Reconstructing femtosecond longitudinal e-beam profiles from Smith-Purcell radiation spectra

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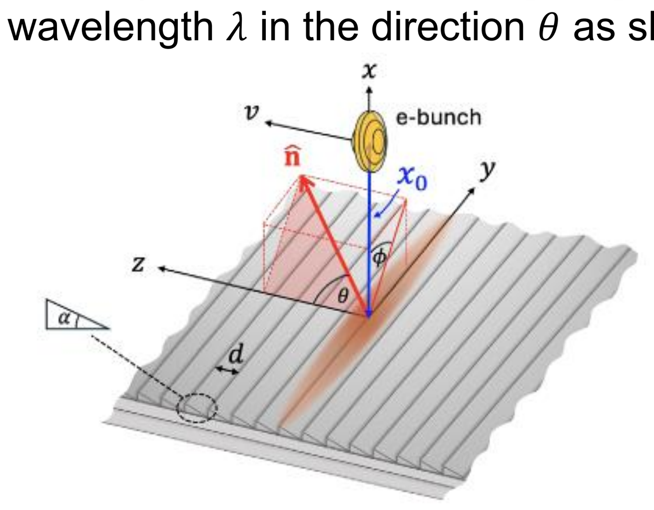
Introduction

With Laser-Wakefield Acceleration (LWFA), it is now possible to generate electron bunches with femtosecond longitudinal envelopes.[1] These fs long beam lengths pose a challenge to previous beam diagnostic methods whose resolution is insufficient in this regime. One promising method is the use of Smith-Purcell radiation (SPR). SPR was first discovered in 1953 to be generated from a periodic metallic surface when a beam of electrons passes close to the surface at velocity $\beta = \frac{v}{c}$.[2] These electrons induce a charge on the surface of the grating, whose periodic motion through it generates radiation of wavelength λ in the direction θ as shown in Figure 1.



$$\lambda = \frac{d}{n}(\beta^{-1} - \cos\theta)$$

Figure 1: Diagram of grating apparatus

The *nth* order of the radiation becomes coherent at specific viewing angles (θ, φ) for electron beam lengths equal to or less than the period of the grating d. Therefore, if the grating period is at least as long as the beam length, coherent Smith-Purcell radiation (CSPR) can be used as a noninvasive diagnose tool for the longitudinal profile of the beam.[3]

Research Goal

Explore known methods for reconstructing the longitudinal profile of electron beams from coherent Smith-Purcell radiation.

Methods

From the induced surface current model, the energy from CSPR for a total of N_e electrons is

$$\left(\frac{dW}{d\Omega}\right)_{N_e} = N_e^2 \left(\frac{dW}{d\Omega}\right)_1 S_{coh}(\theta,\phi)$$

where $\left(\frac{dW}{d\Omega}\right)_1$ is the energy generated by an idealized single electron passing over an infinitely wide grating,

$$\left(\frac{dW}{d\Omega}\right)_1 = 2\pi \frac{Zd}{n\lambda^3} R^2 e^{-2x_0/\lambda_e}$$

λ_e is the evanescent wavelength,

$$\lambda_e = \frac{\beta\gamma\lambda}{2\pi\sqrt{1+\beta^2\gamma^2 sin^2\theta sin^2\phi}}$$

and R^2 is the grating factor that determines how efficient the grating is at generating an electromagnetic field.[4] For an echelle grating that has two facets, one at a -90^o angle, the grating factor approaches the following value for $\phi \approx 0$ and $\gamma \gg 1$.[5]

$$R^2 \to \left(\frac{1}{sin\,\theta - tan\,\alpha\,(1+cos\,\theta)}\right)^2 tan^2\,\alpha\ sin^2\,\theta\ sinc^2\left(\frac{kh\sin\theta}{2}\right)$$

Here, k is the wavenumber, α is the angle of the other facet and $h = d tan\theta$. S_{coh} is the coherent factor that determines the proportion of the radiation that is coherent for an uncorrelated beam distribution $D(x,y,z) = X(x)Y(y)Z(z)$.

$$S_{coh}(\theta,\phi) = \left|\int_0^\infty X(x) e^{-x/\lambda_e} \tilde{Y}(k_y) \tilde{Z}(k_z) dx\right|^2$$

$\tilde{Y}(k_y)$ and $\tilde{Z}(k_z)$ are the Fourier transforms (FTs) for the y and z dimensions. To solve for the temporal profile, we can reparametrize the coherent factor with $t = z/v$ so that the FT of $Z(t)$ is replaced with the FT of $T(t)$.

$$\tilde{T}(\omega) = \int_{-\infty}^{\infty} T(t) e^{-i\omega t} dt$$

The resulting coherent factors for an uncorrelated Gaussian electron beam distribution is shown in Figure 2. For small enough solid angle measurements made along $\phi = 0$ and $\theta > 60^o$, changes in the longitudinal coherent factor $S_{coh}^{(z)} = \left|\tilde{Z}(z)\right|^2$ dominates over the other two factors that remain ≈ 1.

$$\left(\frac{dW}{d\Omega}\right)_{N_e} \approx N_e^2 \left(\frac{dW}{d\Omega}\right)_1 \left|\tilde{Z}(z)\right|^2$$

Therefore, a set of measurements done across $\phi = 0$ can be used to isolate the longitudinal coherent factor and reconstruct the longitudinal profile of the beam. If the shape of the beam is simple, then it is possible to extract the parameters of the shape through least squares fitting of possible known coherent factors. An outline of our approach is the algorithm in Figure 3 below.

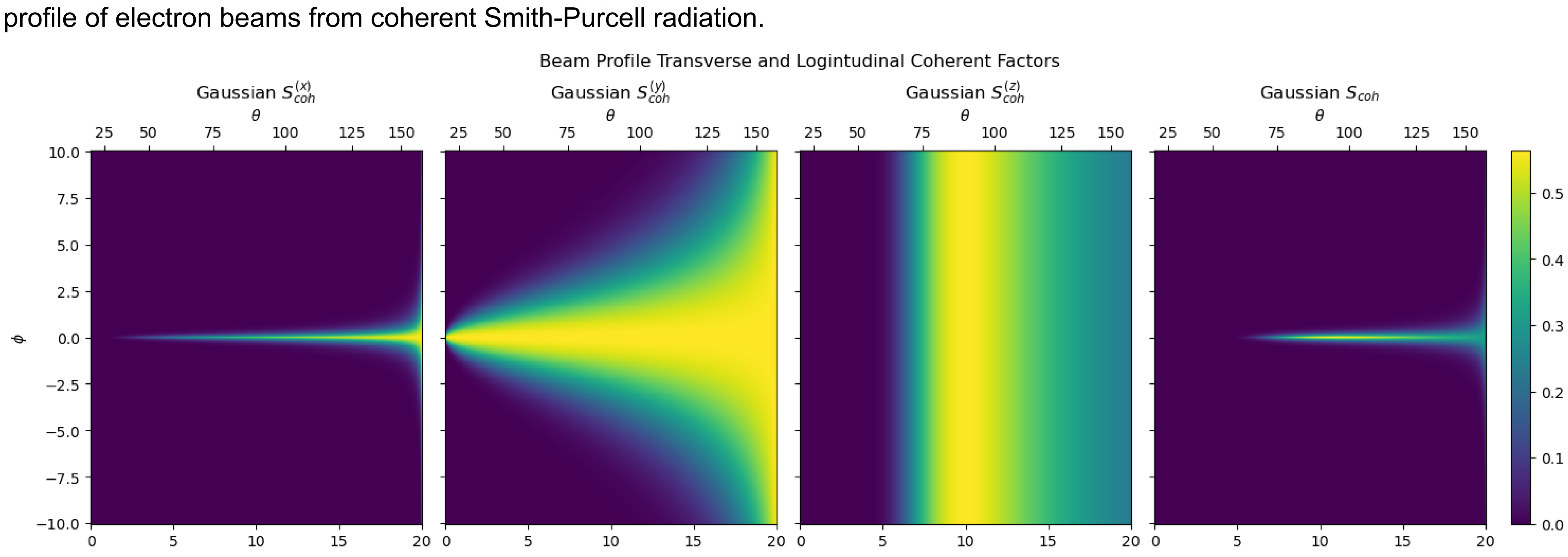


Figure 2: Distribution of coherent factors for an electron beam with uncorrelated Gaussian distributions.

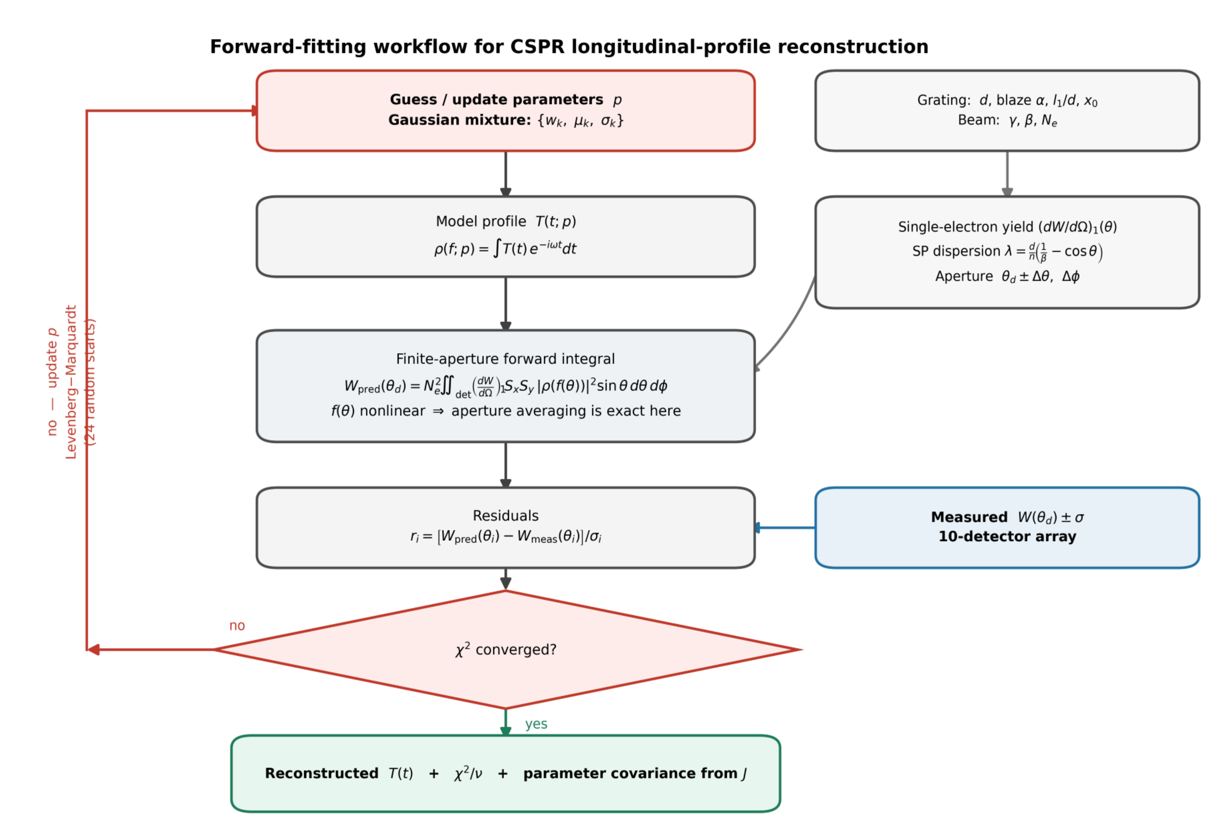


Figure 3: The least-squares algorithm for reconstructing the longitudinal profiles of electron beams.

Results

Forward calculations were completed for a variety of different electron beam distributions to see what should be expected for real measurements. Figure 4a shows these measurements across most of the angular space above the grating for a detector with a diameter of 1^o. To test the least squares method, I extracted 10 measurements from each distribution to simulate an array of 10 detectors. These measurements were used to find a best fit using least squares as outlined in the algorithm in Figure 3. Once the basic parameters are known, it is simple to display how the beams are shaped.

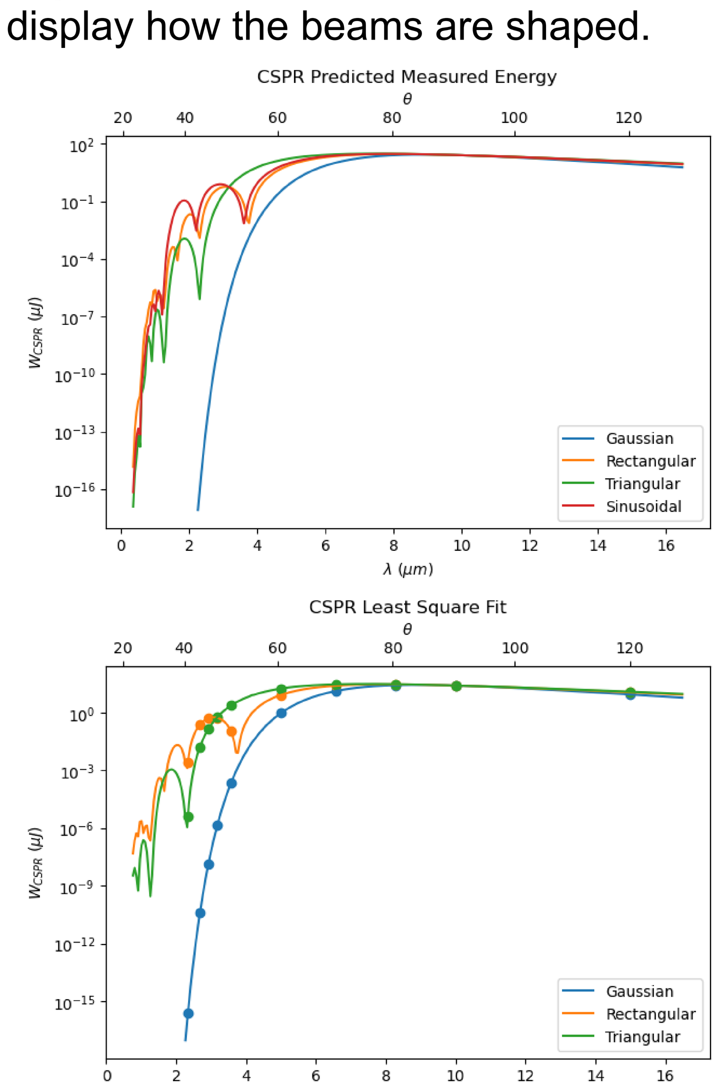


Figure 4: (a) is the top plot where I have calculated the energy that a 1^o diameter idealized detector should measure for Gaussian, rectangular, triangular and sinusoidal beam distributions. (b) is where I have reconstructed the beam profiles for an idealized case least squares.

Conclusion

For more complicated realistic shapes, the method outlined in this poster may be insufficient in accurately determining the longitudinal profile of the beam. We can extend this method in the future by using the Kramers-Kronig (KK) relation to determine the phase, as has been done previously for picosecond long beams.[6,7] This can be done by writing the FT of the temporal profile as

$$\tilde{T}(\omega) = \rho(\omega) e^{i\psi(\omega)}$$

where $\rho^2(\omega) = \left|\tilde{T}(\omega)\right|^2$. The minimal phase can then be found by the formula

$$\psi(\omega_0) = \frac{2\omega_0}{\pi}\int_0^\infty \frac{ln[\rho(\omega)/\rho(\omega_0)]}{\omega_0^2 - \omega^2} d\omega$$

where ω_0 is the frequency at which the temporal profile diverges. The resulting longitudinal profile is then

$$T(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \rho(\omega) cos[\omega t + \psi(\omega)] d\omega$$

However, the KK method has its own limitations as it is impossible to measure up to infinite frequency. There may also be additional singularities in the phase that cannot be known beforehand and are difficult to deal with.[6,8] Determining the longitudinal profile from coherent radiation has been an ongoing problem for decades in accelerator physics. Current research shows that it is possible to apply previous numerical methods to CSPR for femtosecond long beams generated by LWFAs. Future experiments are planned to be conducted at HZDR and ELI-NP to get measurements of CSPR from relativistic electron beams, which can then be used to test the described methods for extracting a longitudinal profile in the femtosecond regime.

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References

1. R. Rudzinksy et al, "Coherent terahertz Smith–Purcell radiation from laser-wakefield-accelerated electron bunches," Optica 13, 5 (2026)
2. S. J. Smith and E. M. Purcell, "Visible light from localized surface charges moving across a grating," Phys. Rev. 92, 1069 (1953).
3. M. C. Lampel, "Coherent Smith-Purcell radiation as a pulse-length diag- nostic," Nucl. Instrum. Meth. Phys. Res. A 385, 19–25 (1997).
4. J. H. Brownell, J. Walsh, and G. Doucas, "Spontaneous Smith-Purcell radiation described through induced surface currents," Phys. Rev. E 57, 1075 (1998).
5. J. H. Brownell and G. Doucas, "Role of the grating profile in Smith- Purcell radiation at high energies," Phys. Rev. ST Accel. Beams 8, 091301 (2005).
6. R. Lai, A.J. Sievers, "On using the coherent far IR radiation produced by a charged-particle bunch to determine its shape: I Analysis," Nuc. Inst. and Meth. in Phys. Res. A 37 221 (1997).
7. V. Blackmore, G. Doucas, C. Perry, et al., "First measurements of the longitudinal bunch profile of a 28.5 GeV beam using coherent Smith-Purcell radiation," Phys. Rev. ST Accel. Beams 12, 032803 (2009).
8. H. M. Nussenzveig, "Phase Problem in Coherence Theory," Journal of Mathematical Physics 8, 3 (1966).
9. G. Doucas, Smith-Purcell Radiation: Basic Theory and Applications (Oxford University, 2025).