SMEFT with Higgs Field Curvature

Based on “A $\mathbb{CP}^2$ SMEFT”, JHEP 04 (2025) 180 ,

arXiv:2411.09521

Nicholas S. Manton

DAMTP, University of Cambridge
N.S.Manton@damtp.cam.ac.uk

WIN, Brighton, June 2025

Standard Higgs

- The Higgs sector of the SM Lagrangian is

$$\mathcal{L}_{\rm Higgs} = \overline{D_\mu \phi}\, D^\mu \phi - U(\bar{\phi}\phi)$$

  with ϕ the Higgs doublet

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix} .$$

- $D_\mu \phi$ is the $U(2)$ gauge covariant derivative of ϕ, and U the Higgs potential. There are further Yukawa couplings of ϕ to fermions.

$SO(4)$ Symmetry in SMEFT

- SMEFT allows a non-renormalisable Higgs sector. An attractive alternative to the standard Higgs sector is a gauged nonlinear σ-model.
- Alonso, Jenkins and Manohar (2016) focussed on the residual $SO(4) \simeq SU(2)_L \times SU(2)_R$ symmetry of

  $$\overline{\phi}\phi = \phi_0^2 + \phi_1^2 + \phi_2^2 + \phi_3^2 \,,$$

  which adds a global $SU(2)_R$ symmetry to the standard $SU(2)_L$ gauge symmetry. They proposed extending the SM Lagrangian to

  $$\mathcal{L} = f(\overline{\phi}\phi)\, \overline{D_\mu \phi}\, D^\mu \phi - U(\overline{\phi}\phi) \,.$$

- Spontaneous gauge symmetry breaking is by U as usual.

- $\mathcal{L}$ is a gauged nonlinear σ-model with Higgs field (target) manifold having Riemannian metric

$$ds^2 = f(\overline{\phi}\phi)\, d\overline{\phi}\, d\phi\,.$$

- Particularly symmetric is the S^4 (4-sphere) metric

$$ds^2 = \frac{M^4\, d\overline{\phi}\, d\phi}{(M^2 + \overline{\phi}\phi)^2}$$

  with $SO(5)$ symmetry. The $U(2)$ gauge orbits are still 3-spheres, as in SM. $\phi = 0$ is a point orbit (symmetry restoration).
- M is a large, BSM mass parameter. Deviations from the SM are of order v^2/M^2.
- Variant models without a point orbit are called HEFTs. A nice example uses a hyperboloid with $SO(4,1)$ symmetry.
- Curvature modifies masses and scattering amplitudes.

$U(2)$ Symmetry in $\mathbb{CP}^2$ SMEFT

- The gauge group $U(2)$ is so fundamental in SM, that I propose preserving only this symmetry in σ-model extension of SM. The general Higgs Lagrangian is then

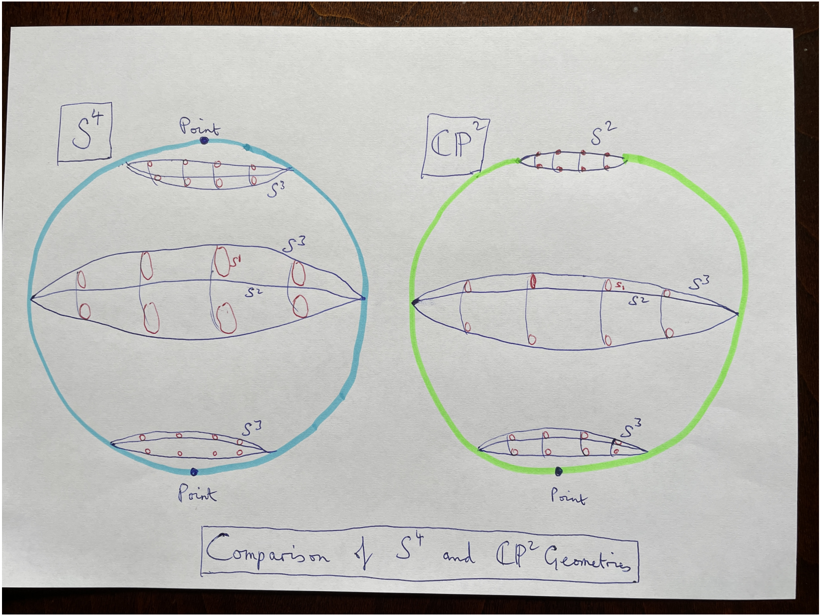
$$\mathcal{L} = f(\bar{\phi}\phi)\,\overline{D_\mu\phi}\,D^\mu\phi + g(\bar{\phi}\phi)\,(\overline{D_\mu\phi}\,\phi)\,(\bar{\phi}\,D^\mu\phi) - U(\bar{\phi}\phi)\,.$$

- A particularly symmetric example exploits the Riemannian metric

$$ds^2 = \frac{M^2(M^2+\bar{\phi}\phi)\,d\bar{\phi}\,d\phi - M^2(d\bar{\phi}\,\phi)\,(\bar{\phi}\,d\phi)}{(M^2+\bar{\phi}\phi)^2}$$

(i.e. $f = M^2(M^2+\bar{\phi}\phi)^{-1}$, $g = M^2(M^2+\bar{\phi}\phi)^{-2}$).

- This is the Fubini–Study metric of the complex projective space $\mathbb{CP}^2$, which has an overall $SU(3)$ symmetry.
- The gauge symmetry is still $U(2)$, but its orbits are now squashed 3-spheres (an S^1 bundle over S^2 with incommensurate radii).



S^4 and $\mathbb{CP}^2$ Geometries

BSM Phenomenology of $\mathbb{CP}^2$ Model

- There are new dimension-6 terms if we expand out, in particular $\frac{1}{M^2}(\overline{D_\mu\phi}\,\phi)\,(\overline{\phi}\,D^\mu\phi)$.
- Spontaneous symmetry breaking occurs in the usual way, at the minimum of the Higgs potential U. Use standard expansion $\phi_0 = v + h$.
- The coupling constants g and g' need to be calibrated using the fermion-gauge boson couplings.
- The W-boson and Z-boson masses are slightly altered. This is the most striking prediction of the $\mathbb{CP}^2$ model. We find

  $$m_{\mathrm{W}}^2 = \left(1 + \frac{v^2}{2M^2}\right) m_{\mathrm{Z}}^2 \cos^2\theta_w\,.$$

  CERN/D0 data imply lower bound $M > 6.6$ TeV, but if CDF W-boson mass confirmed, then $M \simeq 4$ TeV.

- In SM, Higgs-fermion Yukawa couplings (from $\Gamma_f \bar{f} f (v + h)$ term) are proportional to fermion masses. Recent data verifies this linear relationship remarkably well.
- $\mathbb{CP}^2$ model predicts a slightly reduced (universal) slope

$$\kappa_f = 1 - \frac{v^2}{M^2}\,.$$

Currently $\kappa_f = 1 \pm 0.1$. Our model can be tested/challenged with more accurate $\bar{f} f h$ data.

- There are also small changes to Higgs self-couplings, but these are out-of-reach of forthcoming experiments.