# Measurement of Muon Anomalous Precession Frequency in the Muon $g-2$ Experiment at Fermilab

**Yonghao Zeng, on behalf of the Muon $g-2$ collaboration**

**June 10, 2025 | WIN 2025 | University of Sussex, Brighton, UK**

WIN 2025

9–13 June 2025
University of Sussex, Brighton, UK

WiN 25

李政道研究所
TSUNG-DAO LEE INSTITUTE

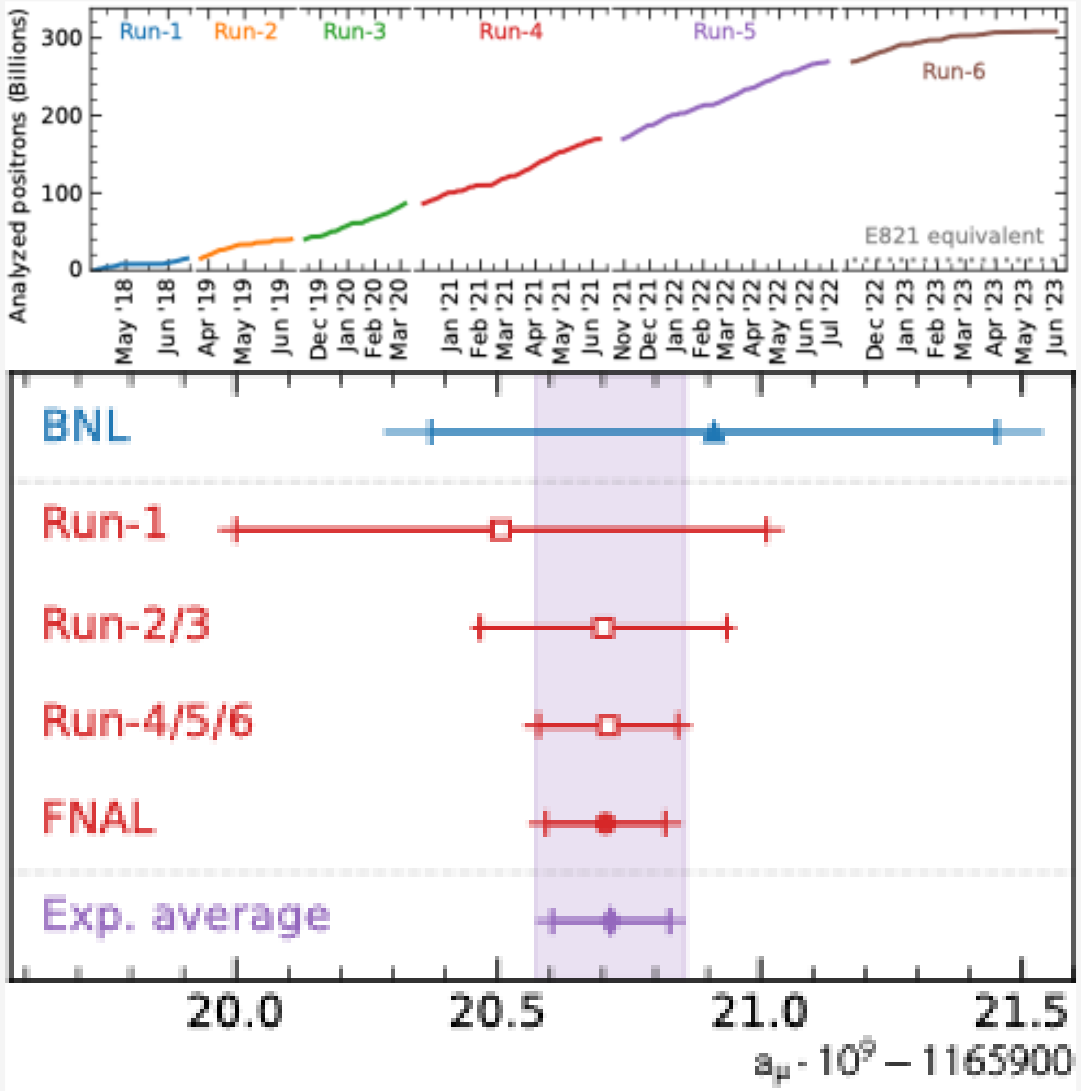
# Final Results of Muon g-2 from Fermilab

- All datasets have been fully analyzed
- **Run-4/5/6 result announced** at Fermilab on June 3rd 2025
- **Perfectly consistent with** previous measurements
- Most precise determination of $a_\mu$ with **127 ppb accuracy**

$$a_\mu(\mathrm{FNAL}) = 0.001165920705(148)\ [127\ \mathrm{ppb}]$$

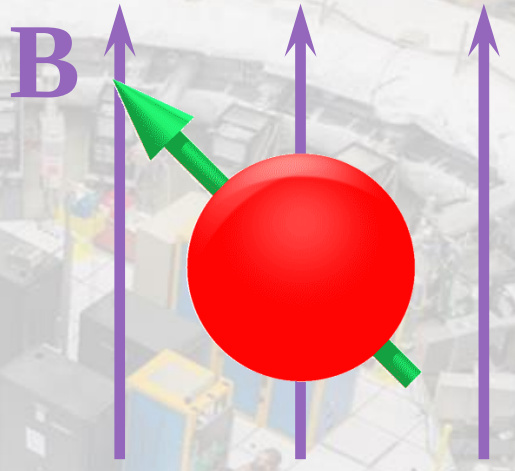
$$a_\mu(\mathrm{Exp.}) = 0.001165920715(145)\ [124\ \mathrm{ppb}]$$

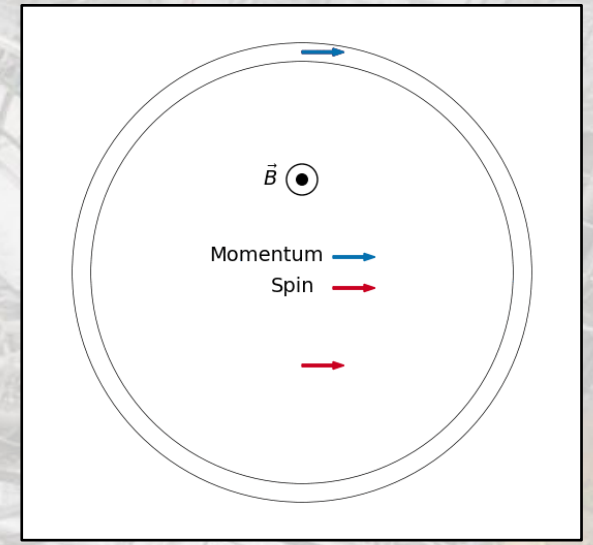
Presented in the previous talk by **Joe Price**

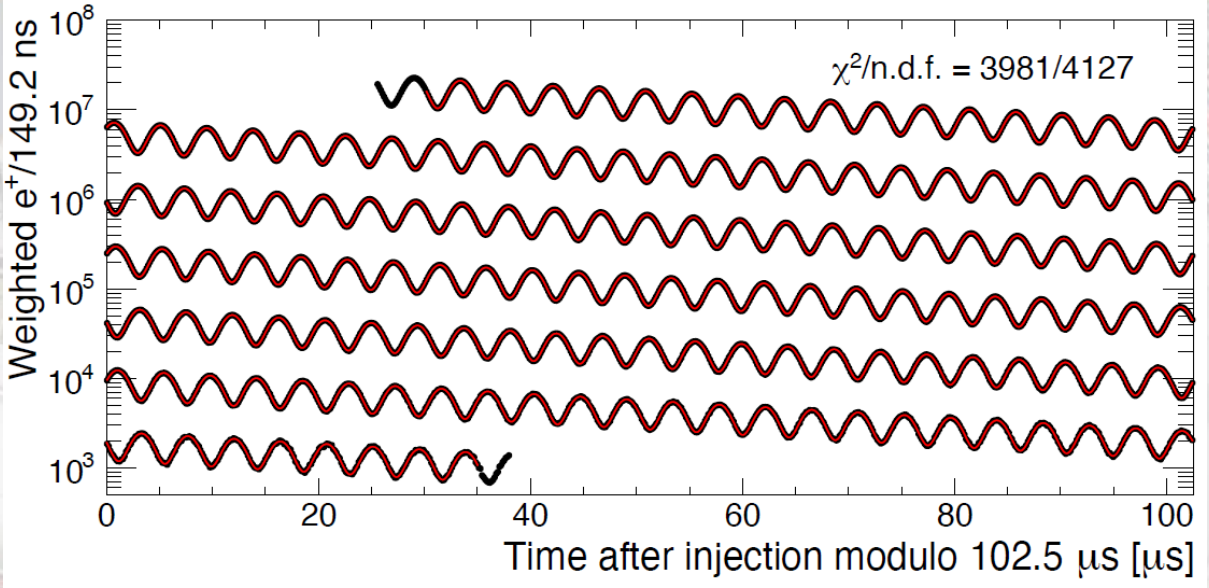


# Outline

- Anomalous precession frequency $\omega_a$ measurement at FNAL
  - Principles
  - Analysis flow
- Analyses combination
- Systematic uncertainties
- Summary

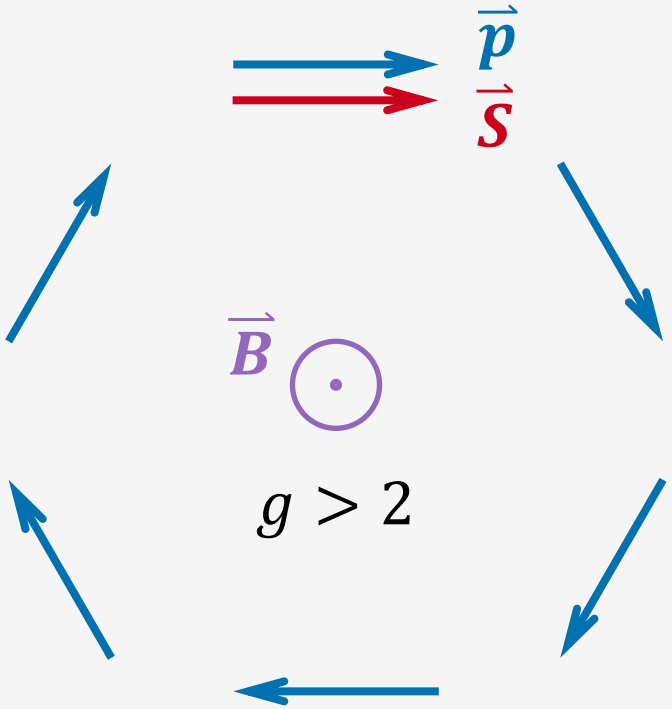


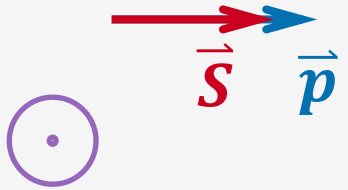
# Spin Precession in a Magnetic Storage Ring

Lab Frame




$g > 2$

Rest Frame

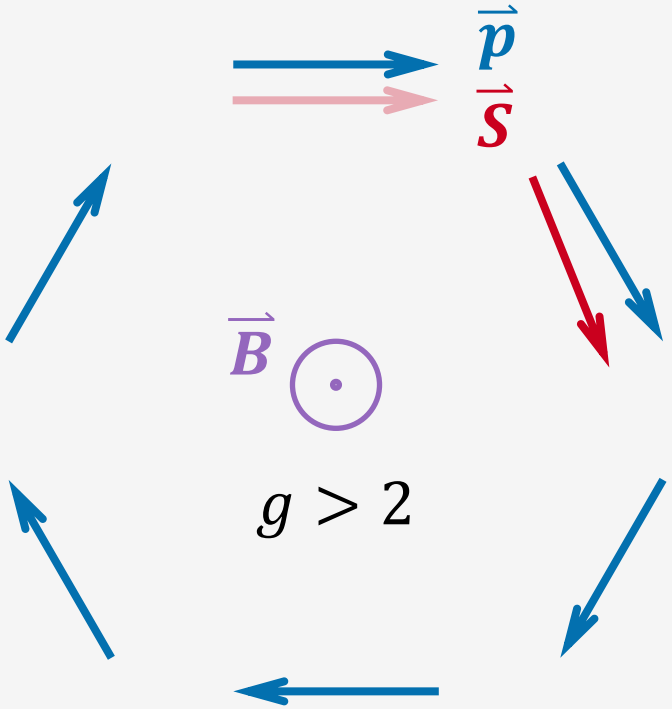



$$\vec{\omega}_a = \vec{\omega}_c - \vec{\omega}_s$$

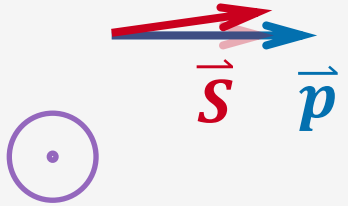
Rate of spin precession with respect to momentum

# Spin Precession in a Magnetic Storage Ring

Lab Frame




Rest Frame

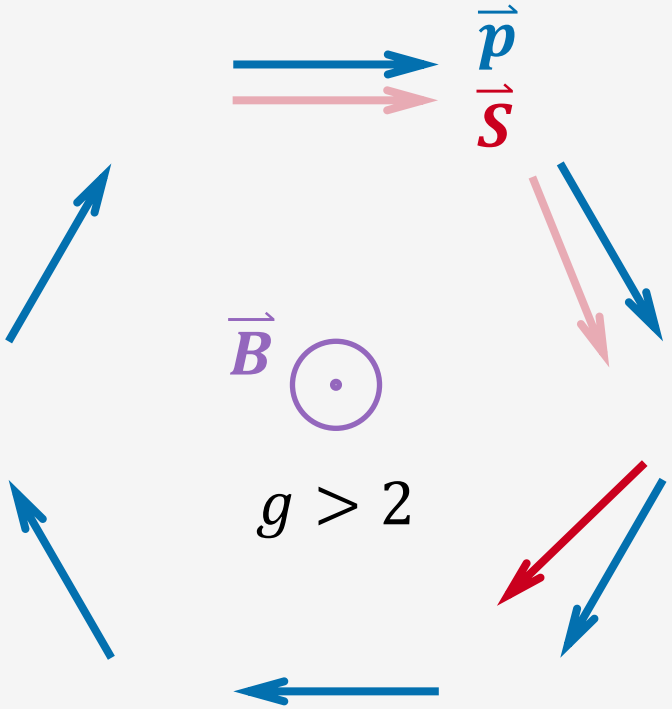



$$\vec{\omega}_a = \vec{\omega}_c - \vec{\omega}_s$$

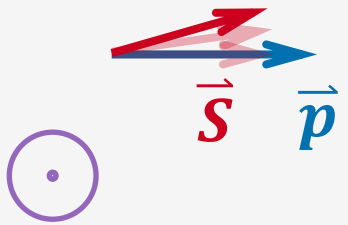
Rate of spin precession with respect to momentum

# Spin Precession in a Magnetic Storage Ring

Lab Frame




Rest Frame

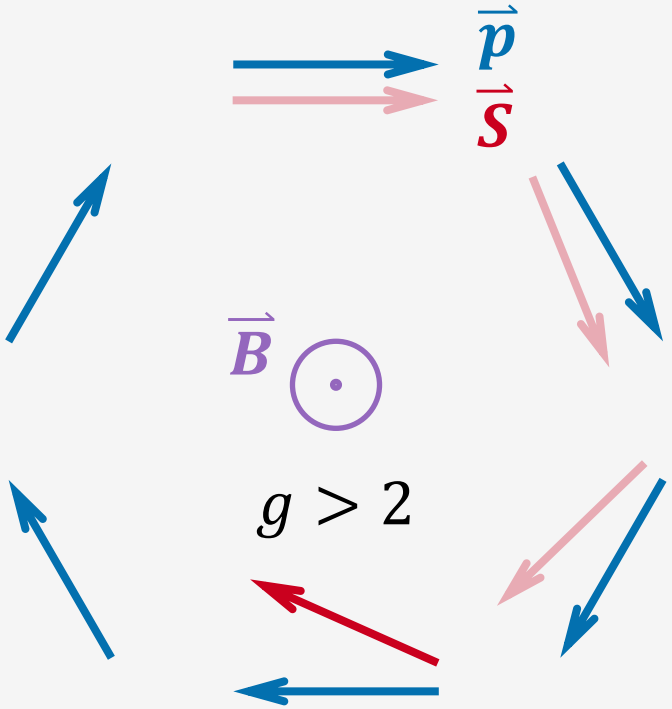



$$\vec{\omega}_a = \vec{\omega}_c - \vec{\omega}_s$$

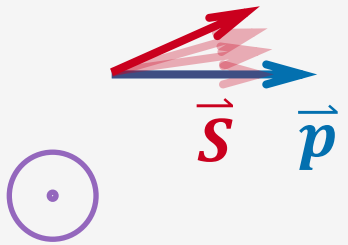
Rate of spin precession with respect to momentum

# Spin Precession in a Magnetic Storage Ring

Lab Frame




Rest Frame




$$\vec{\omega}_a = \vec{\omega}_c - \vec{\omega}_s$$

Rate of spin precession with respect to momentum

# Spin Precession in a Magnetic Storage Ring

This talk

Next talk by
**René Reimann**

**Master Formula**

$$a_\mu = \frac{g_\mu - 2}{2}$$

$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E} - a_\mu \frac{\gamma}{\gamma + 1}(\vec{\beta} \cdot \vec{B})\vec{\beta}\right]$$

$\gamma = 29.3$, p $= 3.094$ GeV/c
*Magic Momentum*

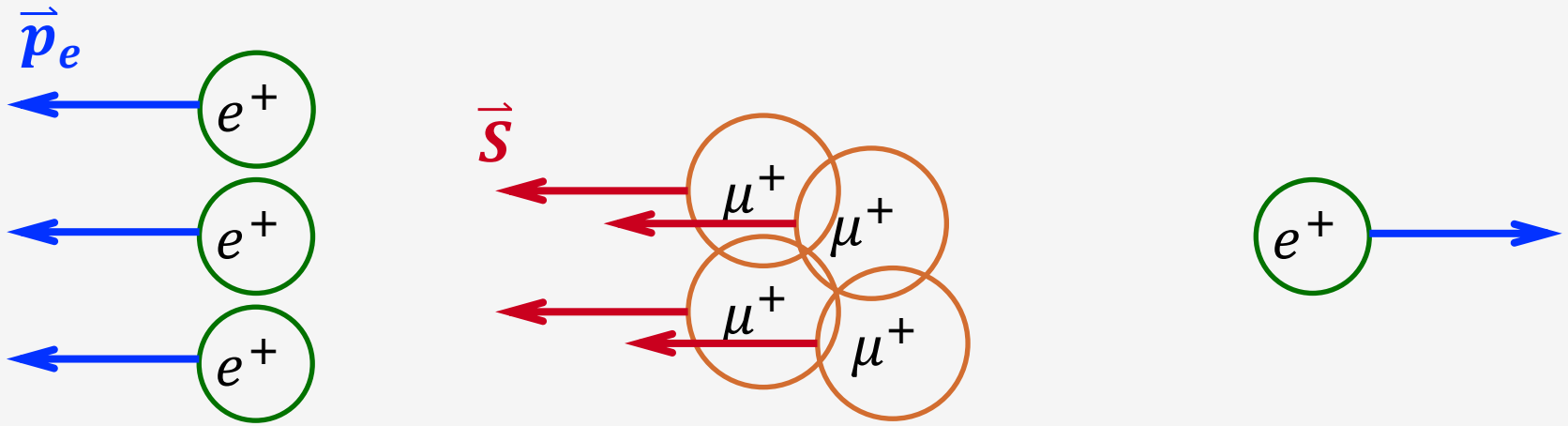
$$\vec{\beta} \cdot \vec{B} = 0$$

Rate of spin precession with respect to momentum

# Principle of $\omega_a$ Measurement

Weak decay violates parity symmetry

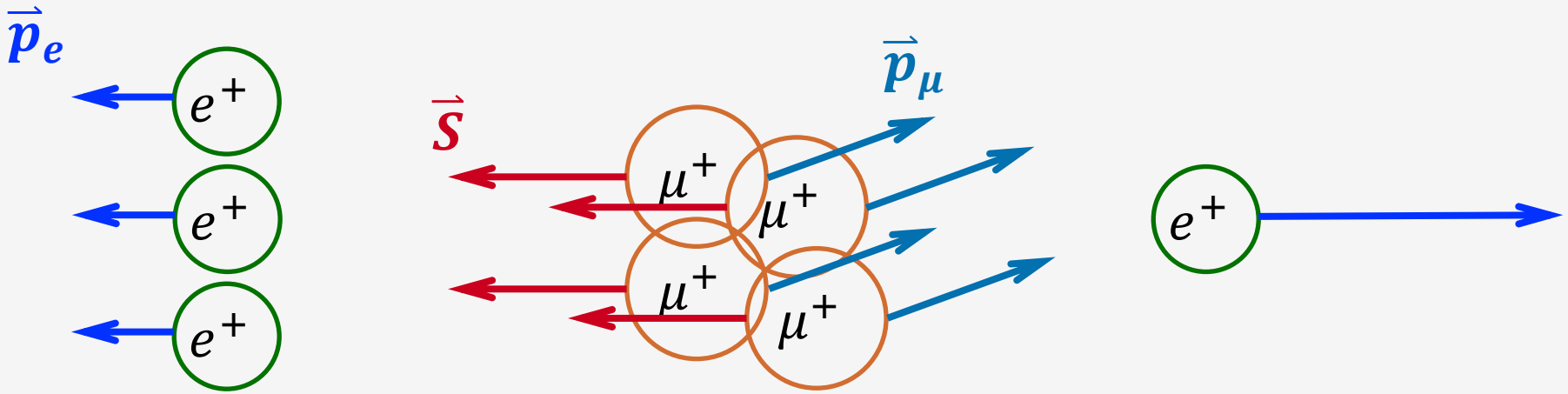
- Positrons are preferentially emitted along the direction of anti-muon spin



# Principle of $\omega_a$ Measurement

Positron energy spectrum encodes muon spin-momentum correlation

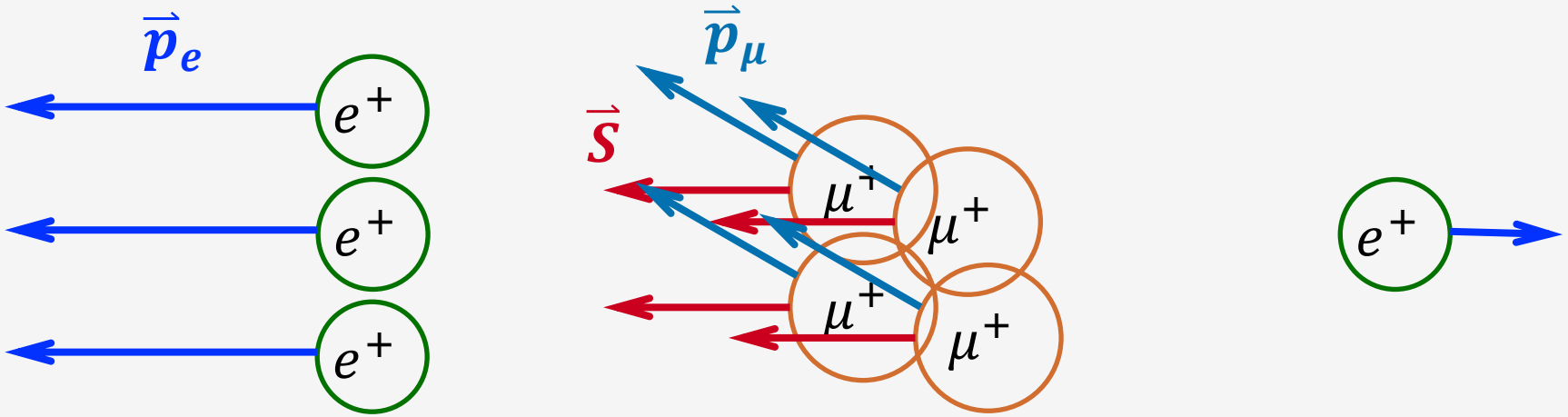
- The most energetic positrons are those emitted along the muon's momentum



# Principle of $\omega_a$ Measurement

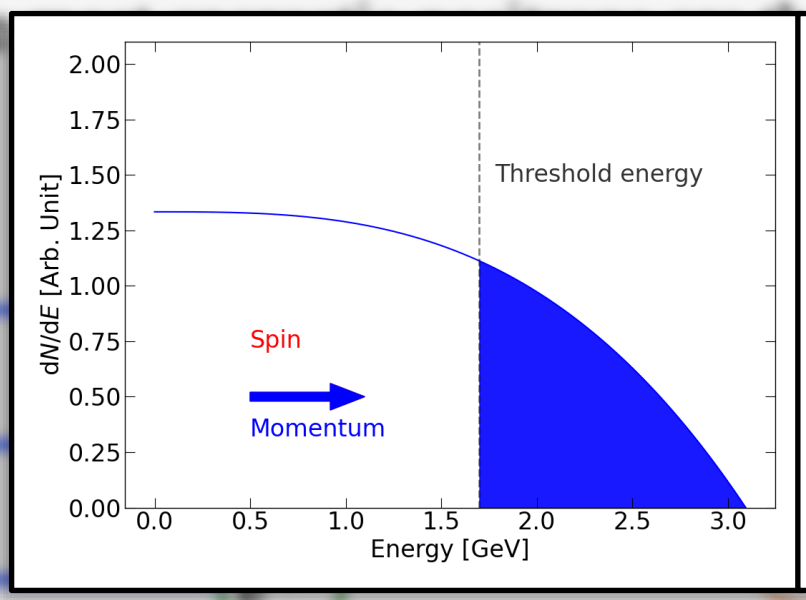
Positron energy spectrum encodes muon spin-momentum correlation

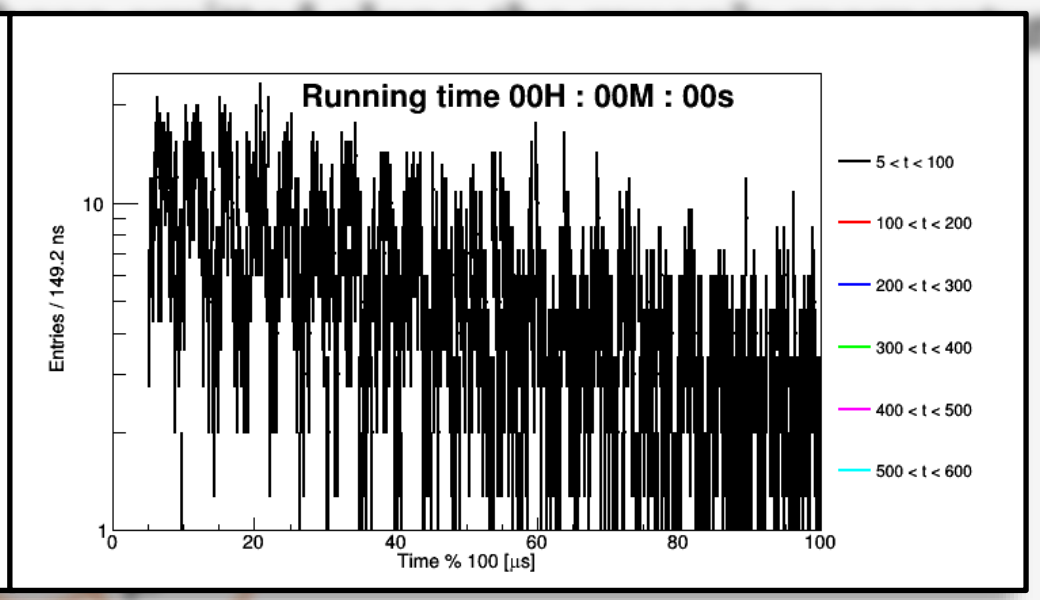
- The most energetic positrons are preferentially emitted along the direction of muon spin



# Principle of $\omega_a$ Measurement

Time-modulation of $e^+$ energy spectrum



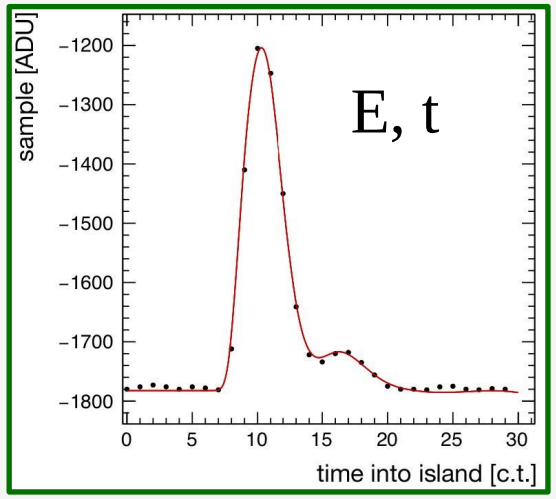

We measure the **energy and time** of decay positrons

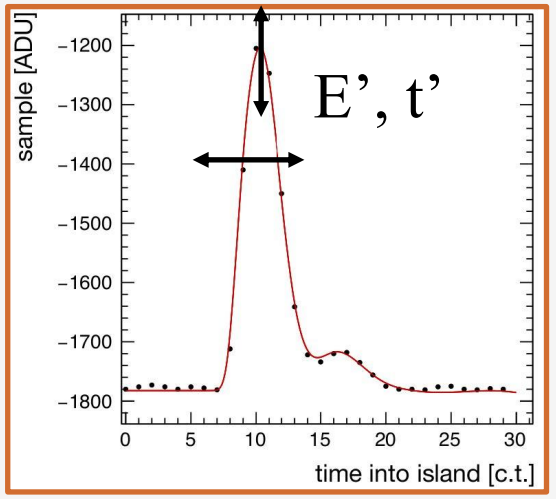
$\boldsymbol{\omega_a}$ signal is encoded in the ***wiggle plot***: $N_e(t) \propto e^{-t/\tau_\mu}[1 + A\cos(\omega_a t + \varphi)]$

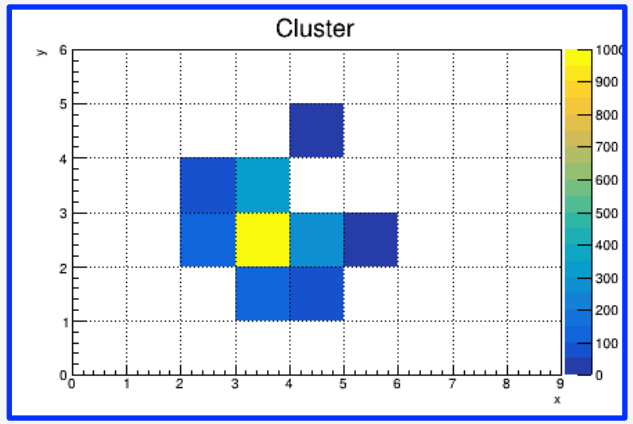
# Analysis Flow

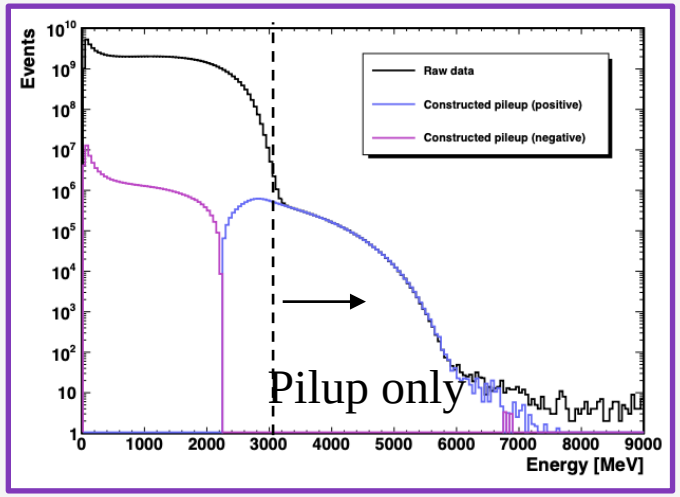
Data Acquisition → Reconstruction → Data Weighting → Wiggle Fitting

Pulse Fitting | Time Correction | Gain Correction | Clustering | Pileup Correction



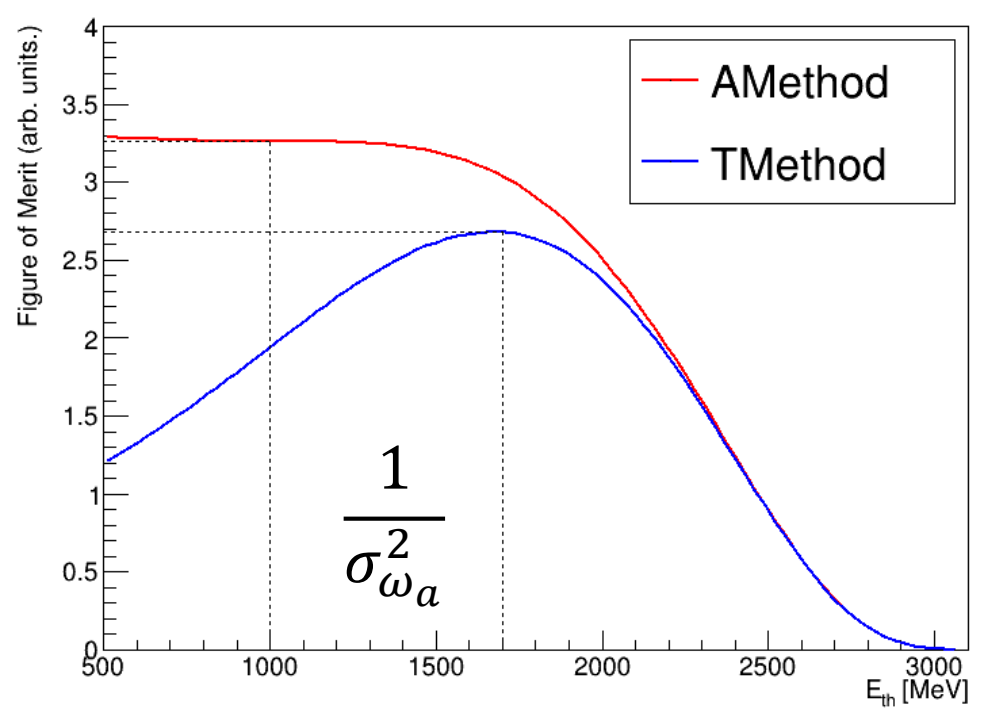



# Analysis Flow

Data Acquisition ⇒ Reconstruction ⇒ Data Weighting ⇒ Wiggle Fitting

- ➤ Enhancing statistical power through asymmetry-weighted events
- ➤ **T-methed**
  - A(E) = 1
  - With a threshold of 1.65 GeV
- ➤ **A-method**
  - Extract A(E) from energy bin scan (50 MeV bin size)
  - With a lower cut of 1 GeV



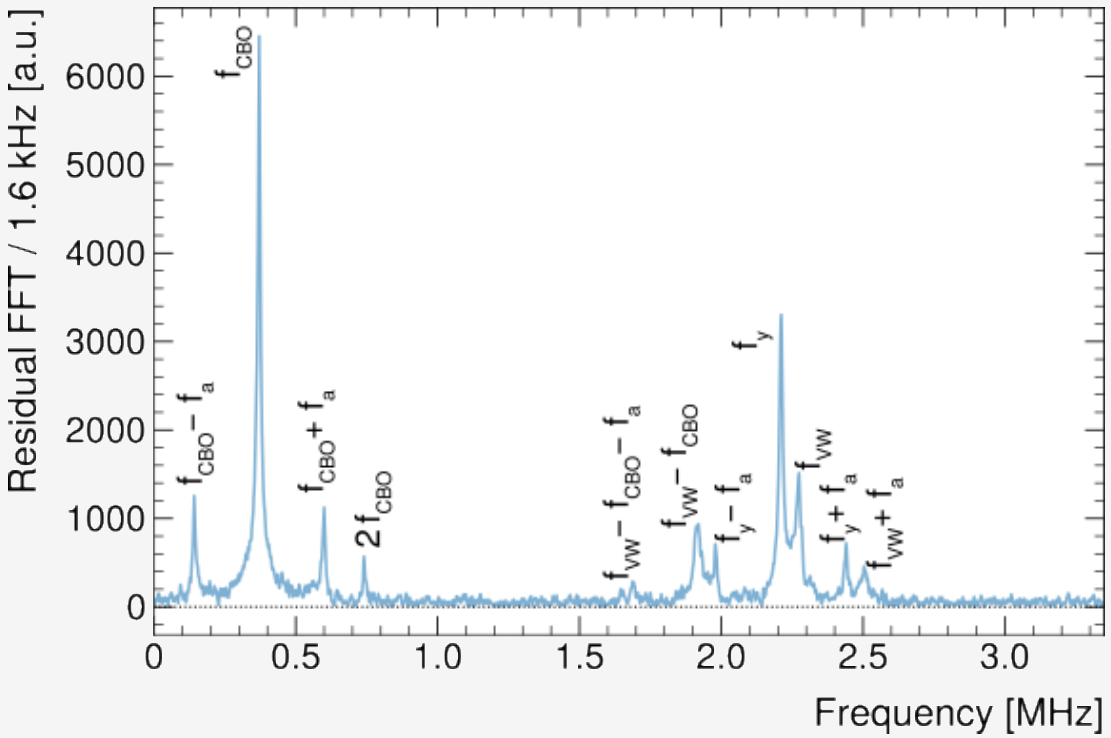
# Analysis Flow

Data Acquisition → Reconstruction → Data Weighting → Wiggle Fitting

- Simple 5-parameters model only captures the exponential decay and $g-2$ oscillation

$$N_e(t) = N_0 e^{-t/\tau_\mu}[1 + A\cos(\omega_a t + \varphi)]$$

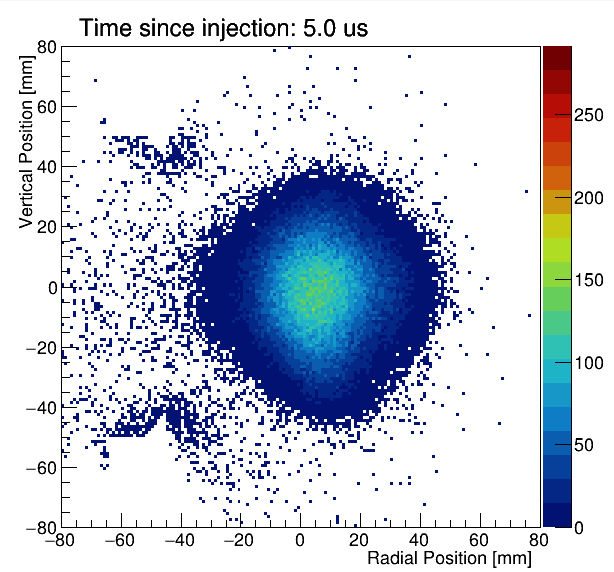
- Must account for **beam oscillations**, **detector effects** and **muon losses**



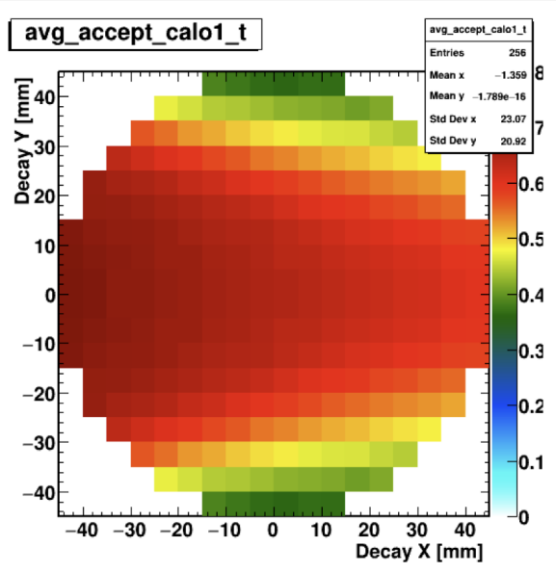
# Coherent Betatron Oscillation (CBO)

- CBO enters the data through coupling between detector acceptance and muon decay position. It **dominated Run-1/2/3 systemactic uncertainty**

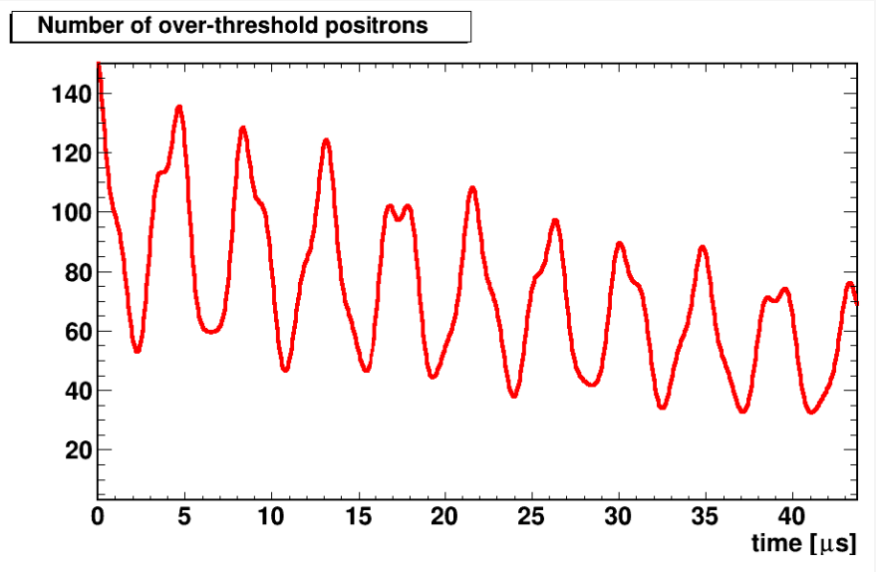
Beam Motion




Detector Acceptance




Distorted Wiggle Plot



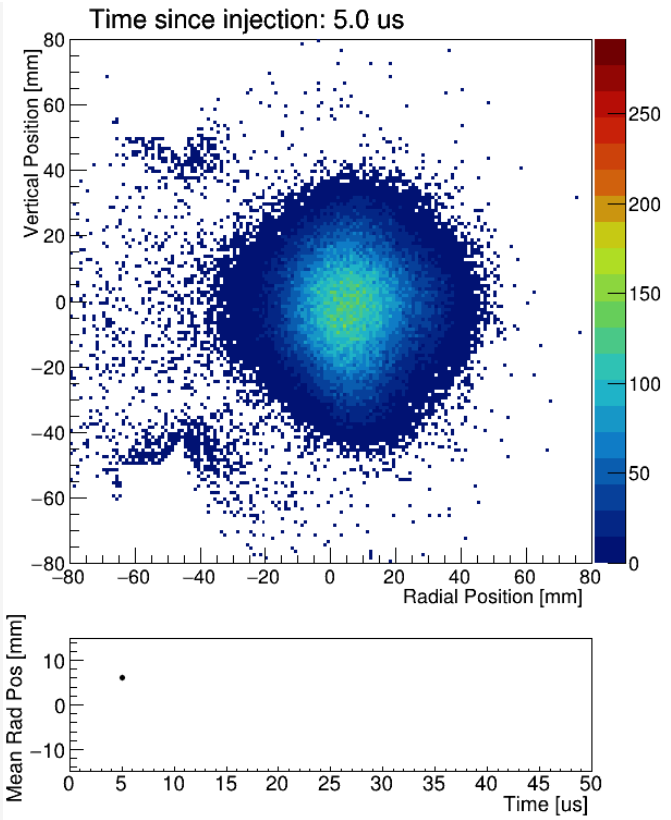
# Coherent Betatron Oscillation (CBO)

- CBO enters the data through coupling between detector acceptance and muon decay position. It **dominated Run-1/2/3 systemactic uncertainty**

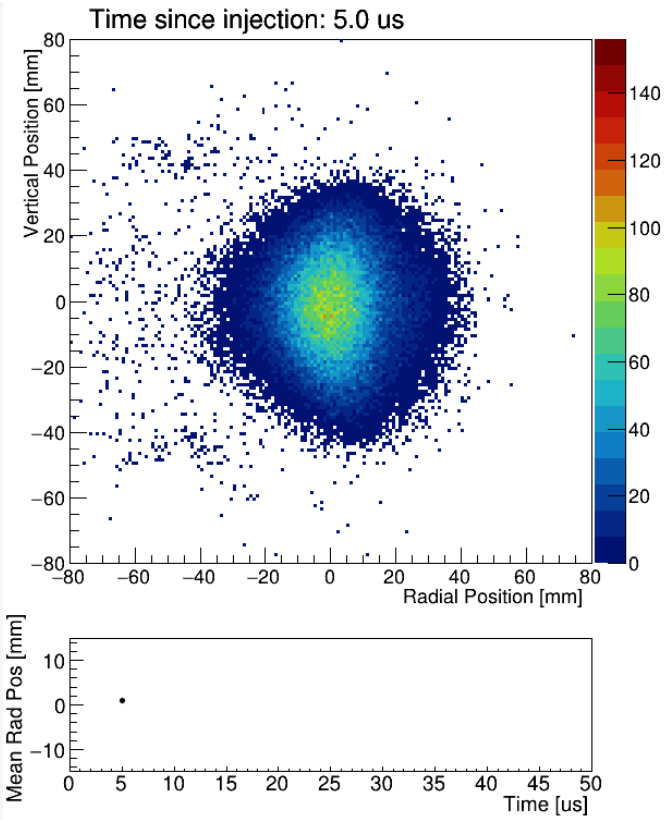
- A radio-frequency (RF) was applied to the electrostatic quadrpole plates to suppress CBO effect in Run-5/6

New!

No Additional RF on ESQ




Additional RF on ESQ



# Coherent Betatron Oscillation (CBO)

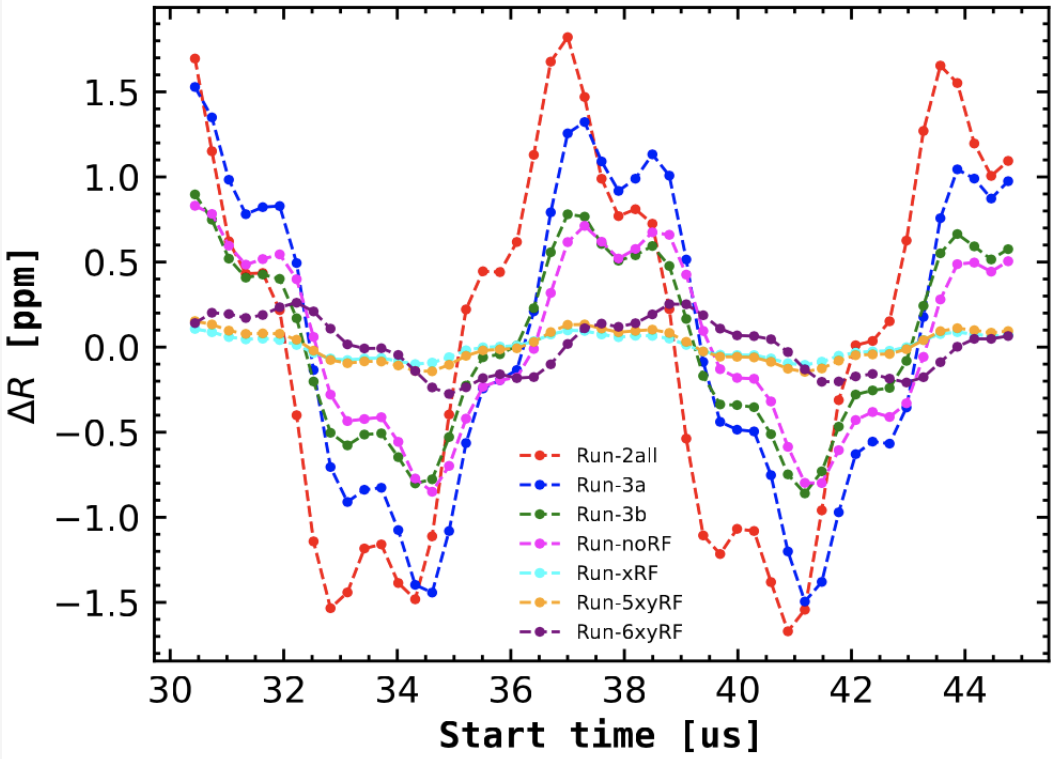
- CBO enters the data through coupling between detector acceptance and muon decay position. It **dominated Run-1/2/3 systemactic uncertainty**

- A radio-frequency (RF) was applied to the electrostatic quadrpole plates to suppress CBO effect in Run-5/6

New!

- The CBO-induced bias: **1.7 ppm (Run-2) → 0.1 ppm (Run-xyRF)**

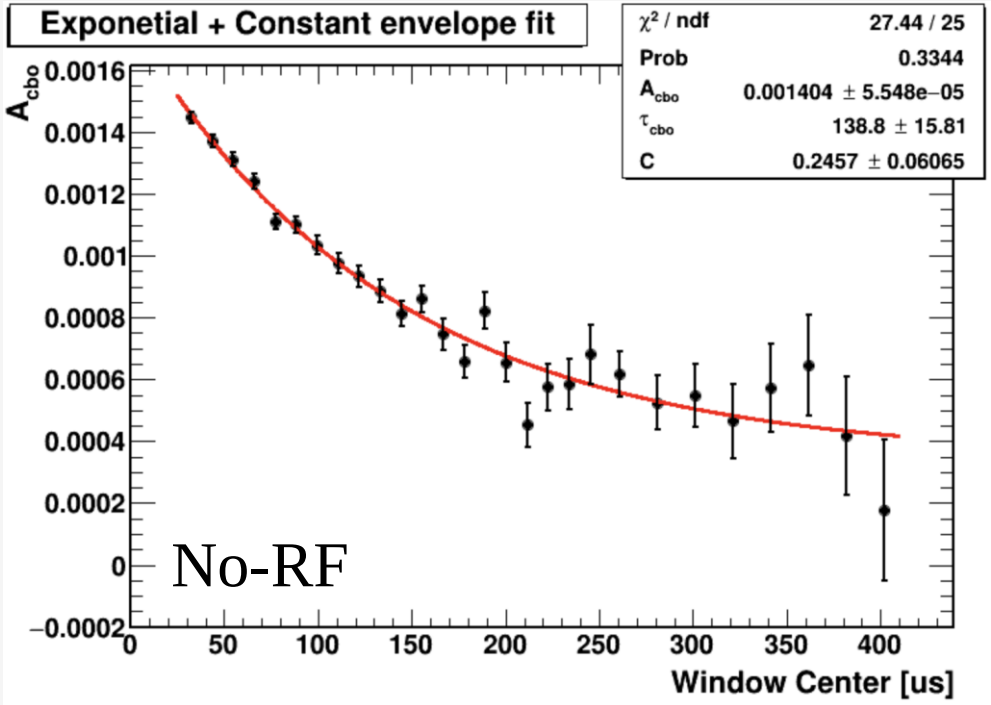
CBO-induced bias as a function of fit start time



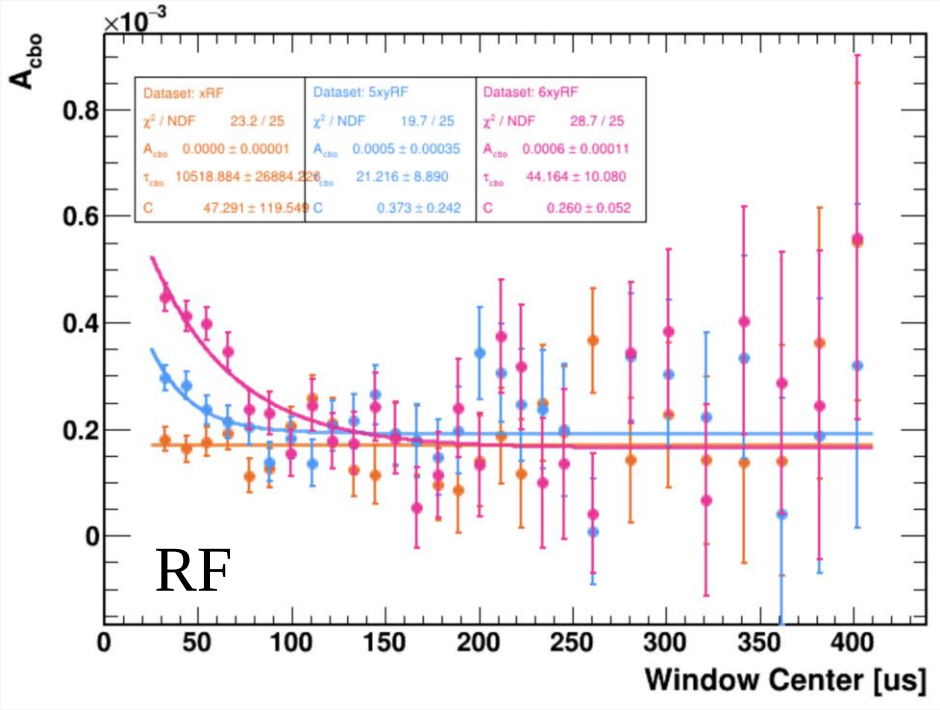
# Coherent Betatron Oscillation (CBO)

- A large CBO (thus a large bias) that can be well modeled, vs. a small CBO (thus a small bias) that is harder to model accurately

CBO separation via sliding window technique

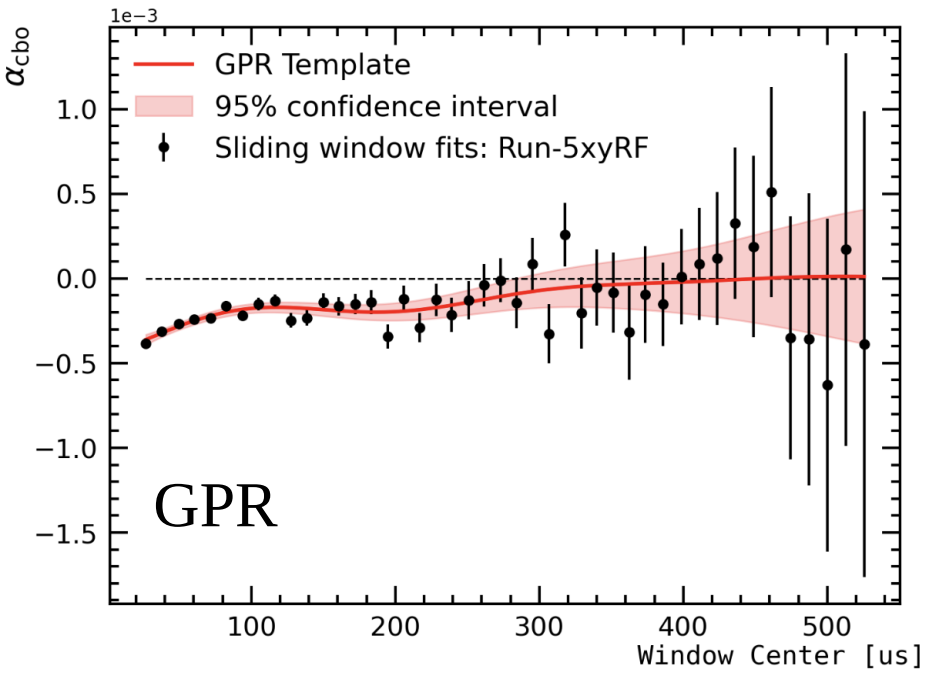


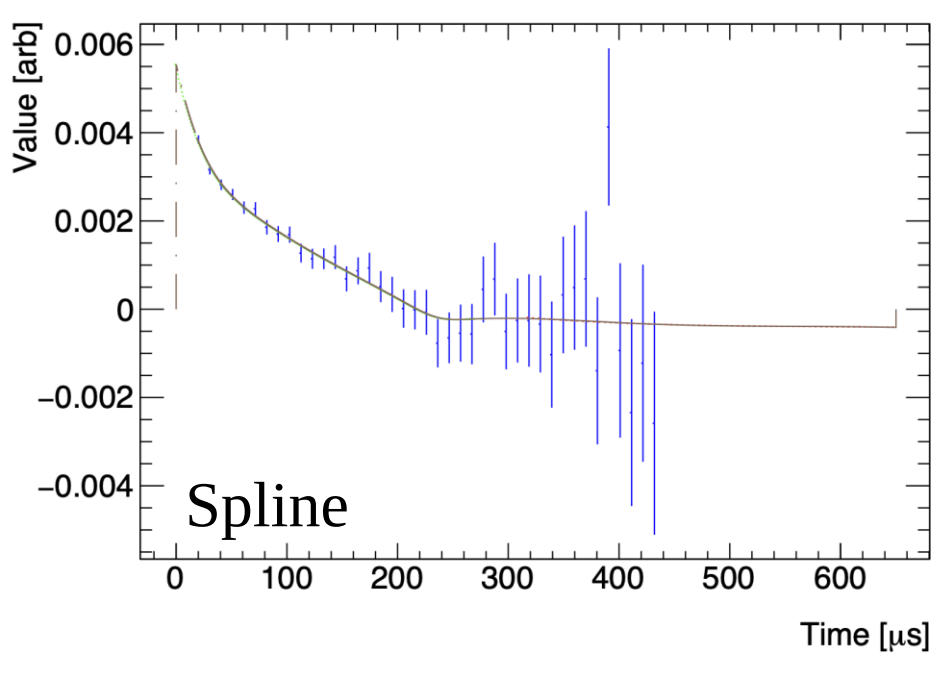

**vs.**



# Coherent Betatron Oscillation (CBO)

- A large CBO (thus a large bias) that can be well modeled, vs. a small CBO (thus a small bias) that is harder to model accurately
- Non-parametric approaches developed: Gaussian Process Regression (GPR), Spline



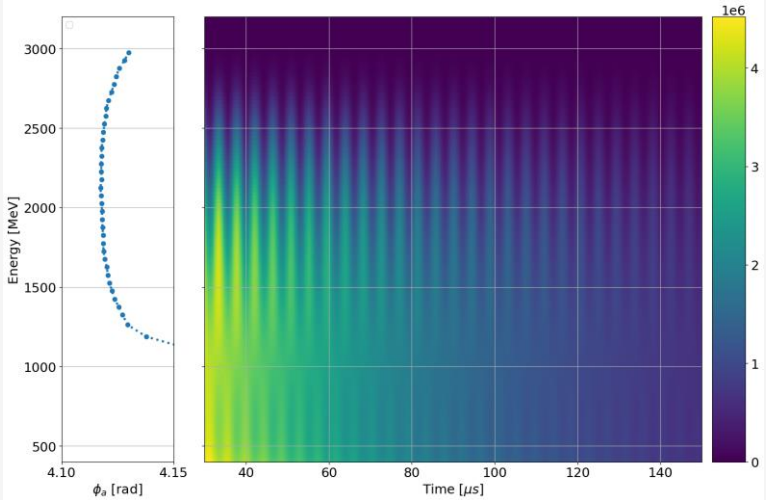

New!

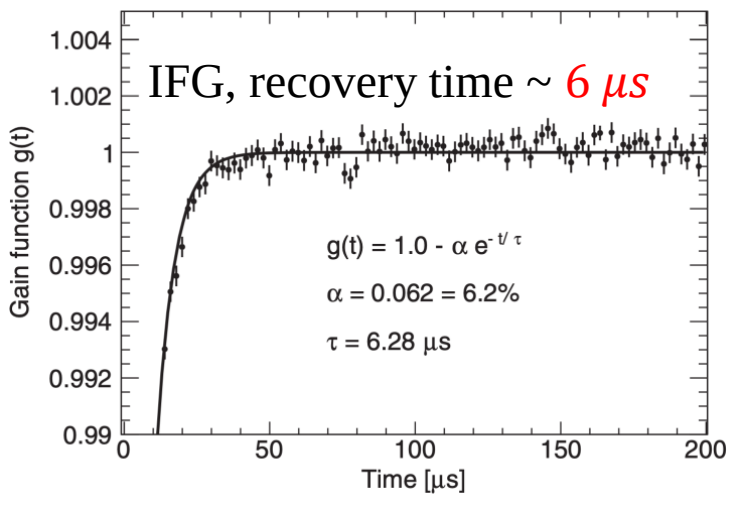
# Residual Slow Effect

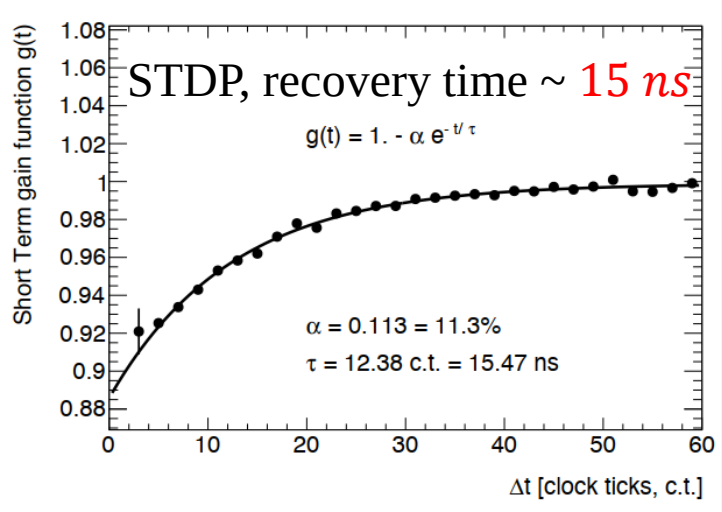
Early-to-late

- Time-dependent phase change leads to a bias: $\omega_a t + \varphi \rightarrow \omega_a t + \varphi(t) = \boxed{(\omega_a + \frac{d\varphi}{dt})} t + \cdots$
- The detector effect because of SiPM gain sag
- Gain Correction in Run-1/2/3:
  - In-Fill Gain (IFG): caused by initial flash
  - Short-Term Double Pulse (STDP): caused by single pulse

Energy-dependent Phase



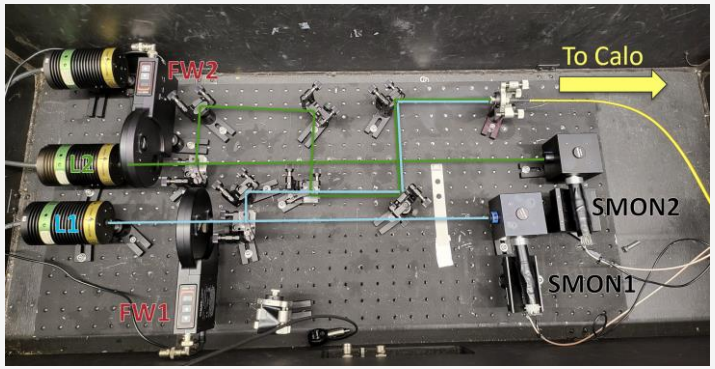



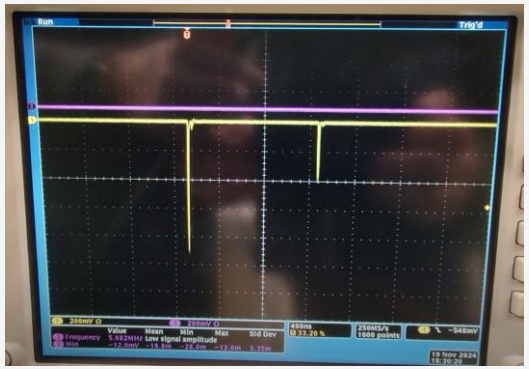



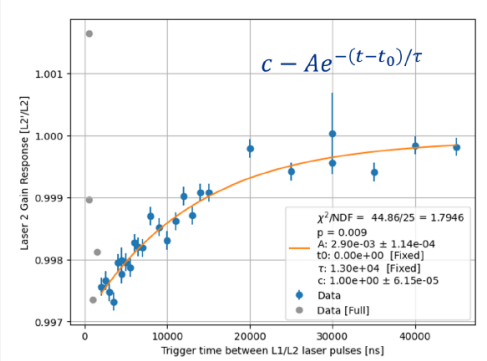
# Residual Slow Effect

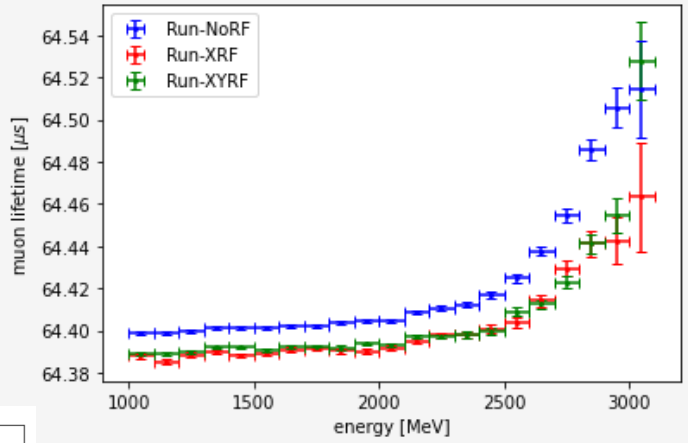
- Still remained in Run-4/5/6
- Identified: Intermediate-Term Double Pulse (ITDP)
  - Similar to STDP, but longer time scale, recovery time ~ $6\ \mu s$
  - Specialized hardware study

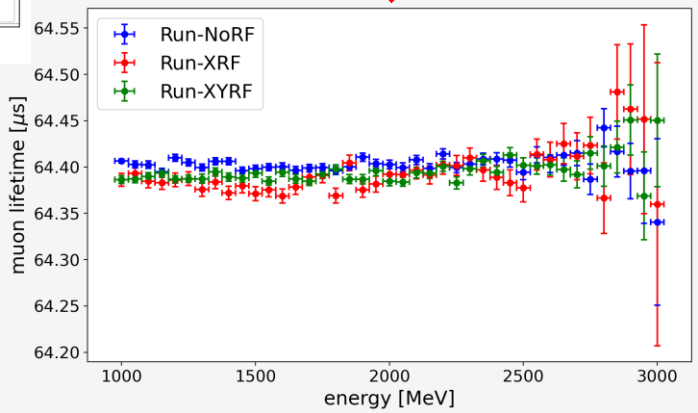
New!






- **20 ppb ~ 40 ppb** effect on $\omega_a$, **10 ppb ~ 20 ppb** uncertainty

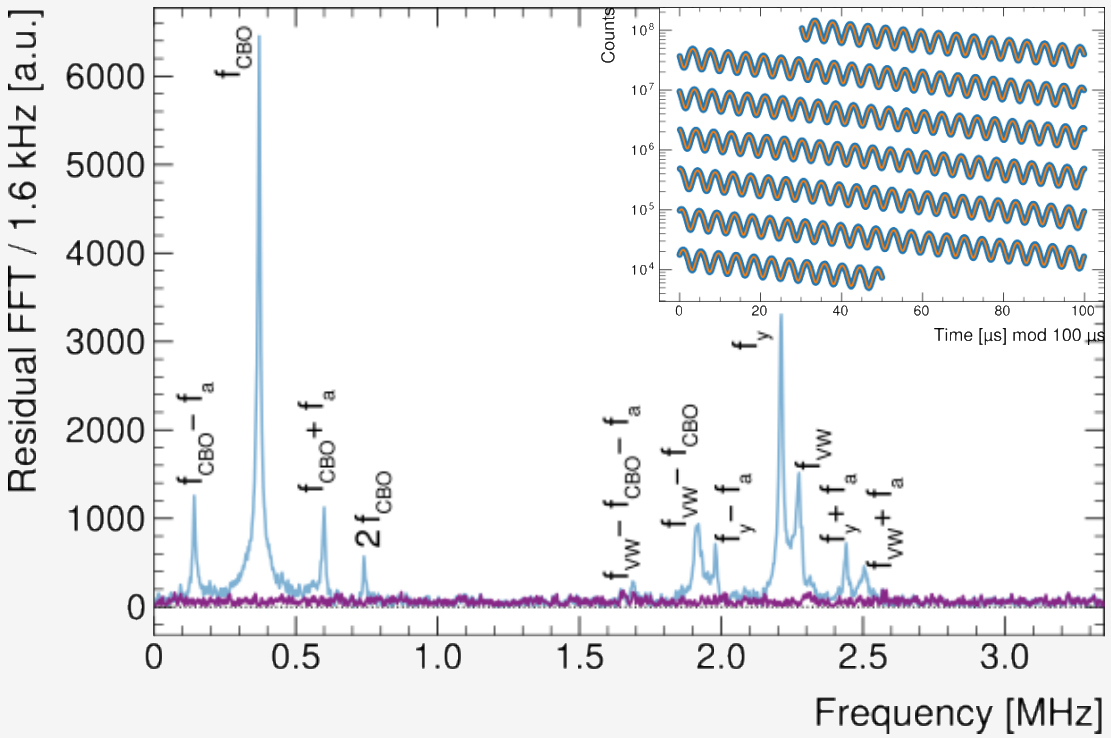
# Analysis Flow

Data Acquisition → Reconstruction → Data Weighting → Wiggle Fitting

- Simple 5-parameters model only captures the exponential decay and $g-2$ oscillation

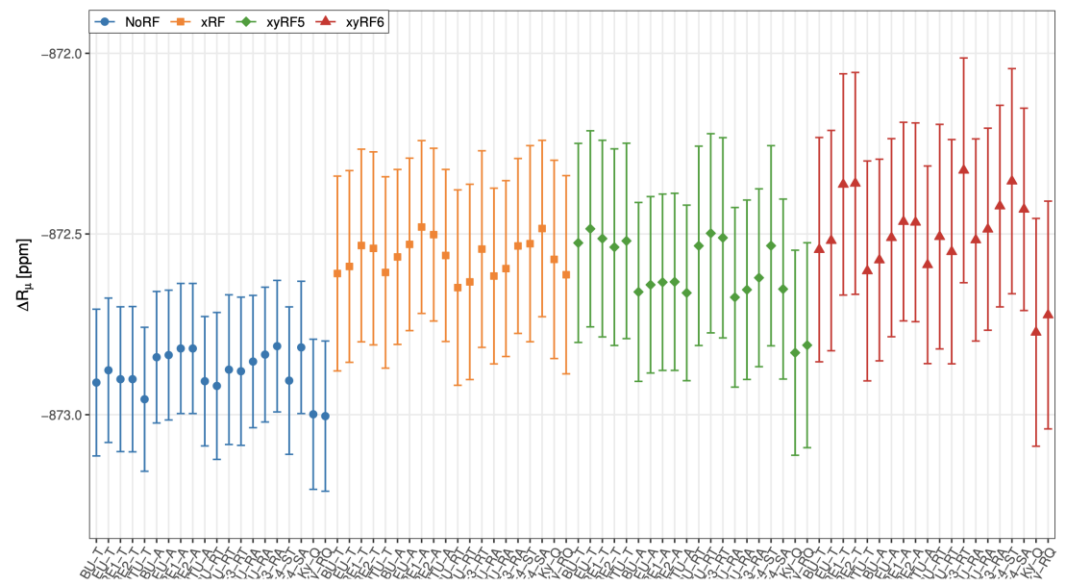
$$N_e(t) = N_0 e^{-t/\tau_\mu}[1 + A\cos(\omega_a t + \varphi)]$$

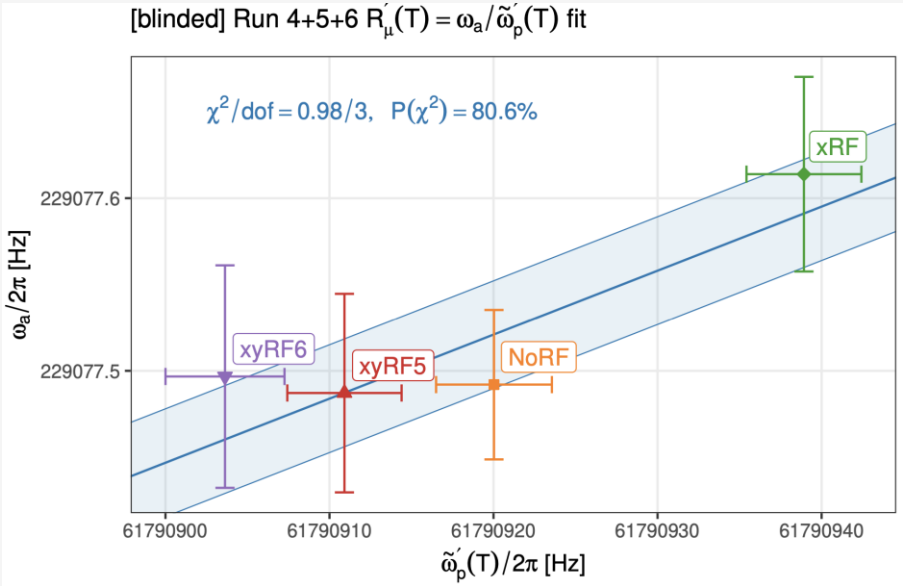
- Must account for **beam oscillations**, **detector effects** and **muon losses**
- The function expands to between 7 and 50 parameters



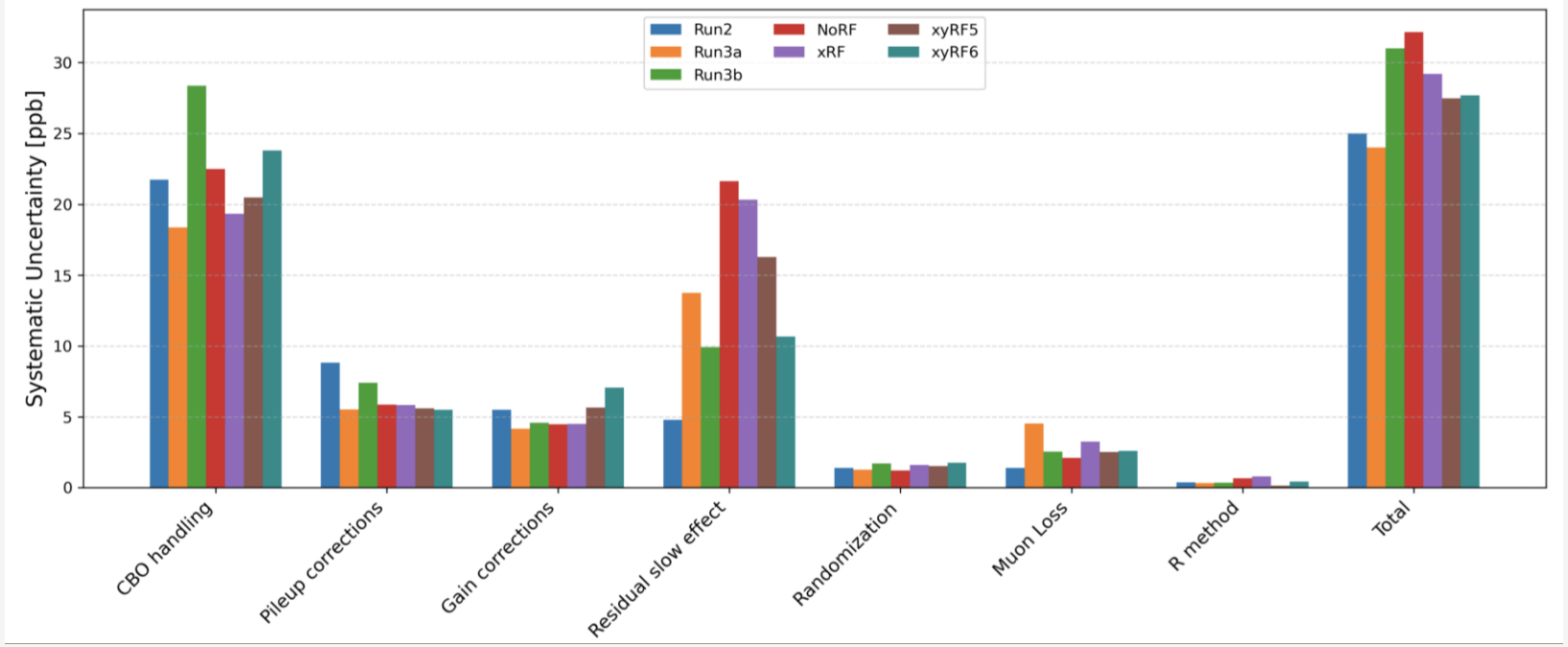
# Analyses Combination

- Acquisition clocks are hardware blinded, analysis is software blinded
- Four major datasets based on the presence or absence of RF: Run-noRF, Run-xRF, Run-5xyRF, and Run-6xyRF
- 5 analysis groups, 20 methods in total
- 7 analyses are used in the average



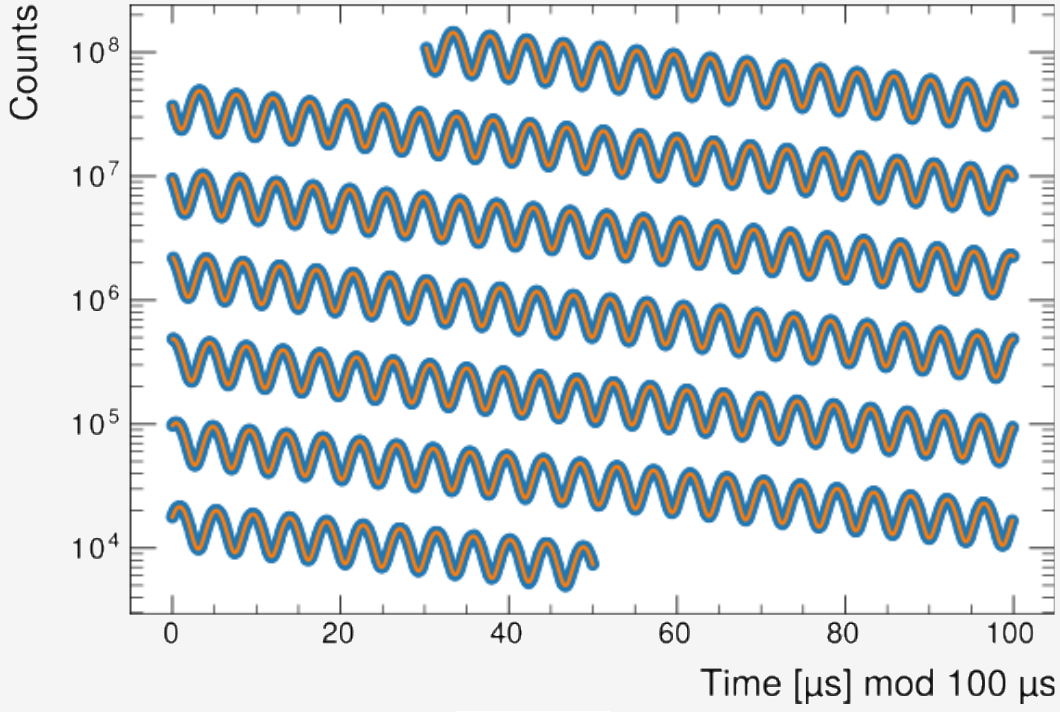
# Systematic Uncertainties

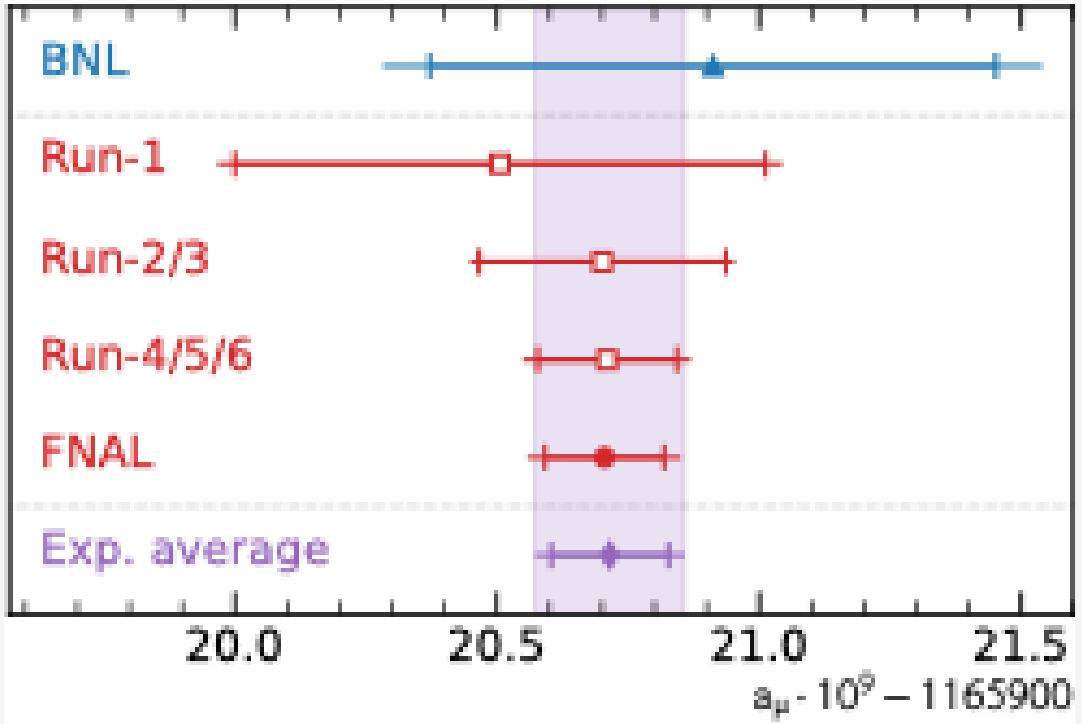



- With respect to Run-2/3:
  - Largest uncertainties are still CBO and Residual Slow Effect
  - Rest of the systematics are stable over datasets
  - Total systematic uncertainty for Run-4/5/6 is **30 ppb** with 114 ppb statistical uncertainty

# Summary

- The most precise determination of $a_\mu$ is given with an unprecedented precision of 124 ppb
- I've presented the analysis flow of $\omega_a$ and improvements on CBO and Residual Slow Effect
- Next, you will hear about magnetic field measurement from **René Reimann**



# THANK YOU FOR YOUR ATTENTION!

Fermilab

李政道研究所
TSUNG-DAO LEE INSTITUTE

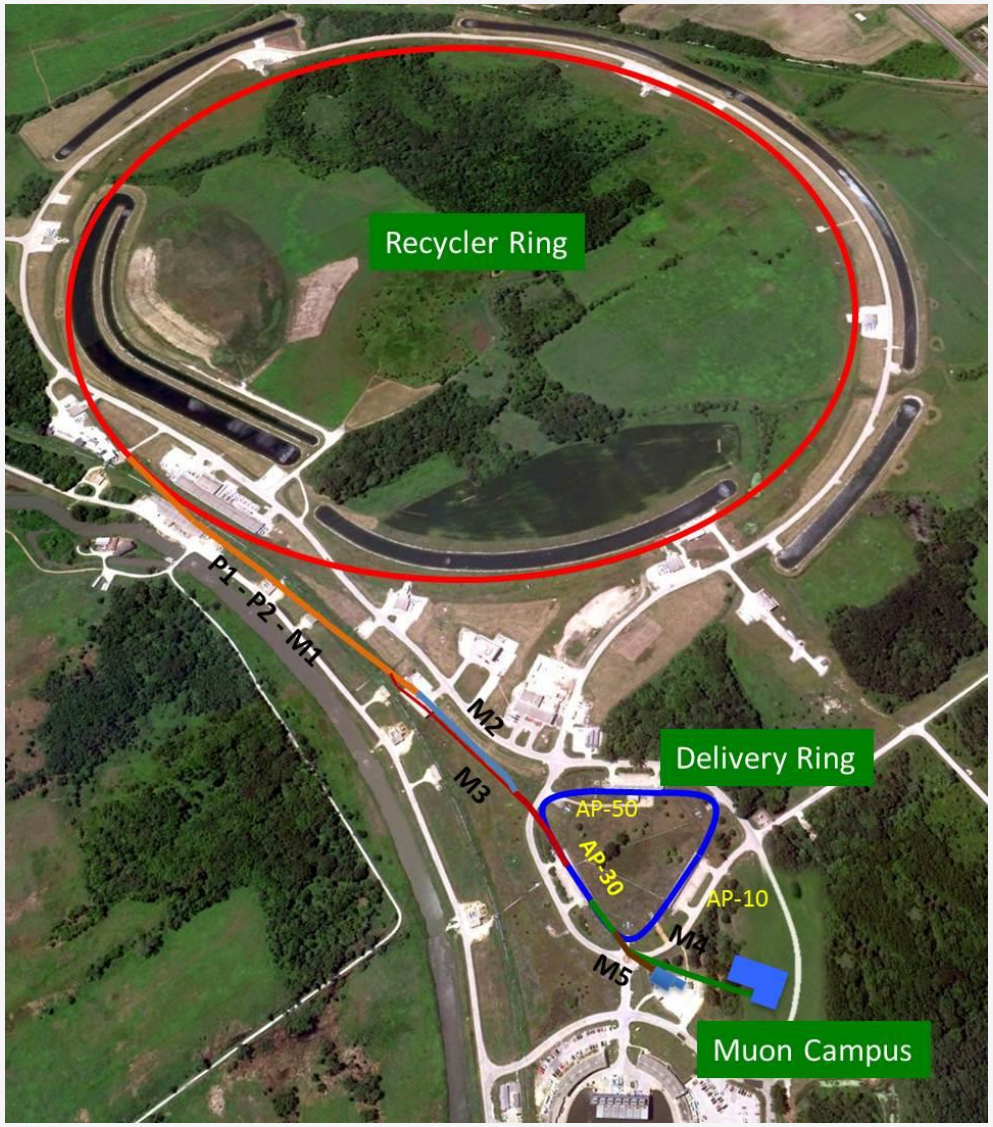
WIN 2025
9–13 June 2025
University of Sussex, Brighton, UK
WiN 25

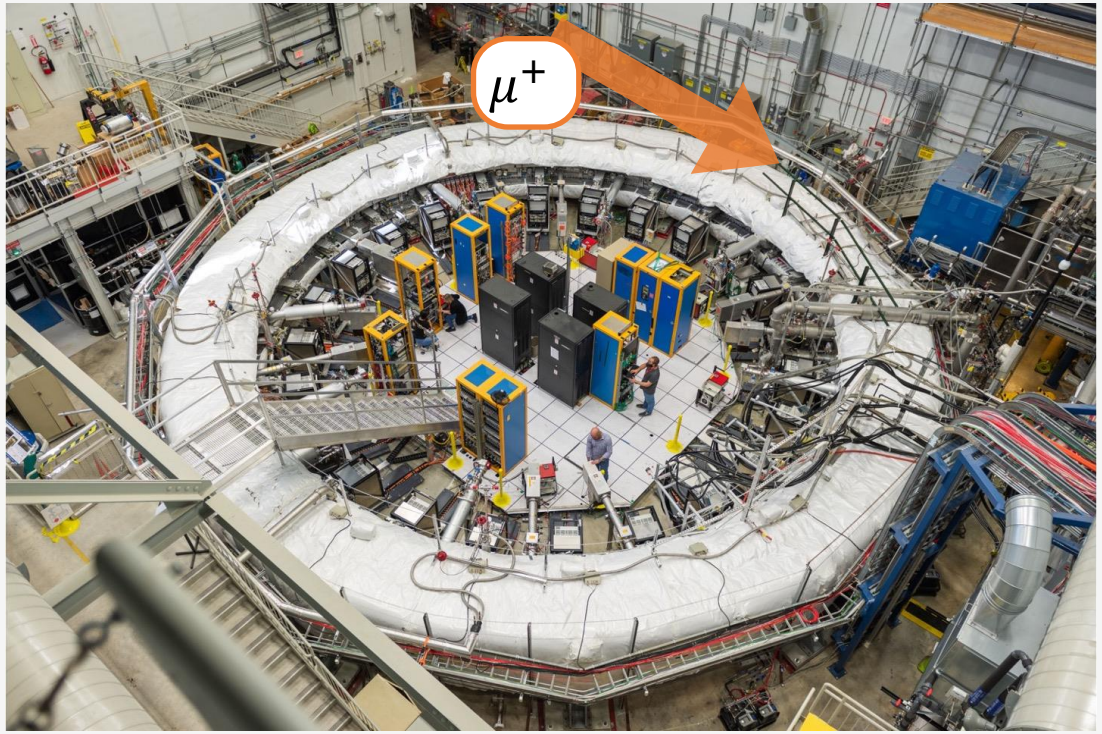
PART

# Backup

TSUNG-DAO LEE INSTITUTE

# Muon g-2 Experiment at Fermilab



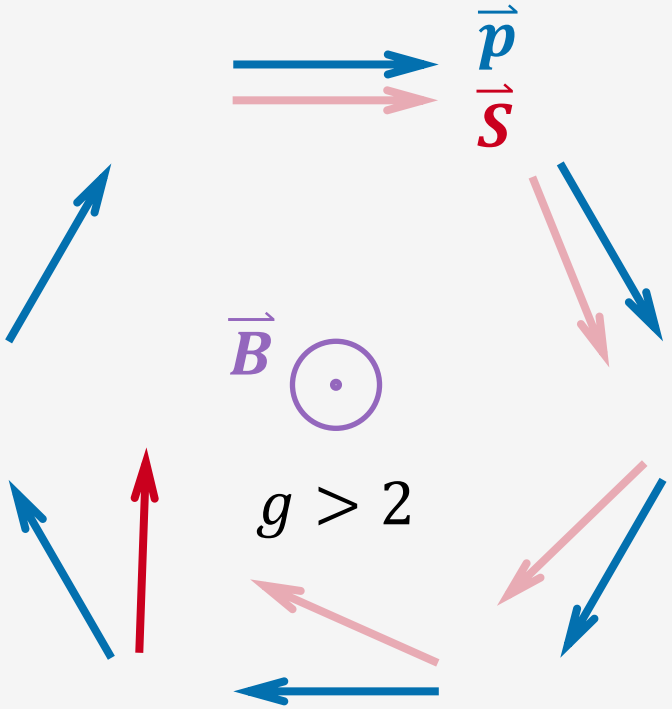




- 8 GeV protons on target for pion production
- Transport line for $\mu$ collection from $\pi \rightarrow \mu + \upsilon_\mu$ decay
- $\mu$ stored in a magnetic storage ring

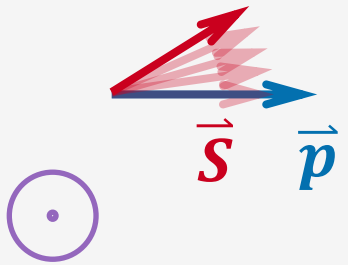
Presented in the previous talk by **Joe Price**

# Spin Precession in a Magnetic Storage Ring

Lab Frame



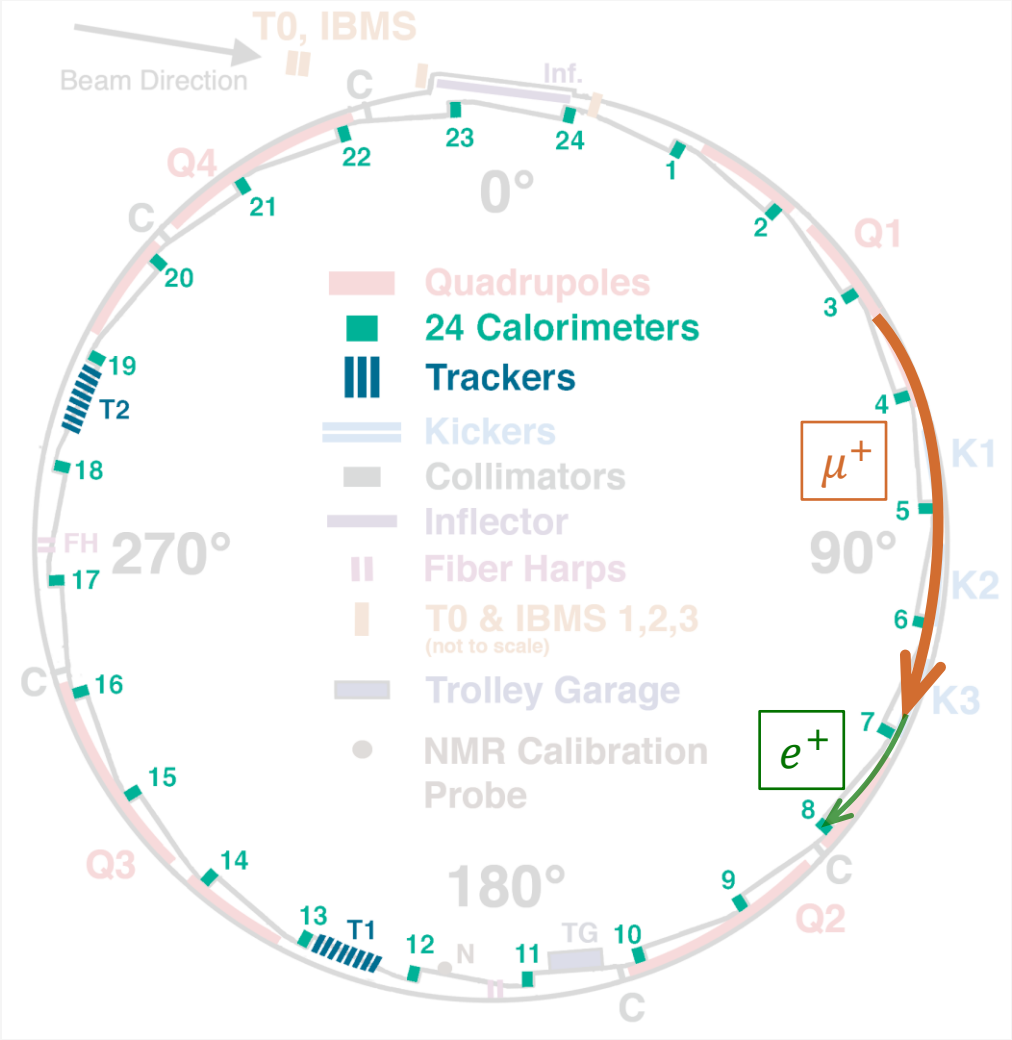

Rest Frame



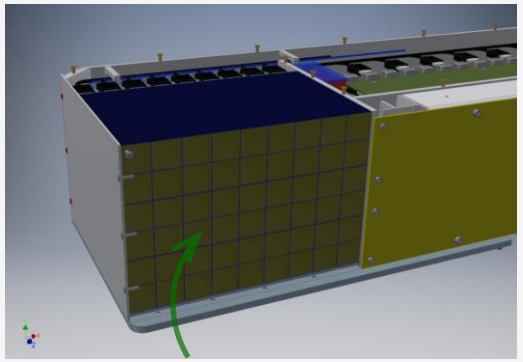

$$\vec{\omega}_a = \vec{\omega}_c - \vec{\omega}_s$$

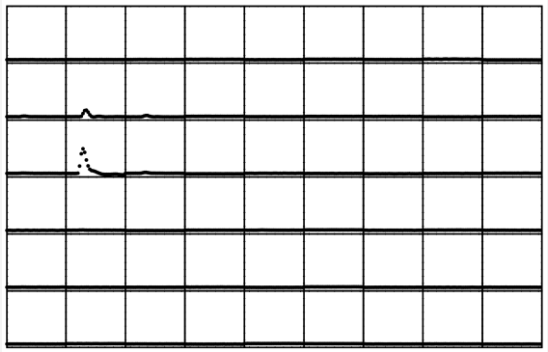
Rate of spin precession with respect to momentum

# Calorimeters




- 24 calorimeters uniformly placed around the inner side of the storage ring
- Each calorimeter has 6x9 $PbF_2$ crystals coupled with SiPMs
- SiPMs collect Cerenkov light from decay positrons
- Extract energy and time from pulse fitting

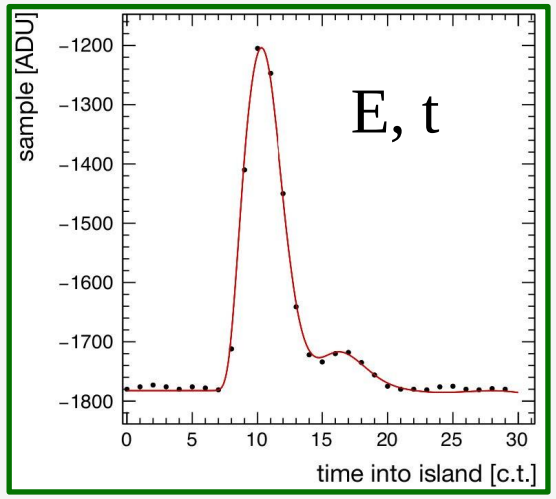


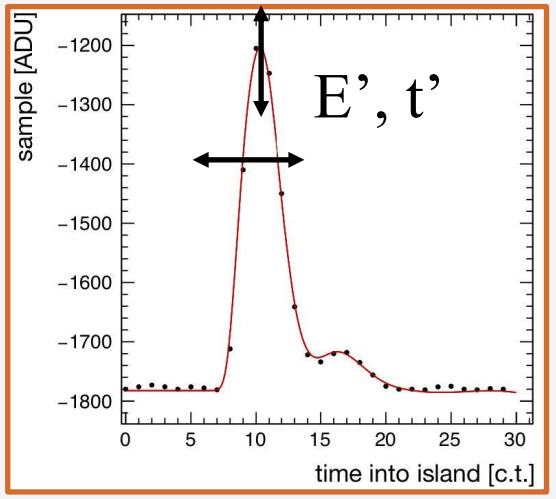


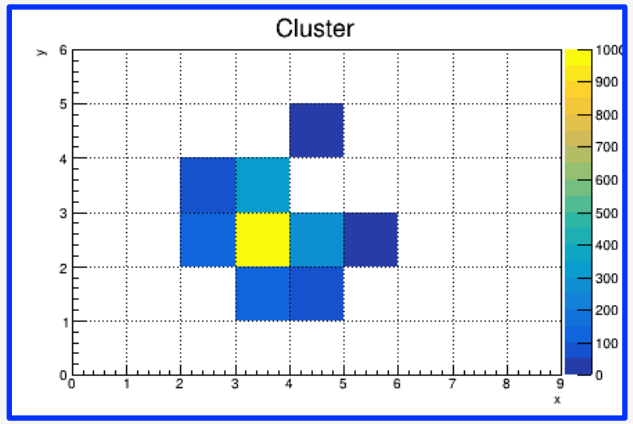
# Analysis Flow

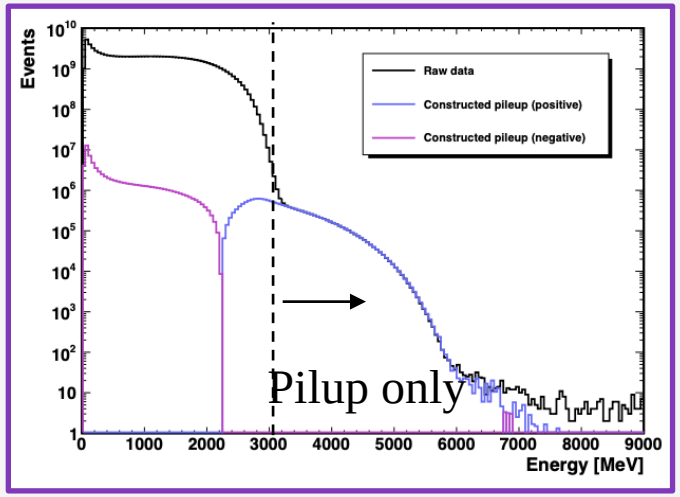
Data Acquisition → Reconstruction → Data Weighting → Wiggle Fitting

Pulse Fitting | Time Correction | Gain Correction | Clustering | Pileup Correction



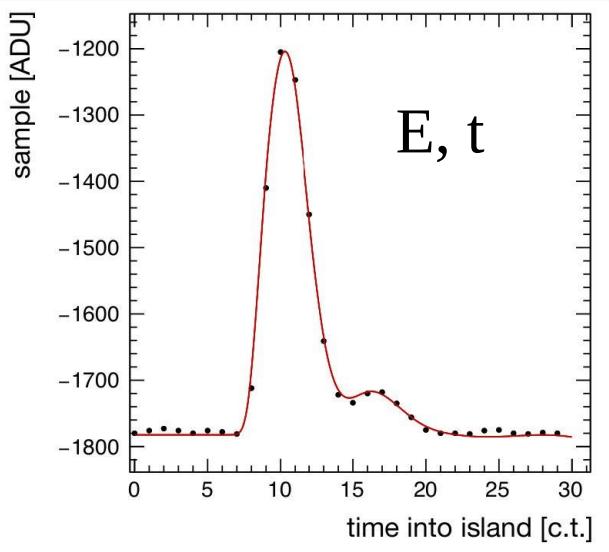



# Analysis Flow

Data Acquisition → Reconstruction → Data Weighting → Wiggle Fitting

Raw Data | Pulse Fitting | Time Correction | Gain Correction | Clustering | Pileup Correction

- Raw ADC pulses are fitted with empirical pulse templates. Each positron generates a pulse in one or more SiPMs
- Two approaches:
  - **Local fitting**: fit each individual crystal pulse separately
  - **Global fitting**: fit multiple crystal traces simultaneously

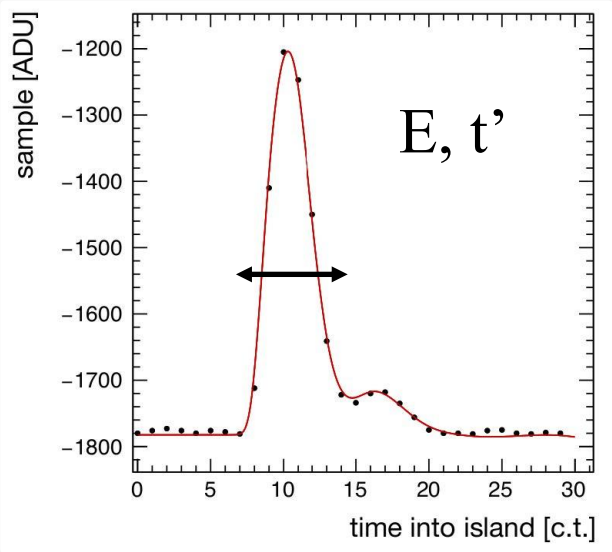


# Analysis Flow

Data Acquisition → Reconstruction → Data Weighting → Wiggle Fitting

Raw Data | Pulse Fitting | Time Correction | Gain Correction | Clustering | Pileup Correction

- 1296 SiPMs are time aligned with a laser pulse sent before beam injection, with ~**50 ps** accuracy

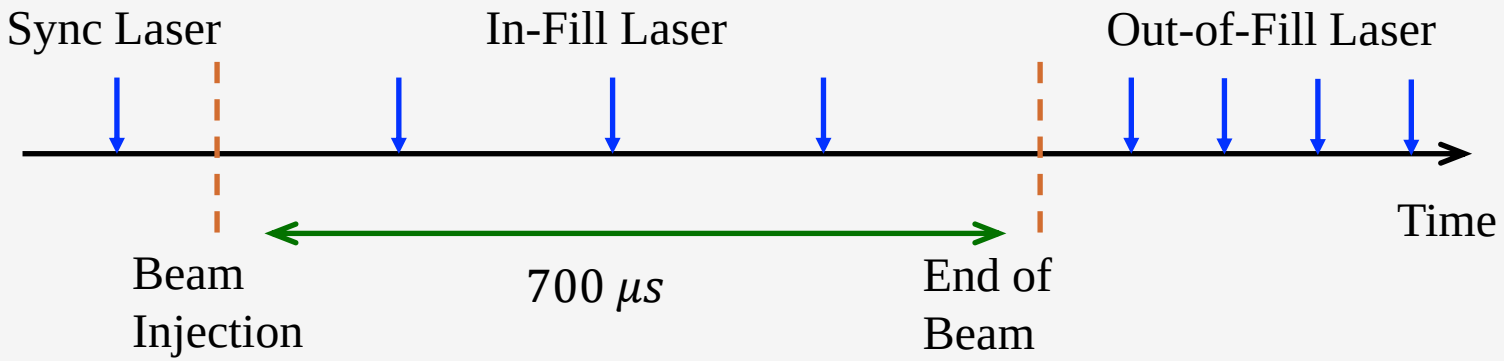


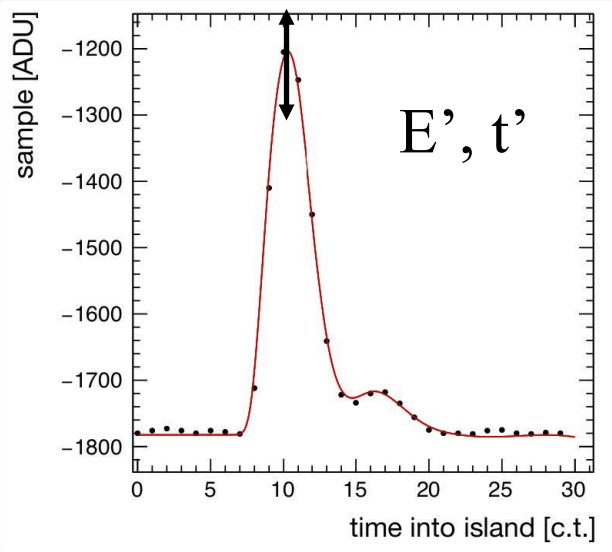
# Analysis Flow

Data Acquisition → Reconstruction → Data Weighting → Wiggle Fitting

Raw Data | Pulse Fitting | Time Correction | Gain Correction | Clustering | Pileup Correction

- 1296 SiPMs gain are calibrated by Laser Calibration System in different time scale



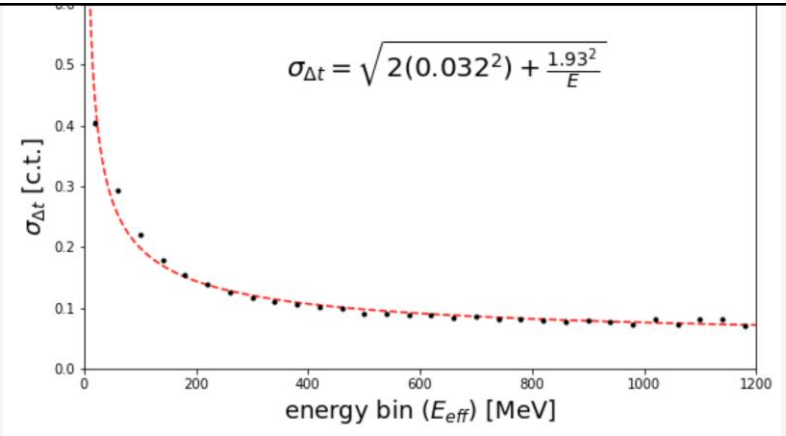
# Analysis Flow

Data Acquisition → Reconstruction → Data Weighting → Wiggle Fitting

Raw Data | Pulse Fitting | Time Correction | Gain Correction | Clustering | Pileup Correction

- Cluster SiPM pulses into individual positron candidate for Local approach
  - Goal: reduce the dead-time of clusters, thus reduce pileup events
  - Considered energy-dependent resolution
  - Reduced the level of pileup by a factor of four

Time resolution as a function of cluster energy

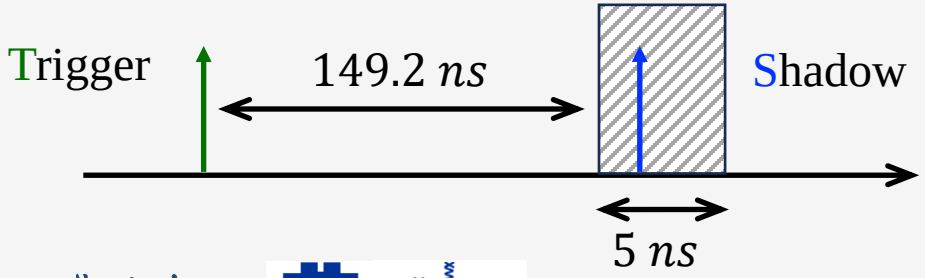


# Analysis Flow

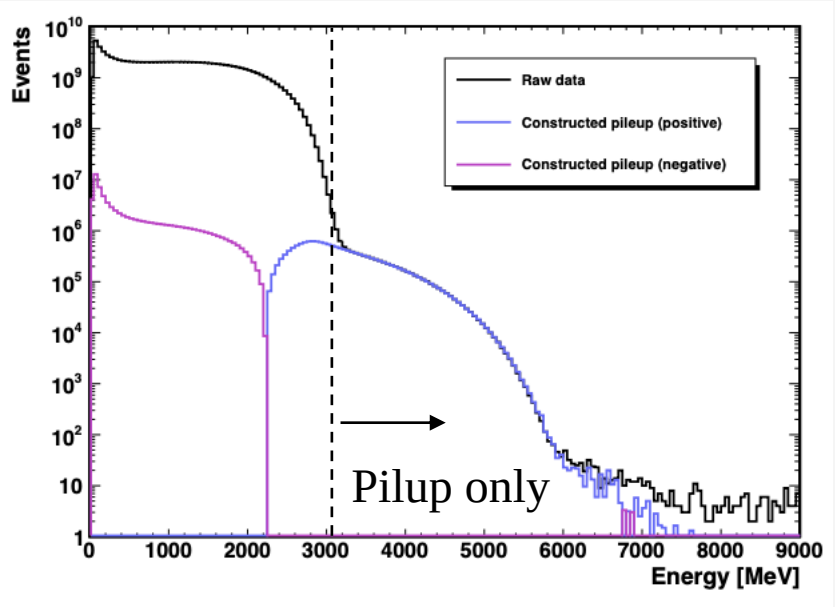
Data Acquisition → Reconstruction → Data Weighting → Wiggle Fitting

Raw Data | Pulse Fitting | Time Correction | Gain Correction | Clustering | Pileup Correction

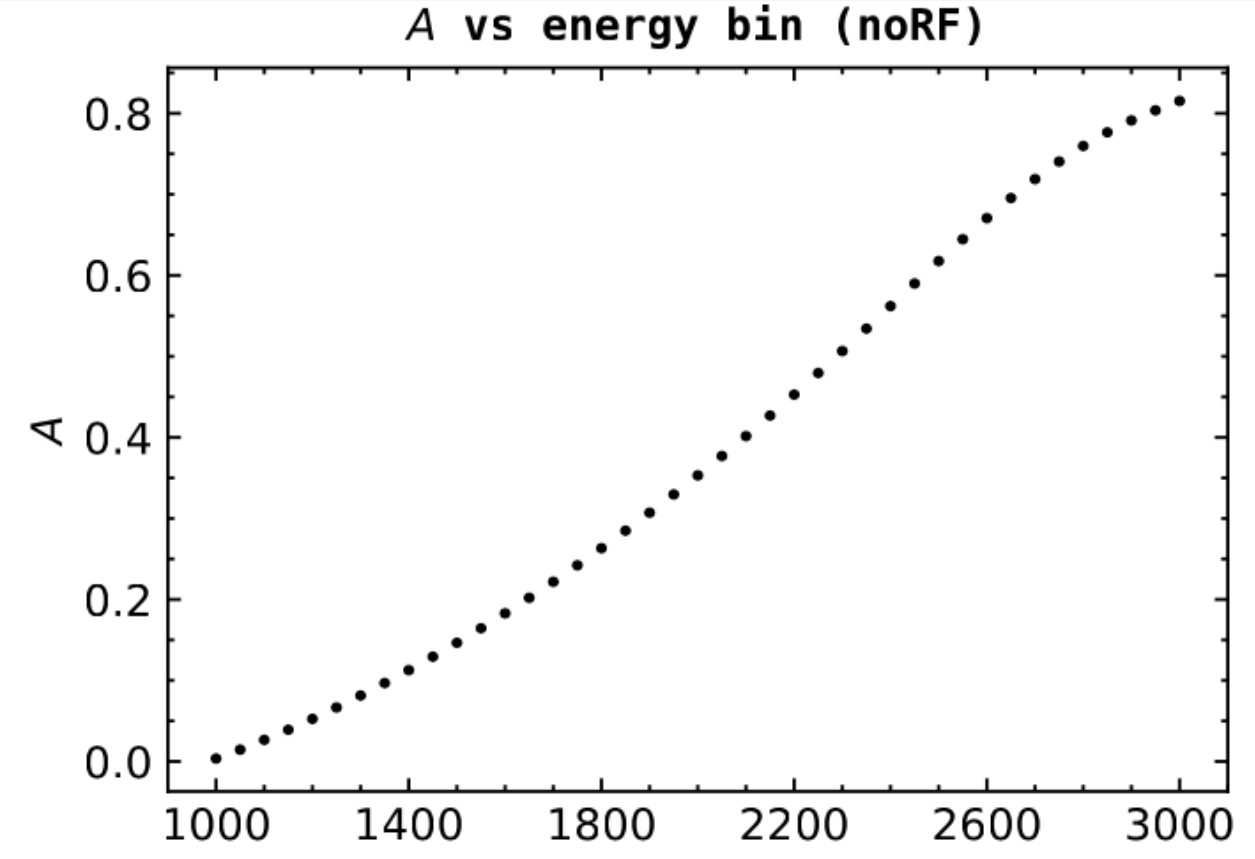
- Pileup occurs when two or more positrons arrive too close in time/space
  - Empirically construct pileup spectrum from data




P(ileup) = T+S



# A-method Weights

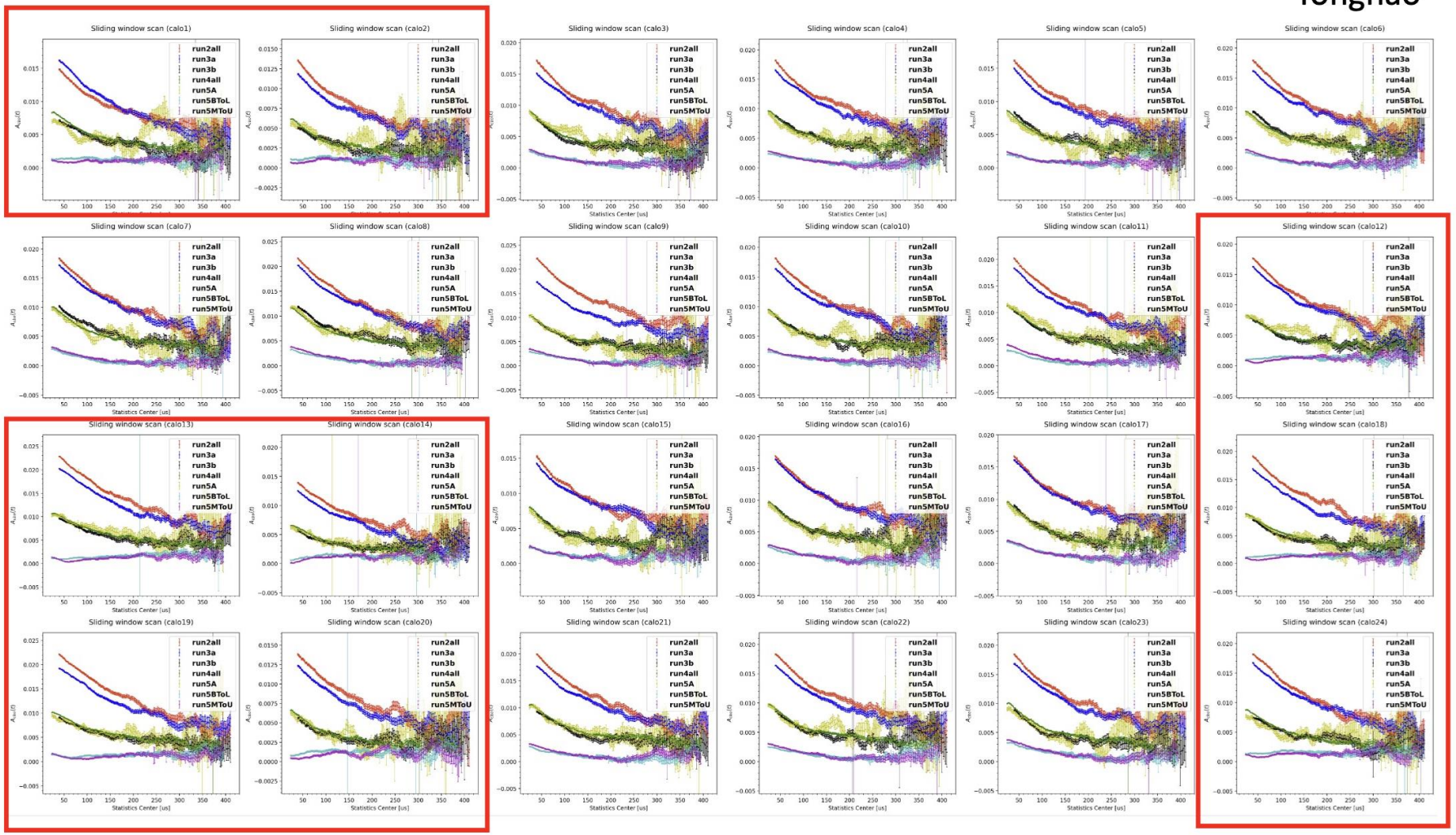


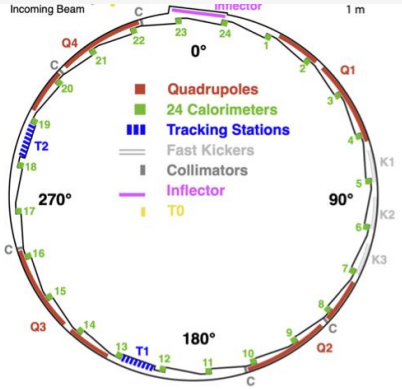

$$\frac{\sigma_{\omega_a}}{\omega_a} = \sqrt{\frac{2 < w(E)^2 >}{N < w(E)A >^2 \gamma^2 \tau^2 \omega_a^2}}$$

Optimal: $w(E) = A(E)$

# Sliding Window for Individual Calorimeter

Yonghao



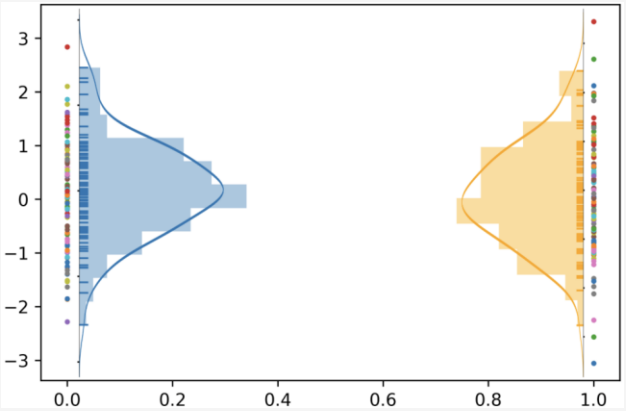



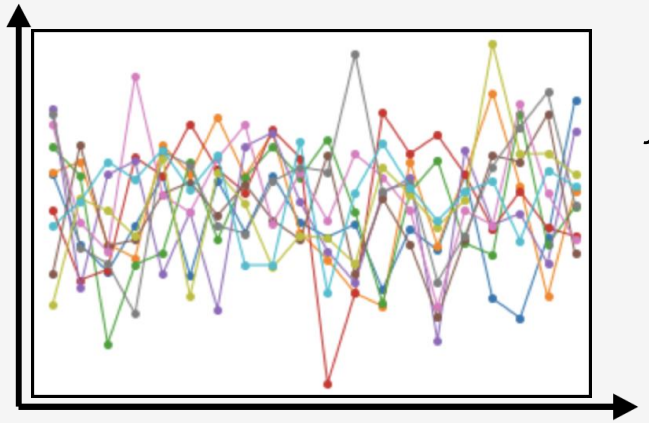

- Inflector
  - 24,1, 2
- Quad1
  - 3,4
- Kicker
  - 5,6,7
- Quad2
  - 8,9,10, 11
- Station12
  - 12,13, 14
- Quad3
  - 15,16, 17
- Station18
  - 18,19, 20
- Quad4
  - 21,22, 23

11

# Intro to GPR

- A Gaussian Process is a collection of random variables (indexed by continuous time or space), of which, any finite number have a joint Gaussian distribution



$$f(x) \sim G(\mu(x), K(x, x'))$$

- Regression is described as given some observed data (training dataset), finding a function that represents the data as close as possible, then using the function to make predictions at new data points

- A GP can help to make predictions, by inputing the prior assumption and observed data, then generating posterior prediction

# GPR

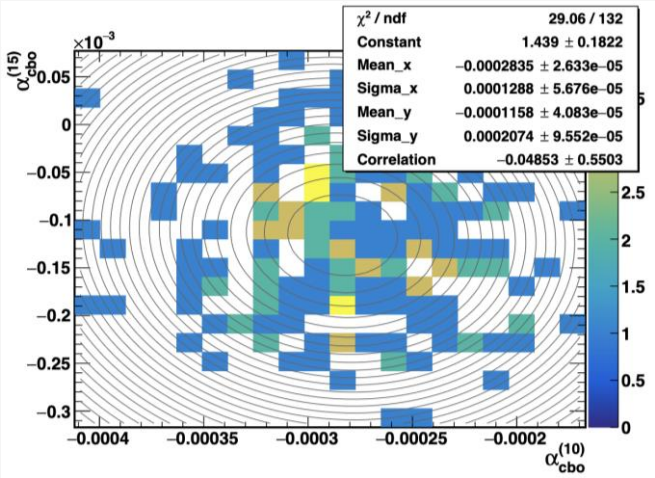
$$N_{CBO}(t) = 1 - A_{CBO}(t)\cos(\omega_{CBO}(t)t + \phi_{CBO})$$

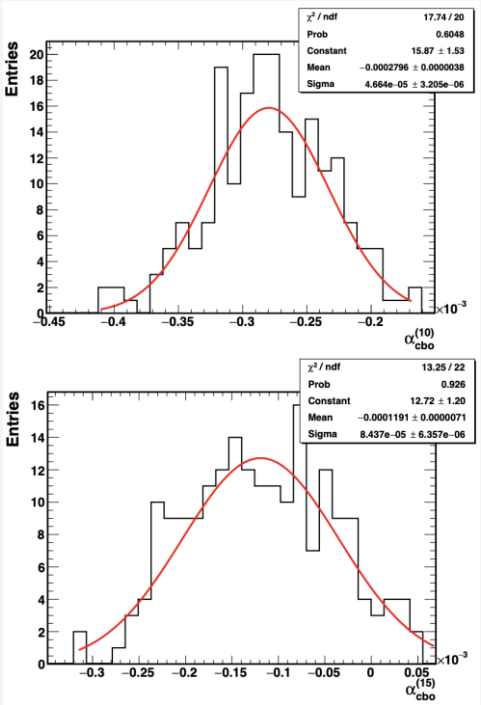
- Alpha-beta representation: model $A(t)$ and $\omega_{CBO}(t)$ ➔ model $\alpha(t)$ and $\beta(t)$

$$N_{CBO}(t) = 1 - [\alpha_{CBO}(t)\cos(\omega_{CBO}t) + \beta_{CBO}(t)\sin(\omega_{CBO}t)]$$

- A Gaussian Process is a collection of random variables (indexed by continuous time or space), of which, any finite number have a joint Gaussian distribution.

- 200 bootstrap wiggle plots provide 200 times sliding window fits. Each specific sliding window point has a Gaussian distribution, and joint distribution between points follow a joint Gaussian distribution.

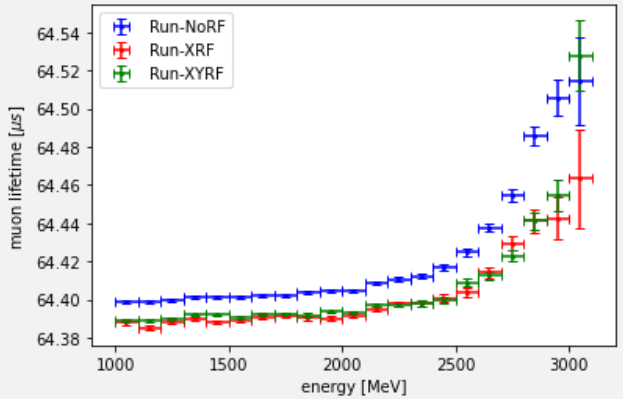


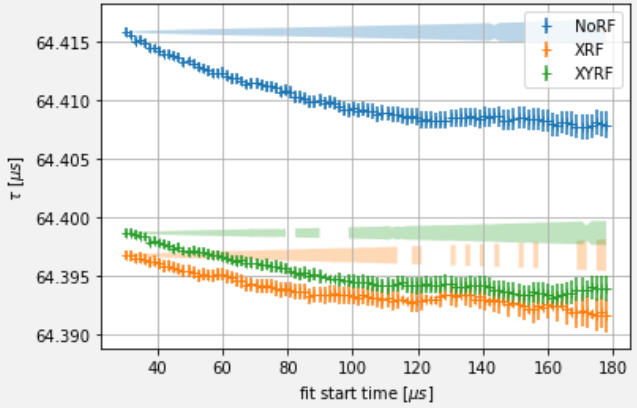



# Residual Slow Effect



❑ **Evidence of residual slow term**:
The fitted muon lifetime will drift with energy and fit start time

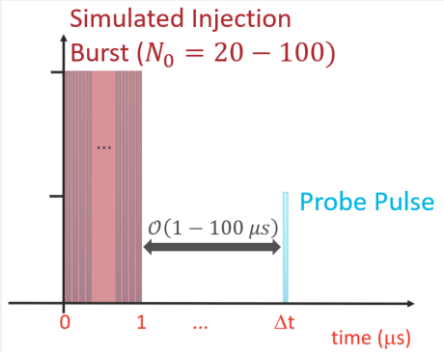


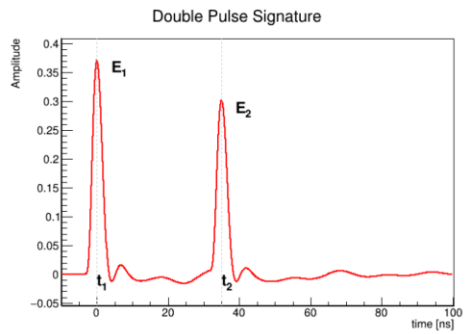

❑ In Run-4/5/6, we found that most residual slow term are caused by the **Intermediate-Term Double Pulse (ITDP effect).**

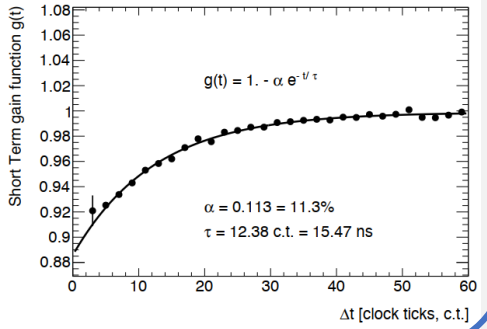
**What we have:**

- **In-fill gain (IFG)** effect: caused by initial flash, recovery time ~6 us



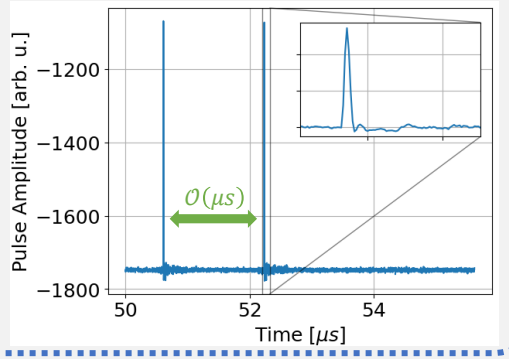

- **Short-term double pulse (STDP)** effect: caused by single pulse, recovery time ~15 ns




**What we miss:**

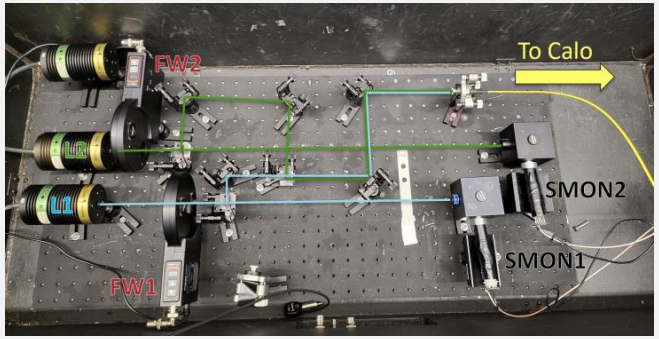
- **Intermediate term double pulse (ITDP)** effect: caused by single pulse, recovery time ~6 us (same as IFG)

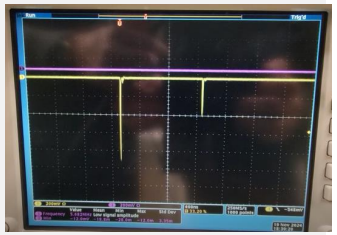


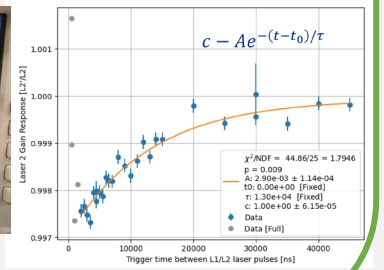
# Residual Slow Effect

## ❑ Validation and measurement of residual slow effect

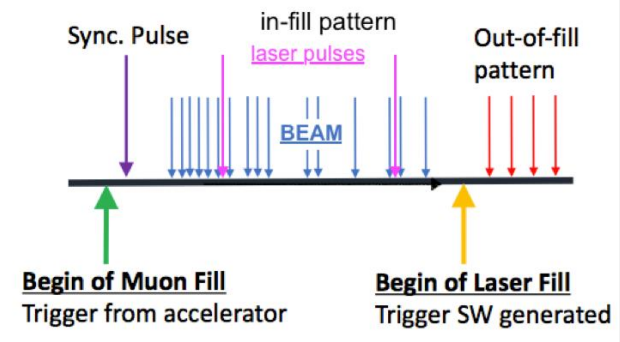
- **Specialized hardware study**
  for measuring the ITDP effect (gain sag amplitude and recovery time).

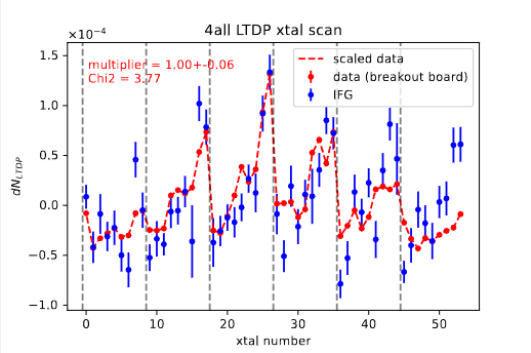





- **Laser gain monitor data**
  Use existed in-fill laser data to validate the hardware measurement result




## ❑ Systematics uncertainty

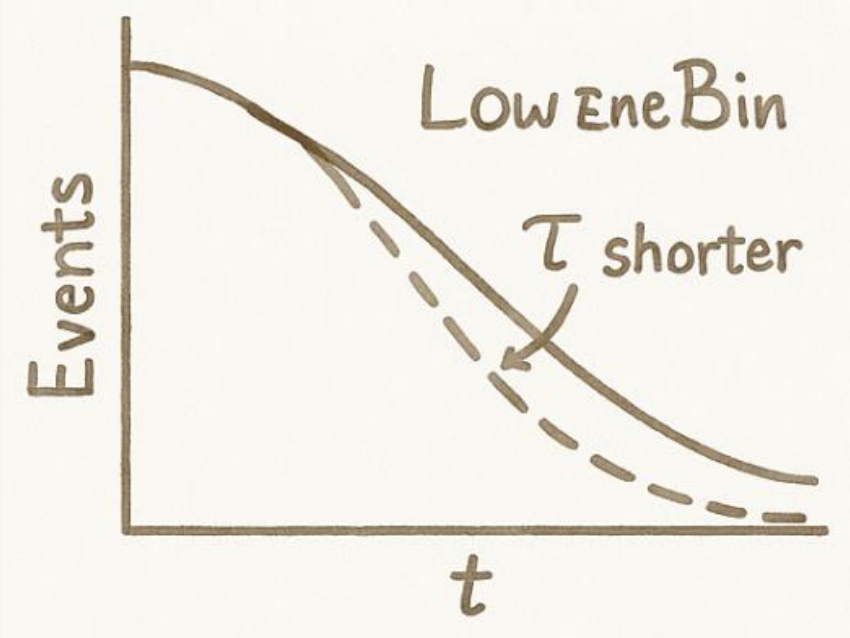
| Category | Subcategory | $\delta R$ [ppb] — RW (A-Method) | | | |
|---|---|---|---|---|---|
| | | NoRF | xRF | xyRF-5 | xyRF-6 |
| ITDP | Lifetime uncertainty | 0.2 | 0.3 | 0.3 | 0.2 |
| | UW Amplitude range | 11.4 | 11.2 | 10.5 | 6.0 |
| | UW Energy Scale | 6.1 | 4.9 | 3.2 | 3.6 |
| | $p_0$ average vs individual | 1.0 | 0.9 | 0.3 | 0.1 |
| Remaining Slow Effect | Phase of the remaining effect | 9.6 | 9.6 | 9.3 | 5.5 |
| Reconstruction term | $N_A$ amplitude | 1.6 | 0.8 | 1 | 0.3 |
| Application | Cluster vs. Histogram | 4.9 | 4.1 | 2.3 | 0.9 |
| | Total | 20.6 | 19.2 | 16.8 | 11.0 |

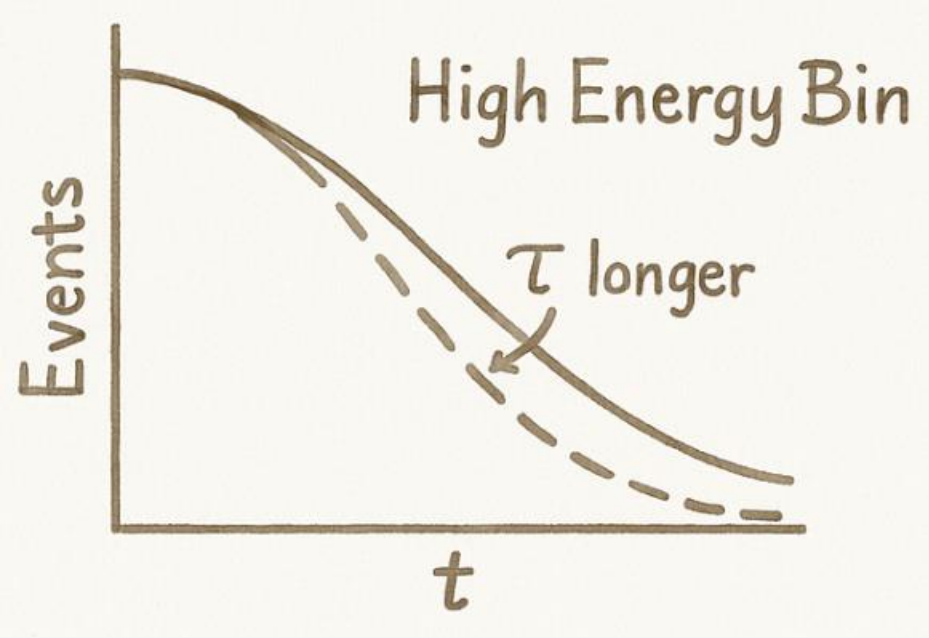
| dataset | ΔR (correction on/off, in ppb) | Systematics (in ppb) |
|---|---|---|
| noRF | 38.5 | **20.6** |
| xRF | 38.7 | **19.2** |
| xyRF-5 | 35.3 | **16.8** |
| xyRF-6 | 27.3 | **11.0** |

- Summary of residual slow term correction effect on $\omega_a$ and systematics uncertainty.

# Dependency on Energy

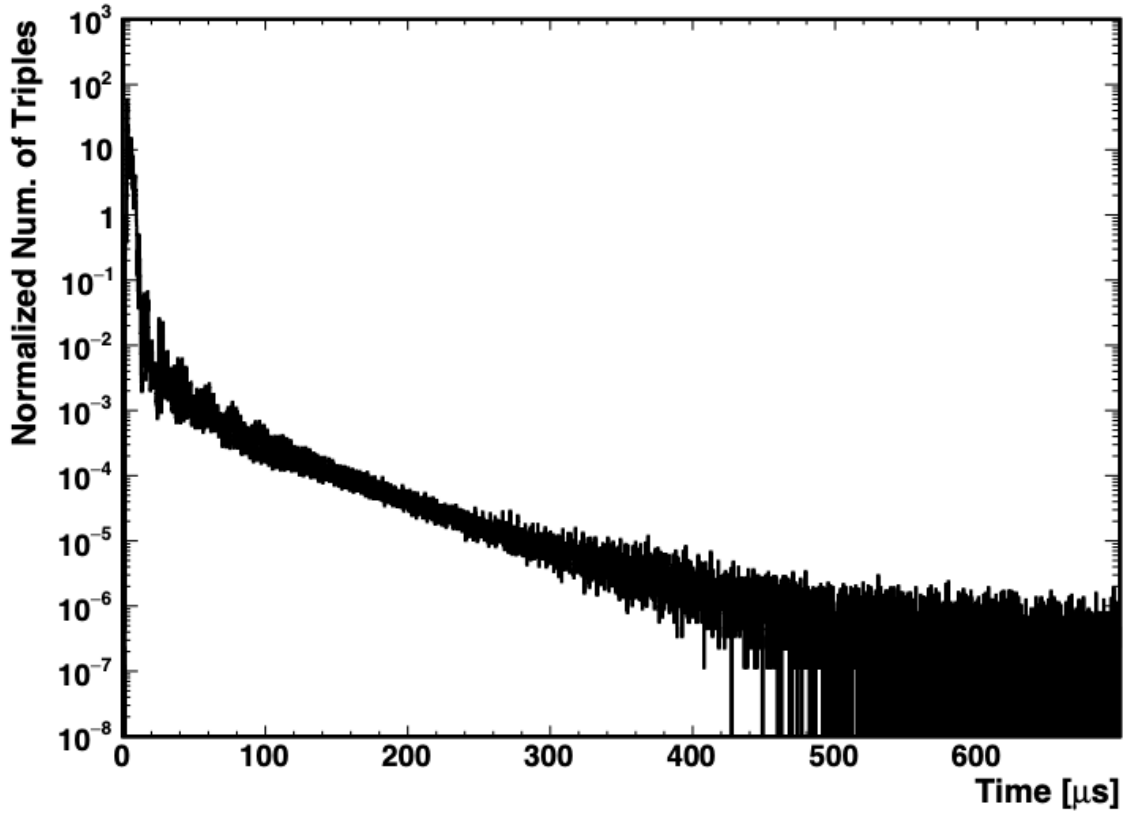
- Gain sag mostly occurs at early-time
- High energy events are underestimated: low energy event becomes more, high energy event becomes fewer



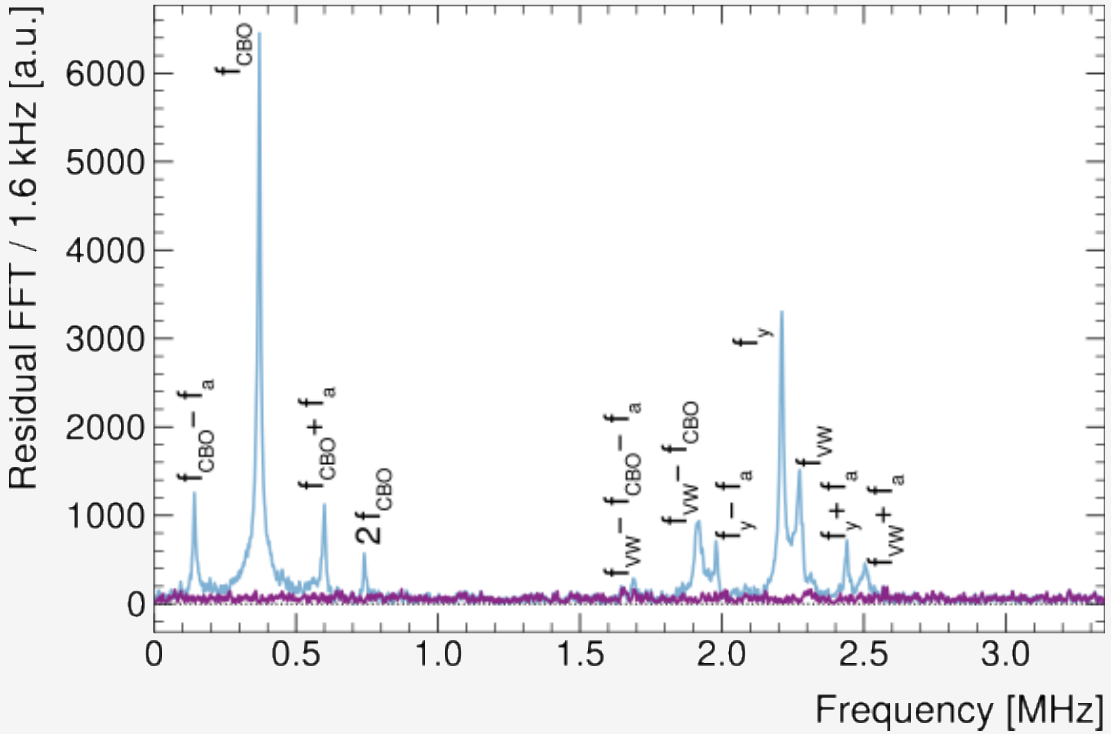
# Muon Losses

- Muon Losses: a fraction of muon drifts out of storage ring over time and hit multiple calorimeters

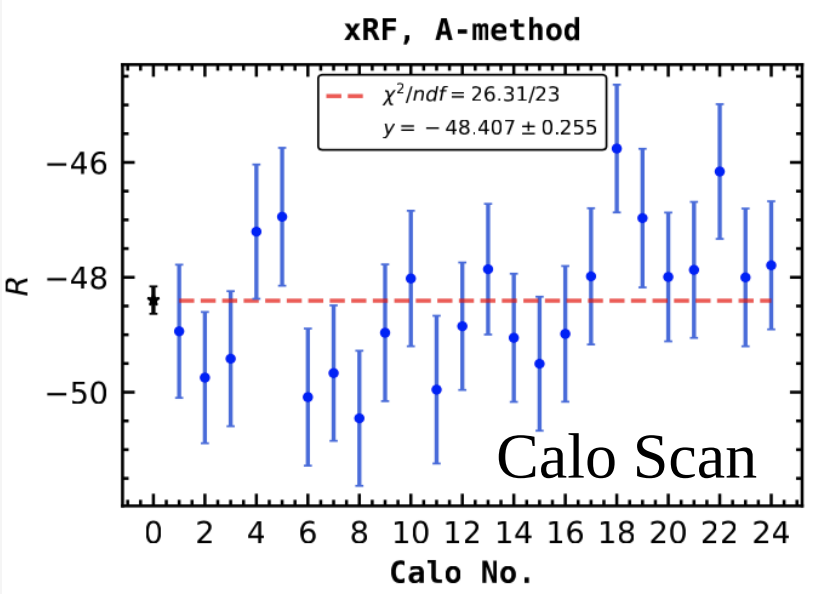


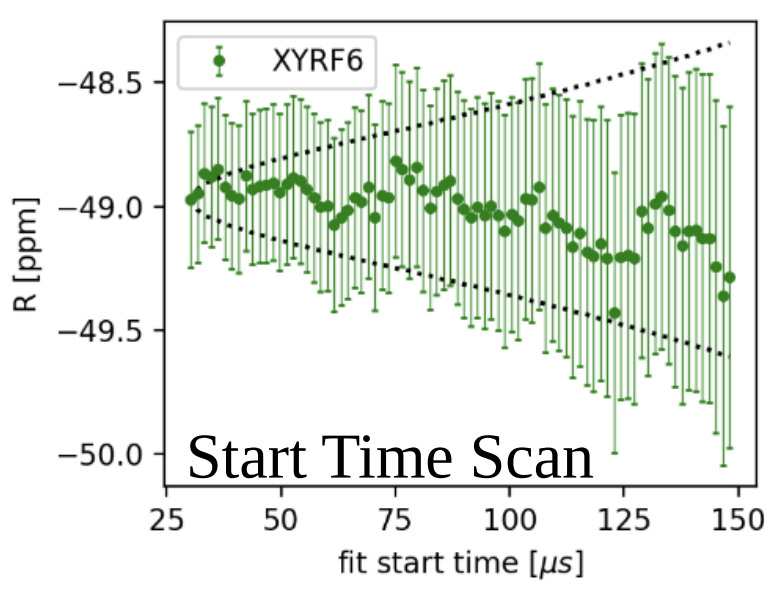
# Mutiple Parameters

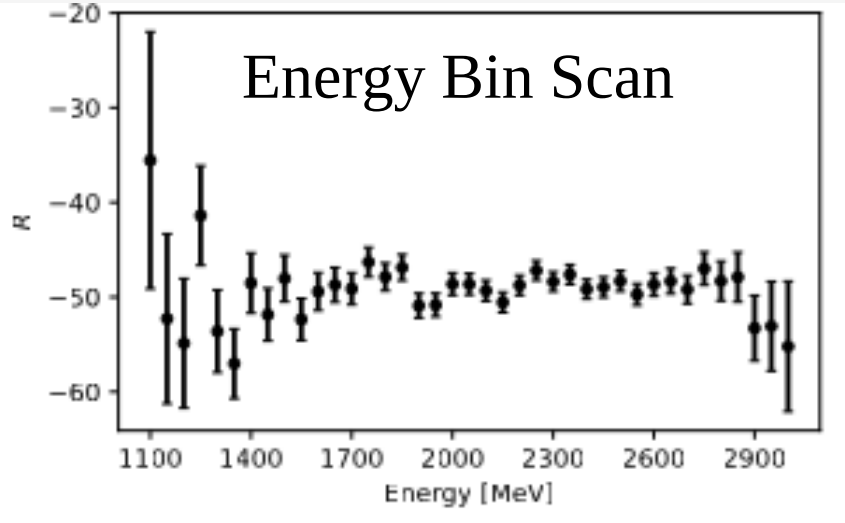
$$
\begin{aligned}
N(t) = Ne^{-t/\tau_\mu}\left[1 + A\cdot\cos(\omega_a t - \phi + \phi_{BO}(t))\right]\cdot \\
\cdot\left(1 + A_{CBO}\cos(\omega_{CBO}t - \phi_{CBO})e^{-\frac{t}{\tau_{CBO}}}\right)\cdot \\
\cdot\left(1 + A_{VW}\cos(\omega_{VW}t - \phi_{VW})e^{-\frac{t}{\tau_{VW}}}\right)\cdot \\
\cdot\left(1 + A_{2CBO}\cos(\omega_{2CBO}t - \phi_{2CBO})e^{-\frac{t}{\tau_{2CBO}}}\right)\cdot \\
\cdot\left(1 + A_{y}\cos(\omega_{y}t - \phi_{y})e^{-\frac{t}{\tau_{y}}}\right)\cdot \\
\cdot\left(1 - k_{LM}\int_0^t L(t')e^{t'/\tau_\mu}dt'\right)\cdot
\end{aligned}
$$



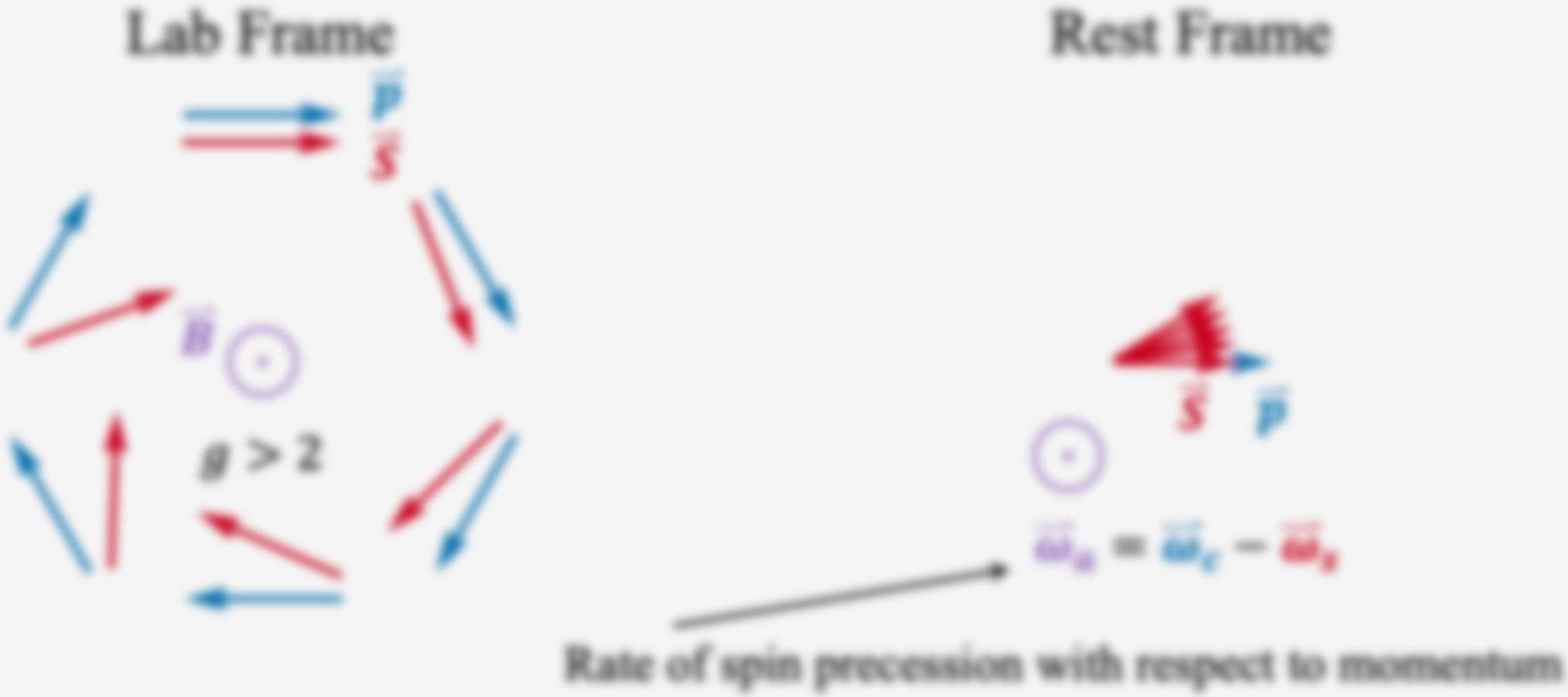
# Consistency Checks



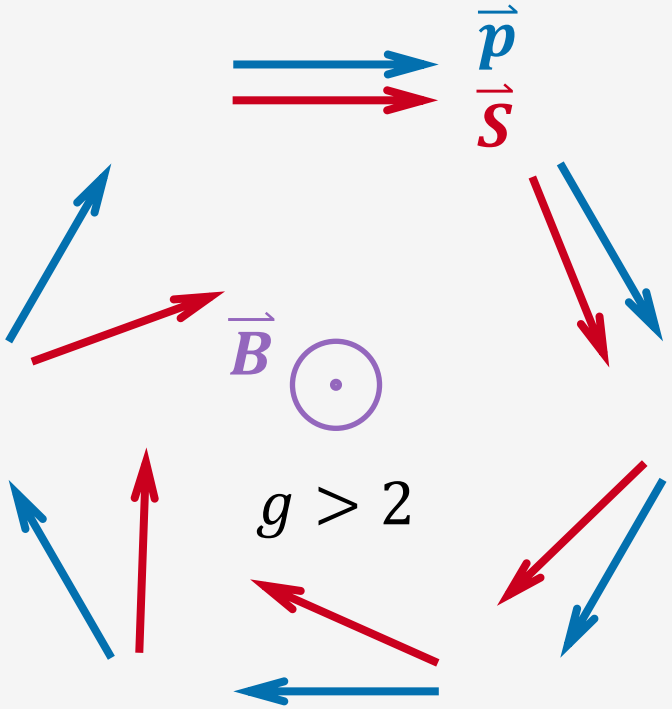



# Spin Precession in a Magnetic Storage Ring

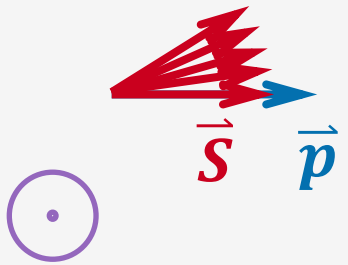


# Spin Precession in a Magnetic Storage Ring

Lab Frame




Rest Frame




$$\vec{\omega}_a = \vec{\omega}_c - \vec{\omega}_s$$

Rate of spin precession with respect to momentum