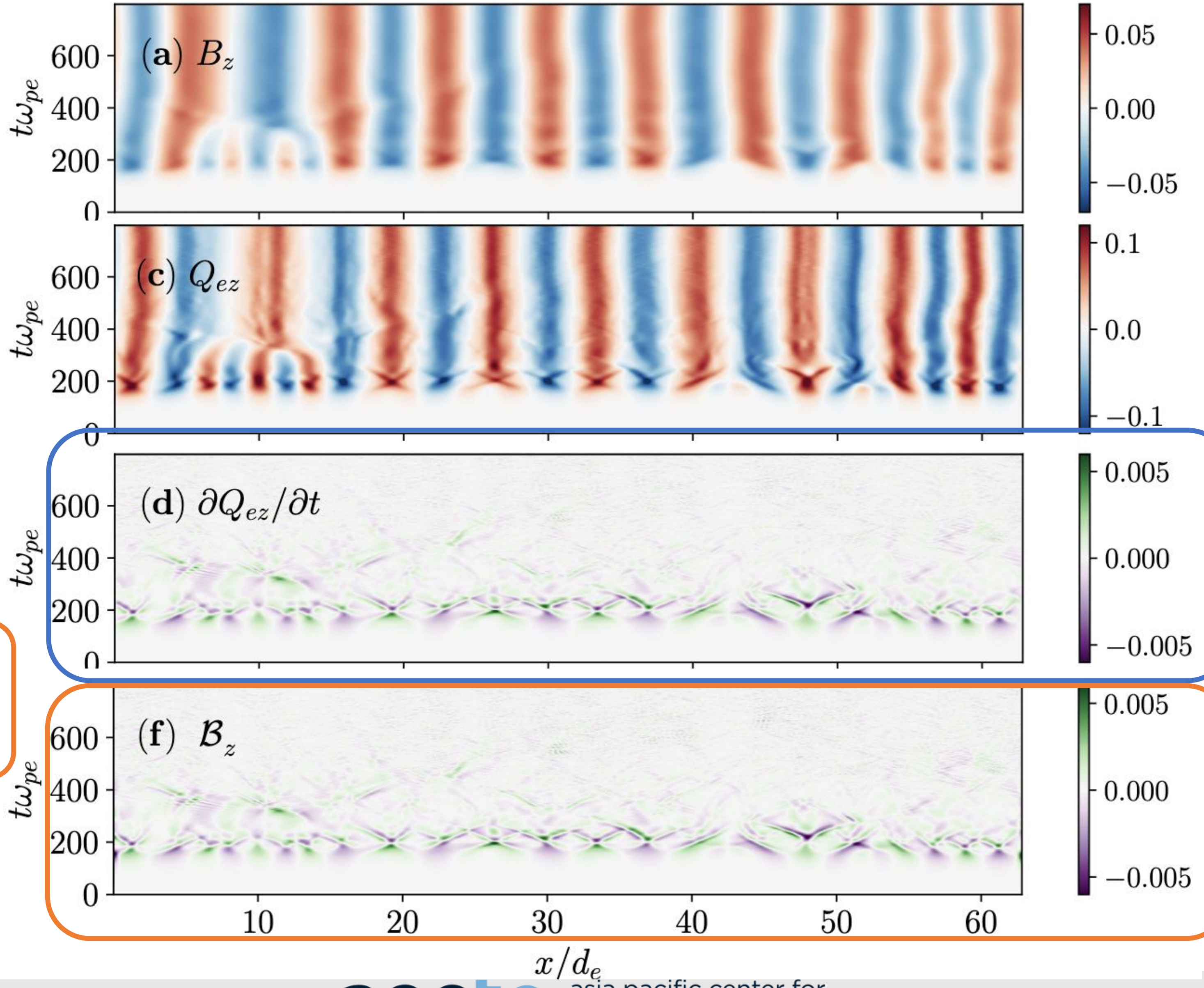



(a) B_z
(c) Q_{ez}
$t\omega_{pe}$
0
200
400
600
10
20
30
40
50
60
x/d_e
0.05
0.00
−0.05
0.1
0.0
−0.1

Weibel Instability



Weibel Instability



$$\frac{\partial \boldsymbol{Q}}{\partial t} = -\nabla \times \left(\frac{\nabla \cdot \overleftrightarrow{\boldsymbol{p}}}{n} \right)$$

Canonical battery is entirely responsible for magnetogenesis

New Mechanisms?

- Canonical battery generalizes Biermann battery ✓
- Canonical battery generalizes Weibel instability ✓
- Can there be other mechanisms?
- Assume 2D ($\frac{\partial}{\partial z} = 0$) and examine the z-component

$$\mathcal{B}_z = -\left(\nabla \times \frac{\nabla \cdot \boldsymbol{p}_e}{n_e}\right) \cdot \hat{z}$$

$$= \hat{z} \cdot \left[-\nabla\left(\frac{1}{n_e}\right) \times \nabla \cdot \boldsymbol{p}_e\right]$$

Biermann-like term

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$$= \hat{z} \cdot \left[-\nabla\left(\frac{1}{n_e}\right) \times \nabla \cdot \boldsymbol{p_e}\right] + \frac{1}{n_e}\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right) p_{exy}$$

Biermann-like term   Weibel term

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$$= \hat{z} \cdot \left[-\nabla\left(\frac{1}{n_e}\right) \times \nabla \cdot \boldsymbol{p}_e\right] + \frac{1}{n_e}\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right) p_{exy} + \frac{1}{n_e}\frac{\partial^2}{\partial x \partial y}\left(p_{exx} - p_{eyy}\right)$$

Biermann-like term — Weibel term — New term (2D-localized pressure anisotropy)

2D-localized Pressure Anisotropy

- Last term

$$\mathcal{B}_z = \frac{1}{n_e}\frac{\partial^2}{\partial x \partial y}\left(p_{exx} - p_{eyy}\right)$$

- Assume

$$p_{exx} - p_{eyy} \sim \exp\left(-\frac{x^2 + y^2}{\sigma^2}\right)$$

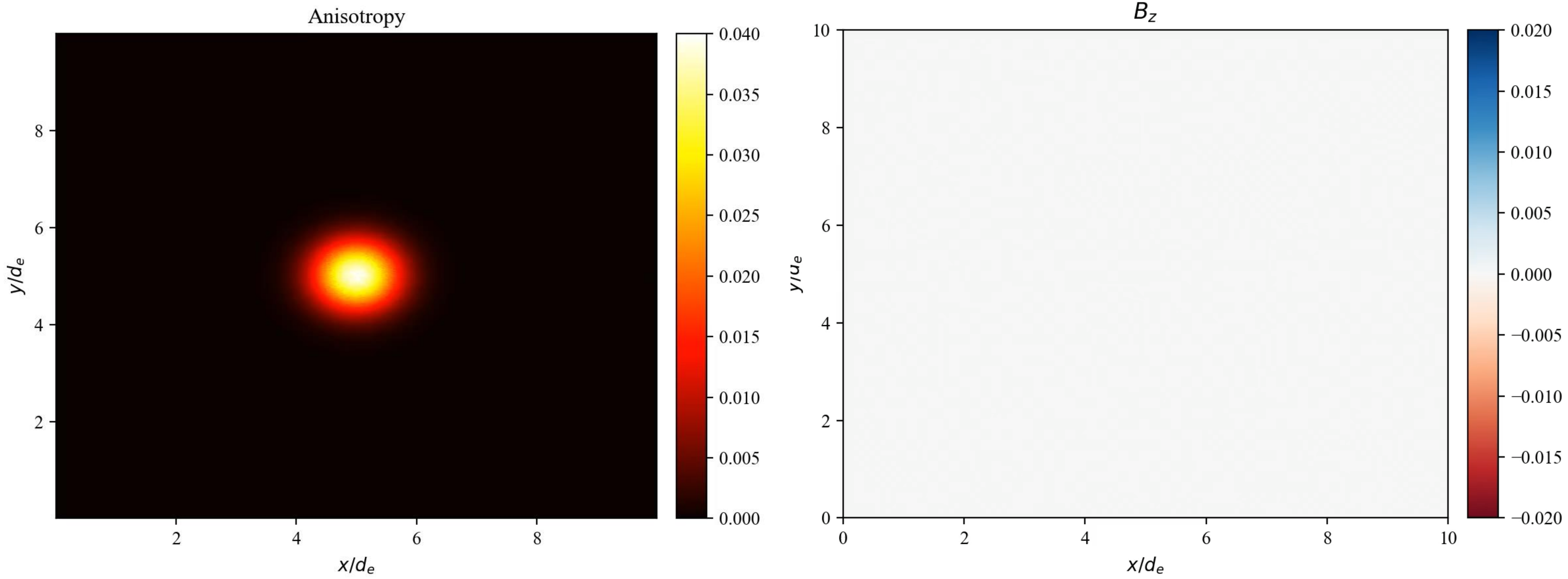
- Then

$$\mathcal{B}_z \sim xy$$

- Quadrupole magnetic fields expected

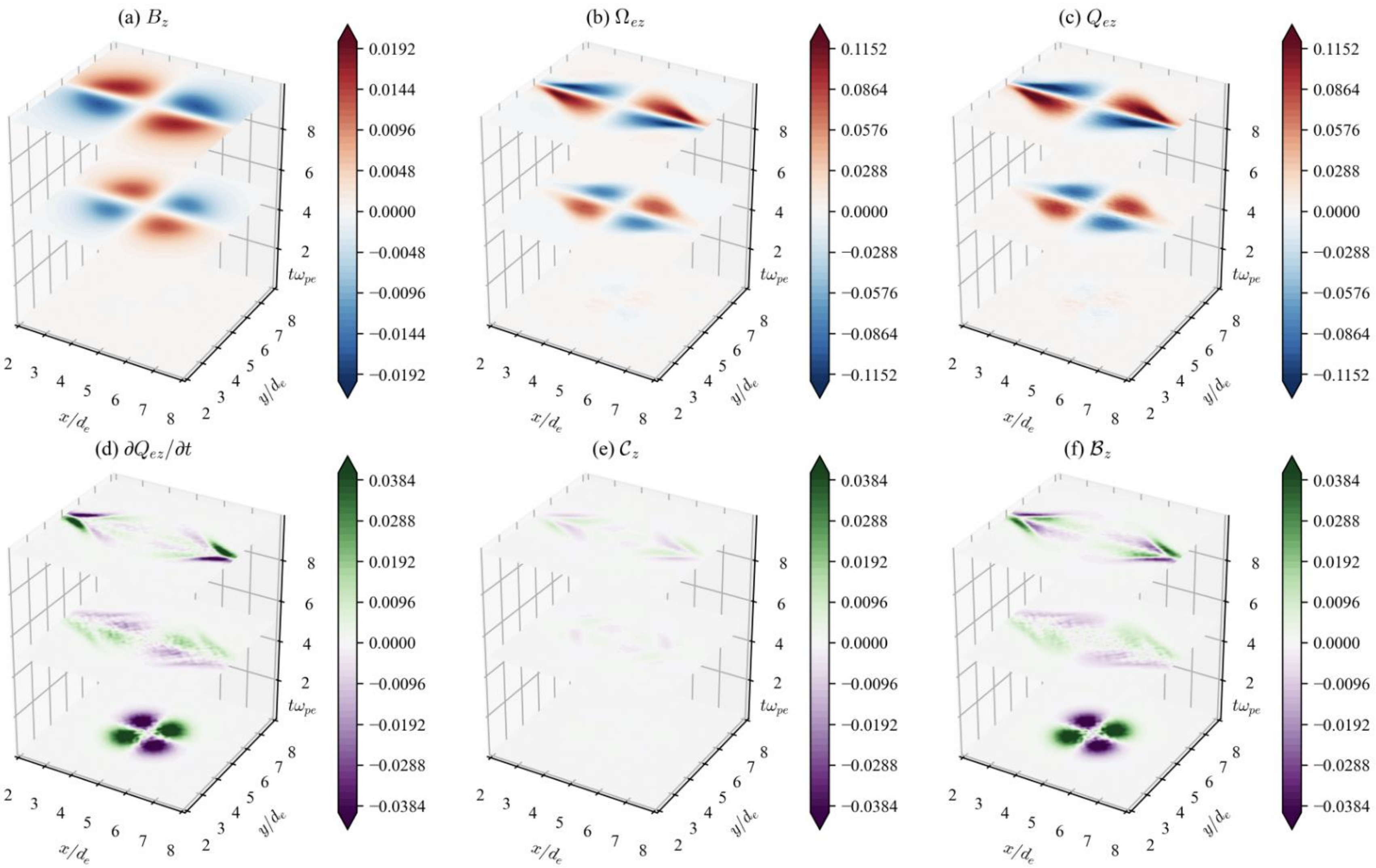
2D-localized Pressure Anisotropy

- Again, verify with particle-in-cell simulations



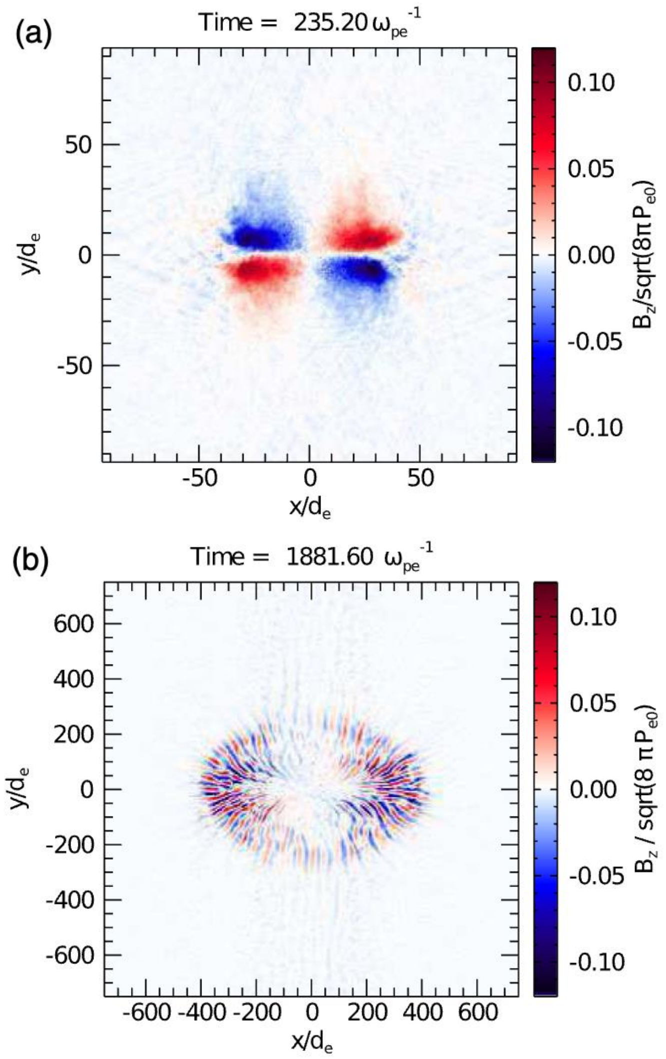
2D-localized Pressure Anisotropy

- Again, canonical battery term is confirmed to be the source!



Usage of the battery term

- Usage 1: Relative Importance among mechanisms



$\frac{L}{d_e} < 100$: Biermann

$\frac{L}{d_e} > 100$: Weibel

Biermann Weibel

$$\left|\frac{\nabla n_e \times \nabla T_e}{n_e}\right| : \frac{1}{n_e}\frac{\partial^2 p_{exy}}{\partial x^2},$$

$$\frac{T_{\rm diag}}{T_{\rm off}} d_e^2 : L^2,$$

Typically, $T_{diag} \gg T_{off}$, so Weibel is important for $L \gg d_e$

[Schoeffler+, PRL (2014)]

Usage of the battery term

- Usage 2: Predict other mechanisms

  - Localized density and pressure anisotropy

$$\mathcal{B}_z \sim \left(\frac{\partial}{\partial y}\frac{1}{n_e}\right)\left(\frac{\partial p_{exx}}{\partial x}\right)$$

  - Localized density and uniform temperature anisotropy

$$\mathcal{B}_z \sim T_{exx}\left(\frac{\partial}{\partial y}\frac{1}{n_e}\right)\left(\frac{\partial n_e}{\partial x}\right)$$

Relativistic Battery

- In the relativistic regime, the canonical induction equation includes a term purely due to relativity

$$\frac{\partial \boldsymbol{Q}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{Q}) - \nabla \times \left(\frac{\nabla \cdot \overleftrightarrow{\boldsymbol{p}}}{n}\right) + \sum_{i=x,y,z} [p_i, u_i] \quad \text{or } \epsilon_{ijk}\partial_i p_l \partial_j u_l \text{ or } \nabla p_l \times \nabla u_l$$

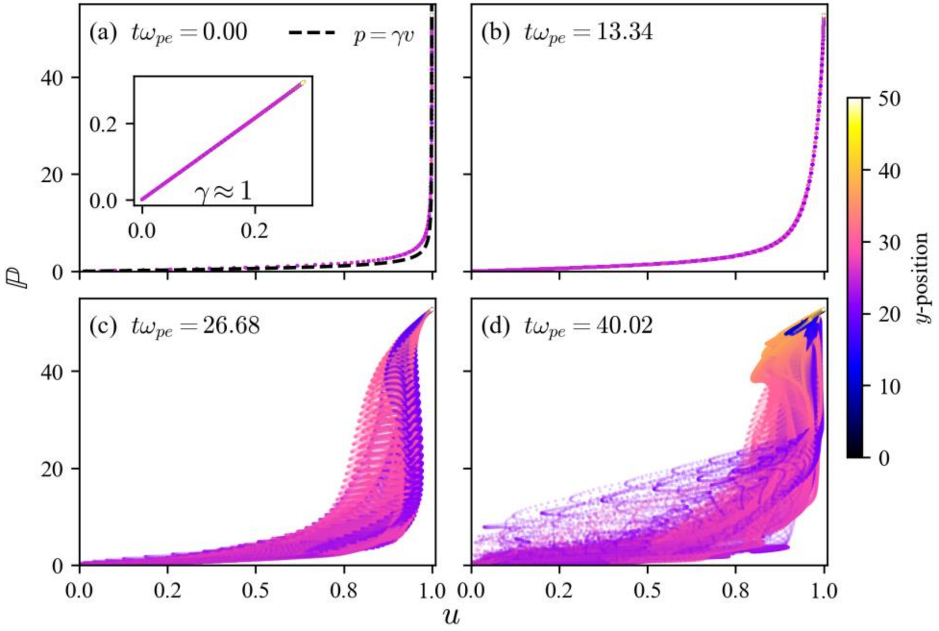
- For a single-particle, velocity and momentum commute

$$\boldsymbol{p} = \boldsymbol{p}(\boldsymbol{v}) = \frac{\boldsymbol{v}}{\sqrt{1 - v^2/c^2}}$$

- For a relativistic fluid, this is not generally the case

$$\langle \boldsymbol{p} \rangle \propto \int \boldsymbol{p} f\, d^3\boldsymbol{p}$$

$$\boldsymbol{u} \propto \int \frac{\boldsymbol{p}}{\gamma} f\, d^3\boldsymbol{p} = \int \frac{\boldsymbol{p}}{\sqrt{1 + p^2}} f\, d^3\boldsymbol{p}$$



[Laishram and Yoon, in prep.]

Other Batteries

- Other non-ideal effects give rise to other terms

$$\frac{\partial \boldsymbol{Q}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{Q}) - \nabla \times \left(\frac{\nabla \cdot \overleftrightarrow{\boldsymbol{p}}}{n} \right) + \hat{z} \sum_{i=x,y} [u_i, p_i] + \nu(\boldsymbol{\Omega} - \boldsymbol{\Omega}_\sigma) + QM + Rad$$

Relativity

Inter-species collisions

QED effects

Radiative effects

Summary

- The pressure tensor in the plasma equation of motion turns into the canonical battery term

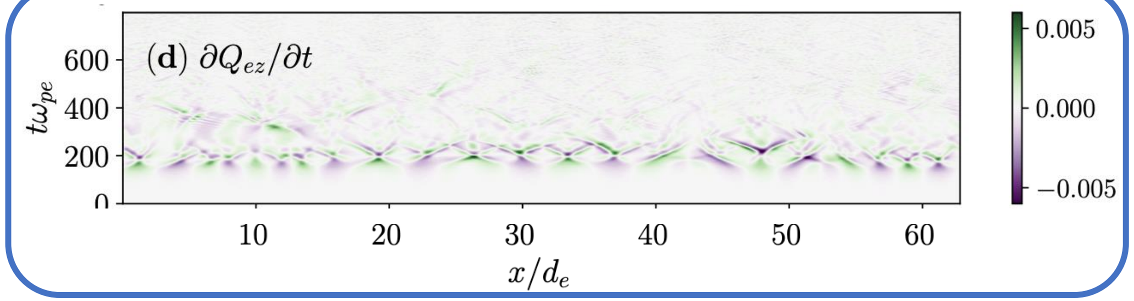
- This term is solely responsible for spontaneously generating canonical vorticity, which is a proxy for the magnetic field

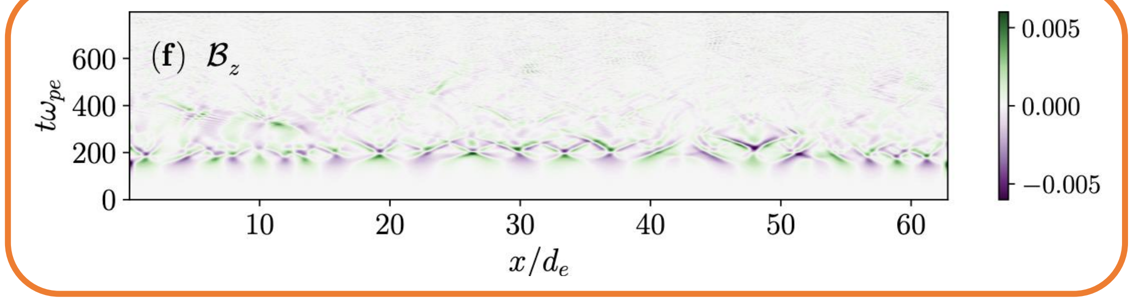
- This term generalizes popular magnetogenesis mechanisms and predicts new ones

- Other effects may provide additional sources

Weibel Instability

$$\frac{\partial \boldsymbol{Q}}{\partial t} = -\nabla \times \left(\frac{\nabla \cdot \overleftrightarrow{\boldsymbol{p}}}{n} \right)$$





Pressure term is entirely responsible for Weibel instability ☑

The canonical induction equation generalizes both 70-year-old magnetogenesis mechanisms (and predicts new ones)

[Laishram, Yun, Yoon, Phys. Rev. Res. (2024)]