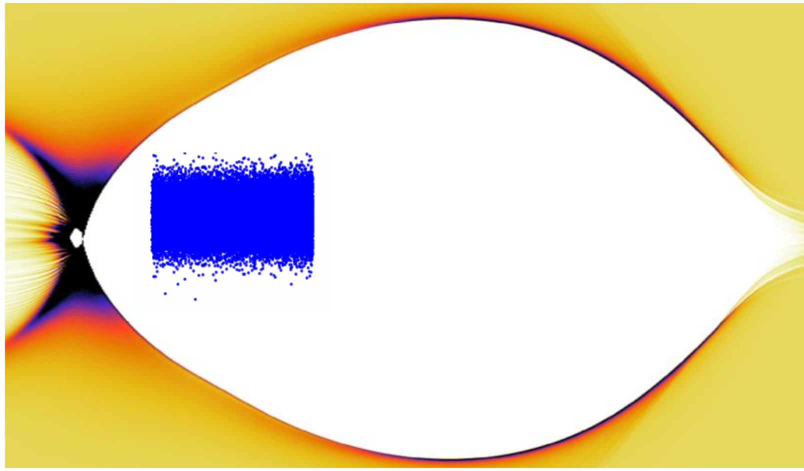
Investigation of transverse instability in efficient plasma-based accelerators



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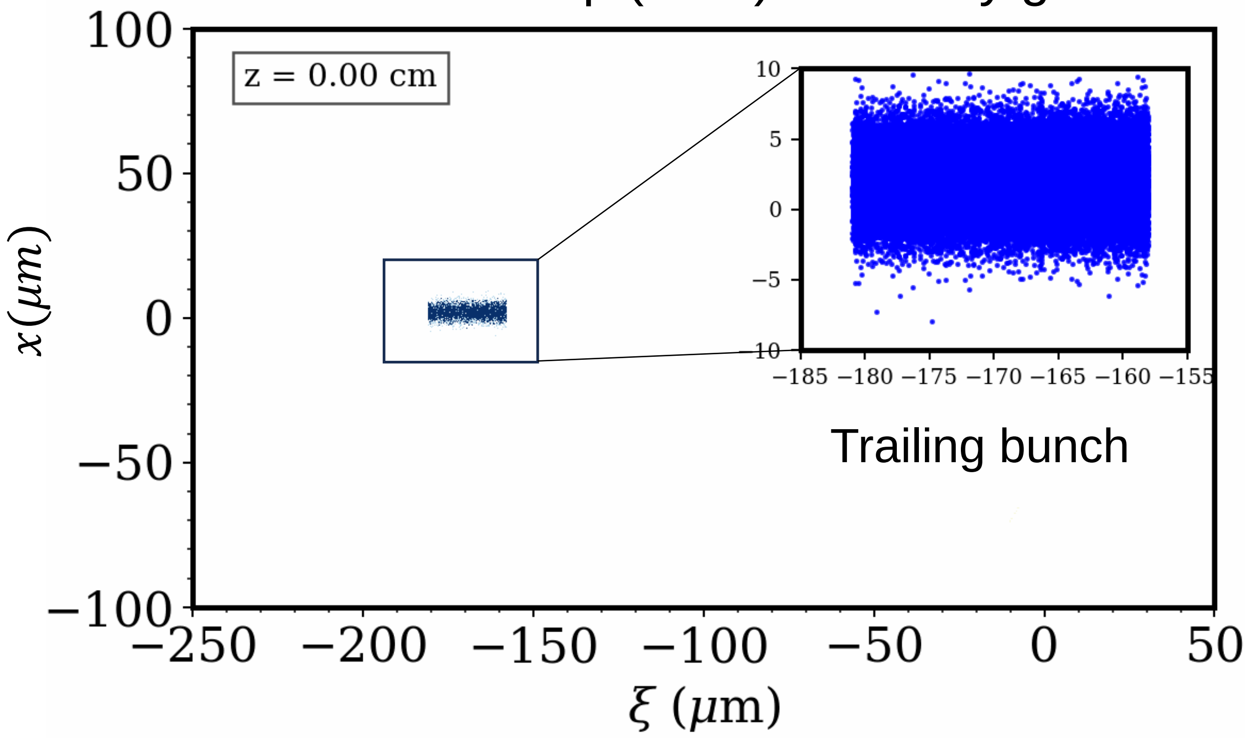
Efficiency vs Instability

- **Goal:** Plasma stages must achieve high power-transfer efficiency while preserving beam phase-space quality (↓ emittance ↓ energy spread)

- **Threat (BBU):** For long-distance laser/plasma wakefield acceleration, strong plasma focusing forces resonantly amplify small transverse offsets into large transverse oscillations
  This is a critical issue of large emittance in prospective plasma-based colliders

- **Trade-off:** Lebedev *et al*., suggested- BBU is unavoidable at high efficiency, mitigation (like BNS damping) compromises collider-level beam quality

Beam-breakup (BBU) instability growth



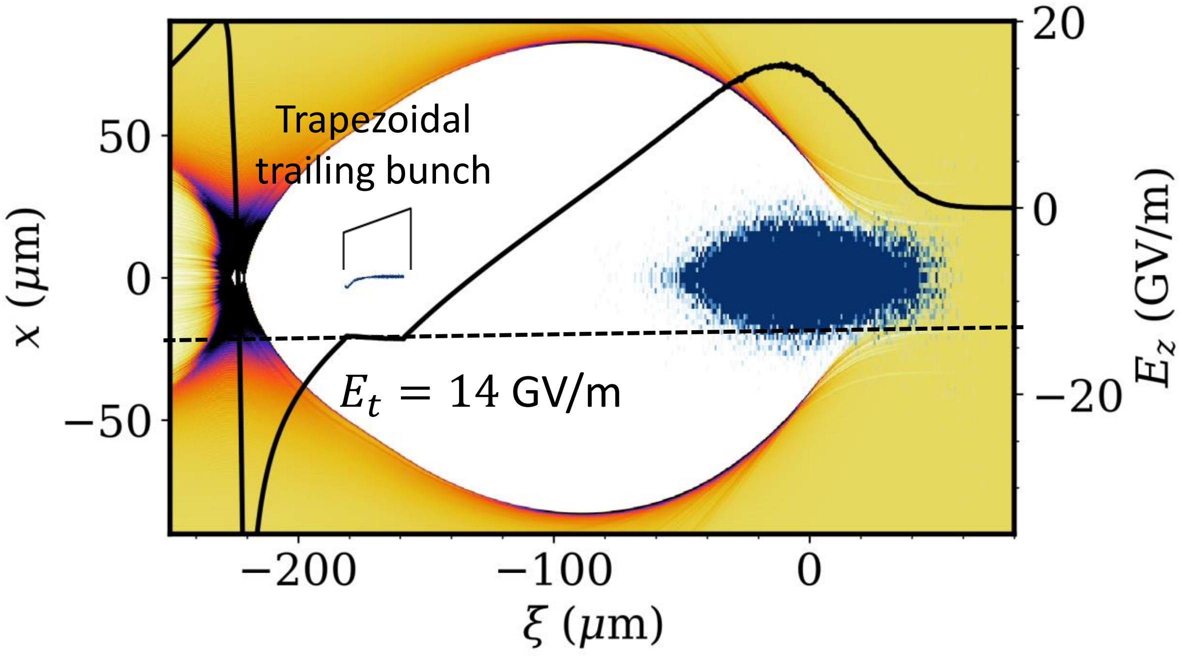
↓ Instability ↑ Efficiency ↓ Energy spread

Lebedev *et al*., PRAB(2017)

Beam loading for achieving low energy spread

Simulation design criteria

- The system targets a blowout cavity radius $R_b \sim 3$ to support collider-relevant charges ~ 1 nC at a loaded accelerating field $E_t > 10$ GV/m.
- Driver: sustain a highly stable plasma cavity over meter-scale propagation allowing the beam-breakup dynamics to be isolated from wake evolution
- Trailing bunch: Trapezoidal current profile flattens the E_z field and minimize energy spread



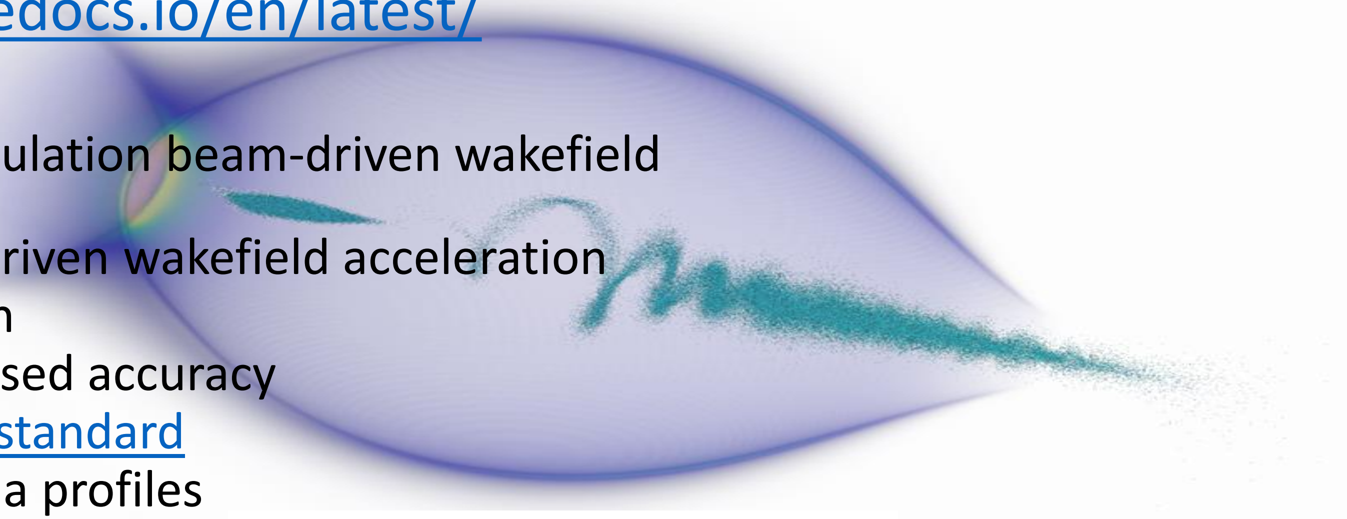
Parameters

- Plasma: $n_0 = 3 \times 10^{16}\ \mathrm{cm}^{-3}$
- Driver beam: $Q = 3.2\ \mathrm{nC}$ $\mathcal{E} = 20\ \mathrm{GeV}$
- Trailing bunch: $Q = 1.1\mathrm{nC}$ $\mathcal{E} = 50\ \mathrm{MeV}$ $\Delta\mathcal{E}/\mathcal{E}$ =0.1% $\sigma_x = \sigma_y = 2\ \mu\mathrm{m}$ $\sigma_z = 23\ \mu\mathrm{m}$ Offset $\mathrm{X}_0 = 2\ \mu\mathrm{m}$

M. Tzoufras *et al.*, PRL(2008) A.Jain *et al.*, PRR(2026) Jiayang Yan PhD thesis (2025)

The code HiPACE++

- 3D, QS PIC for plasma acceleration
- Runs on GPU and (multi-core) CPU
- Open-source:https://github.com/Hi-PACE/hipace
- Documentation: https://hipace.readthedocs.io/en/latest/
- Main features:
  - Multiple beams and plasma species to simulation beam-driven wakefield acceleration
  - A laser envelope solver to simulate laser-driven wakefield acceleration
  - Mesh refinement for efficient computation
  - An advanced explicit field solver for increased accuracy
  - Diagnostics compliant with the openPMD standard
  - Arbitrary profiles for the beams and plasma profiles
  - Readers from files for the beam and laser profiles



S. Diederichs et al., Comput. Phys. Comm. 278: 108421 (2022)

Modelling beam-breakup instability

Transverse motion of beam slice centroid $X(\xi, z)$

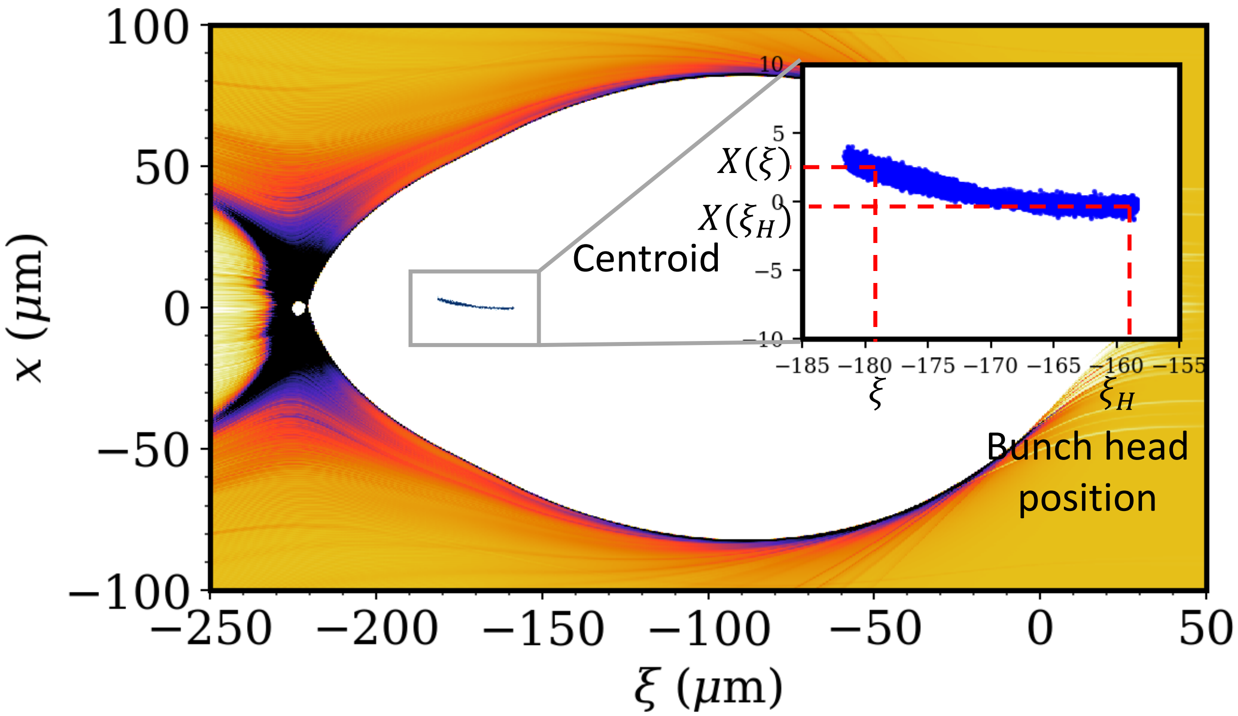
$$\frac{\partial^2 X}{\partial z^2} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial z}\frac{\partial X}{\partial z} + k_\beta^2 X = -\frac{e}{\mathcal{E}}\,\tilde{F}_\perp$$

Adiabatic damping due to energy gain

Ion restoring force

Driving force

$$\tilde{F}_\perp = \frac{F_\perp(\xi, z)}{q} = \int_{\xi_H}^{\xi} W_\perp(\xi' - \xi)\,\lambda(\xi')X(\xi', z)\,d\xi'$$



Precise determination of transverse wake function $W_\perp$ is crucial for accurate modeling of transverse instabilities in a plasma accelerator

Wake potential model

Driving force

$$\tilde{F}_{\perp} = \frac{F_{\perp}(\xi, z)}{q} = \int_{\xi_H}^{\xi} W_{\perp}(\xi' - \xi)\,\lambda(\xi')X(\xi', z)d\xi'$$

Transverse wake function

Short-range wake theorem

$$W_{\perp} = \frac{2}{r_b^2(\xi')q_t}\int_{\xi}^{\xi'} E_z\,du$$

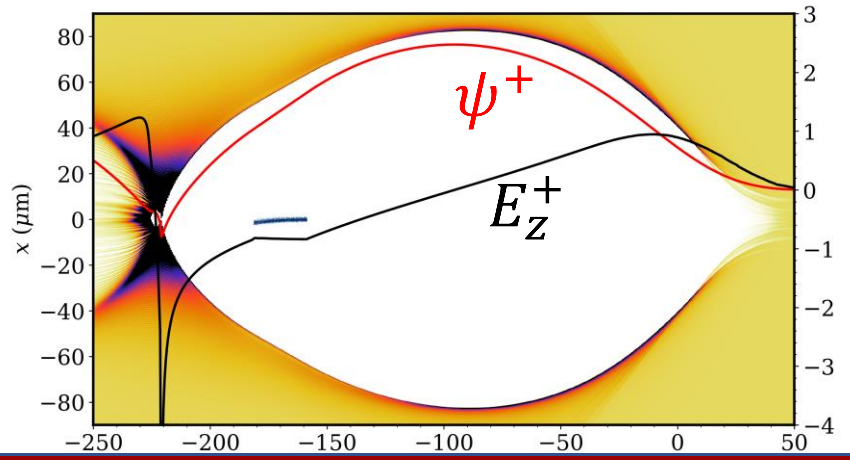
Longitudinal wake function:
E_z generated by the trailing beam

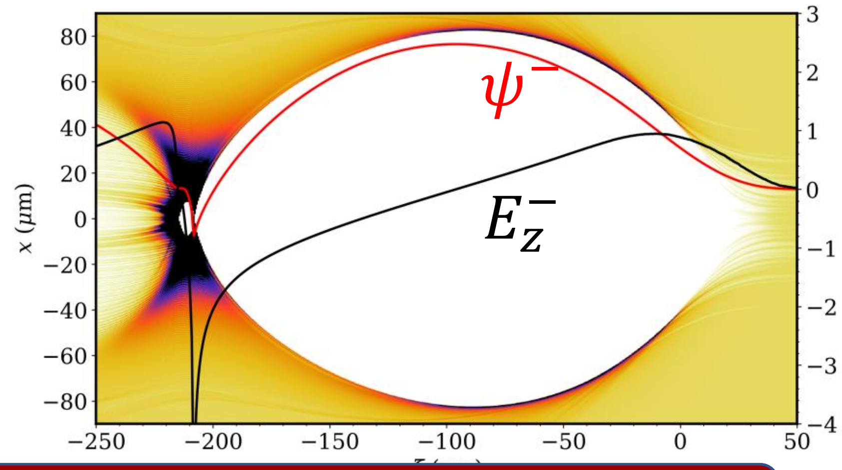
Wake potential superposition

$$E_z = -\frac{1}{q_t}\frac{\partial \psi_t}{\partial \xi}$$
$$\psi = \phi - A_z$$
$$\psi_t = \psi^+ - \psi^-$$

With trailing bunch

Without trailing bunch





Our wake potential model is valid for both LWFA and PWFA

Wake potential model shows excellent agreement with simulation

Previous Models

Lebedev:

$$W_\perp \approx \frac{2}{a(\xi)^4}$$

Stupakov:

$$W_\perp \approx \frac{2}{(a(\xi) + 0.75k_p^{-1})^4}$$

High charge witness beam significantly deforms the bubble boundary, resulting in significant error in this model

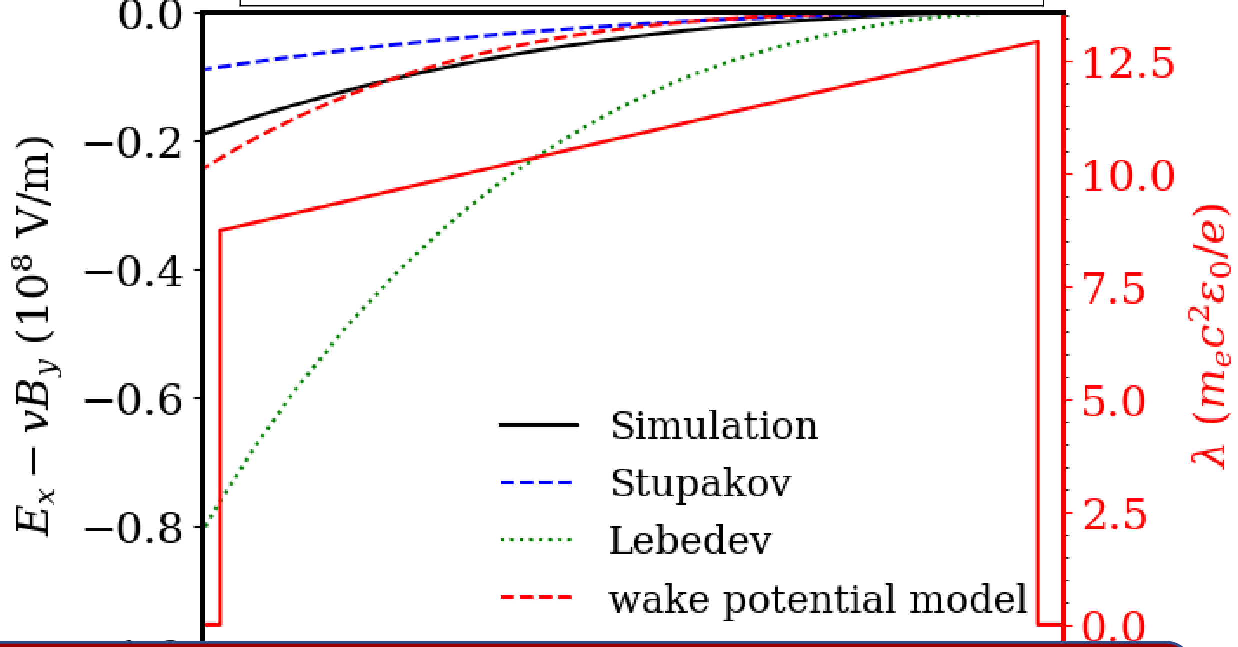
Wake potential model

$$W_\perp = \frac{2}{r_b^2 q_t}(\psi^+ - \psi^-)$$

- Our model naturally incorporates exact, deformed boundaries without any fitting parameters.
- The potential difference mathematically represents the exact integral of the longitudinal field.

Transverse force

$$\tilde{F}_\perp = \int_{\xi_H}^{\xi} W_\perp(\xi' - \xi)\, \lambda(\xi') X(\xi', z) d\xi'$$

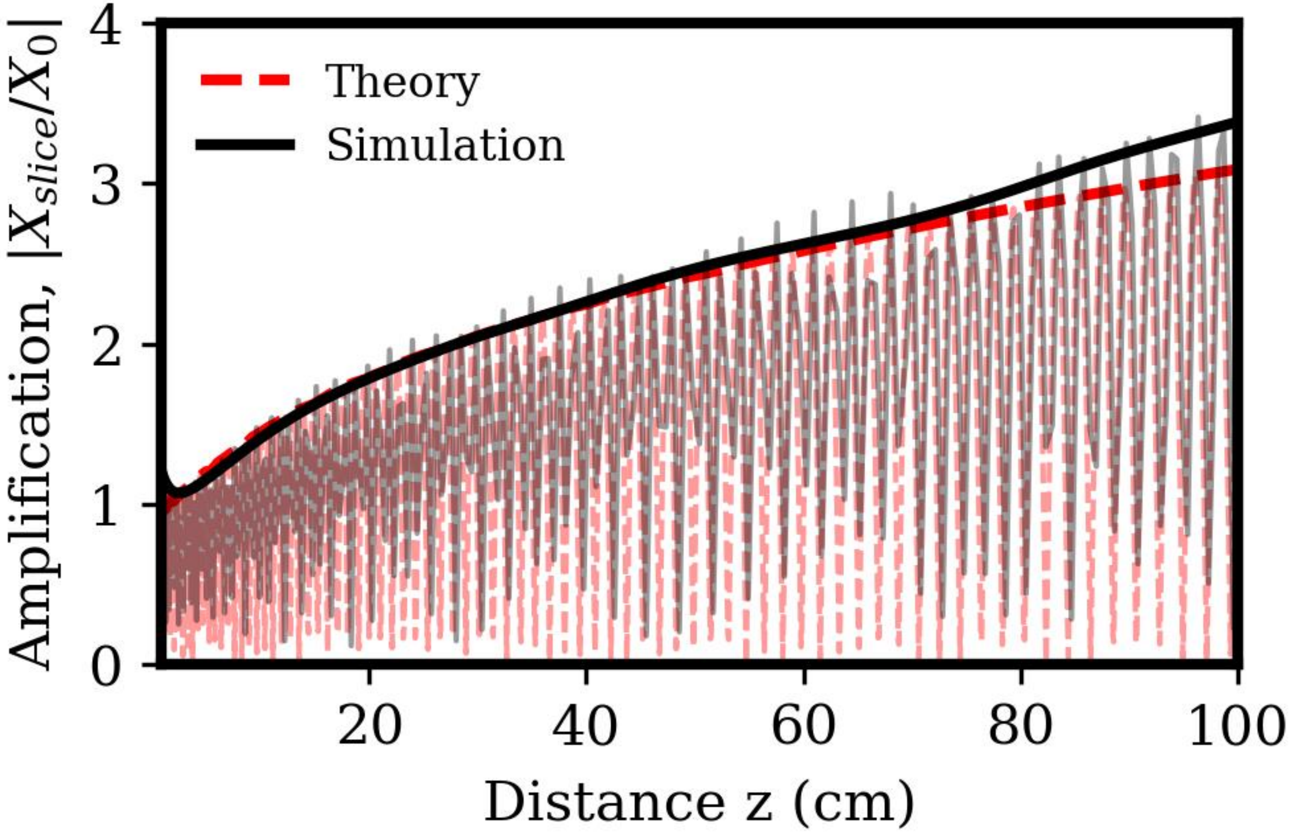


A factor-of-four error can lead to a substantial overestimate of the accumulated transverse instability.

Lebedev *et al.*, PRAB(2017)
Stupakov *et al.*, PRAB(2018)

A. Jain *arXiv:2607.10497* (2026) [under review in PRAB]

Bunch slice centroid evolution



Transverse motion of centroid $X(\xi, z)$
$$\frac{\partial^2 X}{\partial z^2} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial z}\frac{\partial X}{\partial z} + k_\beta^2 X = -\frac{e}{\mathcal{E}}\tilde{F}_\perp$$
Driving force
$$\tilde{F}_\perp = \int_{\xi_H}^{\xi} W_\perp(\xi' - \xi)\,\lambda(\xi')X(\xi', z)d\xi'$$
Transverse wake function
$$W_\perp = \frac{2}{r_b^2 q_t}(\psi^+ - \psi^-)$$

The agreement remains strong over a meter of propagation at 10 GeV/m
Wake potential formalism captures the transverse beam dynamics with predictive accuracy

A. Jain *arXiv:2607.10497* (2026) [under review in PRAB]

Analytical modelling of efficiency and instability

Instability- X/X_0

Transverse wake function

$$W_\perp = \frac{2}{r_b^2 q_t} \int_{\xi}^{\xi'} E_z \, du$$

$E_z = E_t - \frac{\chi\xi}{2}$, $\chi/2$ – Slope of E_z for $R_b < 4$

$$W_\perp = \frac{2}{r_b^2 q_t} \left(E_t(\xi' - \xi) - \frac{\chi}{4}(\xi'^2 - \xi^2) \right)$$

Driving force

$$\tilde{F}_\perp = \int_{\xi_H}^{\xi} W_\perp(\xi' - \xi)\, \lambda(\xi') X(\xi', z) d\xi'$$

Transverse motion of centroid $X(\xi, z)$

$$\frac{\partial^2 X}{\partial z^2} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial z}\frac{\partial X}{\partial z} + k_\beta^2 X = -\frac{e}{\varepsilon}\, \tilde{F}_\perp$$

Efficiency- η_b

Wake-to-trailing-bunch energy transfer efficiency η_b

- Ratio of extracted energy to the maximum energy available in the wake.
- How effectively beam loads the wake.
- How effectively extracts the available wake energy.

Beam loading efficiency η_b

$$\eta_b = \frac{2\pi \int E_t \lambda(\xi)\, d\xi}{\chi\pi R_b^4/16}$$

$$\eta_b = 32 \frac{E_t \Delta\xi}{\chi R_b^4} \left(\sqrt{\left(\frac{\mathrm{E_t}}{\chi}\right)^4 + \frac{R_b^4}{16}} - \frac{E_t}{2\chi}(\Delta\xi) \right)$$

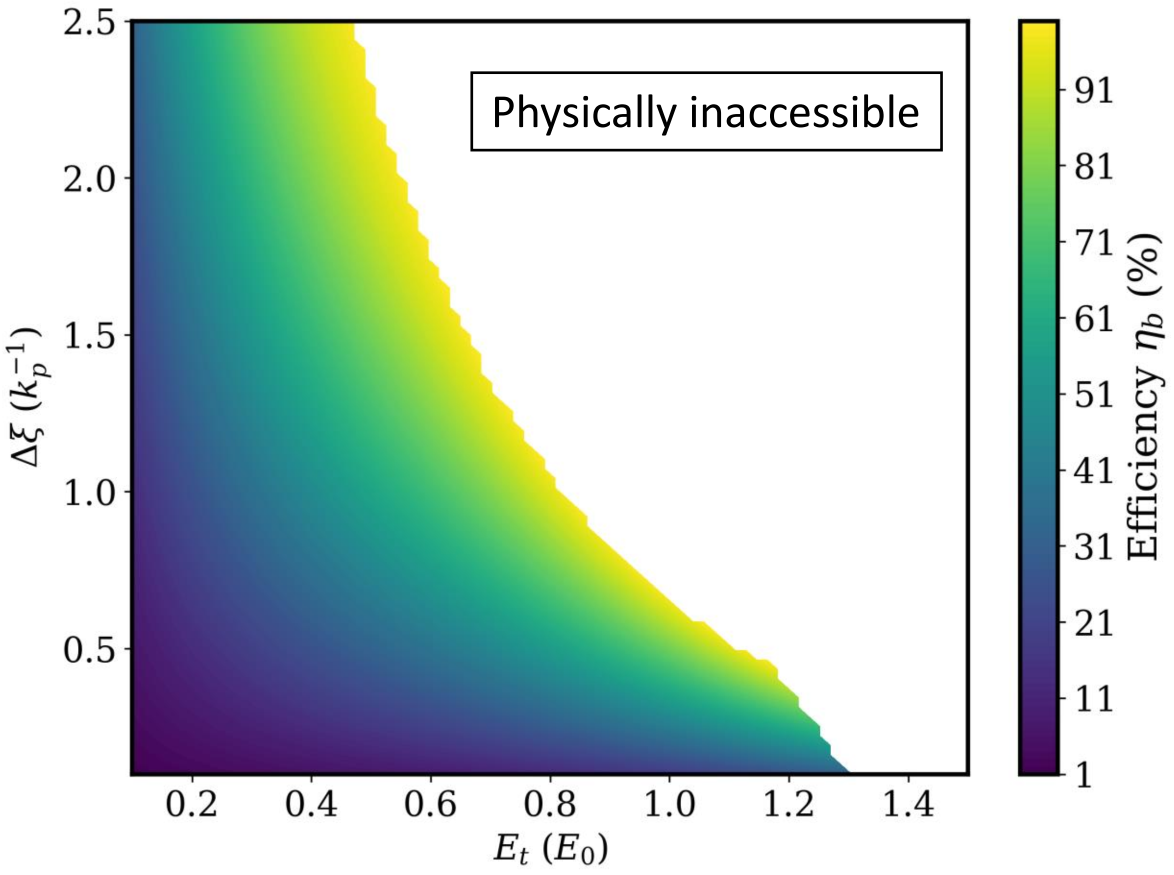
M. Tzoufras *et al.*, PRL (2008)

A. A. Golonov *et al.*,PPCF (2021)

Jiayang Yan PhD thesis (2025)

Mapping the High-Efficiency, High-Stability Operating Window

Bunch Length



Loaded field

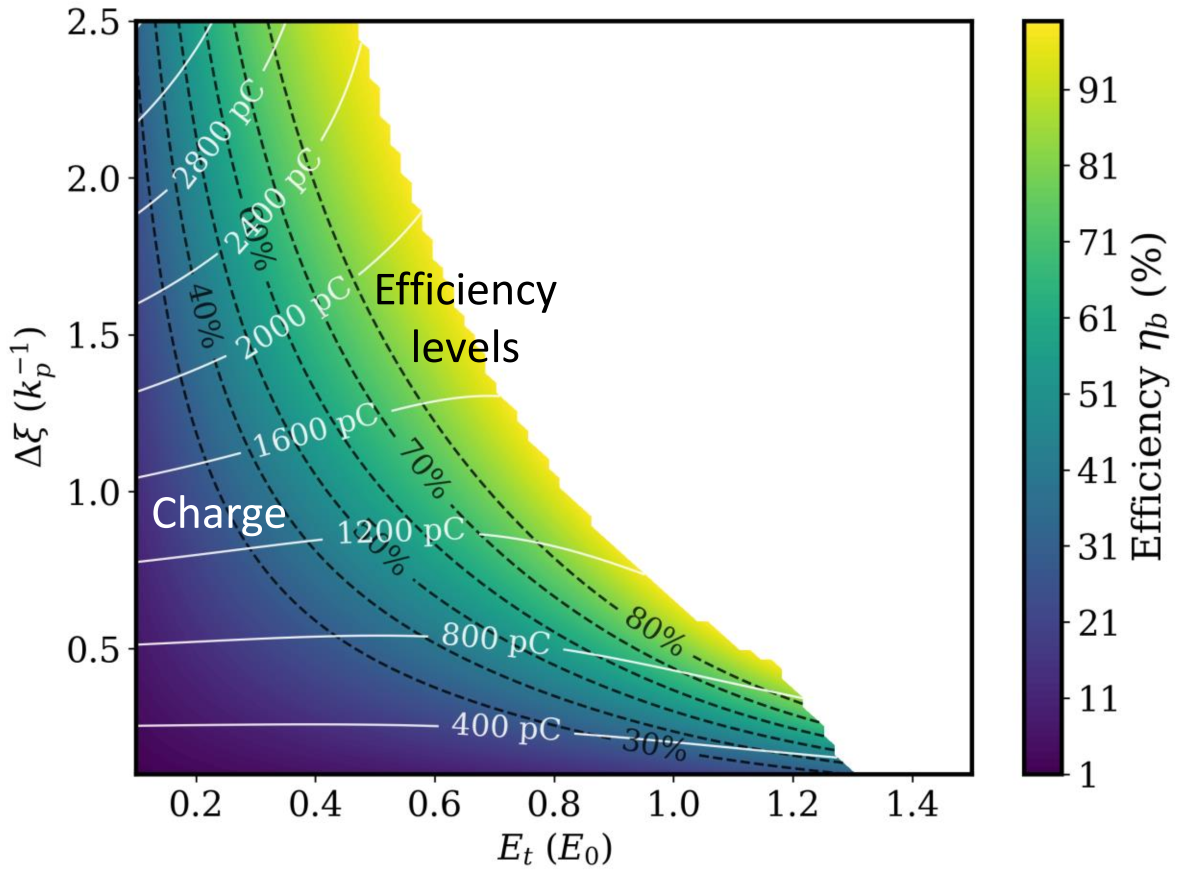
$n_0 = 3 \times 10^{16}$ cm^{-3}

$R_b = 2.7$

$X_0 = \sigma_x = 2$ μm

A. Jain *arXiv:2607.10497* (2026) [under review in PRAB]

Mapping the High-Efficiency, High-Stability Operating Window



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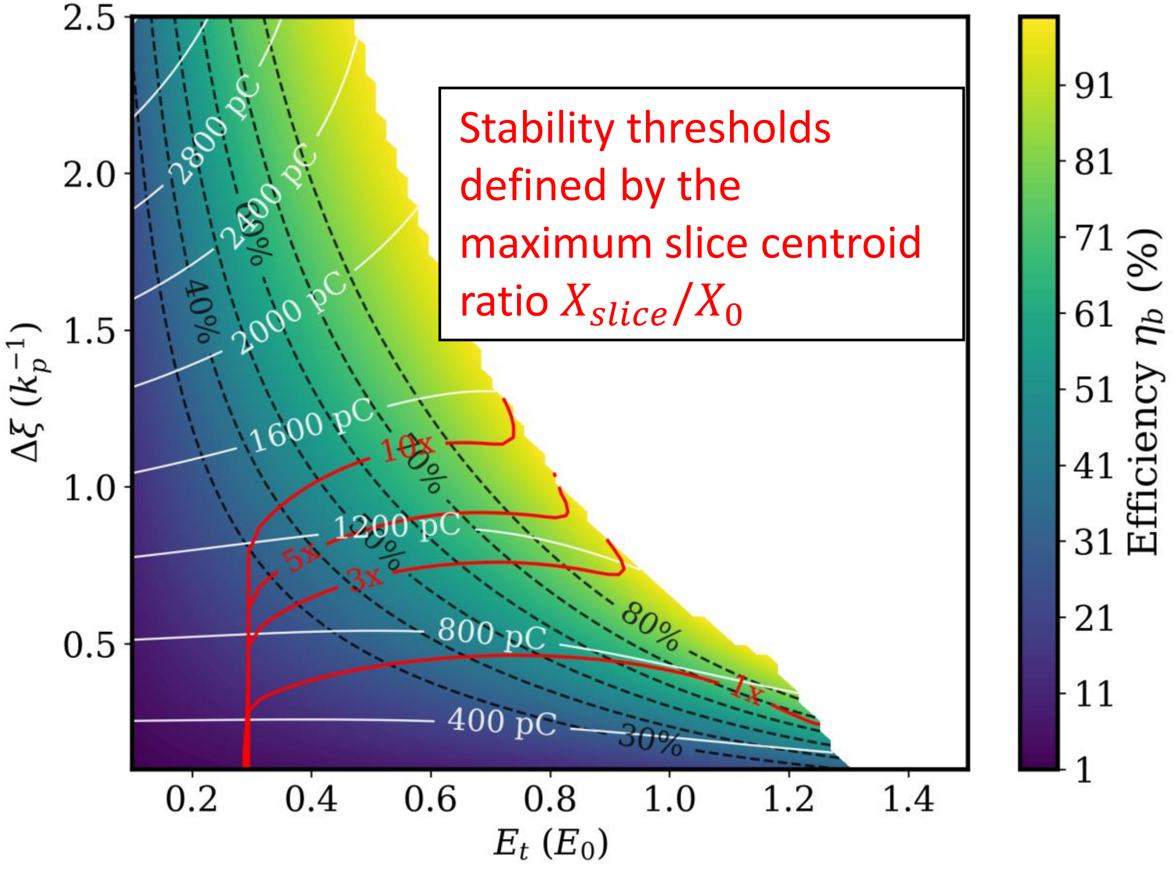

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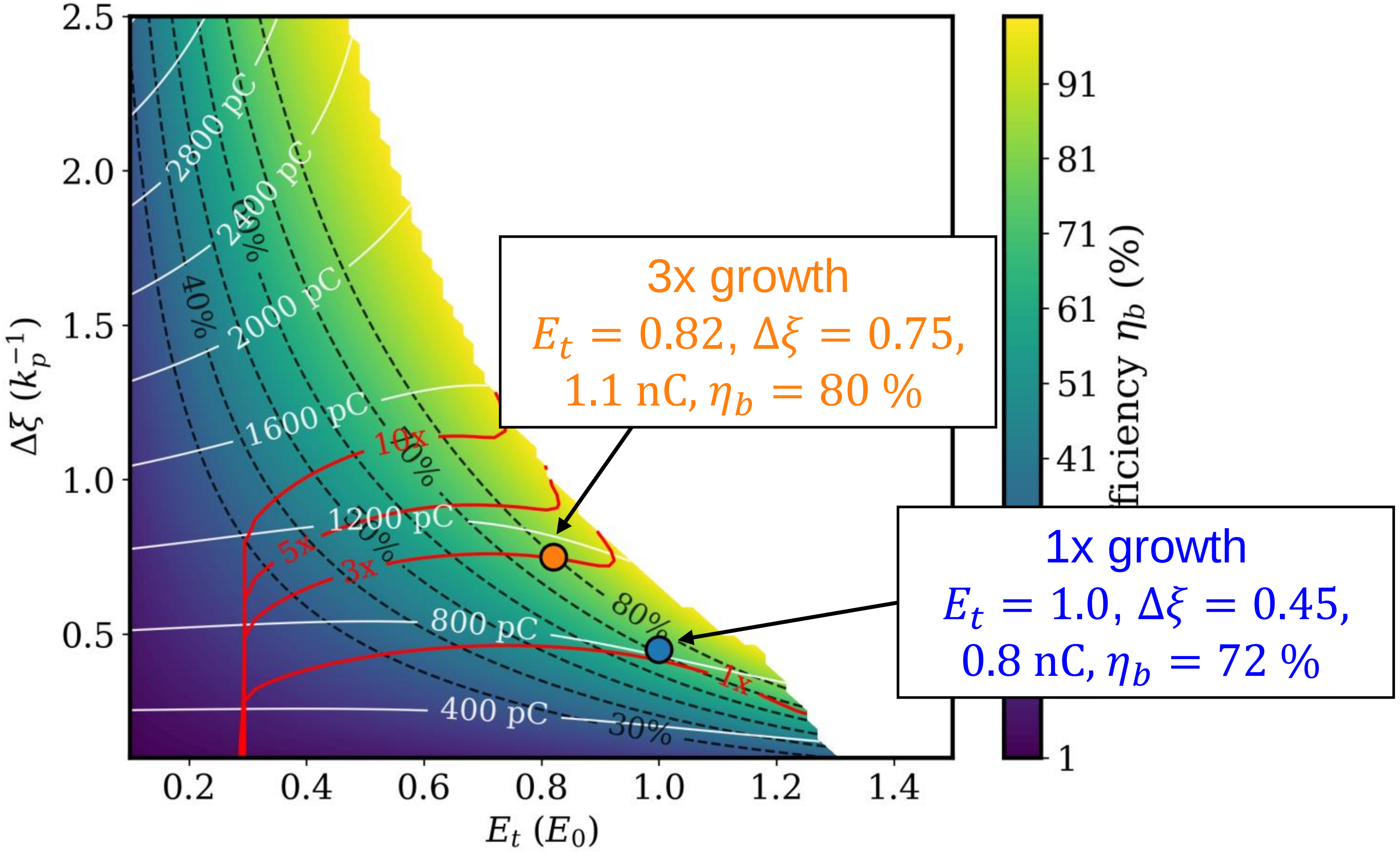


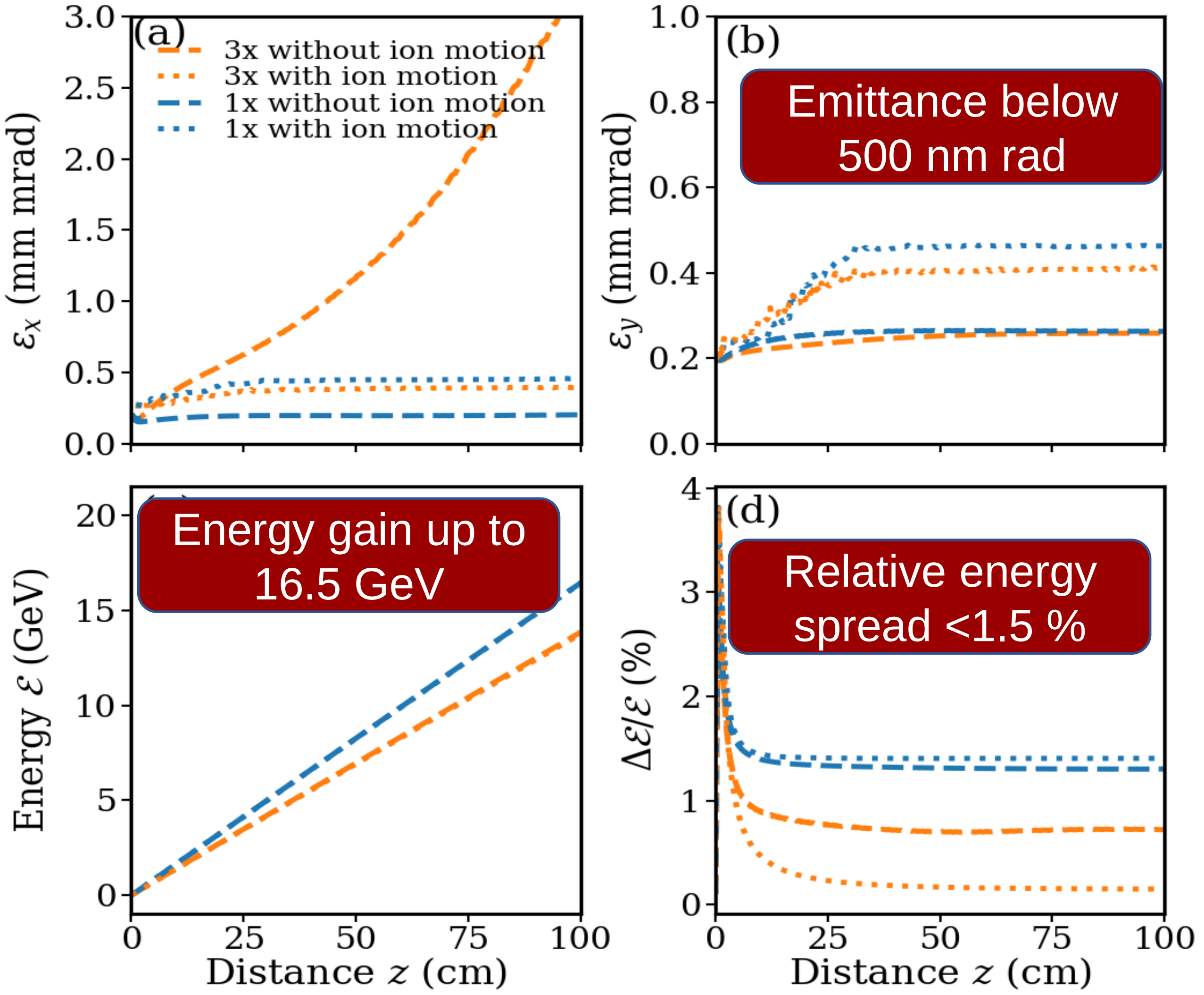
A. Jain *arXiv:2607.10497* (2026) [under review in PRAB]

Preservation of the beam quality over a meter-long plasma

Ion motion (dotted lines)

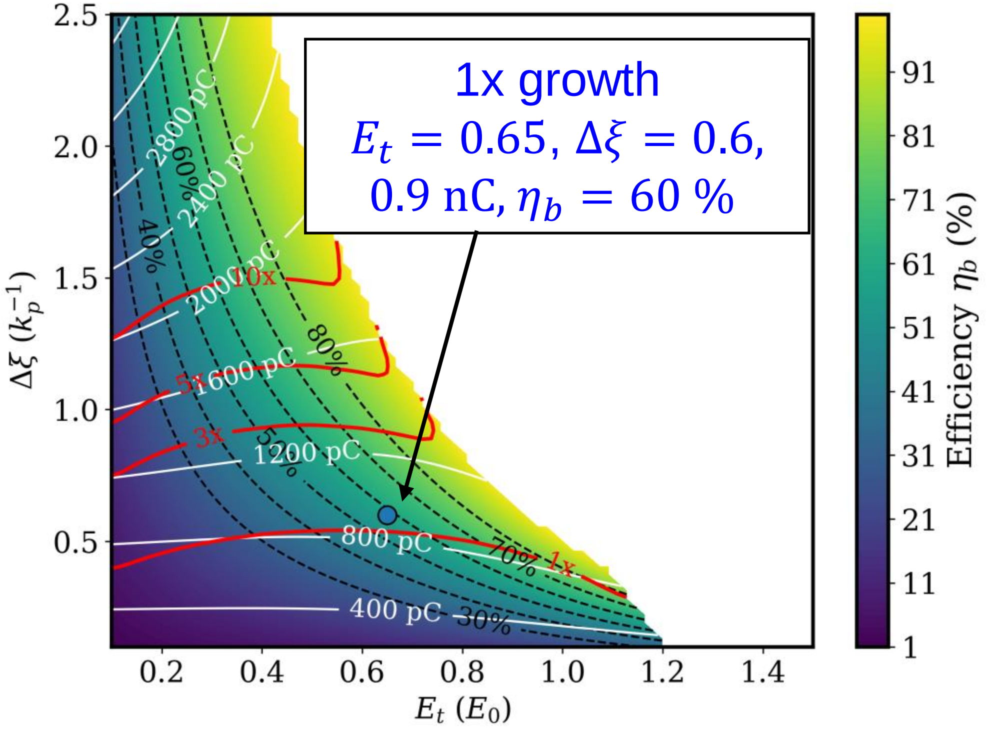
- Well-known mechanism for bounding BBU growth in plasma accelerators
- **Phase Mixing:** Nonuniform focusing along the trailing bunch causes different slices to accumulate differing betatron phases, reducing the BBU growth rate.
- Ion response introduces additional transverse forces modifying projected beam quality in configurations with limited centroid growth.
- Ion motion bounds the emittance growth of 3× while a baseline emittance increase in 1x

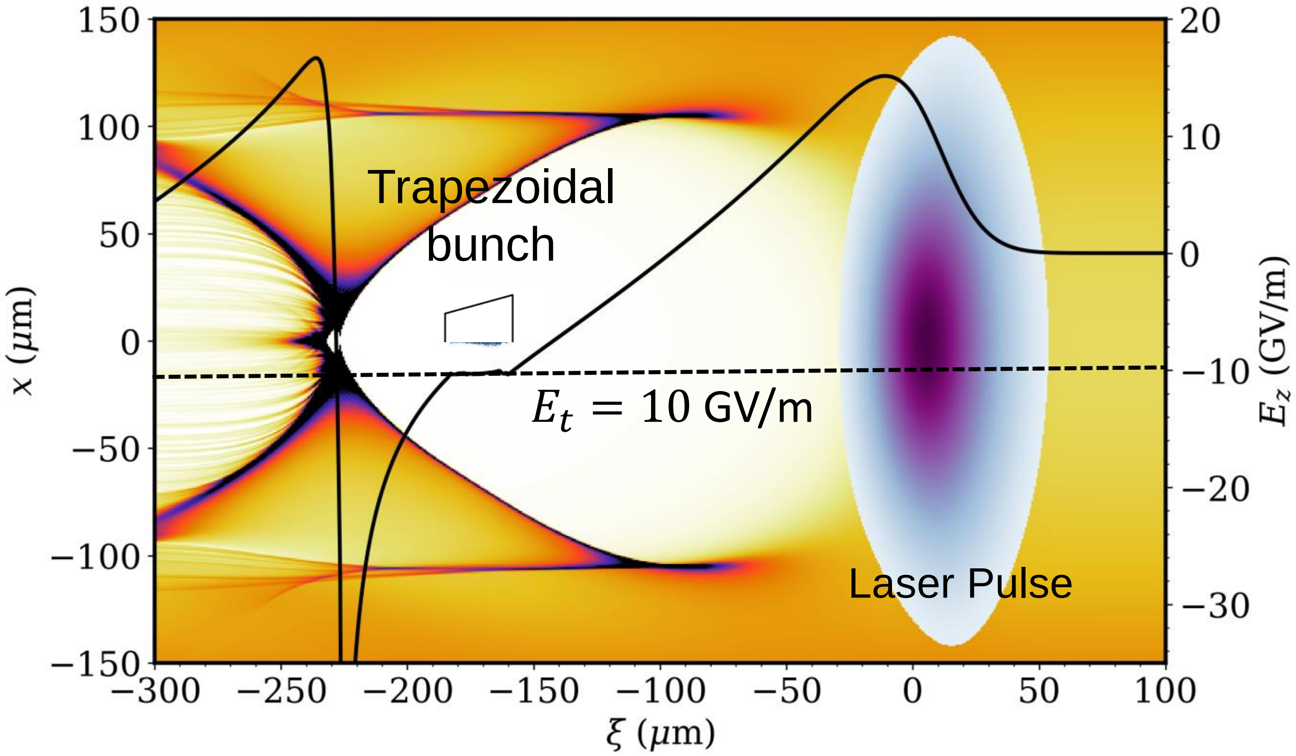
$\eta_b \sim 80\%,\ \ \eta \sim 40\ \%$



T. J. Mehrling *et al.*, PRL (2018)

Laser driven simulation set up

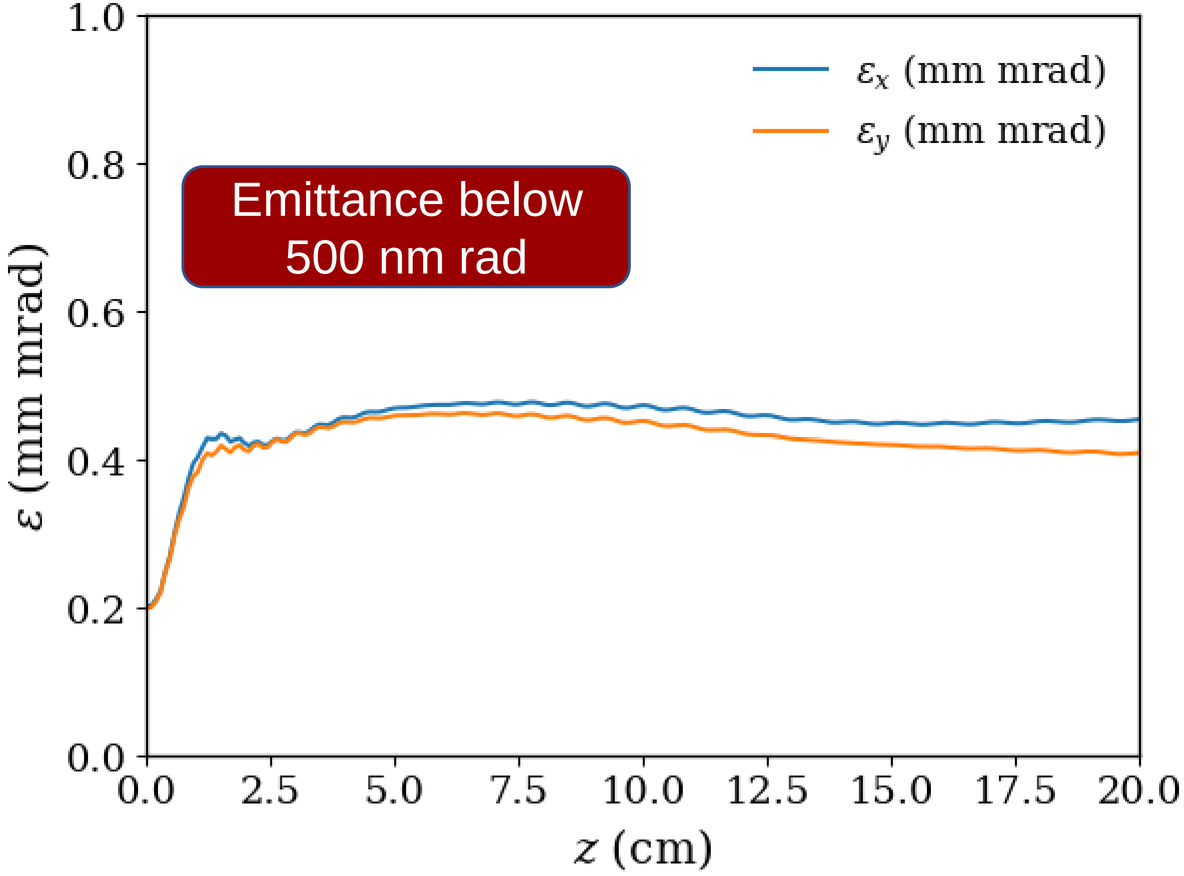


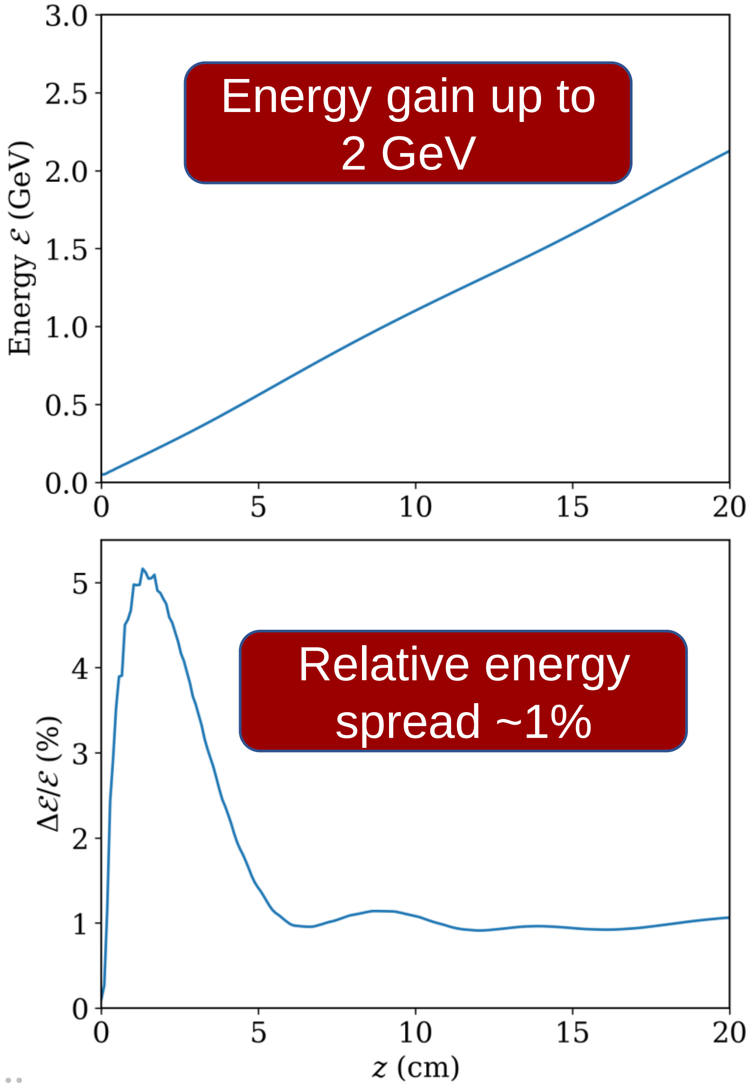


Parameters

- Plasma channel: $n_0 = 2 \times 10^{16}\ \mathrm{cm}^{-3}$ $R_b = 2.5$ Plasma length= 20 cm
- Driver laser : $a_0 = 3, \lambda_0 = 0.8\ \mu$m, $\tau = 100$ fs, $w_0 = 105\ \mu$m
- Trailing bunch: $Q = 0.9$ nC $\mathcal{E} = 50$ MeV $\Delta\mathcal{E}/\mathcal{E}$ =0.1% $\sigma_x = \sigma_y = 2\ \mu$m $\sigma_z = 23\ \mu$m Offset $\mathrm{X}_0 = 2\ \mu$m

Preservation of the beam quality in 20 cm long plasma





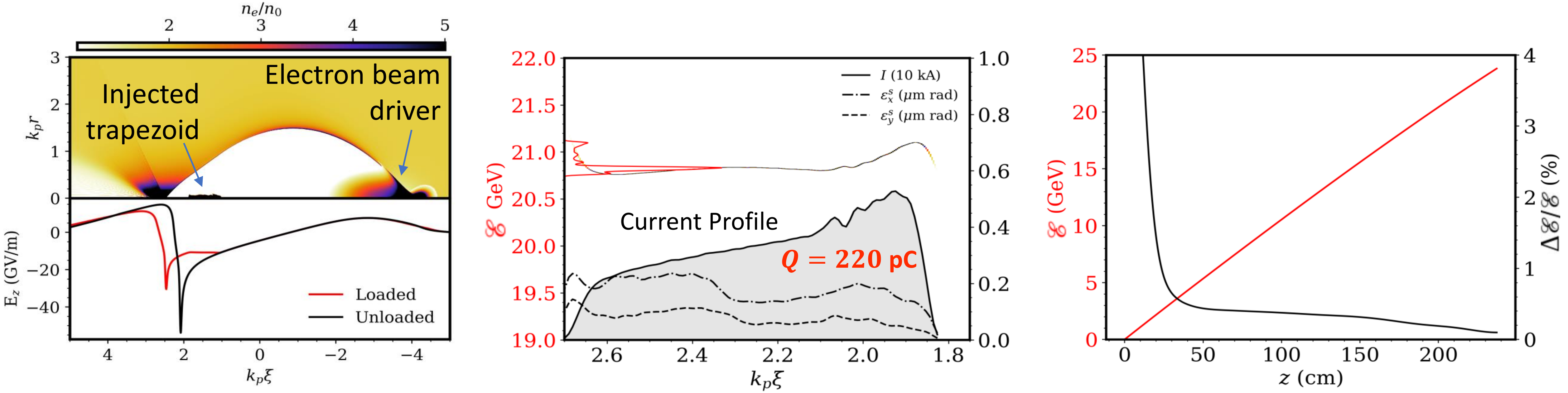
Conclusions

- **Simulation Validation:** Full 3D PIC simulations show excellent quantitative agreement with our analytical transverse force and centroid predictions over a meter-long plasma.
- **Parametric Mapping:** The analytical model maps energy-extraction limits directly against stability thresholds to isolate a highly stable operating window.
- **Beam Quality Preservation:** We successfully map out a highly stable, high-efficiency operating space over a 1 m acceleration distance, simultaneously demonstrating:
  - Near 80% energy transfer efficiency
  - Sub-500 nm rad transverse emittance preservation
  - Up to 16.5 GeV energy gain with $< 1.5\%$ relative energy spread
- **Bypassing the Barrier**: Our study demonstrates that the efficiency-instability trade-off is not an absolute limit. It can be systematically managed and circumvented by pairing precise wake potential modeling with tailored beam loading.
- **Future Work:** This wake-potential formulation provides a path to integrate additional, self-consistent plasma effects like ion motion.

Thank you

Backup slides

Ideal Beam Loading: Achieving Low Energy Spread and High Efficiency for Future Compact Colliders



- Trapezoidal current profile is needed for flattening accelerating field
- Two-color injection experiment at ATF offers a path towards such collider quality trapezoidal beam [1]
- Such a beam also enables high energy extraction efficiency (43% in current simulation, theoretically >90% [2])

[1] A.Jain et al., PRR(2026)

[2] M. Tzoufras, *Phys. Rev. Lett* 101, 14 (2008)

* See backup slides for detailed simulation parameters

Simulation: Beam Loading & Bubble Shape Calculation

This notebook calculates and visualizes the shape of a nonlinear plasma wakefield "bubble" in two distinct regimes:

1. **The Vacuum Regime (Unloaded):** In the absence of a witness bunch, the ions exert a restoring force that closes the electron cavity. In the ultra-relativistic blowout regime, this shape is phenomenologically approximated as a sphere. The vacuum bubble radius r_{vac} as a function of the co-moving coordinate ξ is given by:

$$r_{vac}(\xi) = R_b\sqrt{1 - \left(\frac{\xi}{R_b}\right)^2}$$

Correspondingly, the longitudinal electric field E_z grows linearly along the bubble axis:

$$E_z(\xi) \approx k_p\xi/\chi$$

$$r_b r_b'' + 2(r_b')^2 + 1 = 0 \quad (\text{when } \Lambda = 0)$$

The exact mathematical solution to this specific differential equation is a circle:

$$r_b(\xi) = \sqrt{R_b^2 - \xi^2}$$

$$r_b(\xi)^2 + (2a)^2\xi^2 = R_b^2$$

2. **The Loaded Regime (With Bunch):** When a witness electron bunch is injected, its space charge pushes outward against the ions, modifying the potential.

   - **Goal:** To achieve **constant acceleration** (zero energy spread), the longitudinal field E_z must be flattened to a constant value E_t.
   - **Shape Change:** Since the wake potential ψ scales with the square of the bubble radius ($\psi \sim r_b^2/4$), a constant field ($E_z \propto \partial_\xi \psi = \text{const}$) implies that r_b^2 must decrease **linearly** rather than following the spherical curve.

   The loaded trajectory is calculated by matching the slope of the potential at the injection point ξ_{start}:

$$r_{loaded}^2(\xi) = r_{vac}^2(\xi_{start}) + \left[\frac{d(r_{vac}^2)}{d\xi}\right]_{\xi_{start}} (\xi - \xi_{start})$$

Simulation Logic

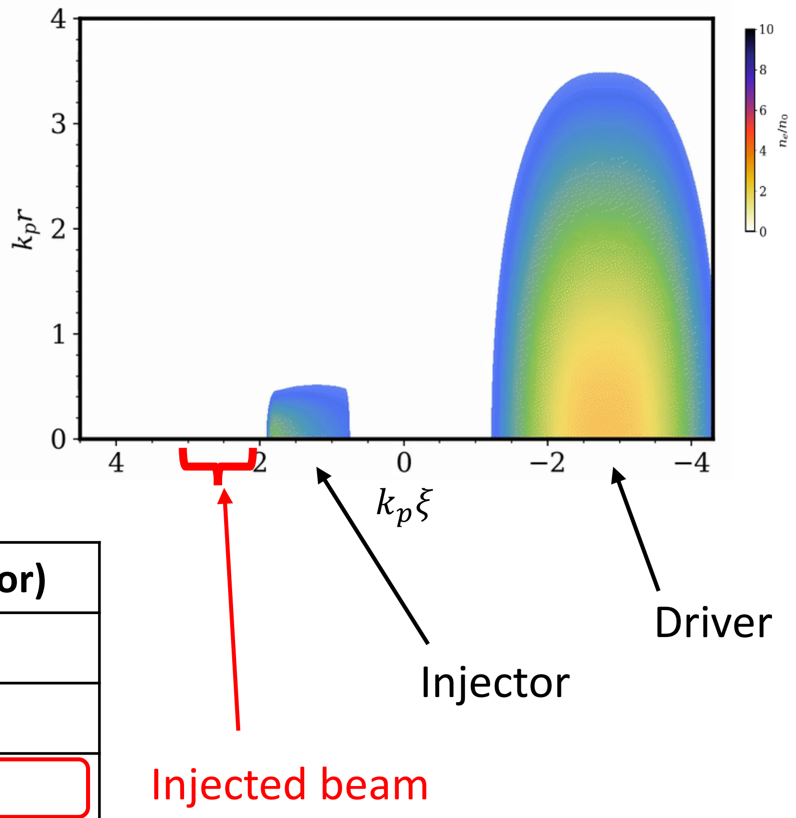
- **Injection Point:** We place the head of the bunch at ξ_{start}, where the vacuum field naturally equals the target loaded field ($E_{vac} = E_t$).
- **Bubble Wall:** Before ξ_{start}, the wall follows the vacuum sphere. After ξ_{start}, the wall follows the linearized r^2 trajectory (the "sheath prop").

Backup: details of previous work

$$n_0 = 2e16\ cm^{-3}$$

Driver	**Injector**
$\lambda_0 = 9.2\ \mu m$	$\lambda_0 = 0.4\ \mu m$
$\tau = 250\ fs$	$\tau = 20\ fs$
$w_0 = 105\ \mu m$	$w_f = 8.6\ \mu m$
$a_0 = 2.7$	$a_0 = 0.17$
	$\beta_f = 1.04$

Ion. level	a_T (Driver)	a_T (Injector)
Kr^{7+} -> Kr^{8+}	1.87	0.06
Kr^{8+} -> Kr^{9+}	$\boxed{3.10}$	0.16
Kr^{9+} -> Kr^{10+}	3.78	$\boxed{0.20}$
Kr^{10+} -> Kr^{11+}	4.51	0.23



Generated beam

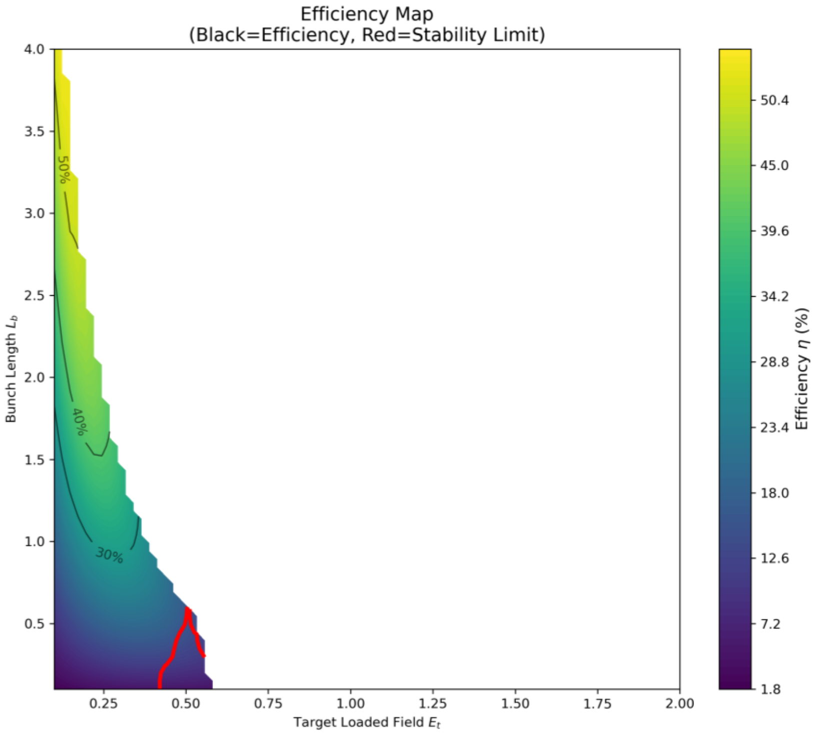
$$Q = 220\ pC$$

$$\epsilon_x = 170\ nm \cdot rad$$

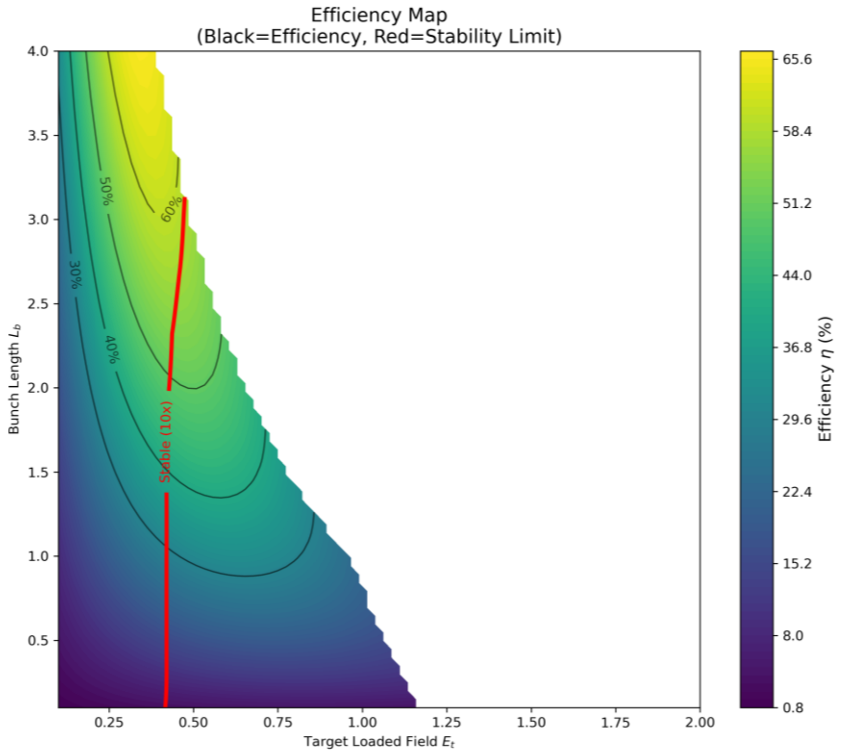
$$\epsilon_y = 76\ nm \cdot rad$$

A. Jain, *et al.*, Phys.l Rev. Research 8.2 (2026): 023213.

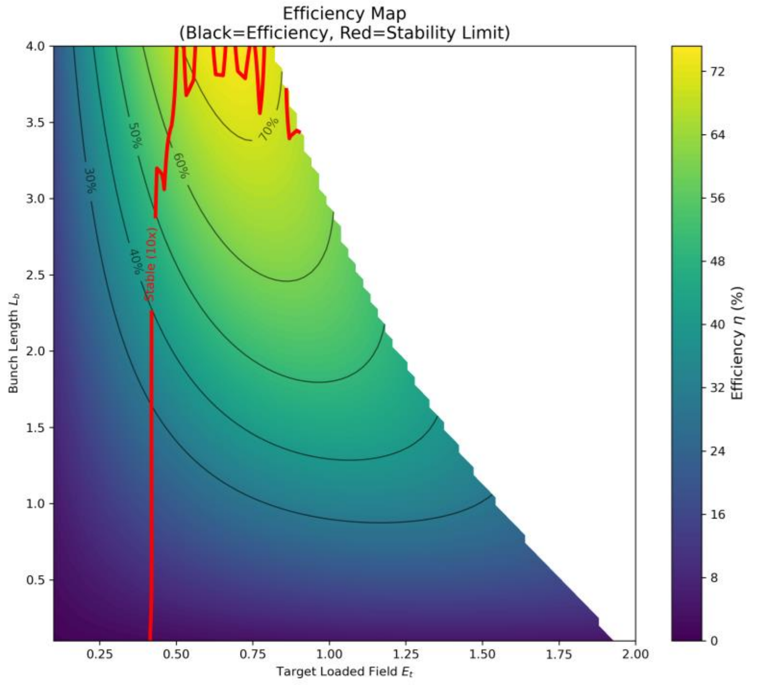
$R_b = 2$



$R_b = 3$



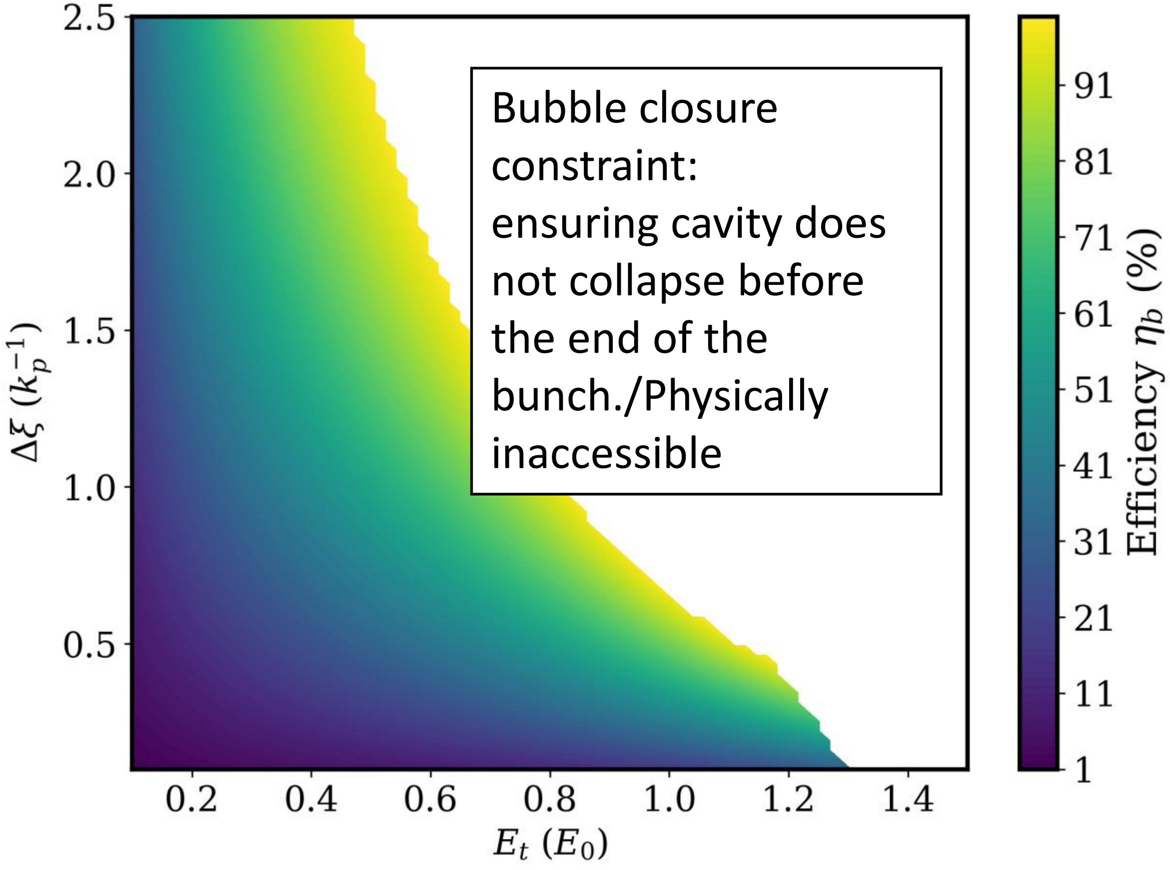
$R_b = 4$



$n_0 = 5 \times 10^{16}\ cm^{-3}, offset = 2.4\ \mu$m, 20 GeV acceleration of the beam

Mapping the High-Efficiency, High-Stability Operating Window

Bunch Length



Loaded field

$n_0 = 3 \times 10^{16}\ \mathrm{cm}^{-3}$
$R_b = 2.7$
$X_0 = \sigma_x = 2\ \mu\mathrm{m}$