Logarithmic EW corrections at one-loop

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The need for precision

$$d\sigma = d\sigma_{\rm LO} + \alpha_S\, d\sigma_{\rm NLO}$$

O(100%) NLO QCD

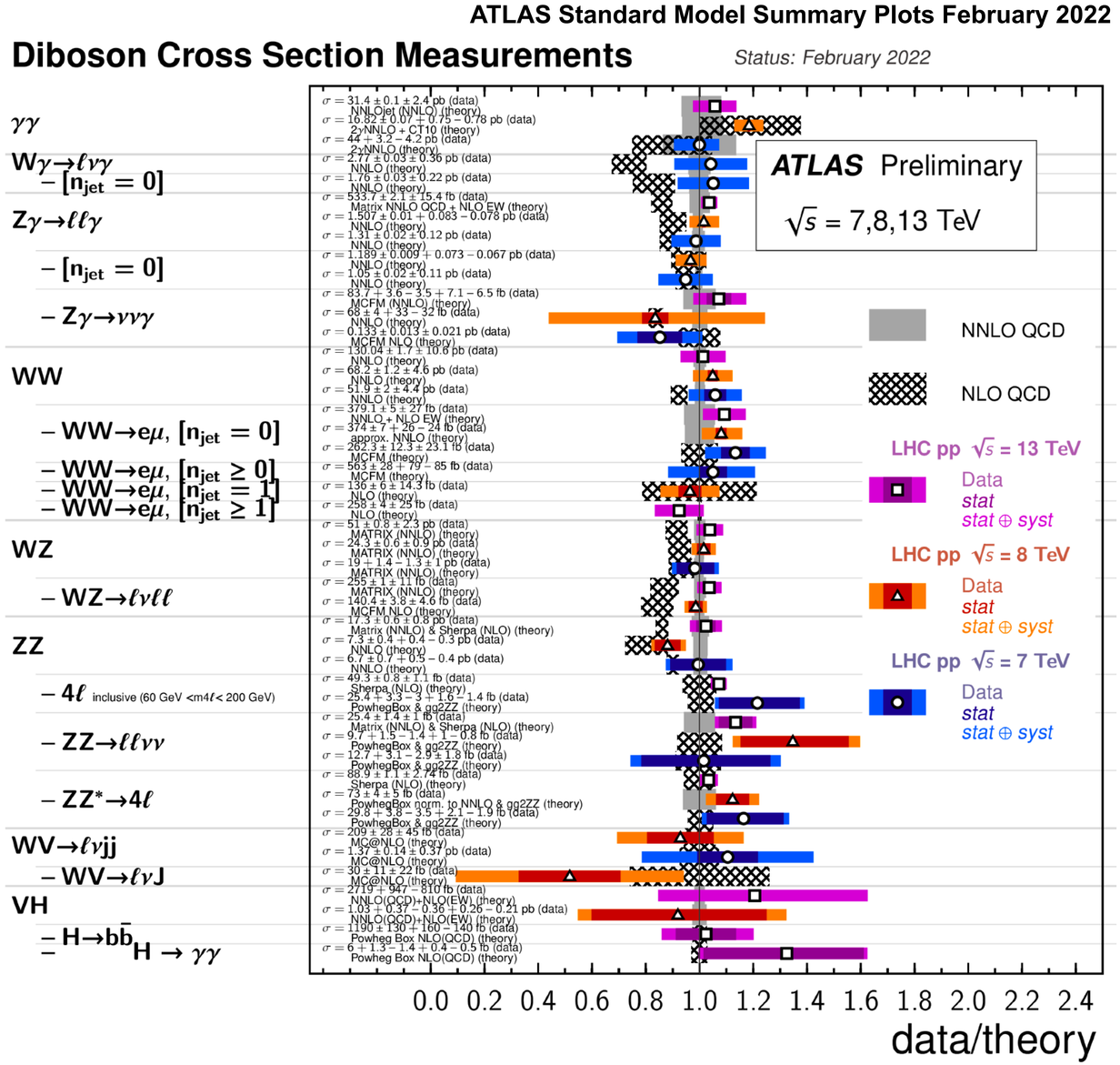
$$+\alpha_S^2\, d\sigma_{\rm NNLO}$$

O(10%) NNLO QCD

$$+\alpha_S^3\, d\sigma_{\rm N3LO+} \ldots$$

O(1%) N3LO QCD

$$\alpha_S \sim 0.1$$



➡Higher-order predictions mandatory for reliable predictions

Perturbative expansion

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_S\, \mathrm{d}\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, \mathrm{d}\sigma_{\mathrm{NLO\,EW}}$$

O(100%) NLO QCD NLO EW

$$+\alpha_S^2\, \mathrm{d}\sigma_{\mathrm{NNLO}}$$

O(10%) NNLO QCD

$$+\alpha_S^3\, \mathrm{d}\sigma_{\mathrm{N3LO}} + \ldots$$

O(1%) N3LO QCD

$\alpha_S \sim 0.1$ $\alpha_{\mathrm{EW}} \sim 0.01$

$\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2) \Rightarrow$ NLO EW ~ NNLO QCD

➡Higher-order predictions mandatory for reliable predictions

Perturbative expansion

aMC@NLO, Sherpa, Herwig… & Recola, Madloop, Gosam, OpenLoops

dedicated MC's: Matrix, MCFM, NNLOjet, …

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_S\, \mathrm{d}\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, \mathrm{d}\sigma_{\mathrm{NLO\,EW}}$$

NLO QCD NLO EW

$$+\alpha_S^2\, \mathrm{d}\sigma_{\mathrm{NNLO}}$$

NNLO QCD

$$+\alpha_S^3\, \mathrm{d}\sigma_{\mathrm{N3LO}} \overset{?}{+} \ldots$$

N3LO QCD

scale variation at NNLO

$$\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2) \Rightarrow$$ NLO EW ~ NNLO QCD

Perturbative expansion

aMC@NLO, Sherpa, Herwig… & Recola, Madloop, Gosam, OpenLoops

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$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_S\, \mathrm{d}\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, \mathrm{d}\sigma_{\mathrm{NLO\,EW}}$$

NLO QCD NLO EW

$$+\alpha_S^2\, \mathrm{d}\sigma_{\mathrm{NNLO}} + \alpha_{\mathrm{EW}}^2\, \mathrm{d}\sigma_{\mathrm{NNLO\,EW}}^{?} + \alpha_S \alpha_{\mathrm{EW}}\, \mathrm{d}\sigma_{\mathrm{NNLO\,QCDxEW}}^{?}$$

NNLO QCD NNLO EW NNLO QCD-EW

$$+\alpha_S^3\, \mathrm{d}\sigma_{\mathrm{N3LO}}^{?} + \ldots$$

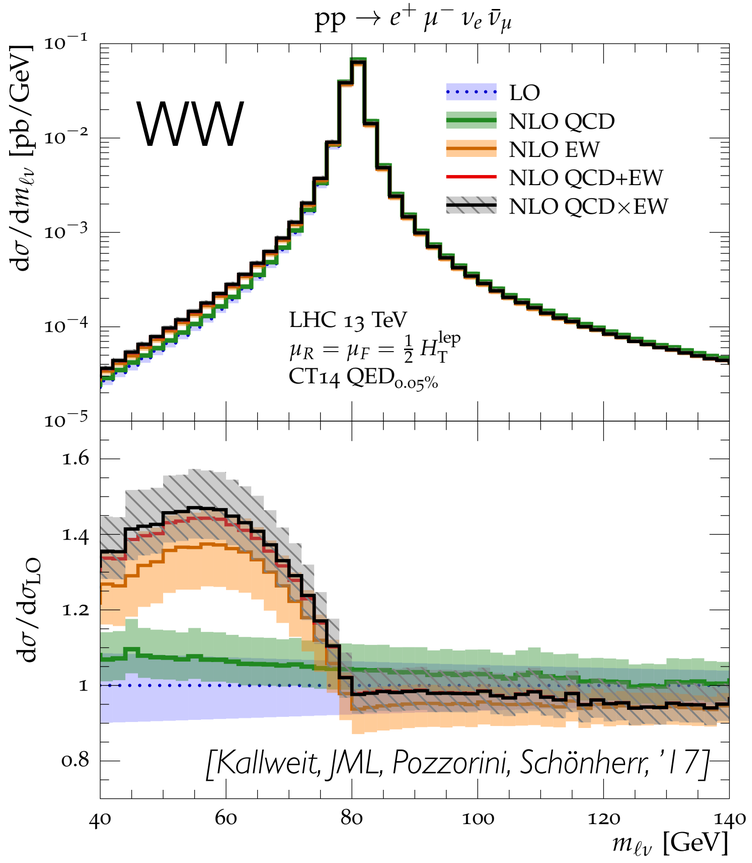
N3LO QCD

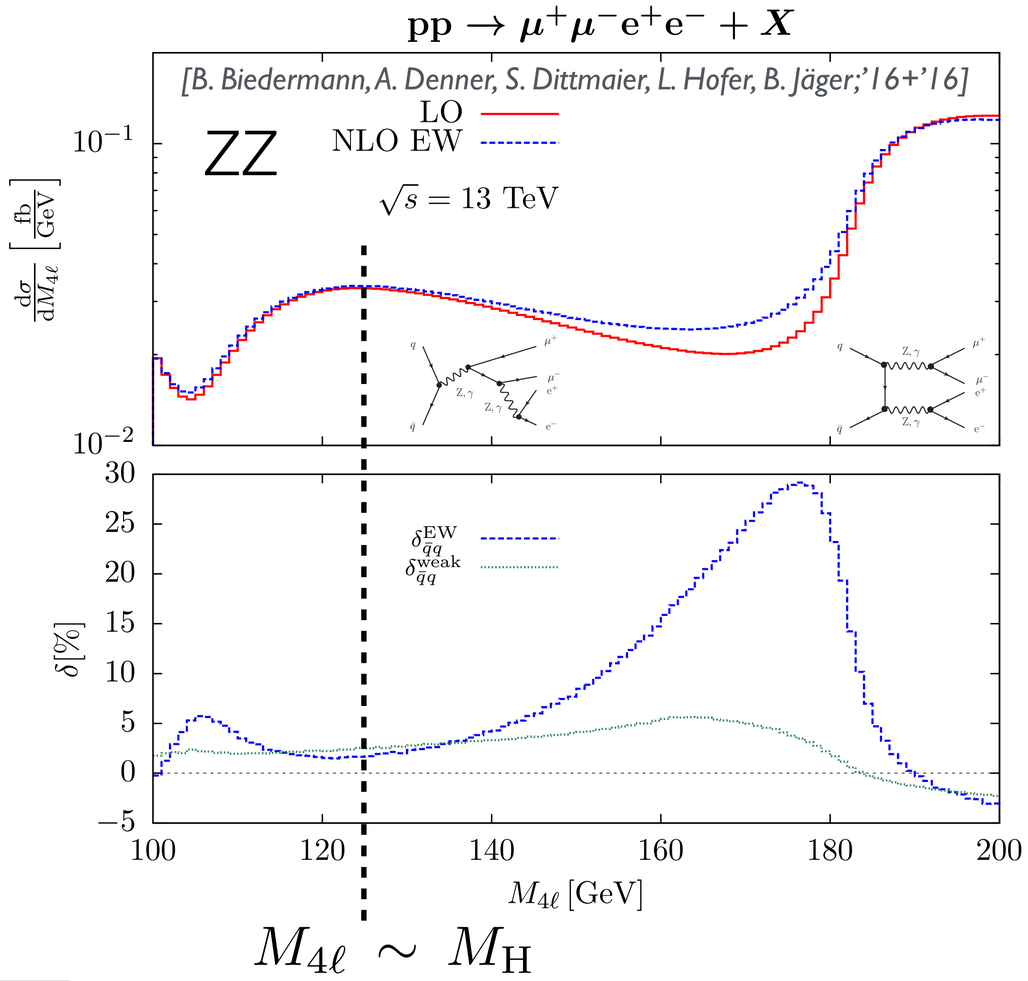
so far only known for DY

scale variation at NNLO

Relevance of EW higher-order corrections: collinear QED radiation

1. Possible large enhancement due to soft/collinear **logs** from photon radiation $\sim \alpha \log\left(\frac{m_f^2}{Q^2}\right)$ in sufficiently exclusive observables.





$M_{4\ell} \sim M_{\mathrm{H}}$

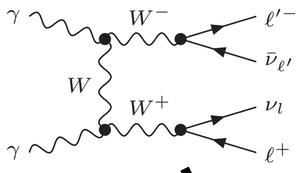
➜ important for radiative tails, Higgs backgrounds etc.

➜typically considered via QED PS (PHOTOS / YFS)

Relevance of EW higher-order corrections: photon-induced channels

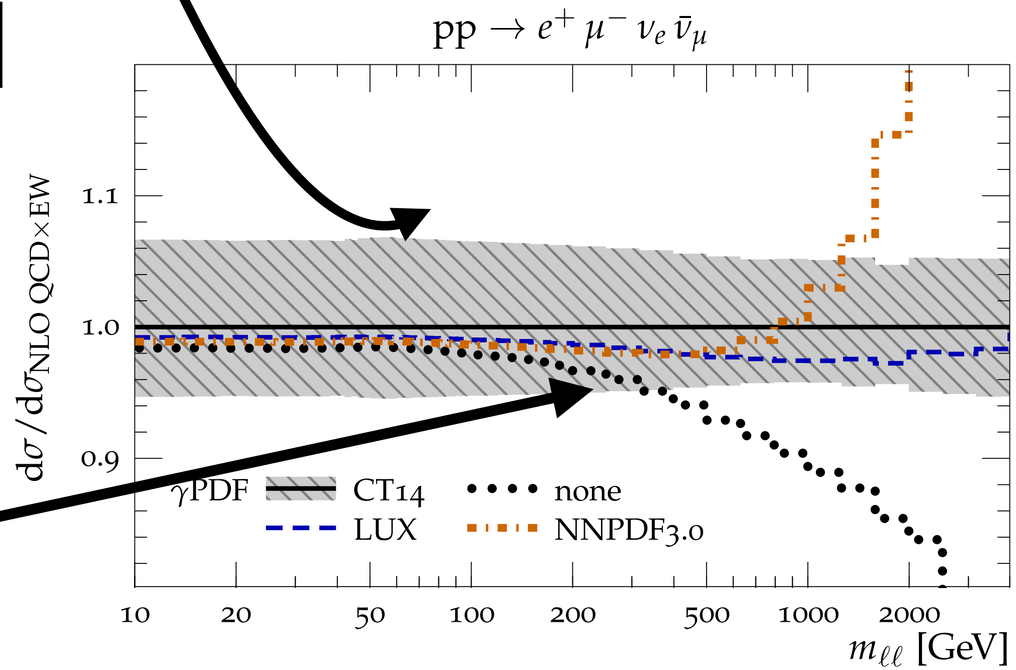
II. QED factorisation and thus photon luminosities needed to absorb IS photon singularities.

➜ Possible large enhancement due to photon-induced channels in the tails of kinematic distributions,

in particular in WW: 

(t-channel enhancement)

$$\mathrm{d}\sigma_{\mathrm{LO}} = \mathrm{d}\sigma_{\mathrm{LO}}^{q\bar{q}} + \mathrm{d}\sigma_{\mathrm{LO}}^{\gamma\gamma}$$



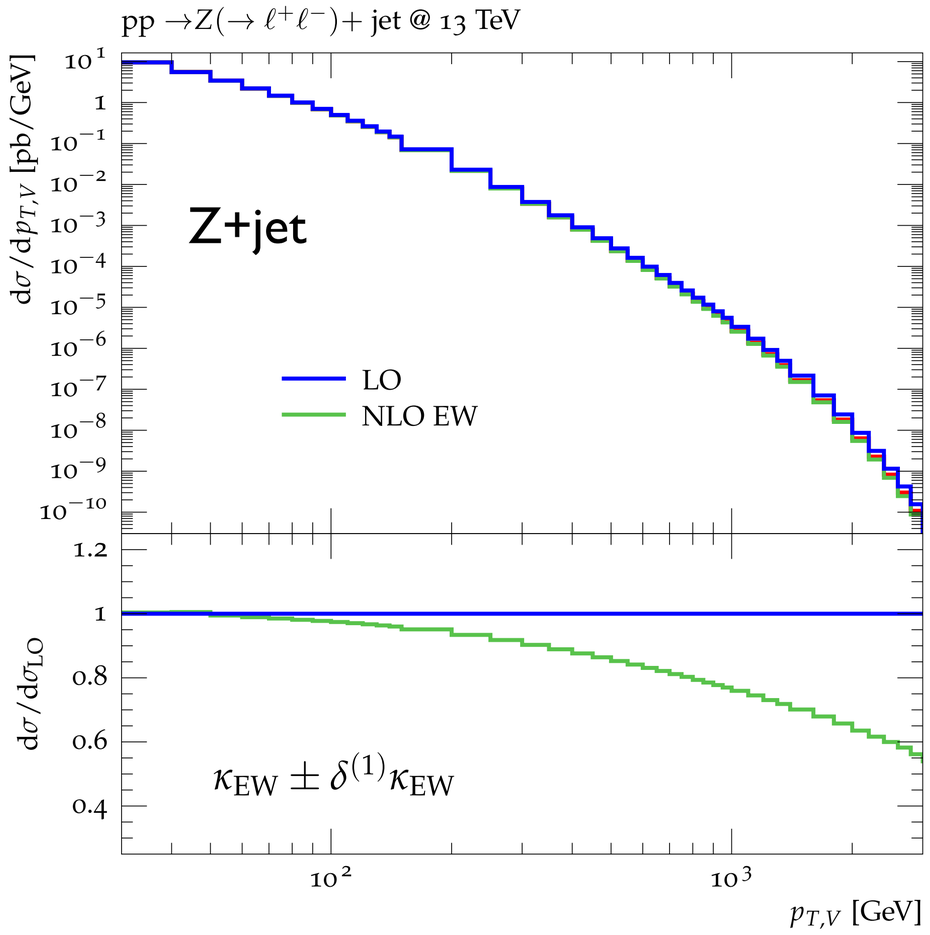
no γ

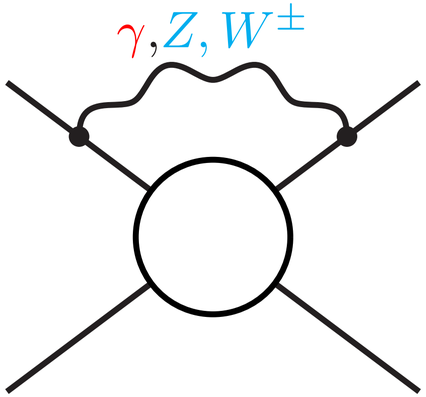
➜ large differences between different photon descriptions. Now settled: LUXqed superior

➜ up to O(10%) contributions from photon-induced channels

Relevance of EW higher-order corrections: **virtual** Sudakov logs in the tails

III. Possible large (negative) enhancement due to soft/collinear **logs** from virtual EW gauge bosons:





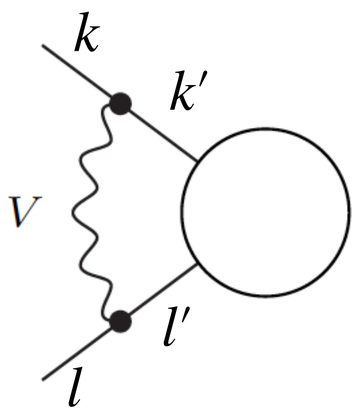
[Ciafaloni, Comelli,'98;
Lipatov, Fadin, Martin, Melles, '99;
Kuehen, Penin, Smirnov, '99;
Denner, Pozzorini, '00]

Universality and factorisation: *[***Denner, Pozzorini***; '01]*

$$\delta\mathcal{M}^{1-\text{loop}}_{\text{LL+NLL}} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\left\{\frac{1}{2}\sum_{l\neq k}\sum_{a=\gamma,Z,W^\pm} I^a(k)I^{\bar{a}}(l)\ln^2\frac{\hat{s}_{kl}}{M^2} + \gamma^{\text{ew}}(k)\ln\frac{\hat{s}}{M^2}\right\}\mathcal{M}_0$$

➜ **overall large (negative) effect in the tails of distributions: p_T, m_{inv}, H_T, … (relevant for BSM searches!)**

Origin of the logs



- **Double Log** (DL) from exchange a soft/collinear (SC) gauge boson V
- In Eikonal/Logarithmic (LA) approximation: factorisation of scalar thee-point function:

SU(2)-rotated trees

$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_k \sum_{l<k} \sum_V \sum_{k',l'} \frac{\alpha}{4\pi} I^V_{kk'} I^{\bar{V}}_{ll'} \underbrace{\left[\log^2\frac{|r_{kl}|}{m_V^2} - 2i\pi\Theta(r_{kl})\log\frac{|r_{kl}|}{m_V^2}\right]}_{\propto C_0|_{\mathrm{LA}}} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{in}}$$

with $r_{kl} = (p_k + p_l)^2$

$$\log^2\left(\frac{|r_{jk}|}{m_V^2}\right) = \log^2\left(\frac{s}{m_V^2}\right) + 2\log\left(\frac{s}{m_V^2}\right)\log\left(\frac{|r_{jk}|}{s}\right) + \log^2\left(\frac{|r_{jk}|}{s}\right)$$

leading (LSC) | subleading (SSC) | sub-subleading (S-SSC)

$$\boxed{\text{LA:}\quad s \sim r_{kl} \equiv (p_k + p_l)^2 \gg M^2_{\mathrm{Z,W}} \qquad \forall k, l}$$

Not fully controlled in LA,
but phenomenologically relevant

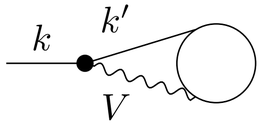
Origin of the logs

Single Logs (SL) have a triple origin: $\hat{\delta}^{\rm SL} = \hat{\delta}^{\rm PR} + \hat{\delta}^{\rm COLL} + \hat{\delta}^{\rm WFRC}$

→ **PR**: running from renormalisation scale (EW scale) to the hard scale

→ **WF**: wave-function renormalisation of external fields $\delta^{\rm WF}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_k \sum_{k'} \delta^{\rm WF}_{kk'} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\rm WF}_{kk'} = \frac{1}{2}\delta Z_{kk'}$

→ **COLL**: collinear-singular diagrams factorise into Born times universal collinear correction factors



Ward identities →

$$\delta^{\rm coll}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_k \sum_{k'} \delta^{\rm coll}_{kk'} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\rm coll}_{kk'} \sim \frac{\alpha}{4\pi}\sum_V I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right)$$

Note 1: approximation applicable for processes that are not mass suppressed at LO: $\mathscr{M}_0^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim E^d$,

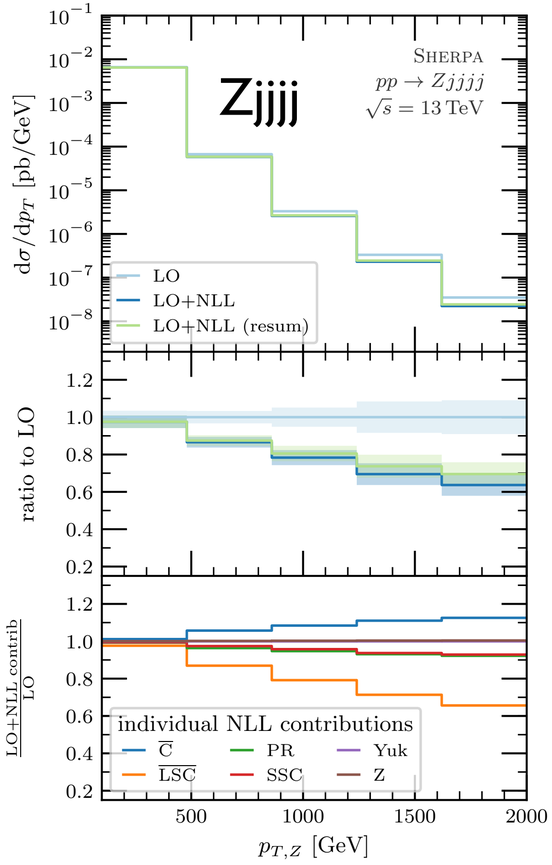
Note 2: for longitudinal modes of external massive gauge bosons use goldstone boson equivalence theorem

Tools for EW Sudakov corrections

also: alpgen *[Chiesa, et. al., '13]*
MCFM *[Campbell et al. '16]*

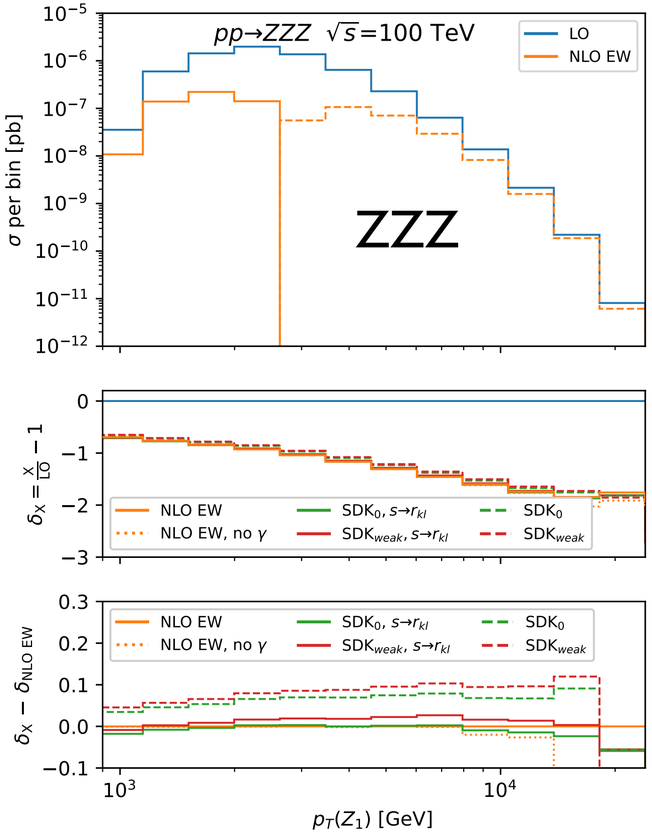
Sherpa

[Bothmann, Napoletano, '20]



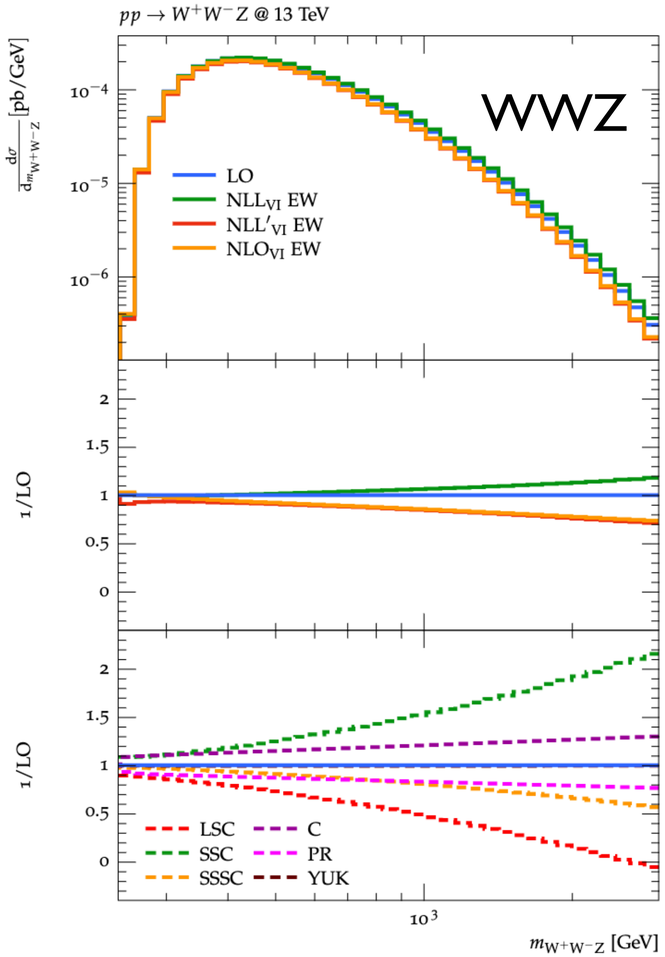
MadGraph5_aMC@NLO

[Pagani, Zaro, '21]



OpenLoops

[JML, Mai, '23]



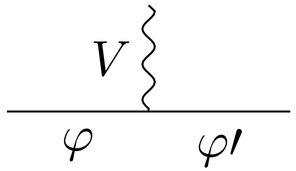
- all based on *[Denner, Pozzorini, '00, '01]*

Implementation in OpenLoops

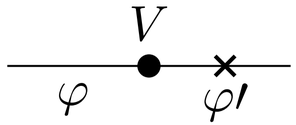
- OpenLoops: automated tool for the calculation of tree and one-loop amplitudes at NLO QCD +NLO EW [Buccioni, JML et. al., 2019]

- Goal: exploit factorisation of Sudakov logs to evaluate one-loop EW corrections at NLL via tree amplitudes → up to two orders of magnitude faster w.r.t. full loop computation

- How? Representation of LA algorithm via <u>helicity-dependent</u> two-point (effective) vertex rules



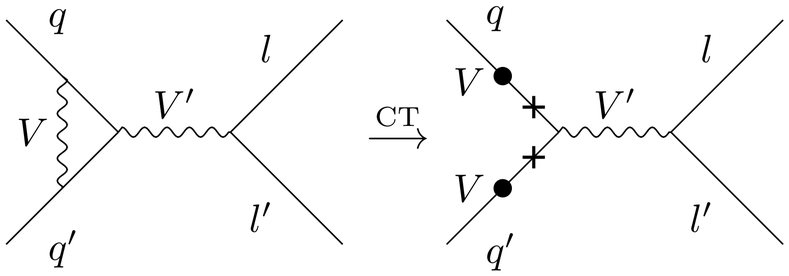
$$= ieI^V_{\varphi\varphi'}T \quad \longrightarrow$$



$$= ieI^V_{\varphi\varphi'}$$

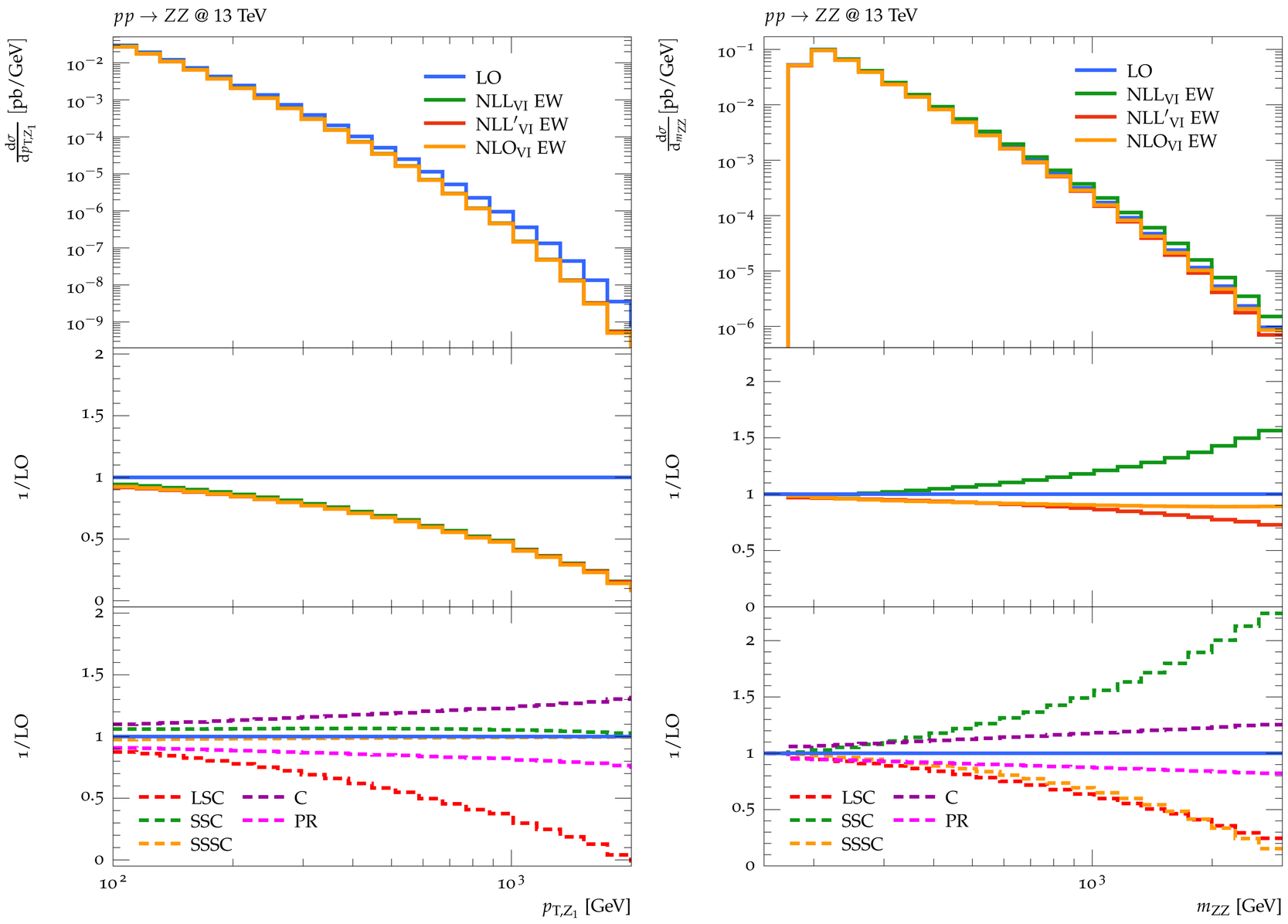
- The virtual soft boson is cut and the internal propagator is removed, while the external particle remains on-shell

- Example: Drell-Yan



$$\rightarrow \quad \mathcal{M}_1^{\mathrm{CT,DL,q\bar{q}}} = \frac{\alpha}{4\pi} \sum_{V=A,Z,W^\pm} \mathcal{I}_q^V \mathcal{I}_{\bar{q}}^V \mathcal{M}_0^{q'\bar{q}'\to\ell^+\ell^-} \left[\log^2\frac{|r_{kk'}|}{m_V^2} - 2i\pi\Theta(r_{kk'})\log\frac{|r_{kk'}|}{m_V^2}\right]$$

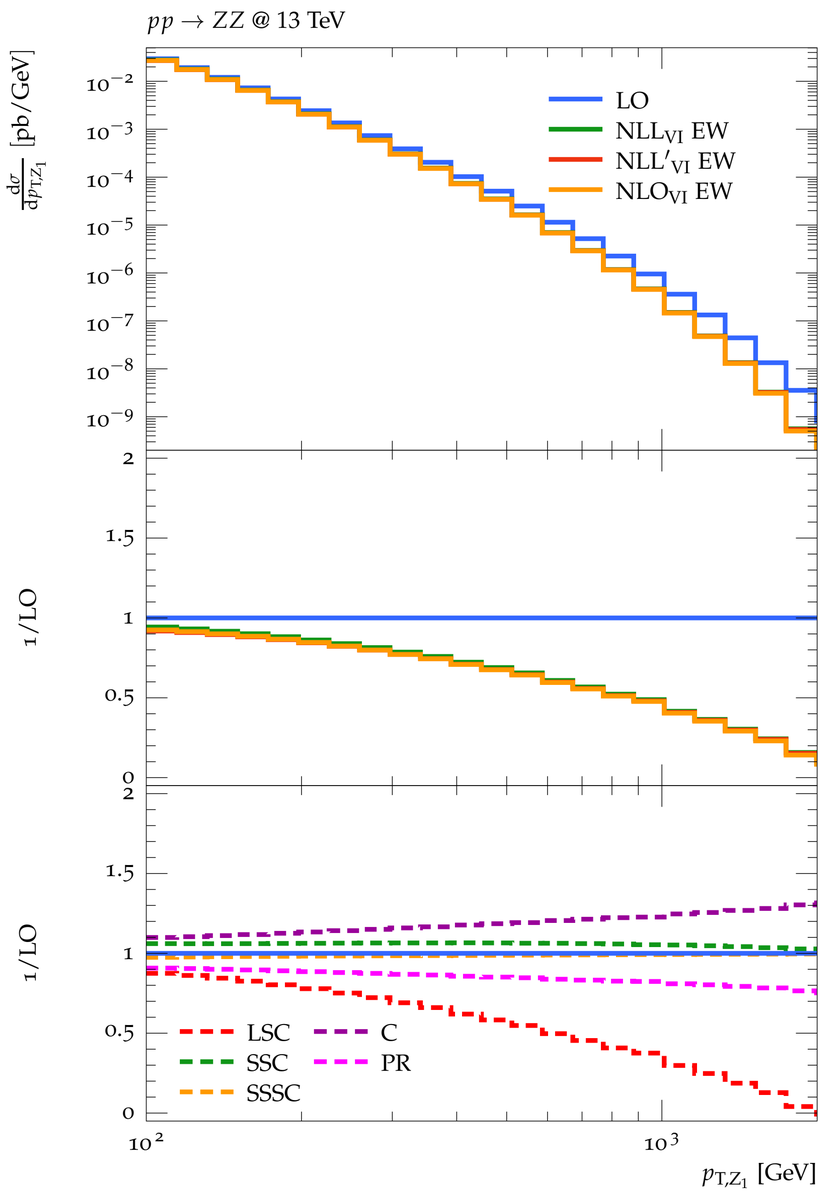
Results: ZZ

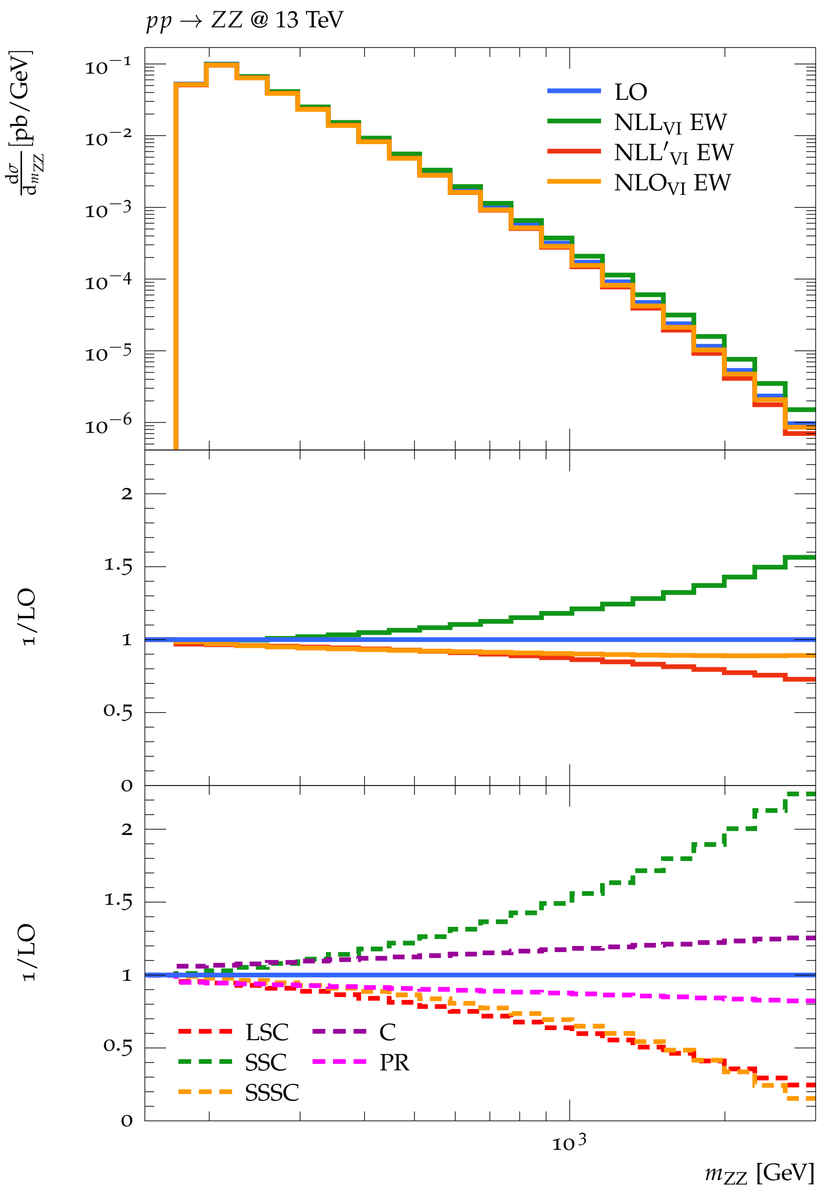


$$\mathrm{NLL_{VI}\ EW} = (1 + \mathrm{LSC} + \mathrm{SSC} + \mathrm{C} + \mathrm{PR} + \mathbf{I})\mathrm{LO}$$

$$\mathrm{NLL'_{VI}\ EW} = (1 + \mathrm{LSC} + \mathrm{SSC} + \mathrm{SSSC} + \mathrm{C} + \mathrm{PR} + \mathbf{I})\mathrm{LO}$$

Results: ZZ





SSC and SSSC become very sizeable for PS regions where LA condition

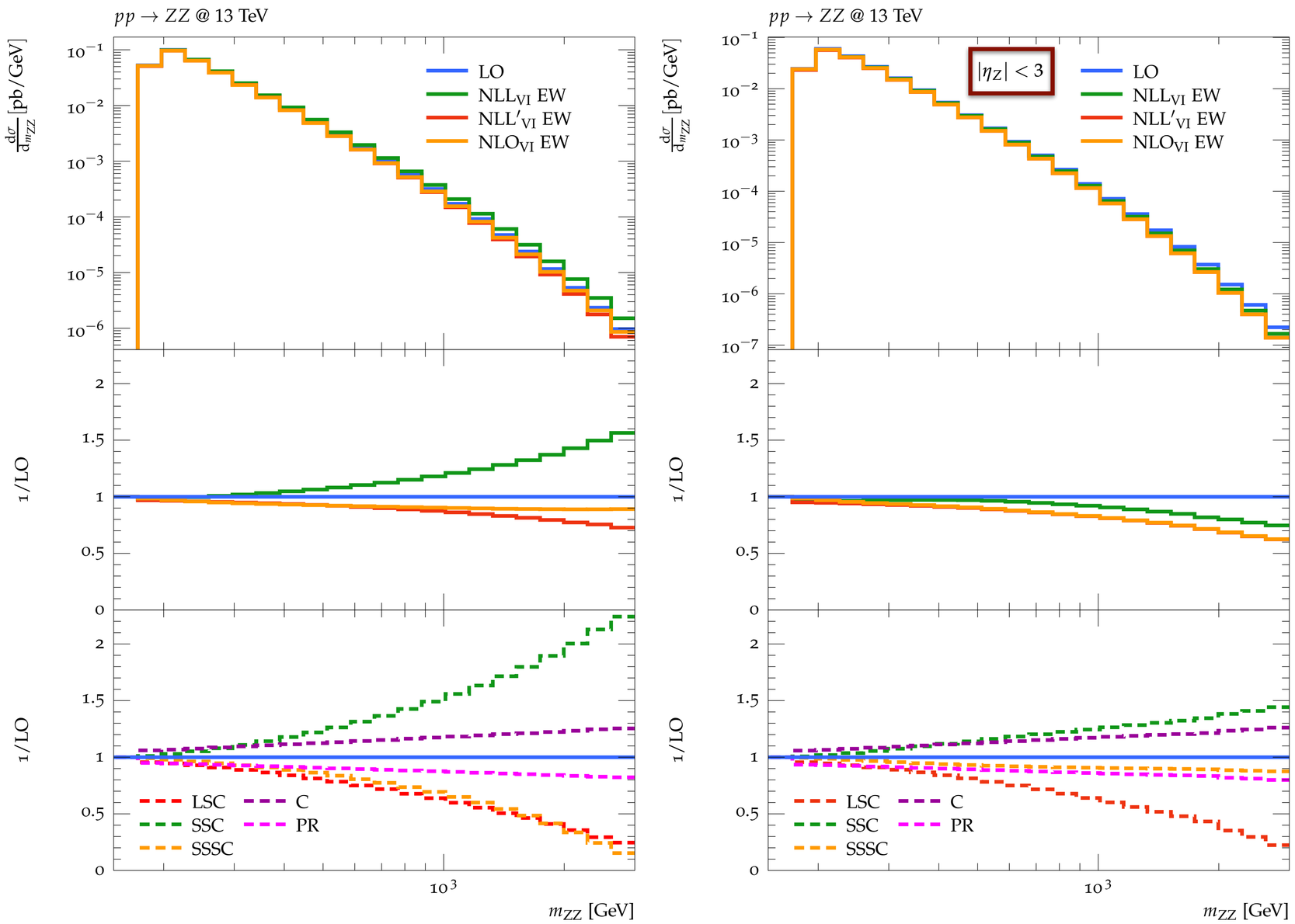
$$s \sim r_{kl} \equiv (p_k + p_l)^2 \gg m^2_{W} \quad \forall k, l$$

is violated, with hierarchy among invariants

$$\delta^{\text{SSC, V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$$

$$\delta^{\text{S-SSC, V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{|r_{kl}|}{s}\right)$$

Results: ZZ

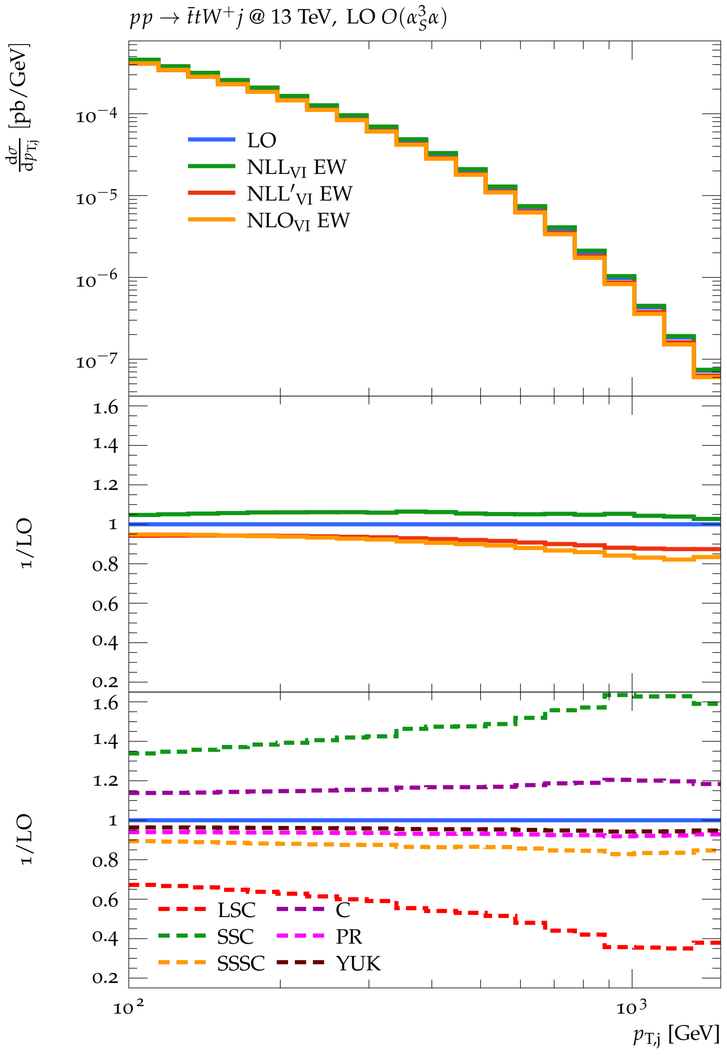


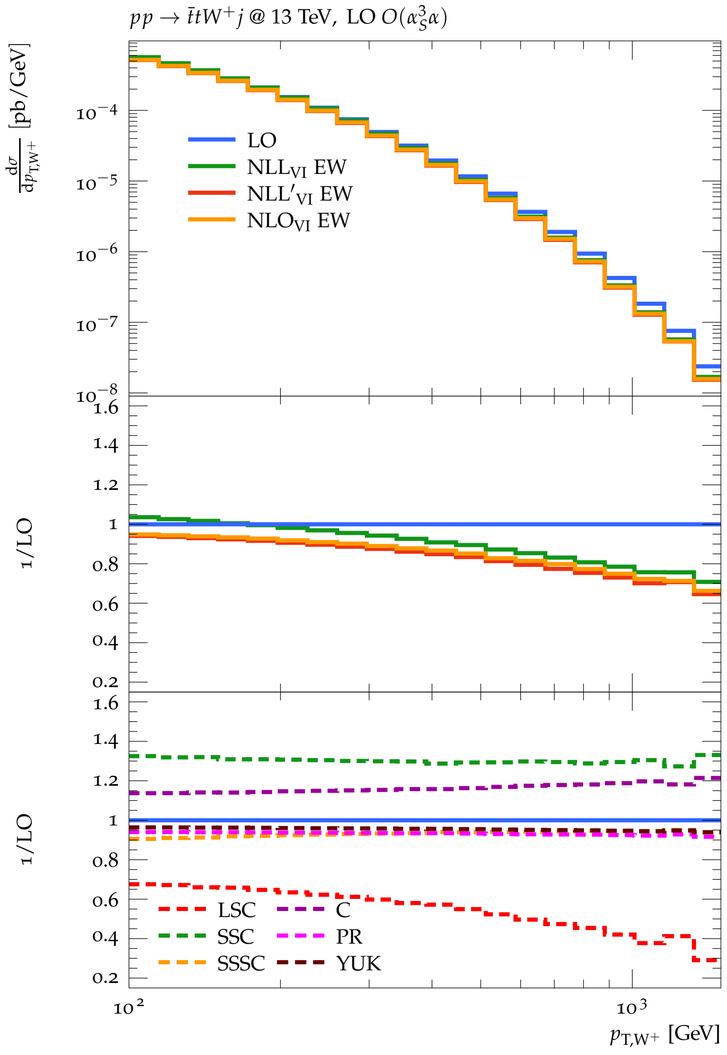
Pseudo-rapidity cut $|\eta_Z| < 3$ avoids pathological very forward configurations which violate LA; such cuts are anyway applied in any realistic analysis

Consider differences between NLL and NLL':

- $\mathrm{NLL}' - \mathrm{NLL} > \mathrm{NLL}' - \mathrm{NLO} \Rightarrow$ SSSC is a reliable estimate of sub-sub-leading angular terms beyond LA
- If NLO is unknown: SSSC might be interpreted as a conservative estimate of uncertainties of LA

Results: ttWj





- applicable to high multiplicity processes at low computational cost

Off-shell decays

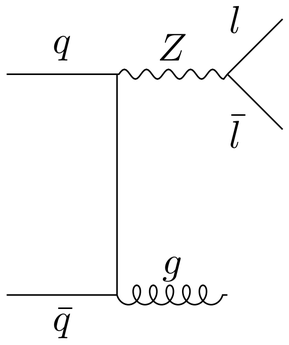
- Standard algorithm not suitable for processes involving decay of an unstable particle as LA is violated in the resonant region: $r_{ij} \approx m_X^2$

- Standard solutions: Narrow Width Approximation (MadSpin, HDH in Sherpa) or Pole Approximation problem: break-down in off-shell regime

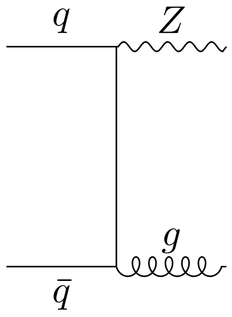
- Out solution: combine off-shell and on-shell processes via probabilities based on kinematic projectors

$$P_{X_i}(k_i) = \left| \frac{\mu_{X_i}^2 - m_{X_i}^2 \Gamma_{X_i}^2}{(k_i^2 - \mu_{X_i}^2)^2 + \mu_{X_i}^2} \right| = \begin{cases} 1 & \text{if} \quad k_i^2 \to m_{X_i}^2 \\ 0 & \text{if} \quad k_i^2 \to \infty \end{cases}$$

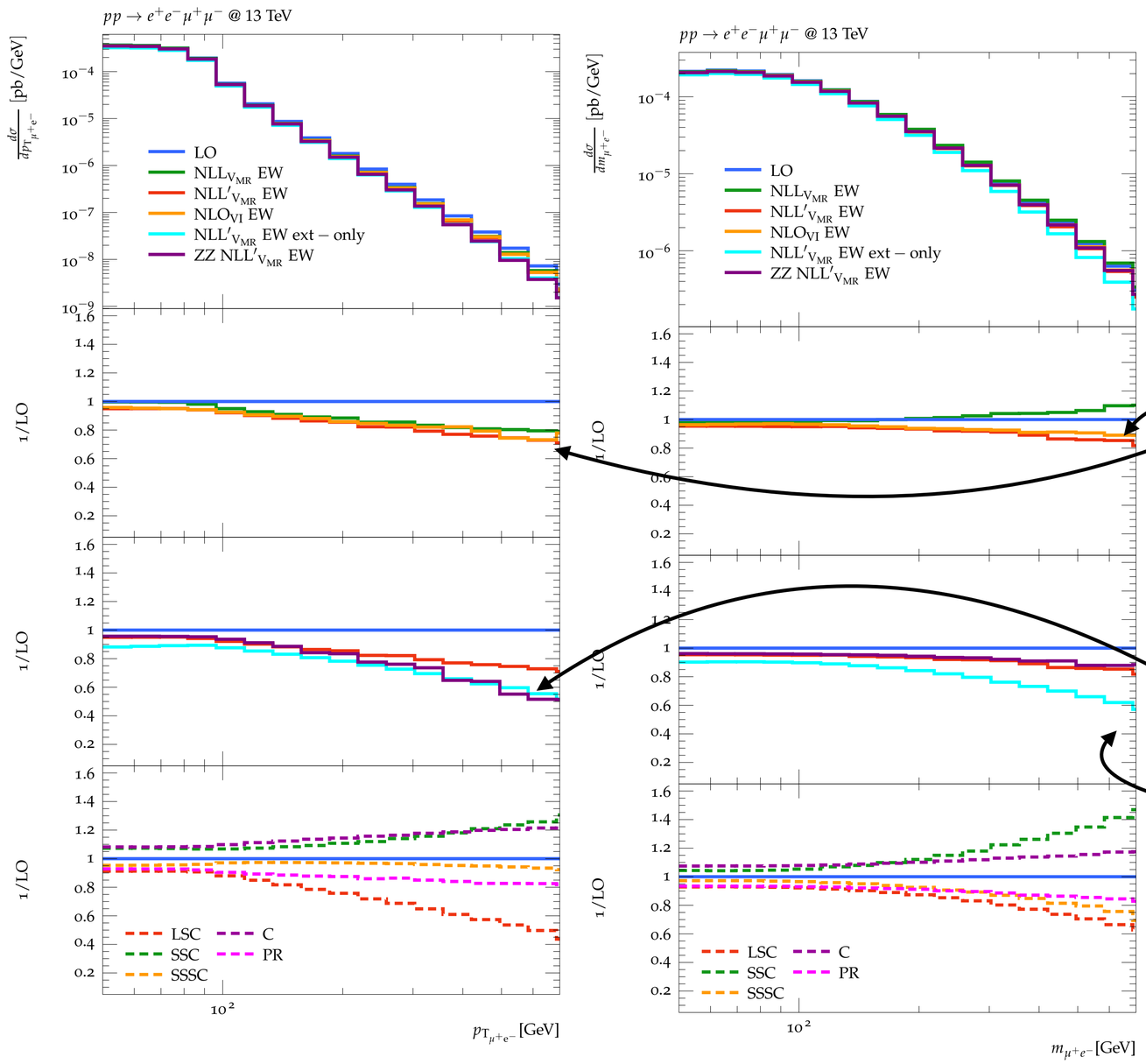
- Example: $pp \to Zj \to \bar{l}lj$

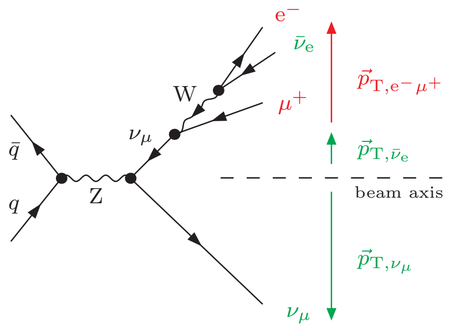


vs.



Off-shell decays





LA for combined process agrees at percent level with exact result

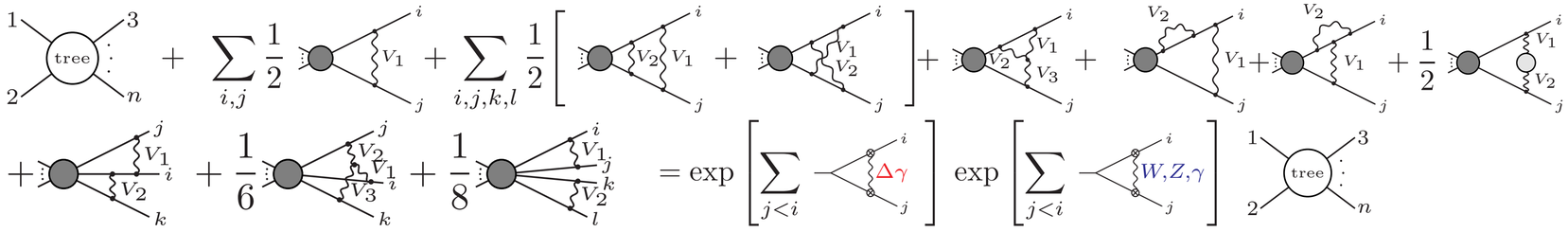
Depending on observable:

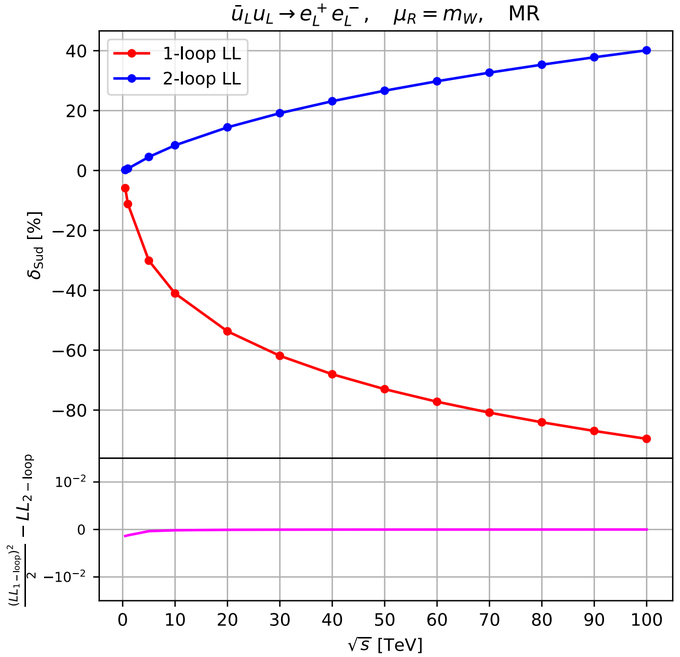
- LO for on-shell process deviates up to 20%
- LA for "external only" deviates up to 20%

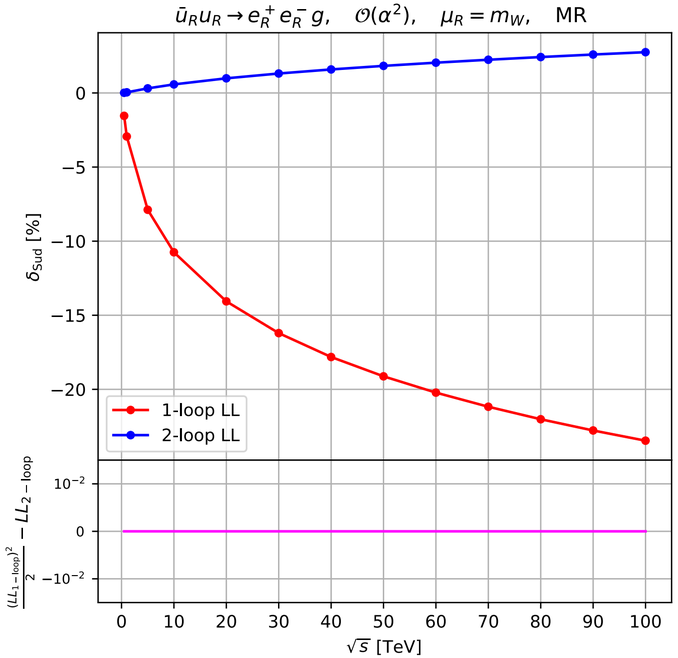
Towards two-loop

[JML, Mai, work in progress]

- Factorisation/universality also holds at two-loop *[Denner, Pozzorini, Jantzen, Melles, Penin, Smirnov, Kühn, …]*
- More complex structure:







- Two-loop corrections tend to partially compensate the large negative one-loop corrections

Conclusions

- NLO EW corrections at high energies are dominated by Sudakov logarithms which are significantly enhanced in tails of kinematic distributions (> 10%)
- Exploiting the universality of Sudakov logs we developed an effective CT vertex approach to implement the NLO EW LA in OpenLoops

  → Reduction of one-loop EW corrections to a tree-level problem with percent level of agreement

- Additional aspects of the implementation:
  - ▸ Largely model independent (applicable to both SM and BSM scenarios)
  - ▸ Direct employment in PS Event Generators (Sherpa, Powheg, HERWIG, ..) via established OpenLoops interfaces
  - ▸ Can be used together with differential QED radiation at NLO
  - ▸ Support EW corrections for resonant processes

- Outlook:
  - ▸ NNLO/two-loop extension (ongoing work): NNLO EW and NLO QCD-EW

Backup