Finite Landscape of 6d N=(1,0) Supergravity

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POSTECH

Based on **2411.19155** with Cumrun Vafa and Kai Xu (Harvard)

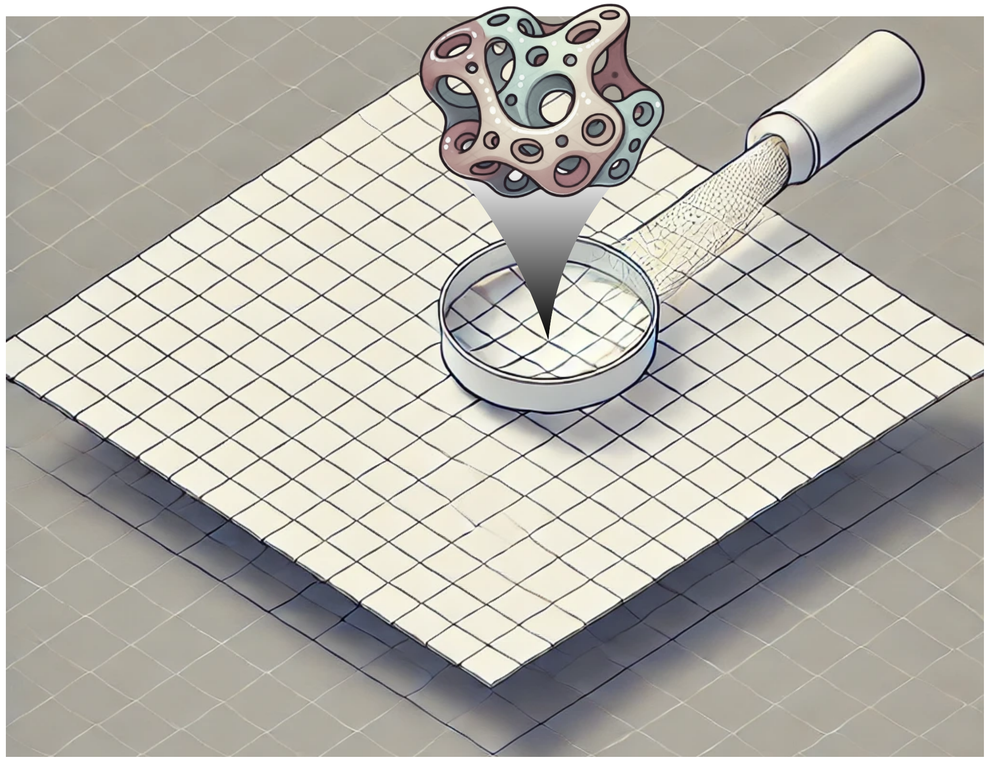
2nd APCTP-INPP Demokritos meeting, *Aug 25, APCTP, Pohang, Korea*

- # of known SUSY string vacua (in Minkowski space) is finite.
- Yau's conjecture : # of CY manifolds in each dim. is finite.
- It has been proven that the number of elliptic CY manifolds is finite.

[Gross '93], [Birkar, Di Cerbo, Svaldi '20]

String compactifications may provide only finite number of consistent quantum gravity (QG) theories.

e.g., # of 6d SUGRAs from F-theory is finite



Swampland conjectures

- Finiteness conjecture [Vafa '05], [Douglas '05], [Acharya, Douglas '06]
  - Set of consistent QG vacua (with fixed cutoff Λ) is finite.
  - Number of massless fields has upper bound.

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- String Lamppost Principle (or String Universality) [Kumar, Taylor '09], [Adams, De Wolfe, Taylor '10], [HCK, Tarazi, Vafa '19], [Montero, Vafa '20], ….

  All consistent quantum gravity theories arise from string theory.

QG Landscape = String Landscape

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  QG Landscape = String Landscape

- For 32 SUSY, consistency & SUSY fully determine the low-energy theories, which are realized as toroidal compactifications of M-theory.

  So, the conjectures are trivially satisfied.

16 Supercharges

- Massless multiplets (in every dim.) : **Gravity multiplet & Vector multiplet**

  Hence, low-energy SUGRA is characterized by the gauge group G.

- Bottom-up argument for the upper bound on the rank of gauge group:

$$\mathrm{rank}(G) \leq 26 - d$$

[HCK, Tarazi, Vafa '19]

Finiteness conjecture holds!

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- In10d, Green-Schwarz constraints fix gauge algebras to

$$E_8 \times E_8\,, \quad SO(32)\,, \quad \underbrace{E_8 \times U(1)^{248}\,, \quad U(1)^{496}}_{\text{No string realizations}}$$

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[Adams, De Wolfe, Taylor '10], [Shiu, HCK, Vafa '19]

String Lamppost Principle (SLP) holds!

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- SLP is also achieved for 16 SUSY theories in 9d, 8d (in terms of rank).

[de Boer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi '01], [Montero, Vafa '20], [Dierigl, Heckman '20], [Cvetic, Dierigle, Lin, Zhang '20], [Font, Fraiman, Grana, Nunez, Parra de Freitas '20, '21], [Hamada, Vafa '21], [Bedroya, Hamada, Montero, Vafa '21], [Fraiman, Parra de Freitas '21, '22], [Parra de Freitas, Montero '22], [Parra de Freitas '22], [Bedroya, Raman, Tarazi '23], …

Q) What about fewer SUSY and less dimensional cases?

This talk will focus on 6d (1,0) SUGRA with 8 SUSY
(in Minkowski space)

Recall that " # of 6d (1,0) SUGRA from F-theory is finite "

This talk

- **Bottom-up argument** to show that the number of massless fields in 6d (1,0) supergravities has upper bound, and the theories saturating the bound are realized in string theory, under two assumptions:

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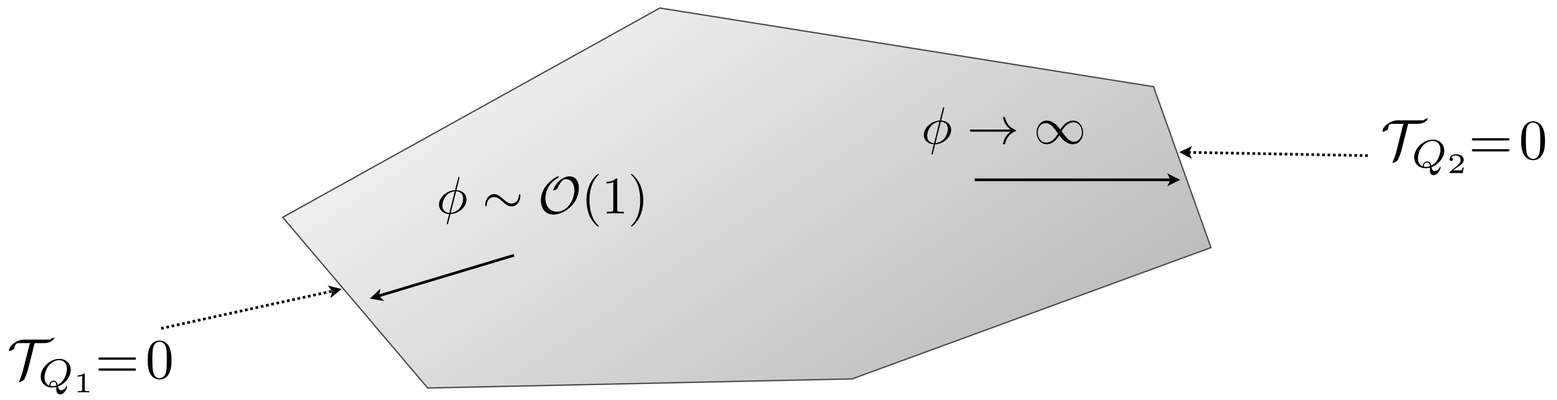
  1. Tensionless BPS strings exist at every point on the boundary of tensor moduli space.



This talk

- **Bottom-up argument** to show that the number of massless fields in 6d (1,0) supergravities has upper bound, and the theories saturating the bound are realized in string theory, under two assumptions:

2. Classification of 6d SCFTs and little string theories (LSTs) based on
   - Top-down : Brane construction and Geometric realization
   - Bottom-up : Gauge theory

   [Hanany, Zaffaroni '97], [Brunner, Karch '97], [Morrison, Taylor '12], [Heckman, Morrison, Vafa '13], [Del Zotto, Heckman, Tomasiello, Vafa '14], [Heckman, Morrison, Rudelius, Vafa '15], [Bhardwaj, Del Zottto, Heckman, Morrison, Rudelius, Vafa '15], [Bhardwaj '15, '19], ….

   is '*complete*'.

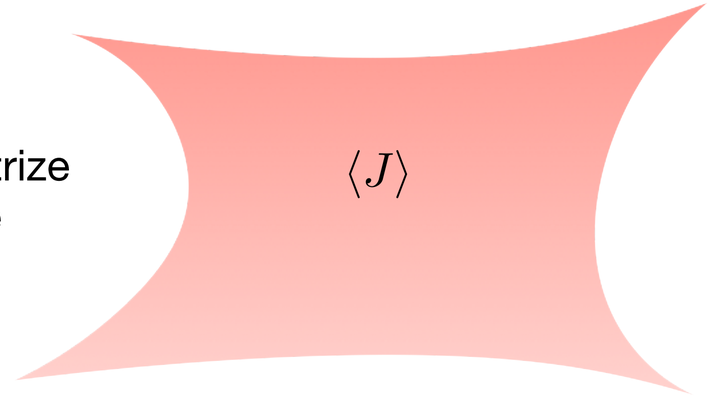
Plan for the talk

- Review on 6d N=(1,0) supergravity
- Properties of tensor moduli space and BPS strings
- Prove the finiteness of 6d N=(1,0) landscape
- Exact bounds on the number of massless fields

6d N=(1,0) Supergravity

- 4 Types of massless supermultiplets
  - Gravity multiplet $(g_{\mu\nu}, B^{+}_{\mu\nu}, \Psi_\mu)$
  - Tensor (T) multiplet $(B^{-}_{\mu\nu}, J, \psi)$
  - Vector (V) multiplet (A_μ, λ)
  - Hyper (H) multiplet (q, Ψ)

Scalar vevs parametrize tensor moduli space



Claim : T, V, H are finite

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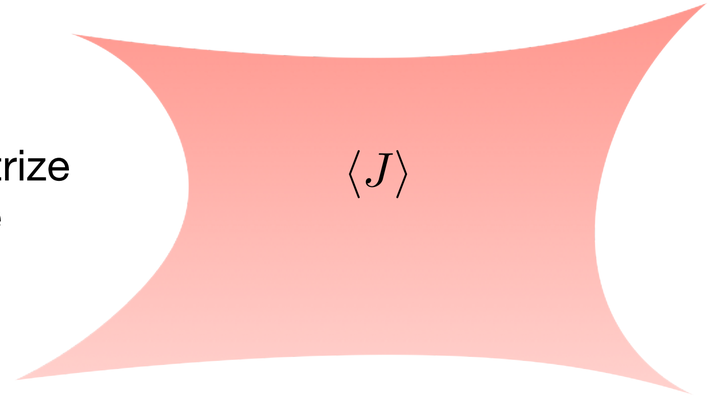

Our sharpe bound $T \leq 193$

Geometric bound $T \leq 567$ [Birkar, Lee '25]

From global anomalies $T \leq 3003$ [Hamada, Loges '25]

Anomaly Free EFTs

- Anomalies can be cancelled if anomaly polynomial factorizes as

[Green, Schwarz '84], [Sagnotti '92]

$$I_8^{1-\text{loop}} = \frac{1}{2}\Omega_{\alpha\beta}\, X_4^\alpha X_4^\beta$$

$$X_4^\alpha = -\frac{1}{2} b_0^\alpha \,\text{tr}R^2 + \frac{1}{4}\sum_i b_i^\alpha \frac{2}{\lambda_i}\text{tr}F_i^2$$

$\Omega_{\alpha\beta}$: intersection form with $(1,T)$ signature

b_0, b_i : anomaly vectors $\in \mathbb{R}^{1,T}$

which constrain the massless spectrum $\Longrightarrow$ $H - V + 29T = 273$

Gravitational anomaly cancellation cond.

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- Infinite families of anomaly free supergravities

Ex) $T = 8k + 9\,,\ G = (E_8)^k$ for arbitrary k [Kumar, Morrison, Taylor '10]

More examples in [Schwarz '96], [Kumar, Taylor '09], [Park, Taylor '11], [Taylor, Turner '18], [Hamada, Loges '23], [Loges '24]

Tensor moduli space & BPS strings

- T-dimensional coset space of $SO(1,T)/SO(T)$ parametrized by scalars $J \in \mathbb{R}^{1,T}$ in the tensor multiplets with conditions

$$J \cdot J > 0\,, \quad J \cdot b_i \geq 0\,, \quad J \cdot b_0 \geq 0$$

(We can fix $J^2 = 1$)

Metric positivity

$1/g_i^2 \sim J \cdot b_i$

Consistency of K3 compactification & more

[Cheung, Remmen '16], [Hamada, Noumi, Shiu '18], [Garcia Etxebarria, et.al. '20], [Ong '22], …

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- BPS strings are 1/2 BPS sources for the 2-form tensor fields
  - Tensions of BPS strings (with charge Q) must be non-negative.

$$T_Q \sim J \cdot Q \geq 0 \;\; \text{for all BPS } Q$$

  - String charges are sitting in a unimodular lattice $\Gamma \subset \mathbb{R}^{1,T}$ with $\Gamma = \Gamma^*$

[Seiberg, Taylor '10]

BPS cone and Generators

- BPS cone is a cone of BPS charges Q.
  - Every BPS charge is a non-negative integral sum of generators $\mathcal{C}_i$

$$Q = \sum_i n_i \mathcal{C}_i \text{ with } n_i \in \mathbb{Z}_{\geq 0} \qquad \left(\text{also, } J = \sum_i a_i \mathcal{C}_i \text{ with } a_i \geq 0 \text{ as } J \cdot J > 0\right)$$

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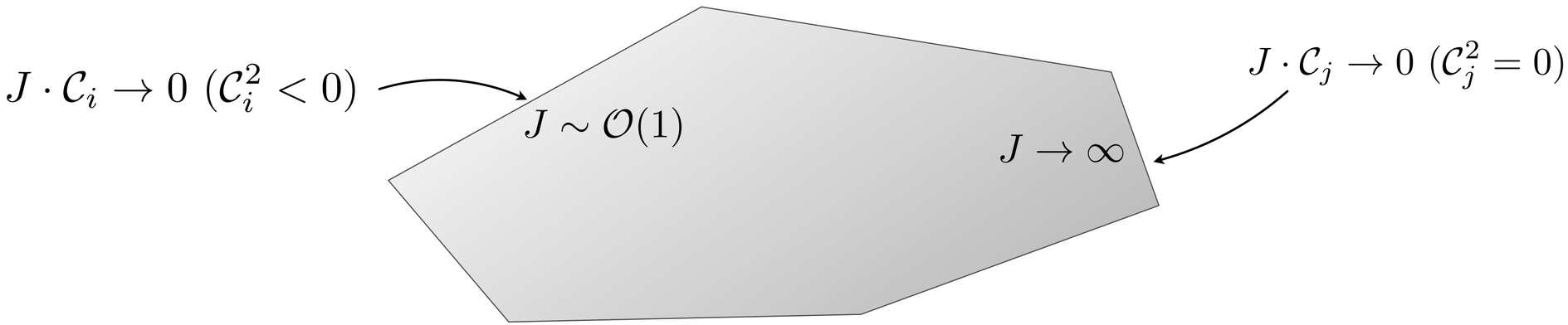
  - Generators are '***shrinkable***' providing tensionless strings at finite distance ($\mathcal{C}_i^2 < 0$) or infinite distance ($\mathcal{C}_i^2 = 0$) boundary in tensor moduli space.



Classification of BPS generators

- BPS generators are ***classified*** in [Morrison, Taylor '12], [Heckman, Morrison, Rudelius, Vafa '15], [Bhardwaj, Del Zotto, Heckman, Morrison, Rudelius, Vafa '15], [Bhardwaj '15, '19], [Hamada, Loges '23, '24]….

$$\begin{cases} \mathcal{C}_i^2 < 0\,, & \mathcal{C}_i \in \text{SCFT} \subset \text{SUGRA} \\ \mathcal{C}_i^2 = 0\,, & \text{little string or critical string} \subset \text{SUGRA} \end{cases}$$

G_i	H_i	$\mathcal{C}_i^2$	$b_0 \cdot \mathcal{C}_i$	Notes
$\mathfrak{g}$	**Adj**	0	0	
$\mathfrak{su}_N$	$(2N) \times \mathbf{N}$	-2	0	
$\mathfrak{su}_N$	$(N-8) \times \mathbf{N} \oplus \frac{\mathbf{N(N+1)}}{\mathbf{2}}$	-1	-1	$N \geq 8$
$\mathfrak{su}_N$	$(N+8) \times \mathbf{N} \oplus \frac{\mathbf{N(N-1)}}{\mathbf{2}}$	-1	1	
$\mathfrak{su}_N$	$16 \times \mathbf{N} \oplus 2 \times \frac{\mathbf{N(N-1)}}{\mathbf{2}}$	0	2	
$\mathfrak{su}_N$	$\frac{\mathbf{N(N-1)}}{\mathbf{2}} \oplus \frac{\mathbf{N(N+1)}}{\mathbf{2}}$	0	0	
$\mathfrak{su}_6$	$15 \times \mathbf{6} \oplus \frac{1}{2}\mathbf{20}$	-1	1	
$\mathfrak{su}_6$	$17 \times \mathbf{6} \oplus \mathbf{15} \oplus \frac{1}{2}\mathbf{20}$	0	2	
$\mathfrak{su}_6$	$18 \times \mathbf{6} \oplus \mathbf{20}$	0	2	
$\mathfrak{su}_6$	$\mathbf{6} \oplus \frac{1}{2}\mathbf{20} \oplus \mathbf{21}$	0	0	
$\mathfrak{so}_N$	$(N-8) \times \mathbf{N}$	-4	-2	$N \geq 8$
$\mathfrak{so}_N$	$(N-7) \times \mathbf{N} \oplus (2^{\lfloor \frac{10-N}{2} \rfloor}) \times \mathbf{2}^{\lfloor \frac{\mathbf{N-1}}{\mathbf{2}} \rfloor}$	-3	-1	$12 \geq N \geq 7$
$\mathfrak{so}_N$	$(N-6) \times \mathbf{N} \oplus (2 \times 2^{\lfloor \frac{10-N}{2} \rfloor}) \times \mathbf{2}^{\lfloor \frac{\mathbf{N-1}}{\mathbf{2}} \rfloor}$	-2	0	$13 \geq N \geq 6$
$\mathfrak{so}_N$	$(N-5) \times \mathbf{N} \oplus (3 \times 2^{\lfloor \frac{10-N}{2} \rfloor}) \times \mathbf{2}^{\lfloor \frac{\mathbf{N-1}}{\mathbf{2}} \rfloor}$	-1	1	$12 \geq N \geq 5$
$\mathfrak{so}_N$	$(N-4) \times \mathbf{N} \oplus (4 \times 2^{\lfloor \frac{10-N}{2} \rfloor}) \times \mathbf{2}^{\lfloor \frac{\mathbf{N-1}}{\mathbf{2}} \rfloor}$	0	2	$14 \geq N \geq 4$
$\mathfrak{sp}_N$	$(2N+8) \times \mathbf{2N}$	-1	1	
$\mathfrak{sp}_N$	$16 \times \mathbf{2N} \oplus (\mathbf{N-1})(\mathbf{2N+1})$	0	2	
$\mathfrak{sp}_3$	$\frac{35}{2}\mathbf{6} \oplus \frac{1}{2}\mathbf{14}'$	0	2	
$\mathfrak{e}_8$		-12	10	
$\mathfrak{e}_7$	$\frac{k}{2} \times \mathbf{56}$	$k-8$	$k-6$	$k \leq 8$
$\mathfrak{e}_6$	$k \times \mathbf{27}$	$k-6$	$k-4$	$k \leq 6$
$\mathfrak{f}_4$	$k \times \mathbf{26}$	$k-5$	$k-3$	$k \leq 5$
$\mathfrak{g}_2$	$(3k+1) \times \mathbf{7}$	$k-3$	$k-1$	$k \leq 3$

H-string

- Among BPS strings, there is a distinguished class of strings, which we call 'H-string', that plays crucial role in our discussions.
- BPS charge f of an H-string satisfies

$$f^2 = 0\,,\ b_0 \cdot f = 2 \quad \Longrightarrow$$ Tensionless limit at infinite distance

- H-strings are critical heterotic strings and little strings with charge f .

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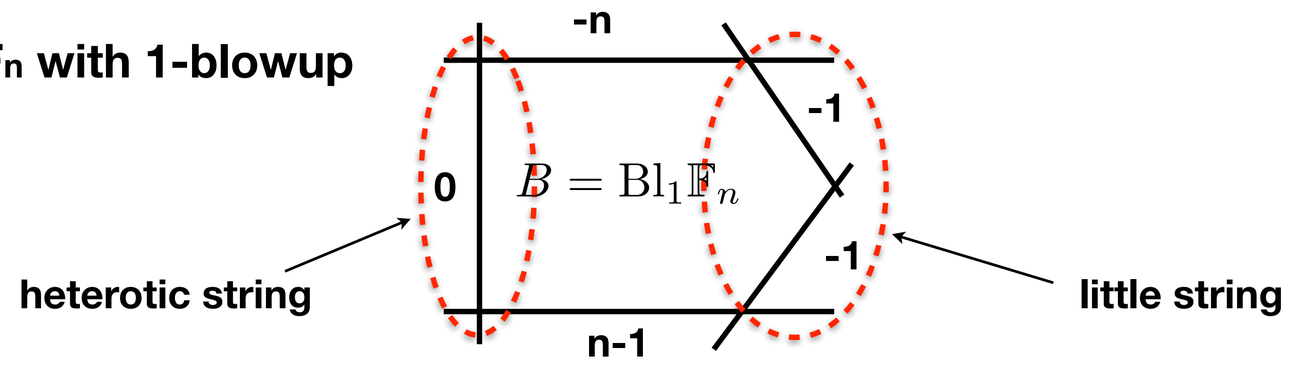
- In F-theory, D3-branes wrapping rational fiber f-curve on base B.

Example: B = Hirzebruch F_n with 1-blowup



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- H-strings are critical heterotic strings and little strings with charge f.
- In F-theory, D3-branes wrapping rational fiber f-curve on base B.
- Unitarity of the worldsheet CFT on H-string requires

$$\sum_{\text{for } b_i \cdot f > 0} \mathrm{rank}(G_i) \leq 20$$ **for '*external*'** G_i **on** $b_i \cdot f > 0$

[HCK, Shiu, Vafa '19]

Existence of H-string

- Consider a **projection to subspace** orthogonal to all e_i with $e_i^2 = -1$

$$\text{Blowdown} \ : \ J \quad \xrightarrow{J\cdot e_i = 0} \quad J' = J|_{J\cdot e_i=0}$$

(our blowdown doesn't necessarily involve small instanton transitions)

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$$J \cdot b_0 \geq 0 \text{ with } J = \sum_i a_i \mathcal{C}_i \text{ and } a_i \geq 0$$ everywhere even after blowdowns

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- Generators $\mathcal{C}'_j$ after the blowdowns satisfy

$$b_0 \cdot \mathcal{C}_j = \begin{cases} 2 & \textbf{H-string} \\ 1 & \text{if } \mathcal{C}_j^2 = -1 \\ 0 & \text{if } \mathcal{C}_j^2 = -2 \text{ or Type II string} \\ \leq -1 & \text{otherwise} \end{cases} \xrightarrow{\text{Blowdowns}} b'_0 \cdot \mathcal{C}'_j = \begin{cases} 2 & \textbf{H-string} \\ 0 & \text{if } \mathcal{C}'^2_j = -2 \text{ or Type II string} \\ \leq -1 & \text{otherwise} \end{cases}$$

- Positivity $J'\cdot b'_0 \geq 0$ requires the existence of at least an H-string!

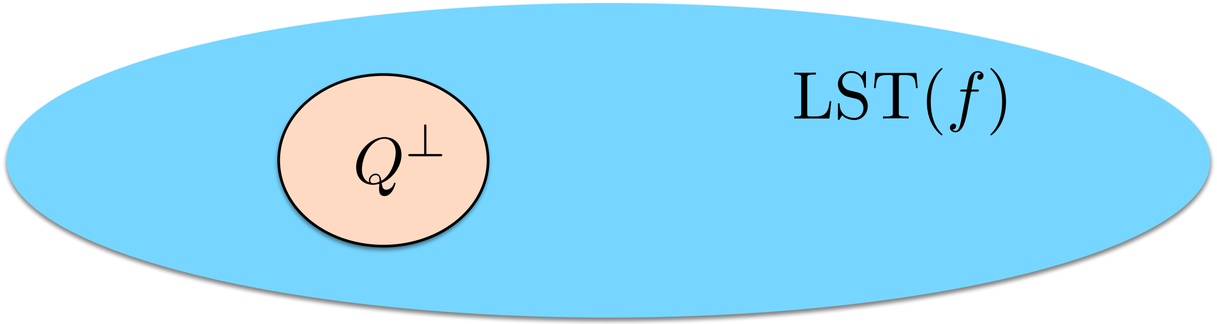
$$\text{Two exceptions}: 1)\; T=0\,, \quad 2)\; b_0 = 0 \text{ (only when } T=9)$$
$$(\text{ex}: \mathbb{P}^2) \qquad (\text{ex}: \text{Enriques surface})$$

SCFTs contained in H-string

- BPS strings with charge $Q^\perp$ satisfying $f \cdot Q^\perp = 0$ are elements in a little string theory (LST) of the H-string charge class f.

$$Q^\perp \in \mathrm{LST}(f)$$

for $f \cdot Q^\perp = 0$



- This containment relation can be proven using the property

$$J \cdot Q^\perp / J \cdot f < \infty \quad \text{for } f \cdot Q^\perp = 0$$ everywhere in tensor moduli space

(together with the classification of LSTs)

Summary of Main Properties

- ✔ *Existence of H-string with charge* f *when* $T \neq 0, 9$
- ✔ $\sum_i \mathrm{rank}(G_i) \leq 20$ *for external gauge algebras on* Q_i *with* $f \cdot Q_i > 0$
- ✔ *Containment relation* $Q_i \in \mathrm{LST}(f)$ *if* $f \cdot Q_i = 0$

These results can be proven solely using EFT arguments.

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- ✔ *Gravitational anomaly constraint* $H - V + 29T = 273$
- ✔ *Classifications of SCFTs and LSTs*

Tensor structure of EFTs

Tensors and their intersections (except $T=9$), as specified by Ω_{ij}, b_0, b_i, can always be realized by topology of $\mathbb{P}^2$ or $\mathbb{F}_n$ or their blowups !

[HCK, Vafa, Xu '24 v.3]

✔ Ω_{ij}, b_0, b_i of EFTs $\subset$ $\mathbb{P}^2$ or $\mathrm{Bl}_p\mathbb{F}_n$ $\sim$ F-theory base

$\mathbb{F}_n$: Hirzebruch surface

This can be proven by using the fact in EFTs that

- H-string eventually becomes a generator after blowdowns.
- BPS cone near charge f is locally polyhedral.

Finiteness of Tensors

- Gravitational anomaly cancellation condition

$$\sum_i \Delta_i = 273 \quad \text{with} \quad \Delta_i = H_i - V_i + 29T_i$$

- Only components having $\Delta_i < 0$ are SCFT 'atoms' of charge $Q_i^2 < 0$

[Hamada, Loges '23]

$\mathfrak{g}_i$		$\mathfrak{f}_4, \mathfrak{e}_6, \mathfrak{e}_7$	$\mathfrak{e}_6, \mathfrak{e}_7$	$\mathfrak{e}_7$	$\mathfrak{e}_7$	$\mathfrak{e}_8$
Q_i^2	:	-5	-6	-7	-8	-12

Note) Gauge anomaly cancellation

$$Q_i^2 = \frac{\lambda_i^2}{3}\left(\sum_{\mathbf{r}} \underbrace{n_{\mathbf{r}}^i C_{\mathbf{r}}^i}_{H_i} - \underbrace{C_{\mathbf{adj}}^i}_{V_i}\right) \qquad \text{where } \mathrm{tr}_{\mathbf{r}} F^4 = B_{\mathbf{r}} \mathrm{tr} F^4 + C_{\mathbf{r}} (\mathrm{tr} F^2)^2$$

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- To achieve $T \to \infty$, EFT must contain infinite number of these atoms.

Finiteness of Tensors

- Only components having $\Delta_i < 0$ are SCFT 'atoms' of charge Q_i with

$\mathfrak{g}_i$		$\mathfrak{f}_4, \mathfrak{e}_6, \mathfrak{e}_7$	$\mathfrak{e}_6, \mathfrak{e}_7$	$\mathfrak{e}_7$	$\mathfrak{e}_7$	$\mathfrak{e}_8$
Q_i^2	:	-5	-6	-7	-8	-12

[Hamada, Loges '23]

- These atoms are either

$$\begin{cases} 1.\ Q_i \cdot f > 0\ , & \sum_i r(\mathfrak{g}_i) \leq 20 \\ 2.\ Q_i \cdot f = 0\ , & Q_i \in \mathrm{LST}(f) \end{cases}$$

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- For the 1st case, their number is finite due to the rank bound.

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- For the 2nd case, the classification of LST tells us that these atoms are always accompanied by additional matters, so that their grav. anomaly contributions eventually become positive!

EX) $\overset{\mathfrak{e}_8}{-12}$ atoms appear as either
$$\begin{cases} \text{conformal matter}: \ [\mathfrak{e}_8]1\ 2\ \overset{\mathfrak{sp}_1}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{sp}_1}{3}\ 2\ 1\ \overset{\mathfrak{e}_8}{\underline{12}} \quad \Delta_{\mathrm{tot}} = 30 \\ \text{small instanton}: \ \overset{\mathfrak{e}_8}{\underline{12}}\ 1\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ \cdots \quad \Delta_{\mathrm{tot}} > 30 \end{cases}$$

Finiteness of Tensors

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$\mathfrak{g}_i$		$\mathfrak{f}_4, \mathfrak{e}_6, \mathfrak{e}_7$	$\mathfrak{e}_6, \mathfrak{e}_7$	$\mathfrak{e}_7$	$\mathfrak{e}_7$	$\mathfrak{e}_8$
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- These atoms are either $\begin{cases} 1.\ Q_i \cdot f > 0\ , & \sum_i r(\mathfrak{g}_i) \leq 20 \\ 2.\ Q_i \cdot f = 0\ , & Q_i \in \mathrm{LST}(f) \end{cases}$

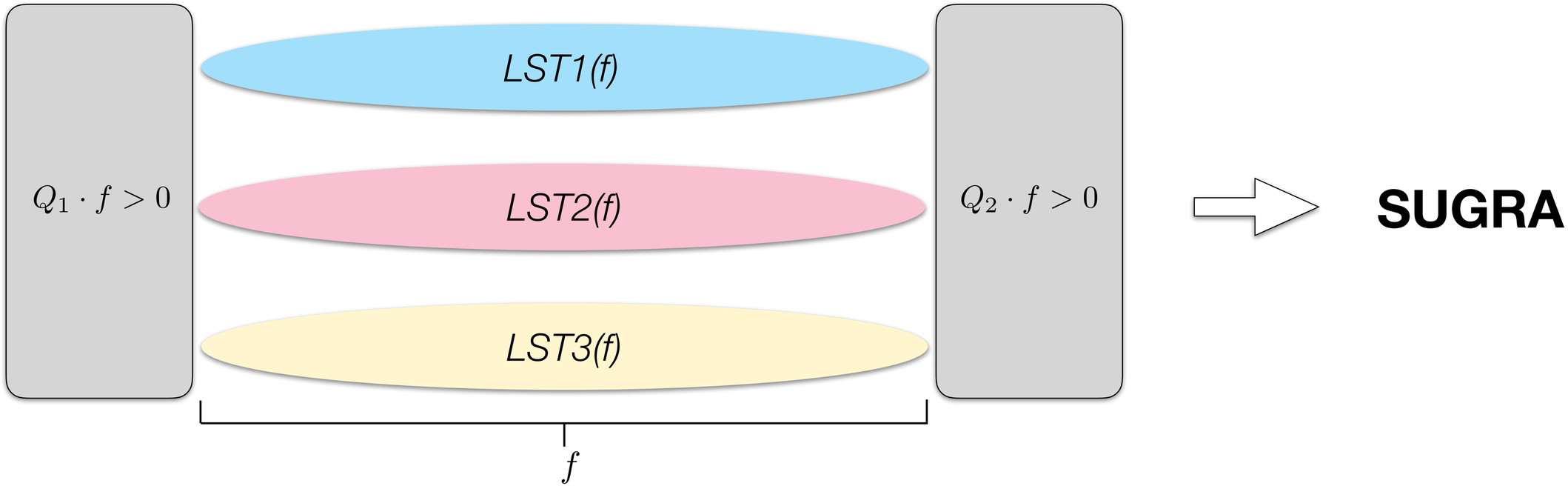
- As a result, ***the number of these atoms is finite*** in both cases.

✔ T is finite

Note: This is a bottom-up argument that accounts for <u>frozen phases and U(1) algebras</u> as well.

Tensor and Vector Bound

- We first identify the composition of 'atoms' for $\mathrm{LST}(f)$ that can host large number of tensors and high rank of gauge algebras.
- Add 'external' matters for $Q_i \cdot f > 0$ properly, and construct supergravity theories with maximum tensors or highest rank of gauge algebras.



Tensor and Vector Bound

- $T_{\max}$ theory : $\overset{\mathfrak{e}_8}{(12)} \otimes \underbrace{\overset{1}{\mathfrak{e}_8} \otimes \mathfrak{e}_8 \otimes \cdots \otimes \mathfrak{e}_8 \otimes \overset{1}{\mathfrak{e}_8}}_{15} \otimes \overset{\mathfrak{e}_8}{(12)}$ **with** $T = 193$

  $G = \mathfrak{e}_8^{17} \times \mathfrak{f}_4^{16} \times \mathfrak{g}_2^{32} \times \mathfrak{sp}_1^{32}$

  $Q^2 = -12,\ Q \cdot f = 1$ $\qquad$ $Q^2 = -12,\ Q \cdot f = 1$

Tensor and Vector Bound

- $T_{\max}$ theory : $\overset{\mathfrak{e}_8}{(12)} \otimes \underbrace{\overset{1}{\mathfrak{e}_8} \otimes \mathfrak{e}_8 \otimes \cdots \otimes \mathfrak{e}_8 \otimes \overset{1}{\mathfrak{e}_8}}_{15} \otimes \overset{\mathfrak{e}_8}{(12)}$ with $T = 193$, $G = \mathfrak{e}_8^{17} \times \mathfrak{f}_4^{16} \times \mathfrak{g}_2^{32} \times \mathfrak{sp}_1^{32}$

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- $r_{\max}(G)$ theory : $\begin{array}{ccccccccc} & & & \mathfrak{sp}_{40} & & & & & \\ & & & 1 & & & & & \\ \mathfrak{so}_{64} & \mathfrak{sp}_{56} & \mathfrak{so}_{176} & \mathfrak{sp}_{72} & \mathfrak{so}_{128} & \mathfrak{sp}_{48} & \mathfrak{so}_{80} & \mathfrak{sp}_{24} & \mathfrak{so}_{32} \\ 4 & 1 & 4 & 1 & 4 & 1 & 4 & 1 & (4) \end{array}$ with $T = 9$, $r(G) = 480$

  $Q^2 = -4,\ Q \cdot f = 1$

- Therefore, we find the bounds:

$$\boxed{T \leq 193\,,\ r(G) \leq 480}$$

(This implies H is also finite.)

Tensor and Vector Bound

- $T_{\max}$ theory : $\overset{\mathfrak{e}_8}{(12)} \otimes \underbrace{\overset{1}{\mathfrak{e}_8} \otimes \mathfrak{e}_8 \otimes \cdots \otimes \mathfrak{e}_8 \otimes \overset{1}{\mathfrak{e}_8}}_{15} \otimes \overset{\mathfrak{e}_8}{(12)}$ with $T = 193$, $G = \mathfrak{e}_8^{17} \times \mathfrak{f}_4^{16} \times \mathfrak{g}_2^{32} \times \mathfrak{sp}_1^{32}$

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  $Q^2 = -4,\ Q \cdot f = 1$

- Therefore, we find the sharp bounds: $\boxed{T \leq 193\,,\ r(G) \leq 480}$

- These theories are realized by 24 instantons in $E_8 \times E_8$ and $SO(32)$ heterotic strings, respectively, on K3 with an E_8 singularity.

[Aspinwal, Morrison '97], [Candelas, Perevalov, Rajesh '97], [Morrison, Taylor '12]

Evidence for string lamppost principle

Maximal SUGRAs from geometry

- $T_{\max}$ theory : elliptic CY3 on $B = \mathrm{Bl}_{192}\mathbb{F}_{12}$ with $h^{1,1} = 491,\ h^{2,1} = 11,\ h^{1,1}(B) = 194$

T-dual $\updownarrow$

- $r_{\max}(G)$ theory : elliptic CY3 on $B = \mathrm{Bl}_{8}\mathbb{F}_{4}$ with $h^{1,1} = 491,\ h^{2,1} = 11,\ h^{1,1}(B) = 10$

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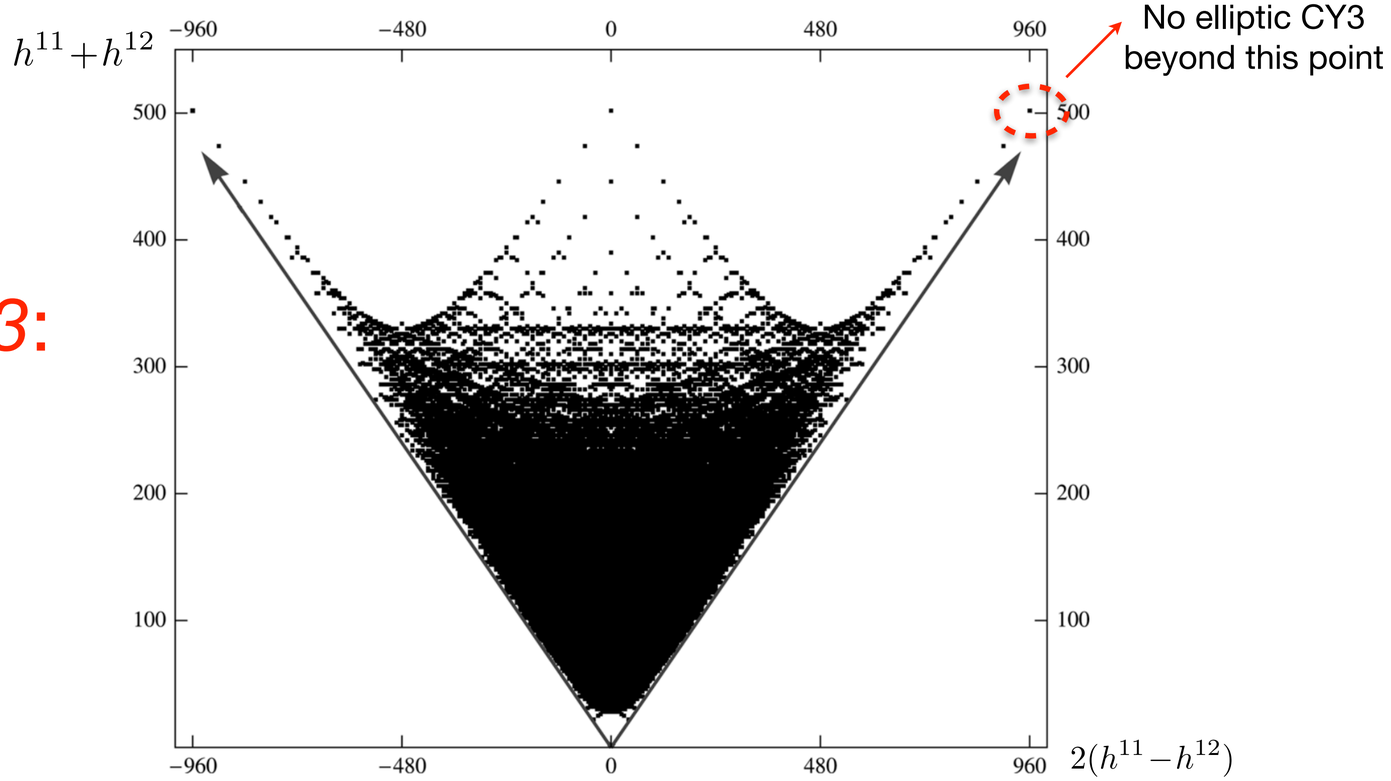
Our result predicts *sharp bounds on Hodge numbers of elliptic CY3*:

$$h^{1,1}(\mathrm{CY}) \leq 491,\ \ h^{1,1}(\mathrm{Base}) \leq 194$$

($h^{2,1}(\mathrm{CY}) \leq 491$ by [Taylor '12])



Hodge numbers of the Kreuzer–Skarke list

Conclusion

- We have demonstrated that, in 6d (1,0) supergravity, the number of tensors and the rank of the gauge algebra are bounded above, based on the consistency of tensor branch physics, the unitarity of BPS strings, and the classification of 6d SCFTs and LSTs:

$$\boxed{T \leq 193\ , \quad r(G) \leq 480}$$

To do:

- Geometric proof for the predictions

$$h^{1,1}(\mathrm{CY}) \leq 491,\ \ h^{1,1}(\mathrm{Base}) \leq 194$$

- Constraints on Abelian gauge charges [HCK, Vafa, Xu in progress]

- Classification of 6d SUGRAs [Hamada, HCK, Jeon in progress]