Status of g-2 theory

Christoph Lehner
(University of Regensburg)

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The anomalous magnetic moment of the muon in the Standard Model: an update

R. Aliberti[1,2], T. Aoyama[3], E. Balzani[4,5], A. Bashir[6,7], G. Benton[8,9], J. Bijnens[10], V. Biloshytskyi[1,2],
T. Blum[11,12], D. Boito[13], M. Bruno[14,15], E. Budassi[16,17], S. Burri[18], L. Cappiello[19],
C. M. Carloni Calame[17], M. Cè[14,15], V. Cirigliano[20,21], D. A. Clarke[22], G. Colangelo[†18], L. Cotrozzi[23],
M. Cottini[18], I. Danilkin[1,2], M. Davier[†24], M. Della Morte[25], A. Denig[1,2,26], C. DeTar[22],
V. Druzhinin[27], G. Eichmann[28], A. X. El-Khadra[†8,9], E. Estrada[29], X. Feng[30,31,32], C. S. Fischer[33,34],
R. Frezzotti[35], G. Gagliardi[36], A. Gérardin[37], M. Ghilardi[16,17], D. Giusti[38,39], M. Golterman[40],
S. Gonzàlez-Solís[41,42], S. Gottlieb[43], R. Gruber[44], A. Guevara[45], V. Gülpers[46], A. Gurgone[47,48],
F. Hagelstein[1,2], M. Hayakawa[49,50], N. Hermansson-Truedsson[10,46], A. Hoecker[51], M. Hoferichter[†18],
B.-L. Hoid[1,2], S. Holz[18], R. J. Hudspith[52], F. Ignatov[23], L. Jin[11], N. Kalntis[18], G. Kanwar[46],
A. Keshavarzi[53], J. Komijani[44], J. Koponen[1,2], S. Kuberski[54], B. Kubis[55], A. Kupich[27], A. Kupść[56,57],
S. Lahert[22], S. Laporta[4,5], C. Lehner[†39], M. Lellmann[1], L. Lellouch[37], T. Leplumey[58,59], J. Leutgeb[60],
T. Lin[30], Q. Liu[61], I. Logashenko[27], C. Y. London[13], G. López Castro[29], J. Lüdtke[62], A. Lusiani[48,63],
A. Lutz[24], J. Mager[60], B. Malaescu[64], K. Maltman[65,66], M. K. Marinković[44], J. Márquez[29],
P. Masjuan[67,68], H. B. Meyer[1,2,26], T. Mibe[†69], N. Miller[26], A. Miramontes[70,71], A. Miranda[67],
G. Montagna[16,17], S. E. Müller[72], E. T. Neil[73], A. V. Nesterenko[27], O. Nicrosini[17], M. Nio[50,74],
D. Nomura[75], J. Paltrinieri[23], L. Parato[44], J. Parrino[39], V. Pascalutsa[1,2], M. Passera[5,76], S. Peris[67,68],
P. Petit Rosàs[23], F. Piccinini[17,77], R. N. Pilato[23], L. Polat[64,24], A. Portelli[46], D. Portillo-Sánchez[29],
M. Procura[62], L. Punzi[48,63], K. Raya[7], A. Rebhan[60], C. F. Redmer[1,2], B. L. Roberts[†78],
A. Rodríguez-Sánchez[71], P. Roig[29,71], J. Ruiz de Elvira[79], P. Sánchez-Puertas[80], A. Signer[81,59],
J. W. Sitison[73], D. Stamen[55], D. Stöckinger[82], H. Stöckinger-Kim[82], P. Stoffer[59,81], Y. Sue[69],
P. Tavella[44], T. Teubner[†23], J.-N. Toelstede[59,81], G. Toledo[83], W. J. Torres Bobadilla[23], J. T. Tsang[54],
F. P. Ucci[16,17], Y. Ulrich[23], R. S. Van de Water[84], G. Venanzoni[23,48], S. Volkov[85], G. von Hippel[1,2],
G. Wang[37], U. Wenger[18], H. Wittig[†1,2,26], A. Wright[23], E. Zaid[23], M. Zanke[55], Z. Zhang[24],
M. Zillinger[18],
C. Alexandrou[86,87], A. Altherr[44], M. Anderson[1,2], C. Aubin[88], S. Bacchio[87], P. Beltrame[23],
A. Beltran[1,2], P. Boyle[89], I. Campos Plasencia[90], I. Caprini[91], B. Chakraborty[92], G. Chanturia[55],
A. Crivellin[59], A. Czarnecki[93], L.-Y. Dai[94], T. Dave[23], L. Del Debbio[46], K. Demory[24],
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L. Flower[23], A. Francis[97], E. Gámiz[98], J. Gogniat[18], A. V. Grebe[84], S. Gündogdu[81,59], M. T. Hansen[46],
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G. S. Huang[101], T. Iijima[102,49], K. Inami[49], A. Jüttner[54,92], R. Kitano[103], M. Knecht[37],
S. Kollatzsch[81,59], A. S. Kronfeld[84], T. Lenz[1,2], G. Levati[18], Q. M. Li[100], Y. P. Liao[100], J. Libby[104],
K. F. Liu[95], V. Lubicz[36], M. T. Lynch[8,9], A. T. Lytle[8,9], J. L. Ma[100], K. Miura[96], K. Möhling[82],
J. Muskalla[1,2], F. Noël[18], K. Ottnad[1,2], P. Paradisi[4,5], C. T. Peterson[105], A. Pich[71], S. Pitelis[1],
S. Plura[1,2], A. Price[106], D. Radic[81,59], A. Radzhabov[27], A. Risch[107], S. Romiti[18], S. Sahoo[108],
F. Sannino[25,19], H. Schäfer[55], Y. Schelhaas[1,2], S. I. Serednyakov[27], O. Shekhovtsova[109,110],
J. N. Simone[84], S. Simula[111], E. P. Solodov[27], F. M. Stokes[66], M. Vanderhaeghen[1,2], A. Vaquero[112,113],
N. Vestergaard[23], W. Wang[1,2], K. Yamashita[114], Y. B. Yang[115], T. Yoshioka[116], C. Z. Yuan[100],
A. S. Zhevlakov[27]

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Second whitepaper (WP25) of Muon g-2 Theory Initiative just released

235 authors

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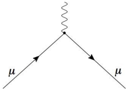
All references of my talk today can be found within this document

The magnetic moment and quantum corrections

The g-factor in $\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}$ describes the strength of coupling to a magnetic field, which can be measured and computed from theory **very** precisely.

Dirac: $g = 2$

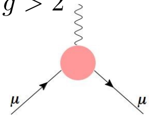


Anomalous magnetic moment

$a = (g-2)/2$

quantum effects

$g > 2$



The quantum effects arise from virtual particle contributions from all known **and unknown** particles.

By comparing high-precision experiments and theory, we have the potential to learn about such contributions of new particles.

- Constrain new physics as

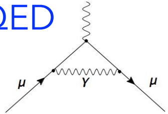
$$a_\mu^{\text{Theory}} = a_\mu^{\text{SM}} + a_\mu^{\text{BSM}} = a_\mu^{\text{Experiment}}$$

  With **spectacular** new result from Fermilab just seen in previous talk!

- Contributions of the Standard Model (SM):

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$$

QED



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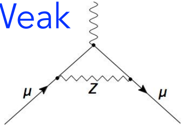
Weak

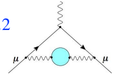


$+\ldots$

Hadronic...

...Vacuum Polarization (HVP)

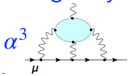
α^2



$+\ldots$

...Light-by-Light (HLbL)

α^3



$+\ldots$

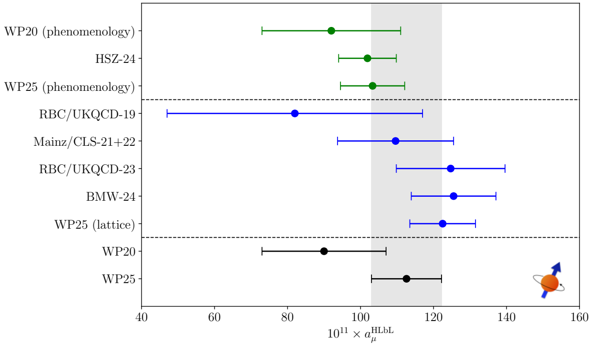
What has happened in the last five years since WP20?

Contributions to $a_\mu^{\mathrm{SM}} \times 10^{11}$

Contribution	WP25	WP20
QED	116 584 718.8(2)	116 584 718.931(104)
EW	154.4(4)	153.6(1.0)
HVP (LO + NLO + NNLO)	. . .	
HLbL (phenomenology + lattice + NLO)	115.5(9.9)	92(18)

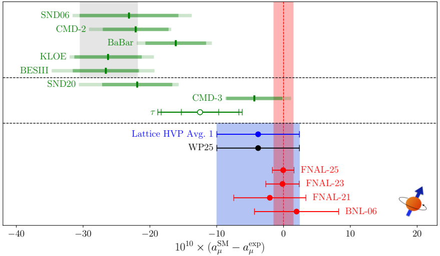
- QED and EW contributions already had been determined in WP20 with sufficient precision to match FNAL E989. Nevertheless important progress:
  - Discrepancy in 5-loop QED contribution has been resolved (S. Volkov; Aoyama, Hayakawa, Hirayama, Nio) $\Rightarrow$ small shift and increased uncertainty due to tensions in fine-structure constant measurements
  - EW contributions more precise due to better knowledge of input parameters and hadronic effects in two-loop EW contributions (Hoferichter, Stoffer, Zillinger)
- HLbL has improved from both data-driven and lattice QCD side (see next slide)
- HVP . . . a story in three acts (see most of the talk)

Status of HLbL



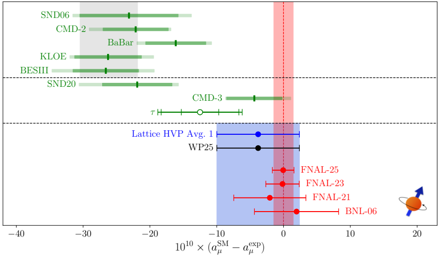
- Agreement between lattice QCD and dispersive method
- Uncertainty halved since WP20
- WP25 consistent with WP20 result
- Uncertainty now already smaller than final Fermilab E989 experimental uncertainty

Major change from WP20 for HVP contribution. The story in a nutshell:



- WP20 result based on hadronic e^+e^- decays
- Since CMD-3 experiment no reasonable average can be provided
- τ has been scrutinized again, critical: IB corrections
- Lattice QCD has substantially matured and can provide a robust estimate
- With lattice QCD the SM is consistent with Fermilab E989

Major change from WP20 for HVP contribution. The story in a nutshell:



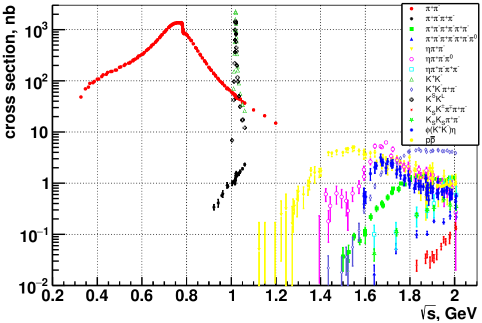
Entire story in three acts:

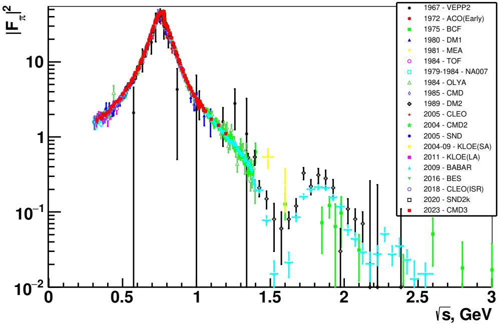
1. HVP from hadronic e^+e^- decays
2. HVP from τ decays
3. HVP from lattice QCD

Act 1: HVP from hadronic e^+e^- decays

Where did the g-2 tension go?

HVP from hadronic e^+e^- decays - method

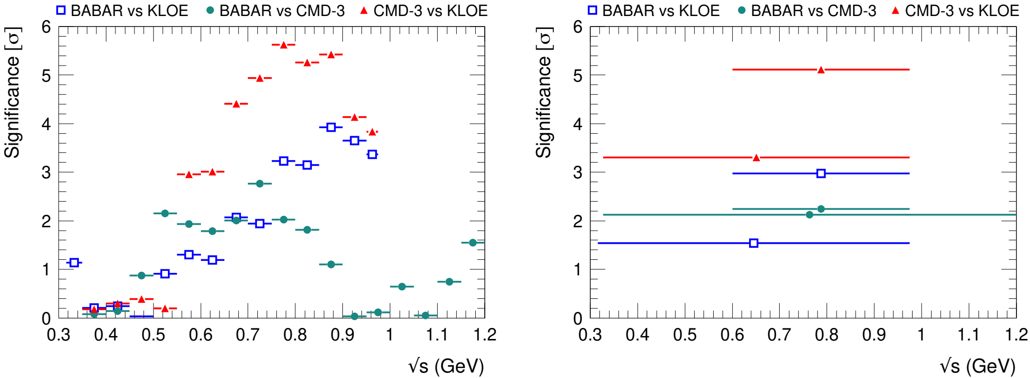




$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s)\,, \qquad R_{\text{had}}(s) = \frac{3s}{4\pi\alpha^2}\sigma[e^+e^- \to \text{hadrons}(+\gamma)]$$

with known function $\hat{K}(s)$

HVP from hadronic e^+e^- decays - tensions between the data sets

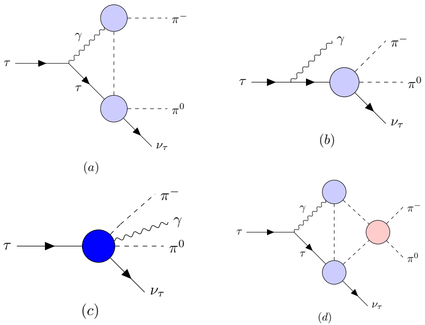


- Tensions are highly statistically significant
- TI organized dedicated workshops to scrutinize these tensions
- Intensified studies of event generators
- Origin of tensions is not understood at this point
- No averaging of results attempted for WP25!

Act 2: HVP from hadronic τ decays

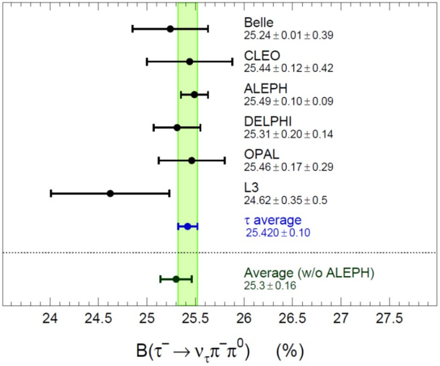
Didn't the τ data say there is no new physics all along?

HVP from hadronic τ decays - methodology



- Relate $\tau \to \nu_\tau \pi^0 \pi^-$ decays to $\gamma^* \to \pi^+\pi^-$ amplitudes
- This requires consistent experimental data sets for the τ decays and
- precise control of isospin-breaking corrections including
  - a consistent renormalization scheme from the EW scale to the effective theory (or lattice QCD) within which one computes the IB corrections
  - Control of long-distance QED effects

HVP from hadronic τ decays - data situation



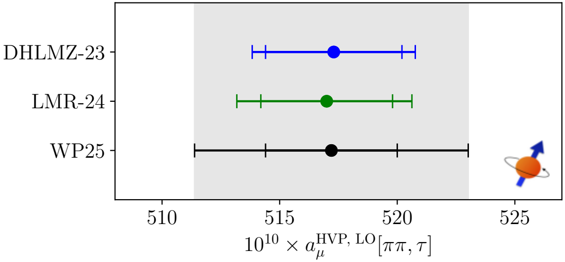
- Available data sets are consistent
- Precision suffices for a competitive result

HVP from hadronic τ decays - isospin breaking corrections

		Refs. [168, 196]	Ref. [211]	Refs. [239, 249]	Our estimate
Phase space		-7.88	-7.52	–	$-7.7(2)$
S_{EW}		$-12.21(15)$	$-12.16(15)$	–	$-12.2(1.3)$
G_{EM}		$-1.92(90)$	$-1.67^{+0.60}_{-1.39}$	–	$-2.0(1.4)$
FSR		$4.67(47)$	$4.62(46)$	$4.42(4)$	$4.5(3)$
ρ–ω mixing		$4.0(4)$	$2.87(8)$	$3.79(19)$	$3.9(3)$
	ΔM_ρ	$0.20(^{+27}_{-19})(9)$	$1.95^{+1.56}_{-1.55}$	–	
	$\Delta\Gamma_\rho(\Delta M_\pi)$	$4.09(0)(7)$	3.37	–	
$\frac{F_\pi^V}{f_+}$ (w/o ρ–ω)	$\Delta\Gamma_\rho(\pi\pi\gamma)$	$-5.91(59)(48)$	$-6.66(73)$	–	
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	–	
	Total	$-1.62(65)(63)$	$(-1.34)^{+1.72}_{-1.71}$	–	$-1.5(4.7)$
Sum		$-14.9(1.9)$	$(-15.20)^{+2.26}_{-2.63}$	–	$-15.0(5.1)$

- IB corrections still make the τ-based evaluation of the HVP model dependent
- In future lattice QCD and dispersive approaches could provide first-principle results
- Difficult to quantify how well we know the IB breaking corrections at this point! Quoted uncertainty was a compromise reached during the WP25 process. Larger than previous uncertainties.

HVP from hadronic τ decays - summary



Act 3: HVP from lattice QCD + QED

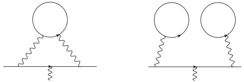
Why should one trust lattice QCD?

HVP from lattice QCD + QED - method

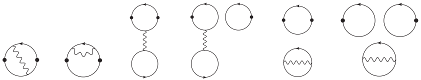
- First-principles calculation with QCD + QED Lagrangian
- Discretized Euclidean 4d space-time (finite volume, nonzero lattice spacing)
- Continuum limit of different discretizations give universal result
- Peturbative expansion in α_{QED} but non-perturbative in α_s, organized in Feynman diagrams without gluons
- Uses leadership-class supercomputers (exascale machines) to perform gluonic path-integral calculations as statistical samples from a Markov chain, therefore has statistical uncertainties as well

Diagrams

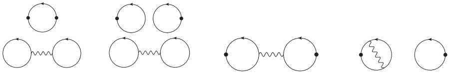
Isospin limit



QED corrections

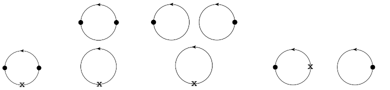


(a) V (b) S (c) T (d) T_d (e) D1 (f) $D1_d$



(g) D2 (h) $D2_d$ (i) F (j) D3

Strong isospin breaking



(a) M (b) R (c) R_d (d) O

HVP from lattice QCD + QED - isospin and flavor decomposition

- Typically organized as an expansion around isospin symmetric world

$$a_\mu^{\mathrm{HVP,\,LO}} = a_\mu^{\mathrm{HVP,\,LO}}(\mathrm{iso}) + \delta a_\mu^{\mathrm{HVP,\,LO}}$$

- Definition of isospin symmetric point adopted for WP25:

$$M_{\pi^+} = 135.0\,\mathrm{MeV}\,, \quad M_{K^0} = M_{K^+} = 494.6\,\mathrm{MeV}\,, \quad M_{D_s^+} = 1967\,\mathrm{MeV}\,, \quad \text{and} \quad w_0 = 0.17236\,\mathrm{fm}$$

- Choice of point irrelevant as long as isospin breaking corrections remain small. Consistency is important for comparison!

- It is possible to also separately provide different quark flavor contributions:

$$a_\mu^{\mathrm{HVP,\,LO}}(\mathrm{iso}) = a_\mu^{\mathrm{HVP,\,LO}}(ud) + a_\mu^{\mathrm{HVP,\,LO}}(s) + a_\mu^{\mathrm{HVP,\,LO}}(c) + a_\mu^{\mathrm{HVP,\,LO}}(b) + a_\mu^{\mathrm{HVP,\,LO}}(\mathrm{disc})$$

HVP from lattice QCD + QED - time-momentum representation

Symmetries allow for the reduction of the HVP leading-order diagram to an integral

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\tilde{f}(x_0)$$

over zero-momentum projected Euclidean correlators

$$C(x_0) = -\frac{1}{3}\sum_{k=1}^{3} \int d^3x\, C_{kk}(x_0, \mathbf{x})$$

of two vector currents

$$C_{\mu\nu}(x) = \langle j_\mu(x) j_\nu(0)\rangle$$

with all quark flavor fields included

$$j_\mu(x) = \sum_{f=1}^{N_f} Q_f\, \bar{q}_f(x)\gamma_\mu q_f(x)$$

HVP from lattice QCD + QED - Euclidean time windows

- RBC/UKQCD-18 PRL 121, 022003 (2018) introduced Euclidean time windows

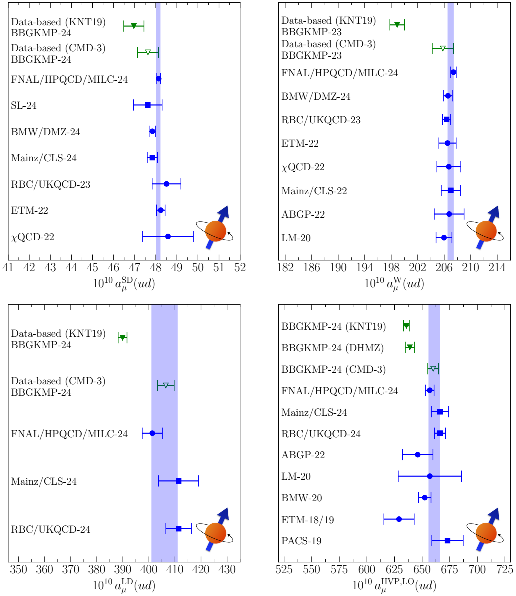
$$
\begin{aligned}
a_\mu^{\text{HVP, LO}} &= a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}\,,\\
a_\mu^{\text{SD}} &= \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\tilde{f}(x_0)[1-\Theta(x_0,t_0,\Delta)]\,,\\
a_\mu^{\text{W}} &= \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\tilde{f}(x_0)[\Theta(x_0,t_0,\Delta)-\Theta(x_0,t_1,\Delta)]\\
a_\mu^{\text{LD}} &= \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\tilde{f}(x_0)\Theta(x_0,t_1,\Delta)\,,
\end{aligned}
$$

$$\Theta(t,t',\Delta) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{t-t'}{\Delta}\right)$$

- Allow for taylored approach since each Euclidean time window has specific challenges (short-distance $\leftrightarrow$ discretization errors, long-distance $\leftrightarrow$ statistics, finite-volume errors)

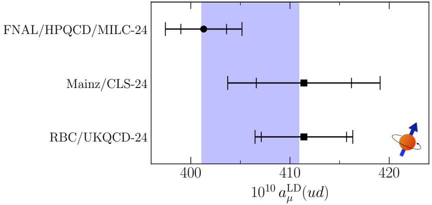
- Each window well-defined irrespective of regulator, can take some windows from, e.g., hadronic e^+e^- decays. This was proposed already in RBC/UKQCD-18 and recently picked up by BMW/DHMZ-24 for distances beyond $t_1 = 2.8$ fm. Tensions of e^+e^- data need to be negligible in this window.

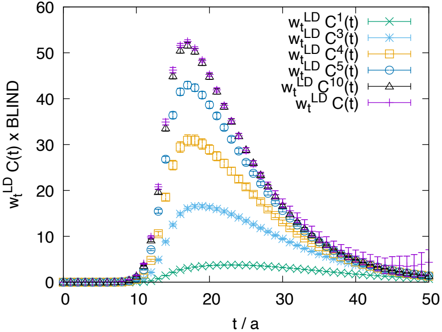
HVP from lattice QCD + QED - Euclidean time windows - results



- Windows allowed for high-precision consolidation over last years
- Different lattice groups use different discretizations
- Challenging long-distance window results just arrived in 2024
- Blinding procedures are now standard for all recent lattice results!
- Strong tensions in particular in intermediate and long-distance windows (projecting to lower energies) with data-driven hadronic e^+e^- results (pre CMD-3)

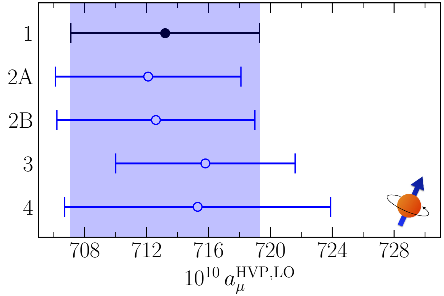
HVP from lattice QCD + QED - Long-distance window





- Uncertainty of HVP dominated by long-distance window uncertainty
- New technology: exclusive state reconstruction, right figure from RBC/UKQCD-24 (PRL 134, 201901 (2025))
- Other challenge: long-distance of QED corrections is currently not well constrained, added entire phenomenological estimate as added uncertainty for WP25

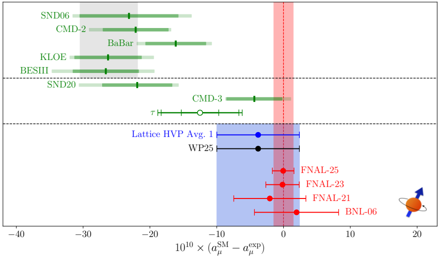
HVP from lattice QCD + QED - Averaging the lattice results



- Consolidation of individual components has led to stability, independent of how averages are performed
- Average 1: For each flavor, average windows first; takes most modern information into account
- Average 2A: Average results for the total flavor contributions
- Average 2B: Average results for total flavor contributions constructed from each collaboration's window results
- Average 3: Average isospin-symmetric total and isospin-breaking corrections separately (BMW, Mainz, RBC/UKQCD enter)
- Average 4: Average total results of each collaboration (BWM, Mainz, RBC/UKQCD enter)

Summary and Outlook

Summary



Contribution	WP25	WP20
HVP LO (lattice)	7132(61)	7116(184)
HVP LO (e^+e^-, τ)	Table 5	6931(40)*
HVP NLO (e^+e^-)	−99.6(1.3)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)	12.4(1)
HLbL (phenomenology)	103.3(8.8)	92(19)
HLbL NLO (phenomenology)	2.6(6)	2(1)
HLbL (lattice)	122.5(9.0)	82(35)
HLbL (phenomenology + lattice)	112.6(9.6)	90(17)
QED	116 584 718.8(2)	116 584 718.931(104)
EW	154.4(4)	153.6(1.0)
HVP (LO + NLO + NNLO)	7045(61)	6845(40)
HLbL (phenomenology + lattice + NLO)	115.5(9.9)	92(18)
Total SM Value	116 592 033(62)	116 591 810(43)

Outlook

- Lattice QCD will continue until exp. precision is matched; no blockers identified so far
- Further scrutiny of hadronic e^+e^- tensions is needed and on-going
- Including hadronic τ decays after IB corrections available without modelling (LQCD, dispersive approach)
- Other approaches to extract the HVP (such as MUonE) are followed as well