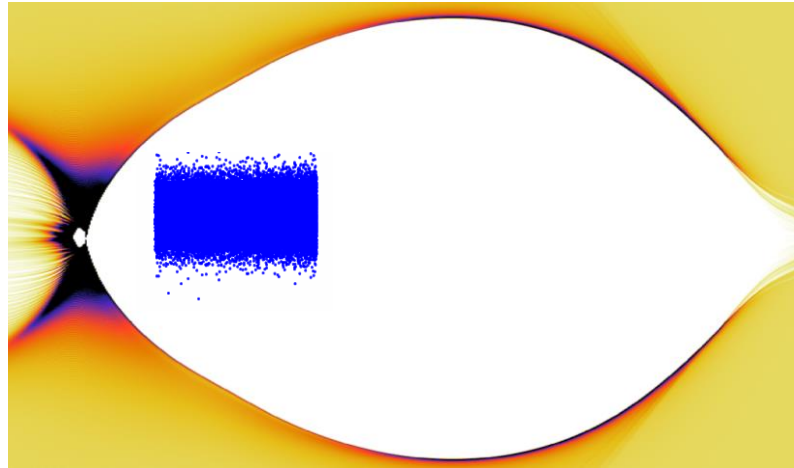


Investigation of transverse instability in efficient plasma-based accelerators



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Plasma
Acceleration
Group



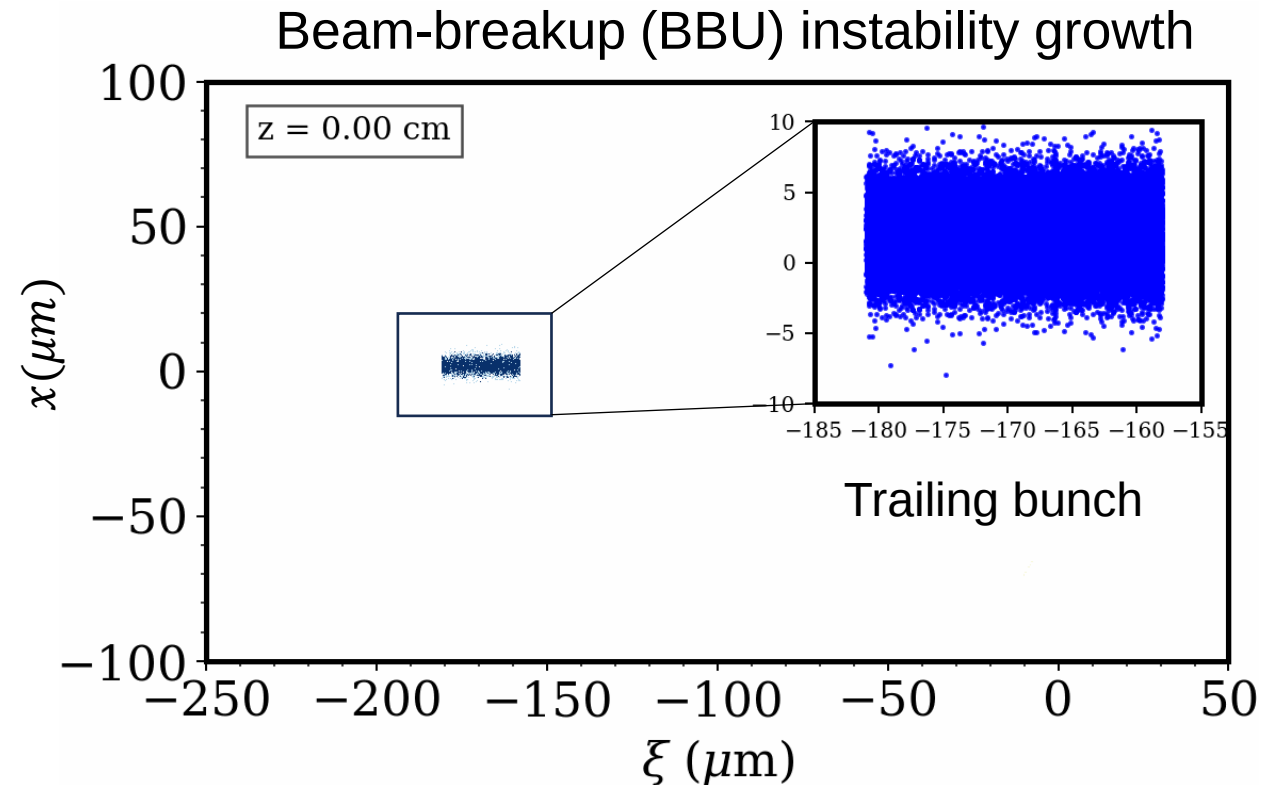
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Efficiency vs Instability

- **Goal:** Plasma stages must achieve high power-transfer efficiency while preserving beam phase-space quality ($\downarrow$ emittance $\downarrow$ energy spread)
- **Threat (BBU):** For long-distance laser/plasma wakefield acceleration, strong plasma focusing forces resonantly amplify small transverse offsets into large transverse oscillations
This is a critical issue of large emittance in prospective plasma-based colliders
- **Trade-off:** Lebedev *et al.*, suggested- BBU is unavoidable at high efficiency, mitigation (like BNS damping) compromises collider-level beam quality

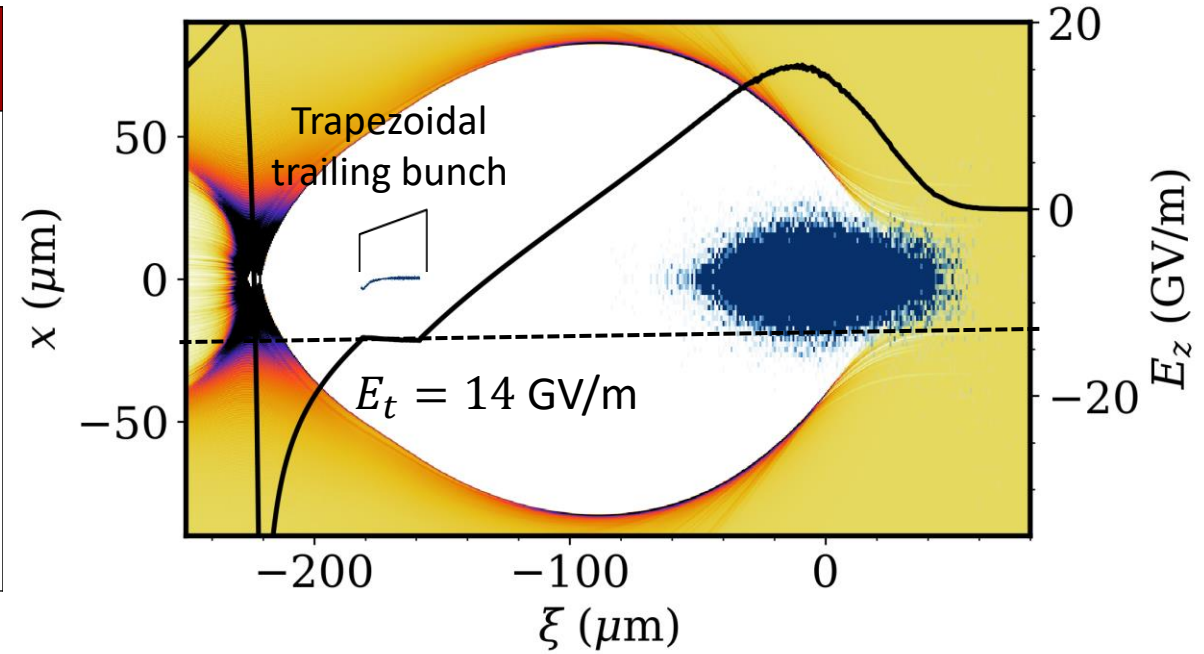


$\downarrow$ Instability $\uparrow$ Efficiency $\downarrow$ Energy spread

Beam loading for achieving low energy spread

Simulation design criteria

- The system targets a blowout cavity radius $R_b \sim 3$ to support collider-relevant charges ~ 1 nC at a loaded accelerating field $E_t > 10$ GV/m.
- Driver: sustain a highly stable plasma cavity over meter-scale propagation allowing the beam-breakup dynamics to be isolated from wake evolution
- Trailing bunch: Trapezoidal current profile flattens the E_z field and minimize energy spread



Parameters

- Plasma: $n_0 = 3 \times 10^{16} \text{ cm}^{-3}$
- Driver beam: $Q = 3.2 \text{ nC}$ $\mathcal{E} = 20 \text{ GeV}$
- Trailing bunch: $Q = 1.1 \text{ nC}$ $\mathcal{E} = 50 \text{ MeV}$ $\Delta\mathcal{E}/\mathcal{E} = 0.1\%$ $\sigma_x = \sigma_y = 2 \mu\text{m}$ $\sigma_z = 23 \mu\text{m}$ Offset $X_0 = 2 \mu\text{m}$

The code HiPACE++

- 3D, QS PIC for plasma acceleration
- Runs on GPU and (multi-core) CPU
- Open-source: <https://github.com/Hi-PACE/hipace>
- Documentation: <https://hipace.readthedocs.io/en/latest/>
- Main features:
 - Multiple beams and plasma species to simulation beam-driven wakefield acceleration
 - A laser envelope solver to simulate laser-driven wakefield acceleration
 - Mesh refinement for efficient computation
 - An advanced [explicit field solver](#) for increased accuracy
 - Diagnostics compliant with the [openPMD standard](#)
 - Arbitrary profiles for the beams and plasma profiles
 - Readers from files for the beam and laser profiles



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VisualPIC

Modelling beam-breakup instability

Transverse motion of beam slice centroid $X(\xi, z)$

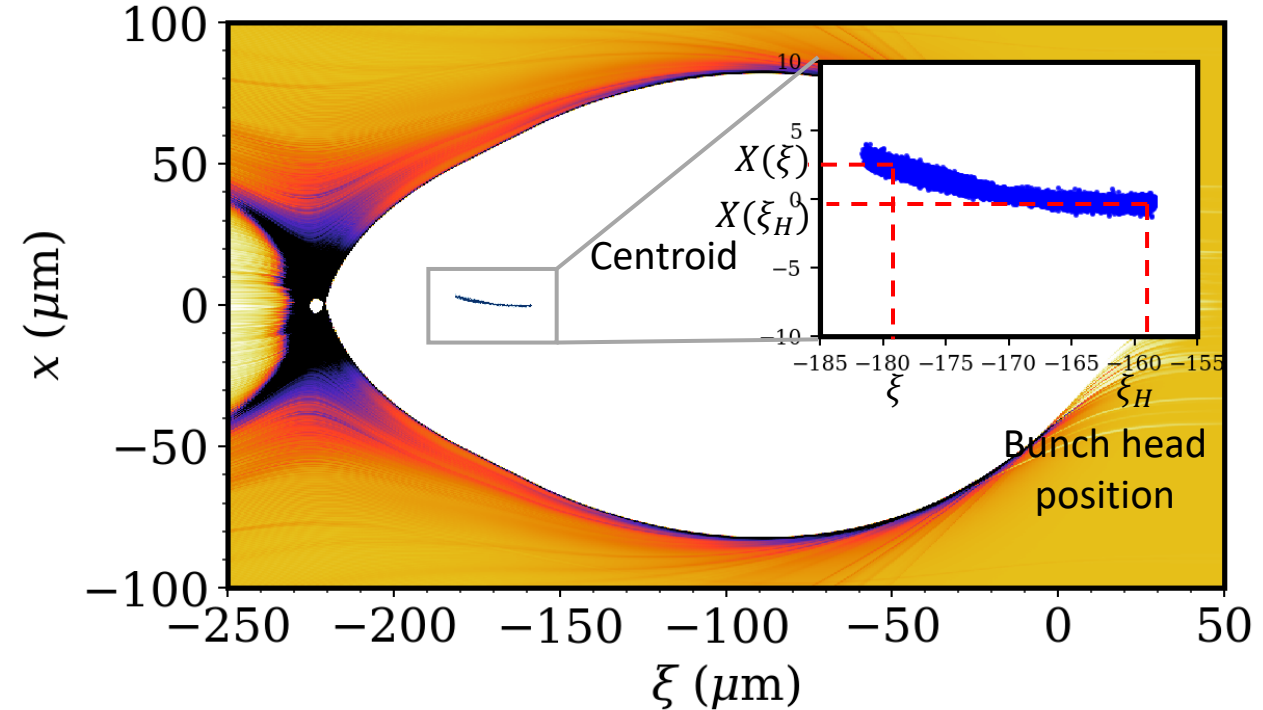
$$\frac{\partial^2 X}{\partial z^2} + \frac{1}{\gamma} \frac{\partial \gamma}{\partial z} \frac{\partial X}{\partial z} + k_\beta^2 X = -\frac{e}{\varepsilon} \tilde{F}_\perp$$

Adiabatic damping due to energy gain

Ion restoring force

Driving force

$$\tilde{F}_\perp = \frac{F_\perp(\xi, z)}{q} = \int_{\xi_H}^{\xi} W_\perp(\xi' - \xi) \lambda(\xi') X(\xi', z) d\xi'$$



Precise determination of transverse wake function $W_\perp$ is crucial for accurate modeling of transverse instabilities in a plasma accelerator

Wake potential model

Driving force

$$\tilde{F}_{\perp} = \frac{F_{\perp}(\xi, z)}{q} = \int_{\xi_H}^{\xi} W_{\perp}(\xi' - \xi) \lambda(\xi') X(\xi', z) d\xi'$$

Transverse wake function

Short-range wake theorem

$$W_{\perp} = \frac{2}{r_b^2(\xi') q_t} \int_{\xi}^{\xi'} E_z du$$

Longitudinal wake function:
 E_z generated by the trailing beam

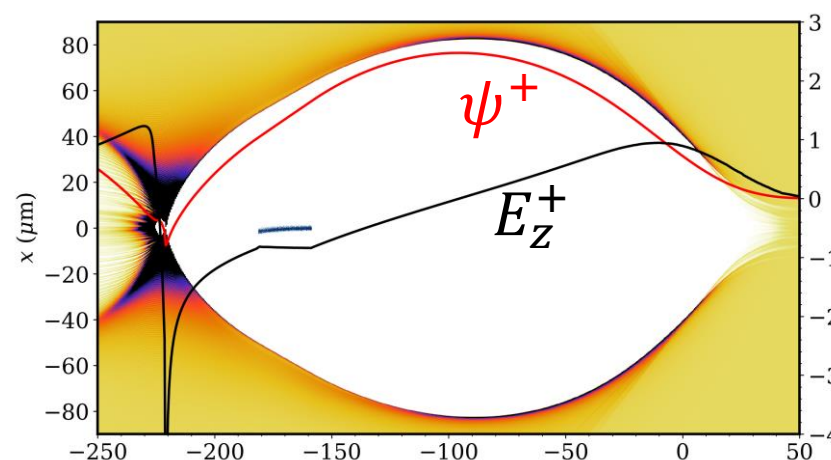
Wake potential superposition

$$E_z = -\frac{1}{q_t} \frac{\partial \psi_t}{\partial \xi}$$

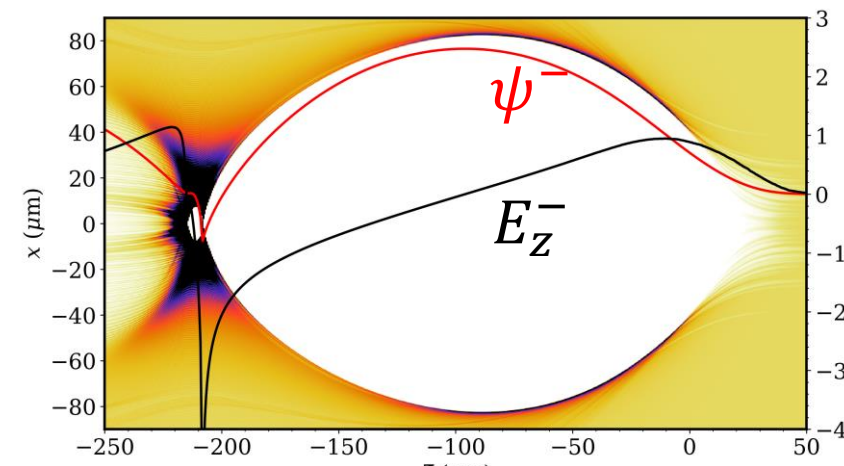
$$\psi = \phi - A_z$$

$$\psi_t = \psi^+ - \psi^-$$

With trailing bunch



Without trailing bunch



Our wake potential model is valid for both LWFA and PWFA

Wake potential model shows excellent agreement with simulation

Previous Models

Lebedev:

$$W_{\perp} \approx \frac{2}{a(\xi)^4}$$

Stupakov:

$$W_{\perp} \approx \frac{2}{(a(\xi) + 0.75k_p^{-1})^4}$$

High charge witness beam significantly deforms the bubble boundary, resulting in significant error in this model

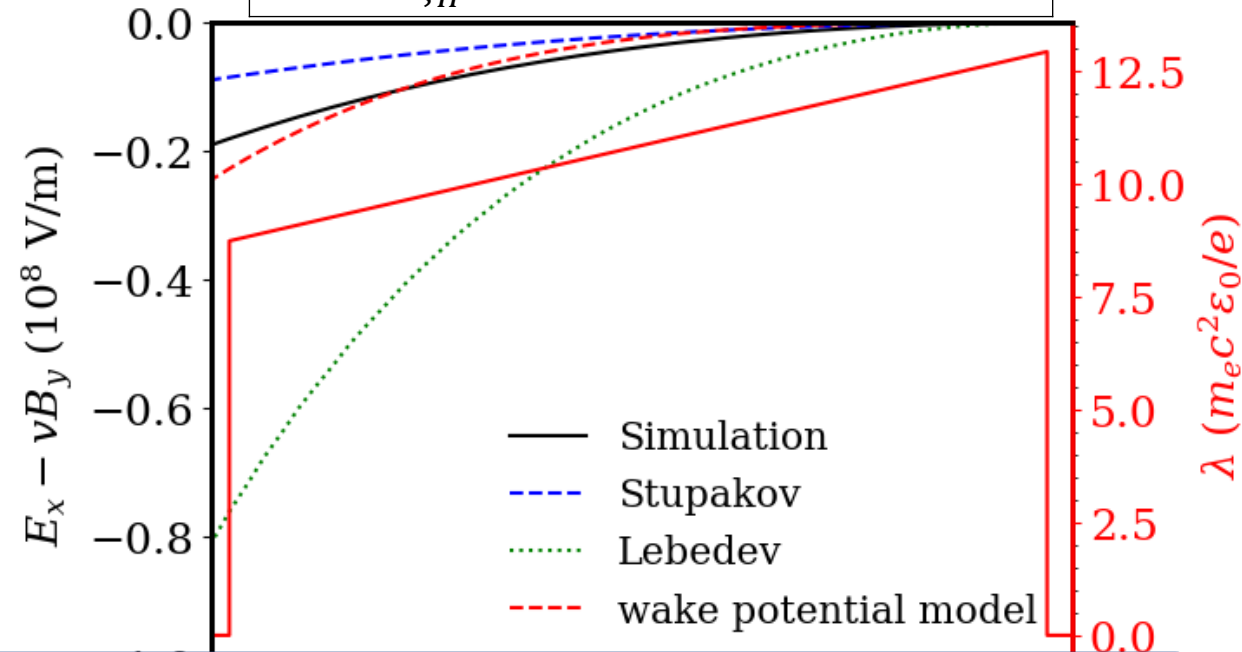
Wake potential model

$$W_{\perp} = \frac{2}{r_b^2 q_t} (\psi^+ - \psi^-)$$

- Our model naturally incorporates exact, deformed boundaries without any fitting parameters.
- The potential difference mathematically represents the exact integral of the longitudinal field.

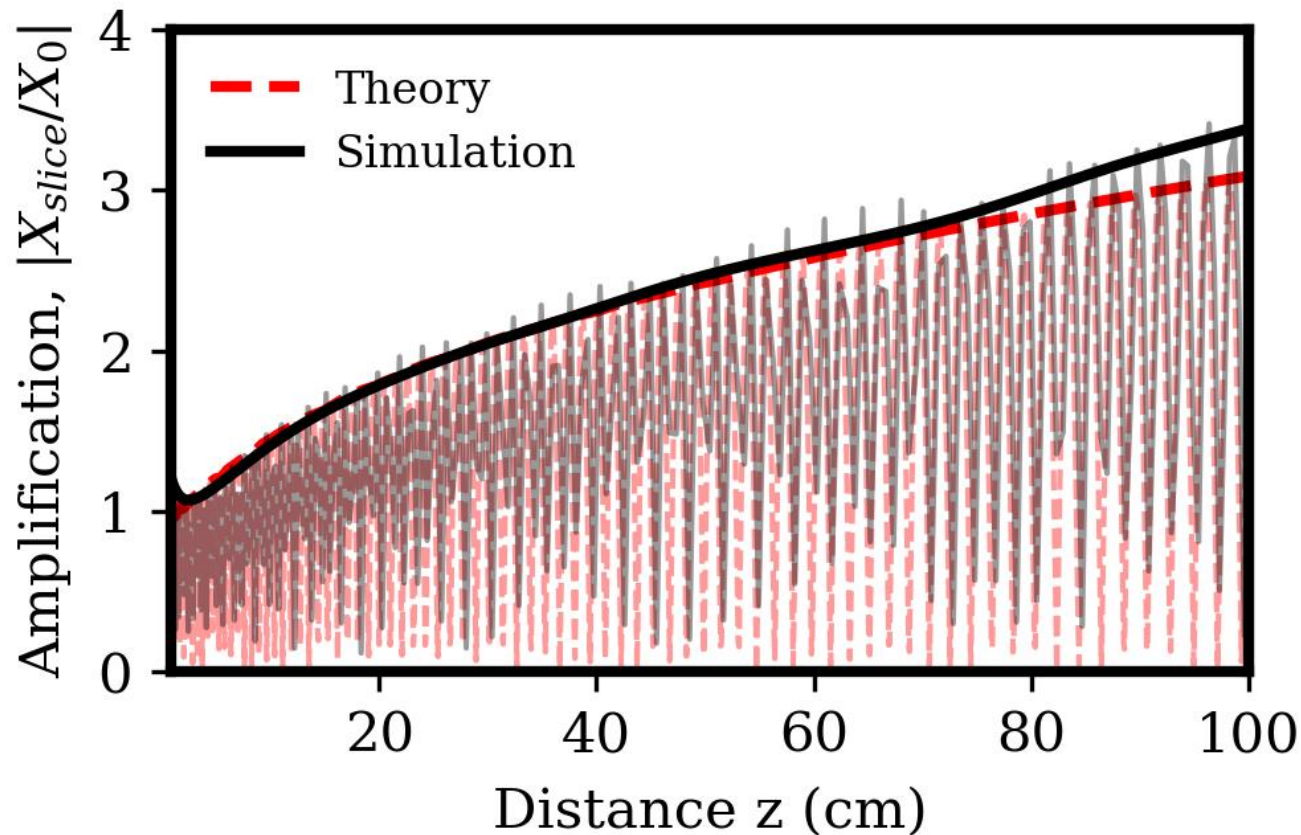
Transverse force

$$\tilde{F}_{\perp} = \int_{\xi_H}^{\xi} W_{\perp}(\xi' - \xi) \lambda(\xi') X(\xi', z) d\xi'$$



A factor-of-four error can lead to a substantial overestimate of the accumulated transverse instability.

Bunch slice centroid evolution



Transverse motion of centroid $X(\xi, z)$

$$\frac{\partial^2 X}{\partial z^2} + \frac{1}{\gamma} \frac{\partial \gamma}{\partial z} \frac{\partial X}{\partial z} + k_\beta^2 X = -\frac{e}{\epsilon} \tilde{F}_\perp$$

Driving force

$$\tilde{F}_\perp = \int_{\xi_H}^{\xi} W_\perp(\xi' - \xi) \lambda(\xi') X(\xi', z) d\xi'$$

Transverse wake function

$$W_\perp = \frac{2}{r_b^2 q_t} (\psi^+ - \psi^-)$$

The agreement remains strong over a meter of propagation at 10 GeV/m
Wake potential formalism captures the transverse beam dynamics with predictive accuracy

Analytical modelling of efficiency and instability

Instability- X/X_0

Transverse wake function

$$W_{\perp} = \frac{2}{r_b^2 q_t} \int_{\xi}^{\xi'} E_z du$$

$$E_z = E_t - \frac{\chi \xi}{2}, \chi/2 - \text{Slope of } E_z \text{ for } R_b < 4$$

$$W_{\perp} = \frac{2}{r_b^2 q_t} \left(E_t (\xi' - \xi) - \frac{\chi}{4} (\xi'^2 - \xi^2) \right)$$

Driving force

$$\tilde{F}_{\perp} = \int_{\xi_H}^{\xi} W_{\perp} (\xi' - \xi) \lambda(\xi') X(\xi', z) d\xi'$$

Transverse motion of centroid $X(\xi, z)$

$$\frac{\partial^2 X}{\partial z^2} + \frac{1}{\gamma} \frac{\partial \gamma}{\partial z} \frac{\partial X}{\partial z} + k_{\beta}^2 X = -\frac{e}{\epsilon} \tilde{F}_{\perp}$$

Efficiency- η_b

Wake-to-trailing-bunch energy transfer efficiency η_b

- Ratio of extracted energy to the maximum energy available in the wake.
- How effectively beam loads the wake.
- How effectively extracts the available wake energy.

Beam loading efficiency η_b

$$\eta_b = \frac{2\pi \int E_t \lambda(\xi) d\xi}{\chi \pi R_b^4 / 16}$$

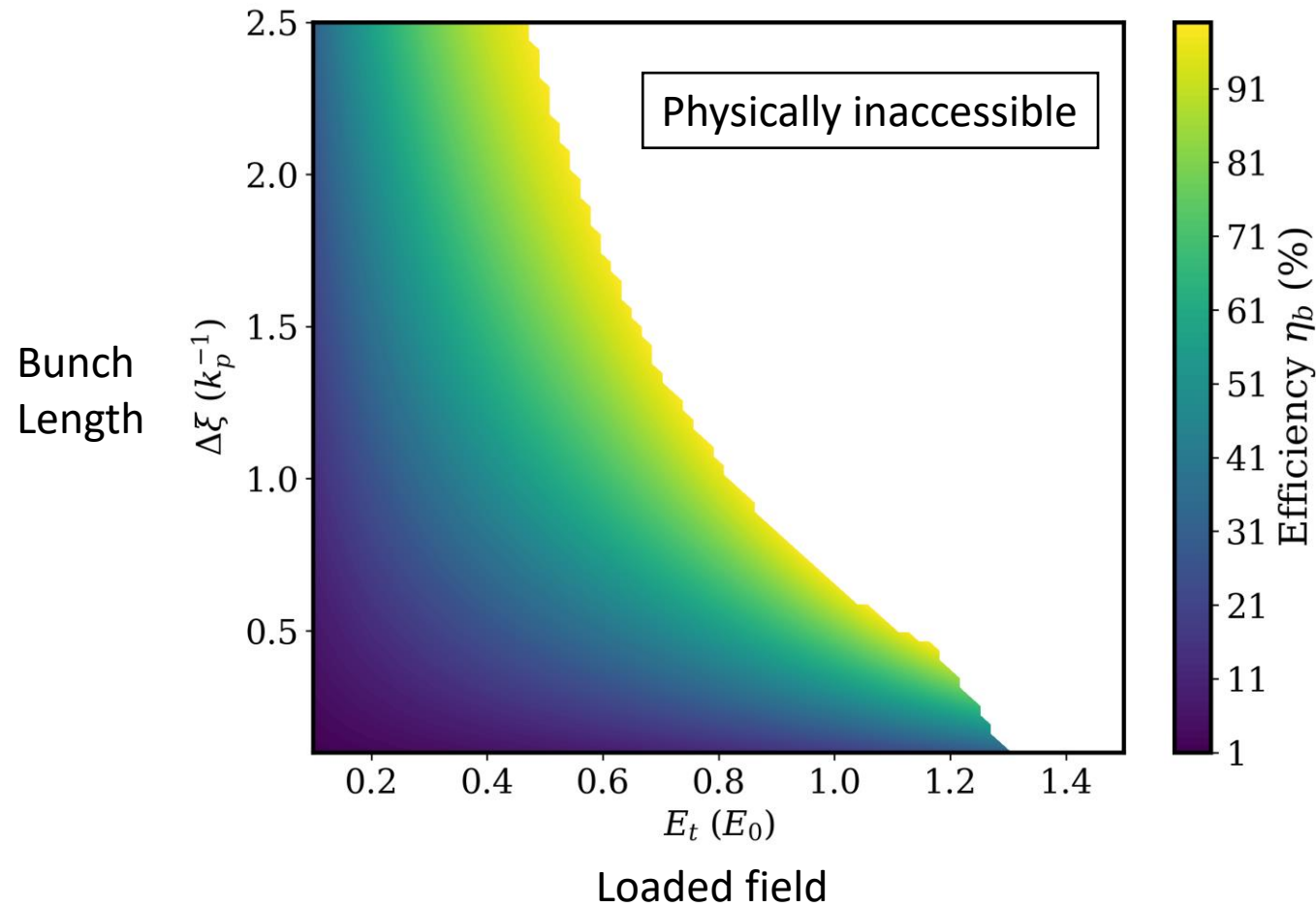
$$\eta_b = 32 \frac{E_t \Delta \xi}{\chi R_b^4} \left(\sqrt{\left(\frac{E_t}{\chi} \right)^4 + \frac{R_b^4}{16}} - \frac{E_t}{2\chi} (\Delta \xi) \right)$$

M. Tzoufras *et al.*, PRL (2008)

A. A. Golonov *et al.*, PPCF (2021)

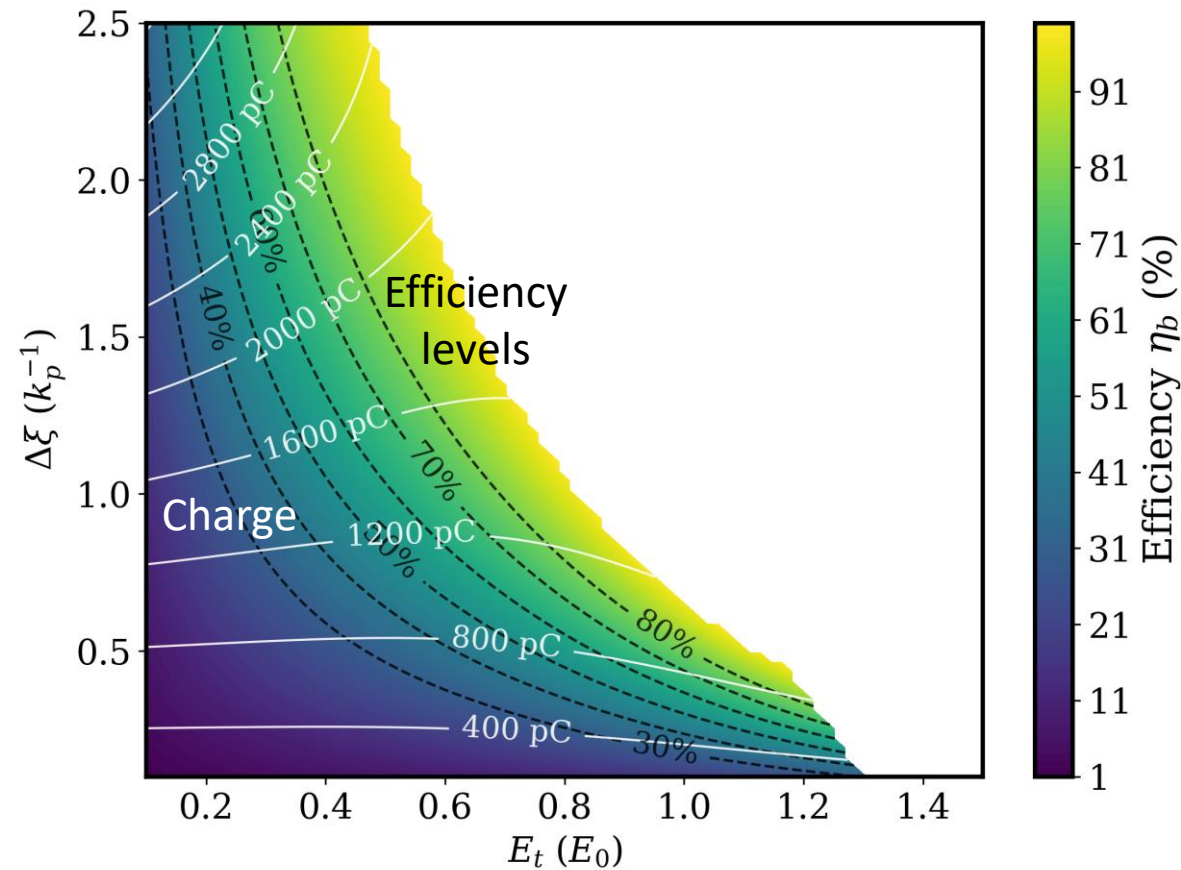
Jiayang Yan PhD thesis (2025)

Mapping the High-Efficiency, High-Stability Operating Window

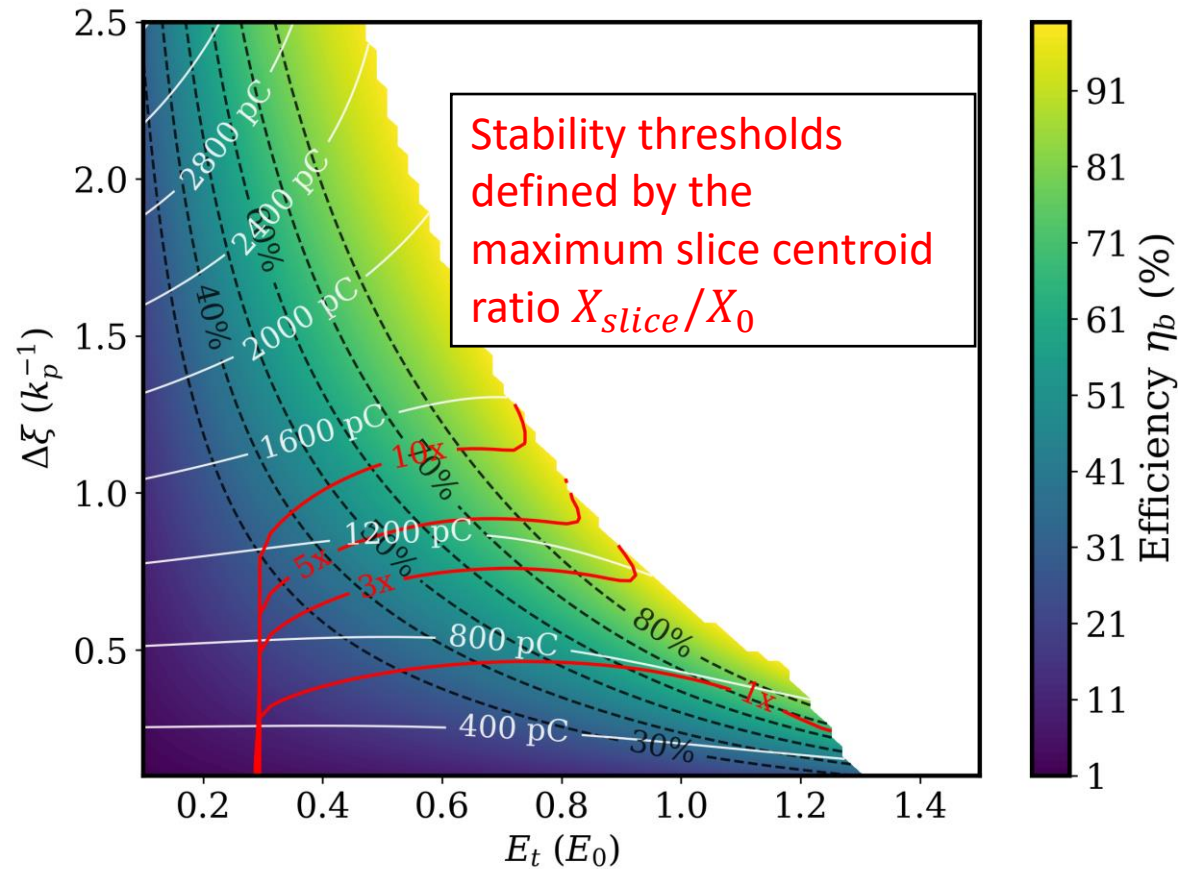


$$\begin{aligned}n_0 &= 3 \times 10^{16} \text{ cm}^{-3} \\R_b &= 2.7 \\X_0 = \sigma_x &= 2 \text{ }\mu\text{m}\end{aligned}$$

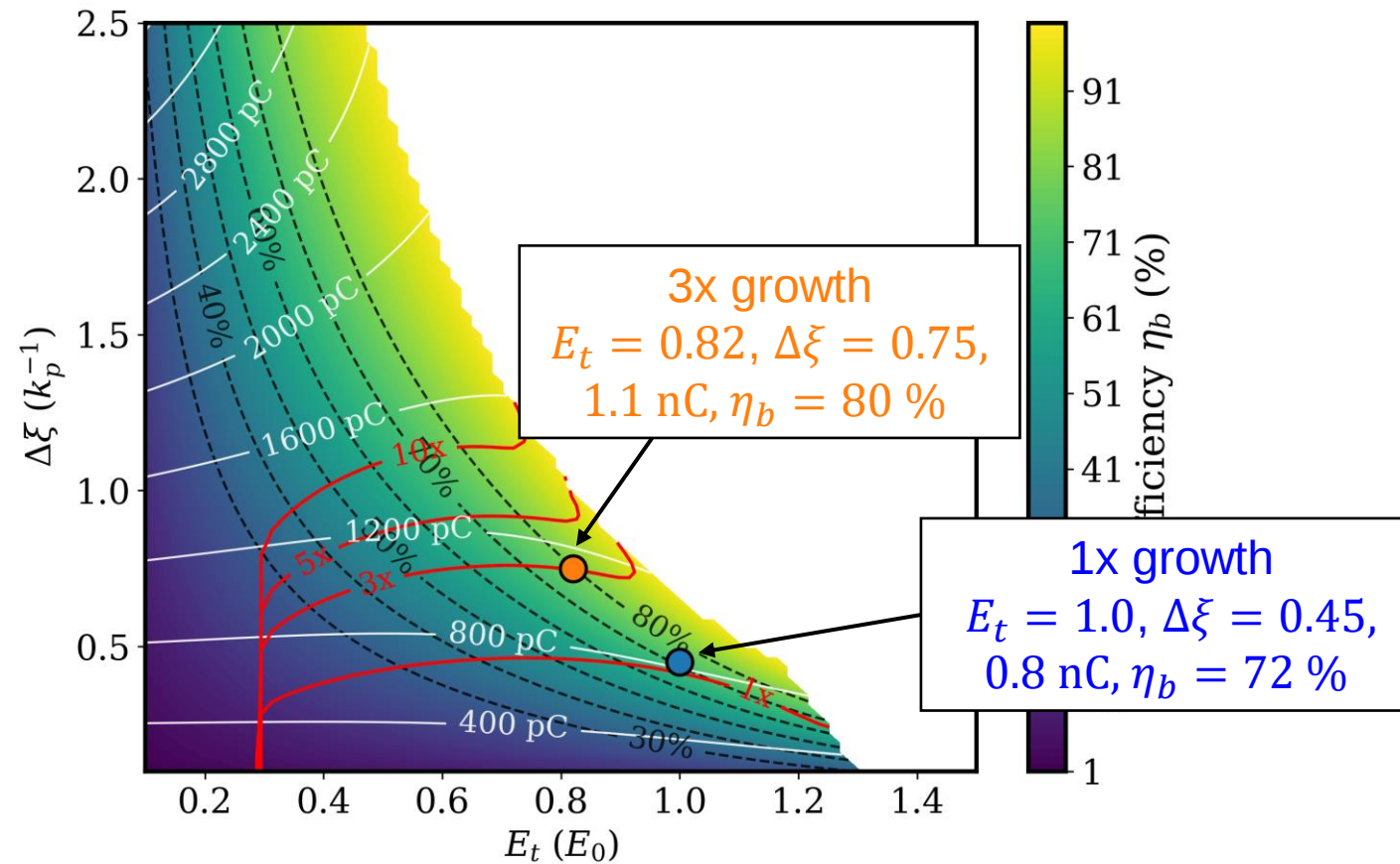
Mapping the High-Efficiency, High-Stability Operating Window



Mapping the High-Efficiency, High-Stability Operating Window



Mapping the High-Efficiency, High-Stability Operating Window

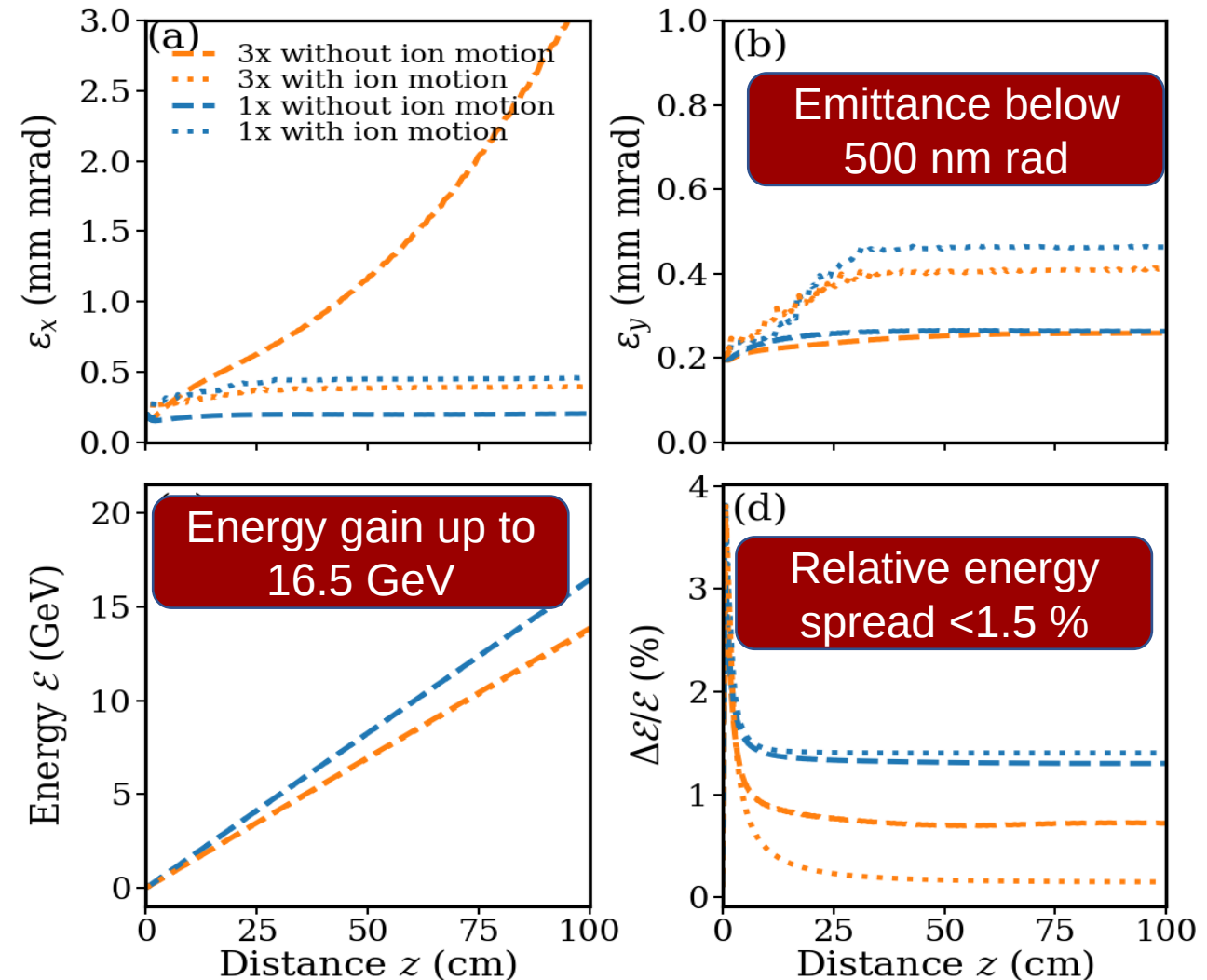


Preservation of the beam quality over a meter-long plasma

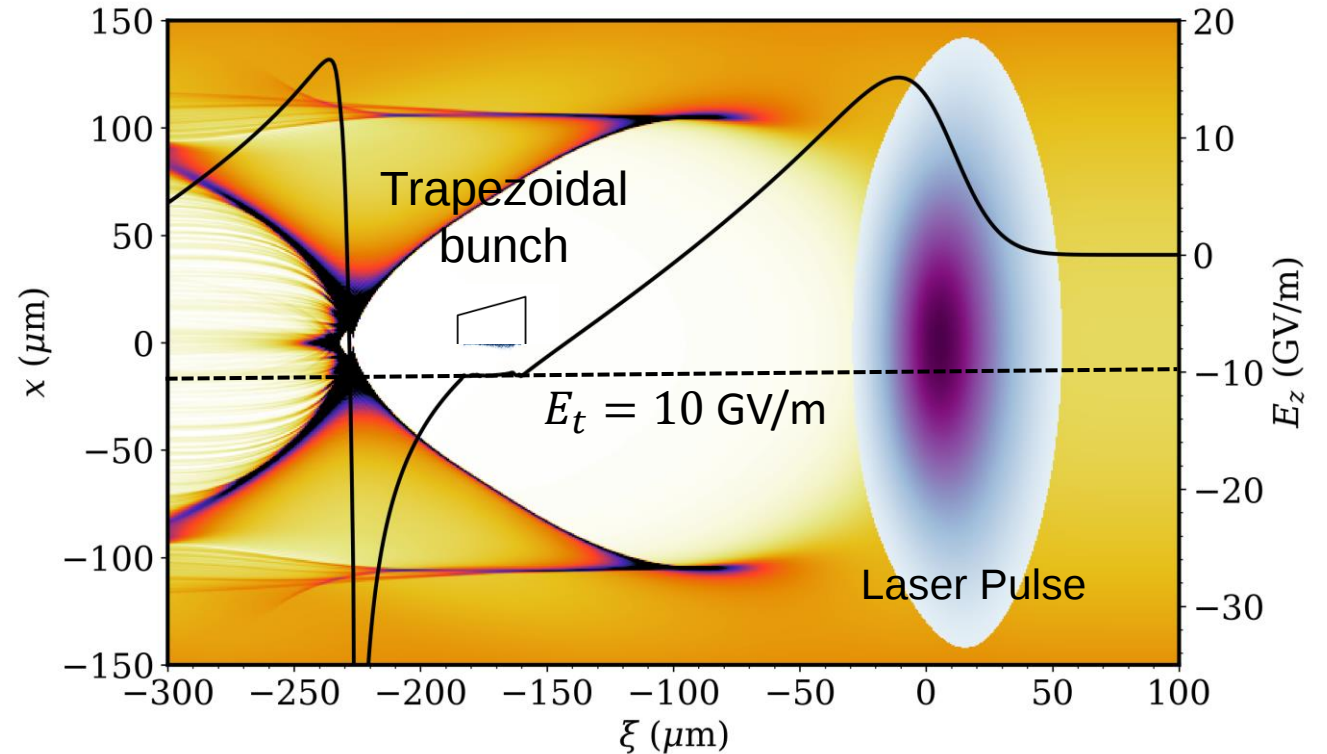
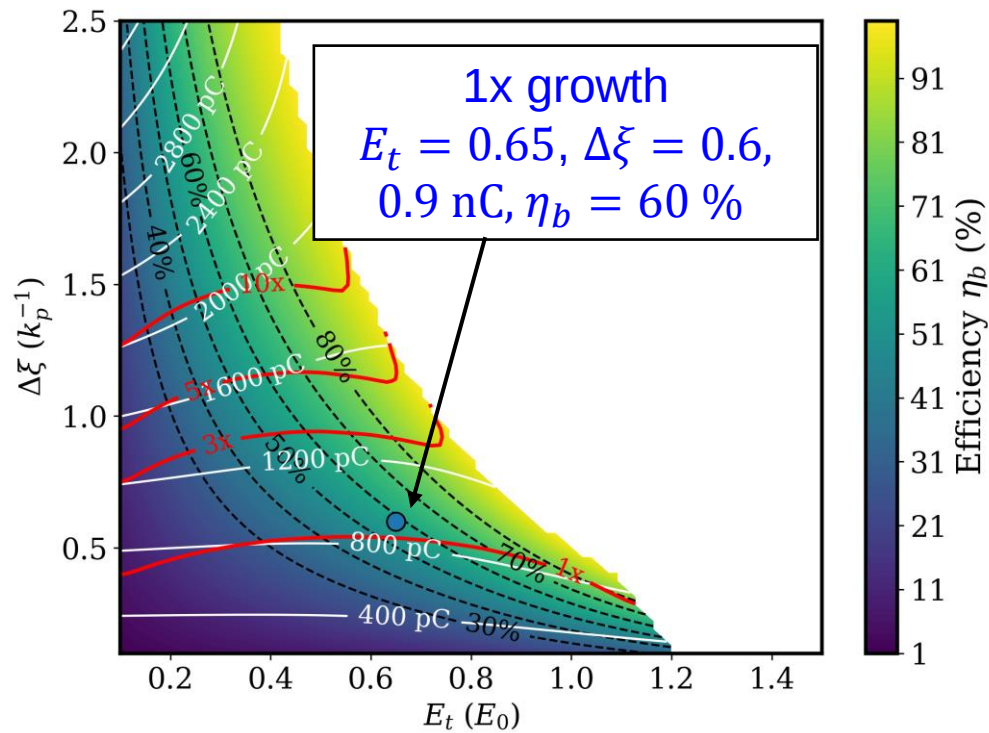
Ion motion (dotted lines)

- Well-known mechanism for bounding BBU growth in plasma accelerators
- **Phase Mixing:** Nonuniform focusing along the trailing bunch causes different slices to accumulate differing betatron phases, reducing the BBU growth rate.
- Ion response introduces additional transverse forces modifying projected beam quality in configurations with limited centroid growth.
- Ion motion bounds the emittance growth of 3x while a baseline emittance increase in 1x

$$\eta_b \sim 80\%, \quad \eta \sim 40\%$$



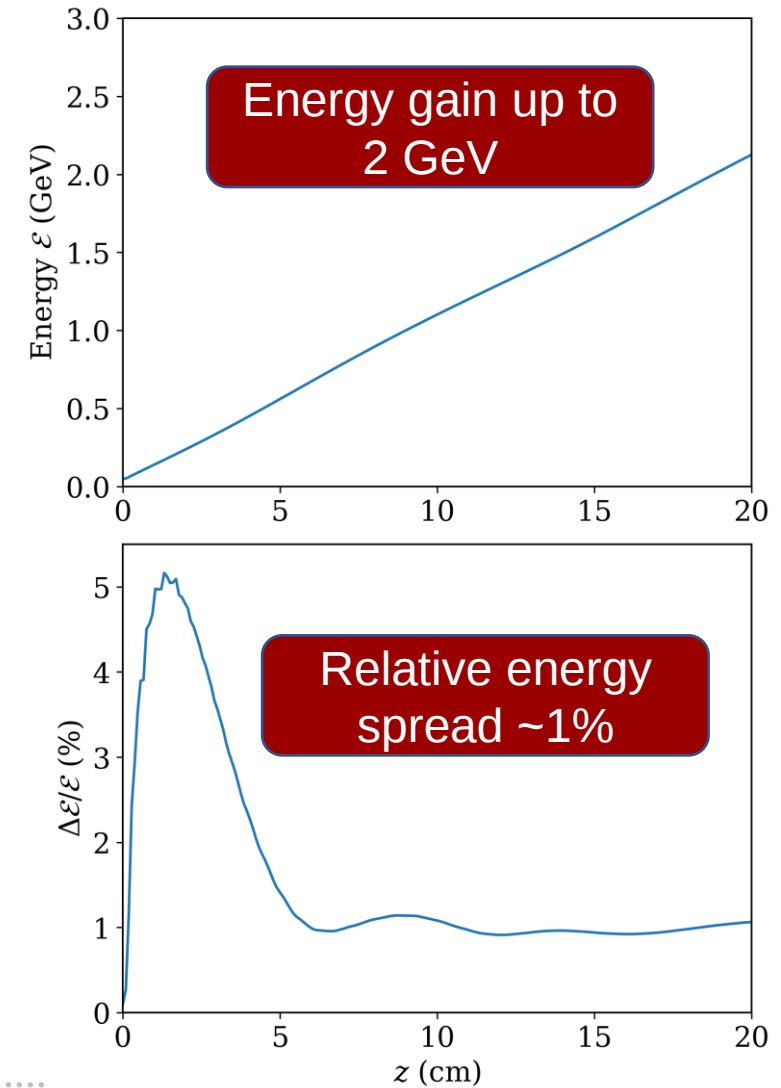
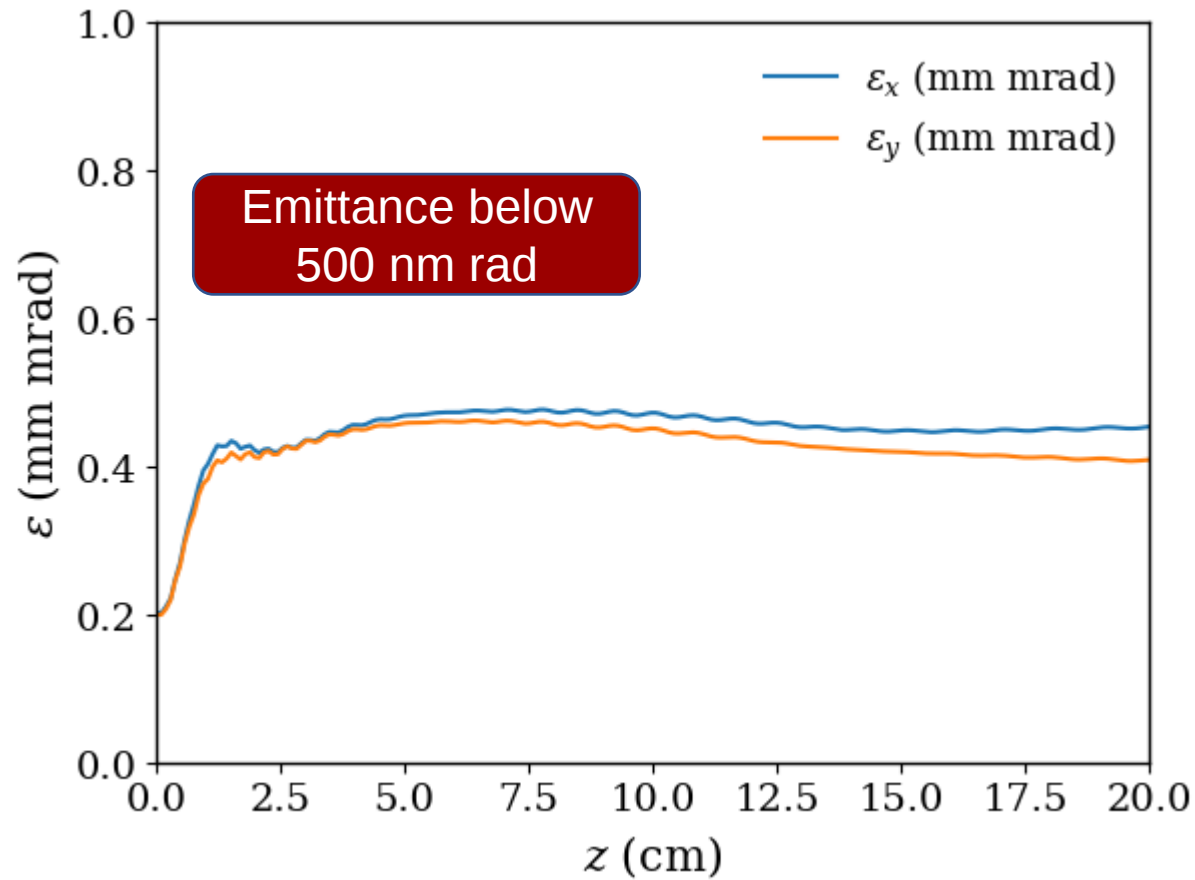
Laser driven simulation set up



Parameters

- Plasma channel: $n_0 = 2 \times 10^{16} \text{ cm}^{-3}$ $R_b = 2.5$ Plasma length= 20 cm
- Driver laser : $a_0 = 3$, $\lambda_0 = 0.8 \mu\text{m}$, $\tau = 100 \text{ fs}$, $w_0 = 105 \mu\text{m}$
- Trailing bunch: $Q = 0.9 \text{ nC}$ $\mathcal{E} = 50 \text{ MeV}$ $\Delta\mathcal{E}/\mathcal{E} = 0.1\%$ $\sigma_x = \sigma_y = 2 \mu\text{m}$ $\sigma_z = 23 \mu\text{m}$ Offset $X_0 = 2 \mu\text{m}$

Preservation of the beam quality in 20 cm long plasma



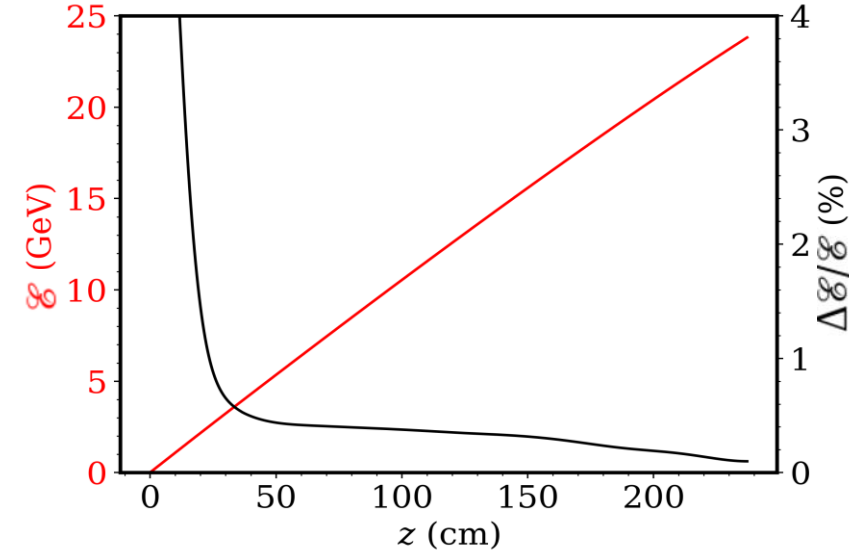
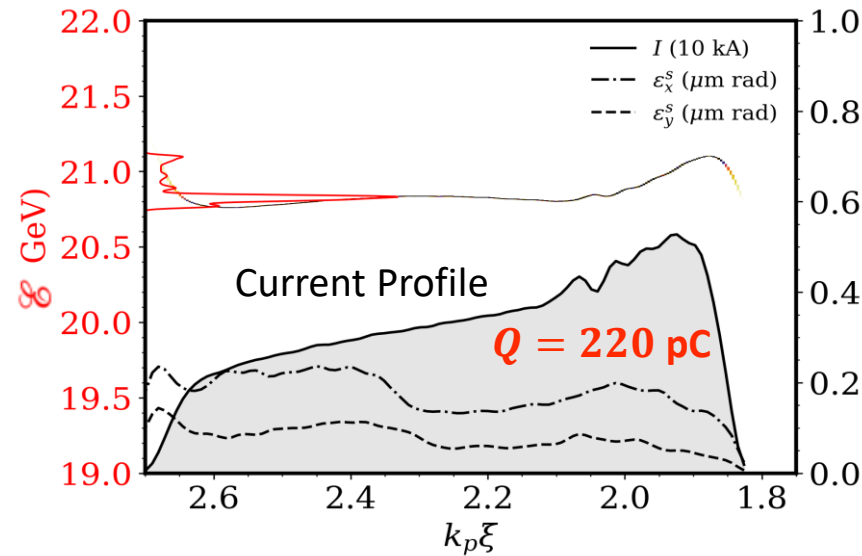
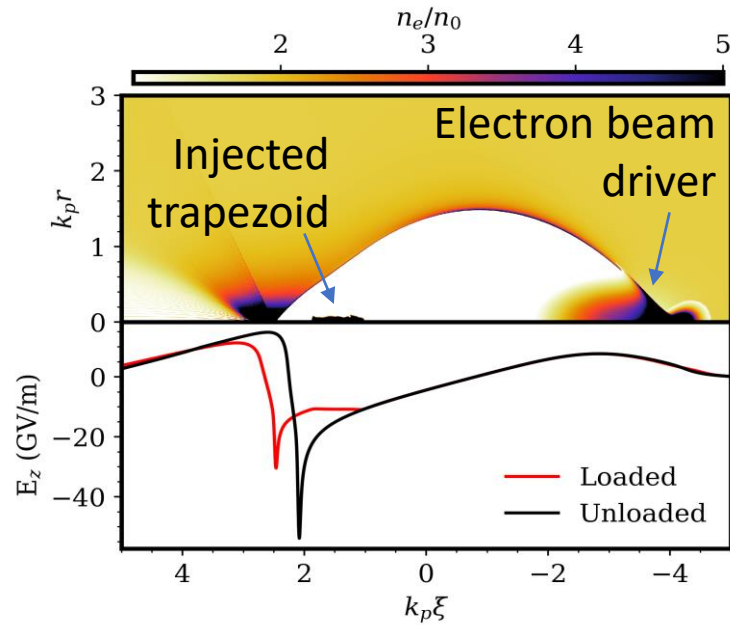
Conclusions

- **Simulation Validation:** Full 3D PIC simulations show excellent quantitative agreement with our analytical transverse force and centroid predictions over a meter-long plasma.
- **Parametric Mapping:** The analytical model maps energy-extraction limits directly against stability thresholds to isolate a highly stable operating window.
- **Beam Quality Preservation:** We successfully map out a highly stable, high-efficiency operating space over a 1 m acceleration distance, simultaneously demonstrating:
 - Near 80% energy transfer efficiency
 - Sub-500 nm rad transverse emittance preservation
 - Up to 16.5 GeV energy gain with $< 1.5\%$ relative energy spread
- **Bypassing the Barrier:** Our study demonstrates that the efficiency-instability trade-off is not an absolute limit. It can be systematically managed and circumvented by pairing precise wake potential modeling with tailored beam loading.
- **Future Work:** This wake-potential formulation provides a path to integrate additional, self-consistent plasma effects like ion motion.

Thank you

Backup slides

Ideal Beam Loading: Achieving Low Energy Spread and High Efficiency for Future Compact Colliders



- Trapezoidal current profile is needed for flattening accelerating field
- Two-color injection experiment at ATF offers a path towards such collider quality trapezoidal beam [1]
- Such a beam also enables high energy extraction efficiency (43% in current simulation, theoretically >90% [2])

[1] A.Jain et al., PRR(2026)

[2] M. Tzoufras, *Phys. Rev. Lett* 101, 14 (2008)

* See backup slides for detailed simulation parameters

Simulation: Beam Loading & Bubble Shape Calculation

This notebook calculates and visualizes the shape of a nonlinear plasma wakefield "bubble" in two distinct regimes:

1. **The Vacuum Regime (Unloaded):** In the absence of a witness bunch, the ions exert a restoring force that closes the electron cavity. In the ultra-relativistic blowout regime, this shape is phenomenologically approximated as a sphere. The vacuum bubble radius r_{vac} as a function of the co-moving coordinate ξ is given by:

$$r_{vac}(\xi) = R_b \sqrt{1 - \left(\frac{\xi}{R_b}\right)^2}$$

Correspondingly, the longitudinal electric field E_z grows linearly along the bubble axis:

$$E_z(\xi) \approx k_p \xi / \chi$$

$$r_b r_b'' + 2(r_b')^2 + 1 = 0 \quad (\text{when } \Lambda = 0)$$

The exact mathematical solution to this specific differential equation is a circle:

$$r_b(\xi) = \sqrt{R_b^2 - \xi^2}$$

$$r_b(\xi)^2 + (2a)^2 \xi^2 = R_b^2$$

2. **The Loaded Regime (With Bunch):** When a witness electron bunch is injected, its space charge pushes outward against the ions, modifying the potential.

- **Goal:** To achieve **constant acceleration** (zero energy spread), the longitudinal field E_z must be flattened to a constant value E_t .
- **Shape Change:** Since the wake potential ψ scales with the square of the bubble radius ($\psi \sim r_b^2/4$), a constant field ($E_z \propto \partial_\xi \psi = \text{const}$) implies that r_b^2 must decrease **linearly** rather than following the spherical curve.

The loaded trajectory is calculated by matching the slope of the potential at the injection point ξ_{start} :

$$r_{loaded}^2(\xi) = r_{vac}^2(\xi_{start}) + \left[\frac{d(r_{vac}^2)}{d\xi} \right]_{\xi_{start}} (\xi - \xi_{start})$$

Simulation Logic

- **Injection Point:** We place the head of the bunch at ξ_{start} , where the vacuum field naturally equals the target loaded field ($E_{vac} = E_t$).
- **Bubble Wall:** Before ξ_{start} , the wall follows the vacuum sphere. After ξ_{start} , the wall follows the linearized r^2 trajectory (the "sheath prop").

Backup: details of previous work

$$n_0 = 2e16 \text{ cm}^{-3}$$

Driver

Injector

$$\lambda_0 = 9.2 \text{ }\mu\text{m}$$

$$\lambda_0 = 0.4 \text{ }\mu\text{m}$$

$$\tau = 250 \text{ fs}$$

$$\tau = 20 \text{ fs}$$

$$w_0 = 105 \text{ }\mu\text{m}$$

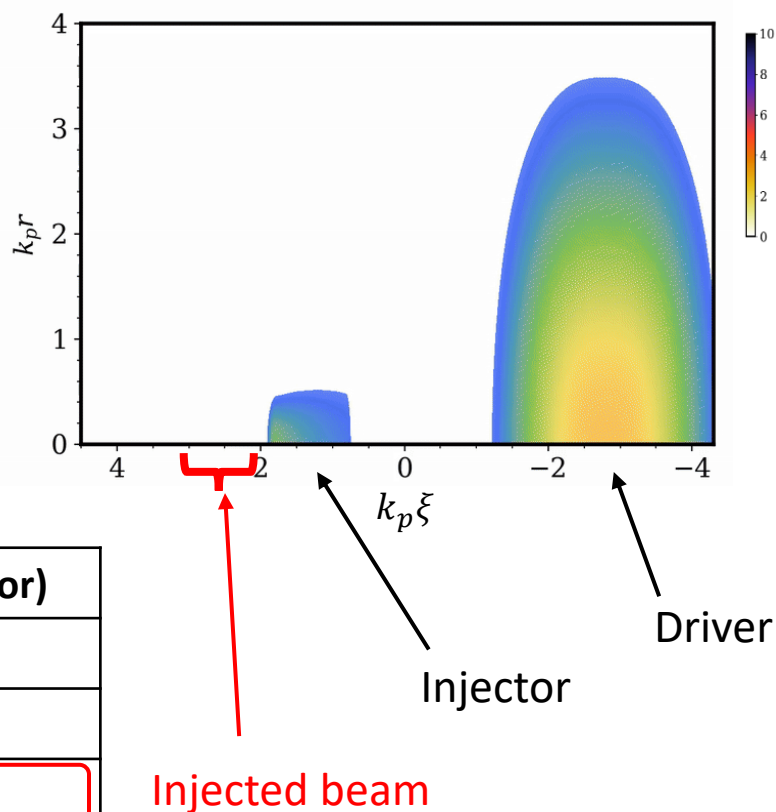
$$w_f = 8.6 \text{ }\mu\text{m}$$

$$a_0 = 2.7$$

$$a_0 = 0.17$$

$$\beta_f = 1.04$$

Ion. level	a_T (Driver)	a_T (Injector)
Kr ⁷⁺ -> Kr ⁸⁺	1.87	0.06
Kr ⁸⁺ -> Kr ⁹⁺	3.10	0.16
Kr ⁹⁺ -> Kr ¹⁰⁺	3.78	0.20
Kr ¹⁰⁺ -> Kr ¹¹⁺	4.51	0.23



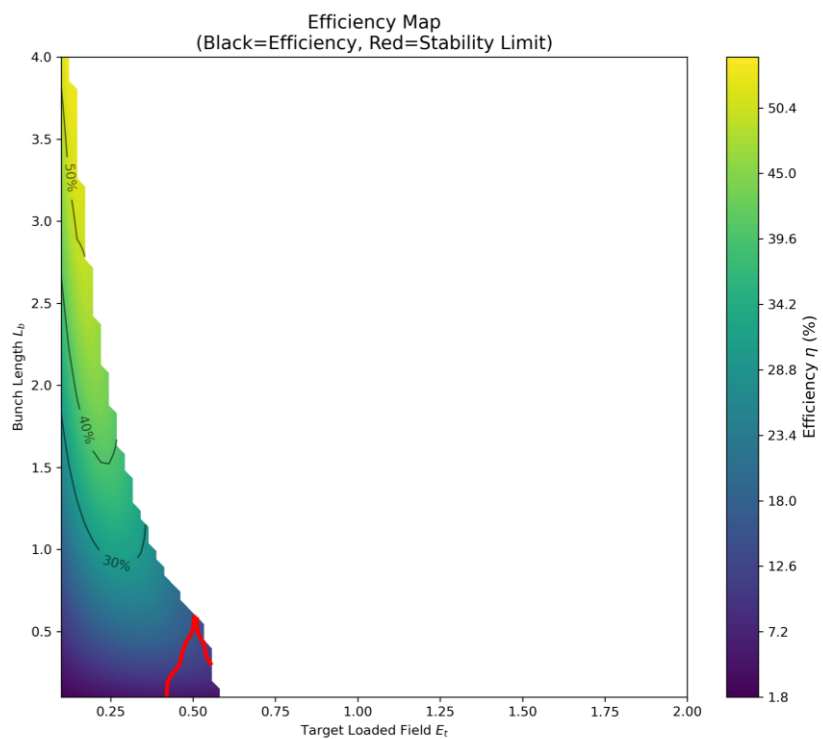
Generated beam

$$Q = 220 \text{ pC}$$

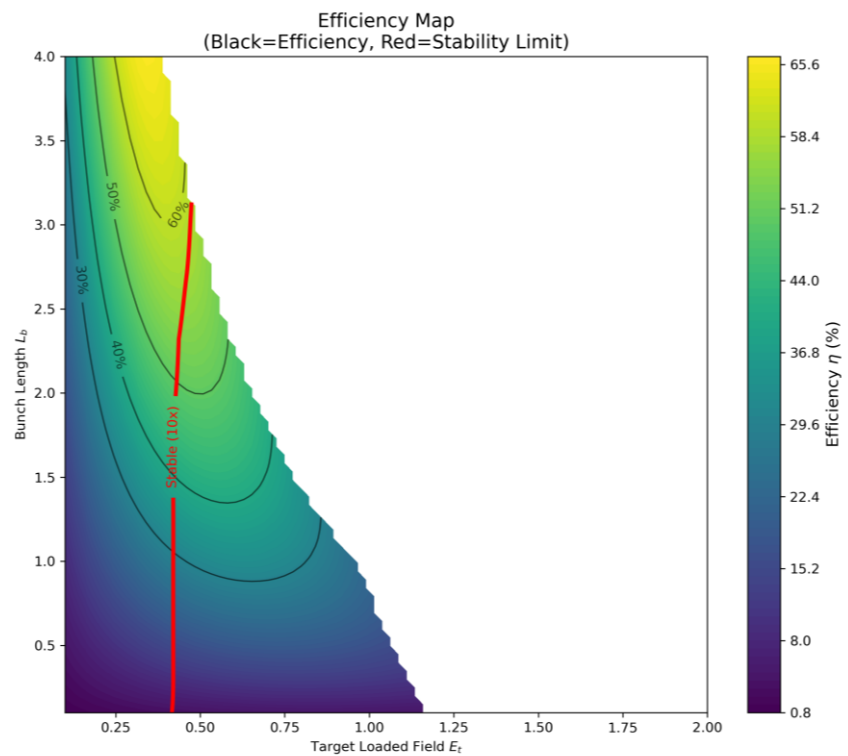
$$\epsilon_x = 170 \text{ nm} \cdot \text{rad}$$

$$\epsilon_y = 76 \text{ nm} \cdot \text{rad}$$

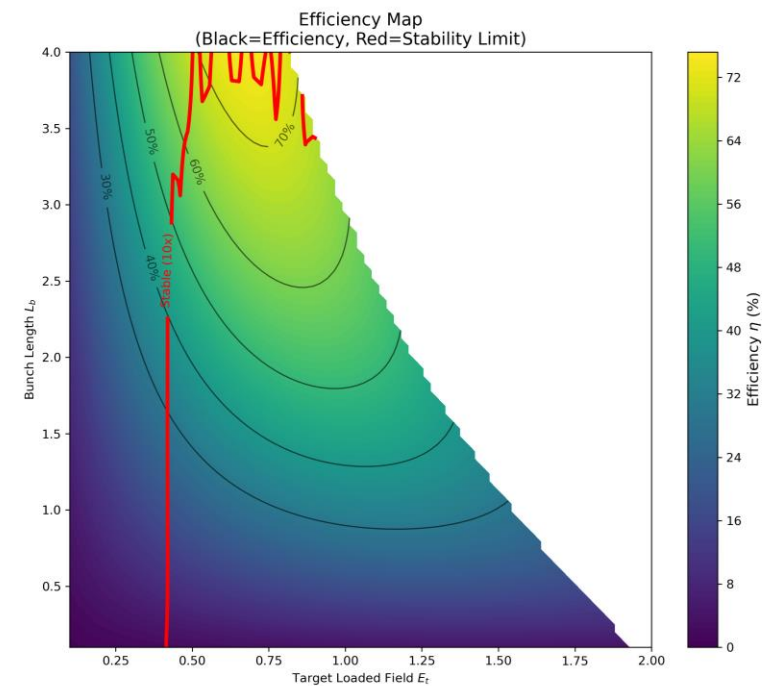
$$R_b = 2$$



$$R_b = 3$$



$$R_b = 4$$



$n_0 = 5 \times 10^{16} \text{ cm}^{-3}$, $offset = 2.4 \text{ } \mu\text{m}$, 20 GeV acceleration of the beam