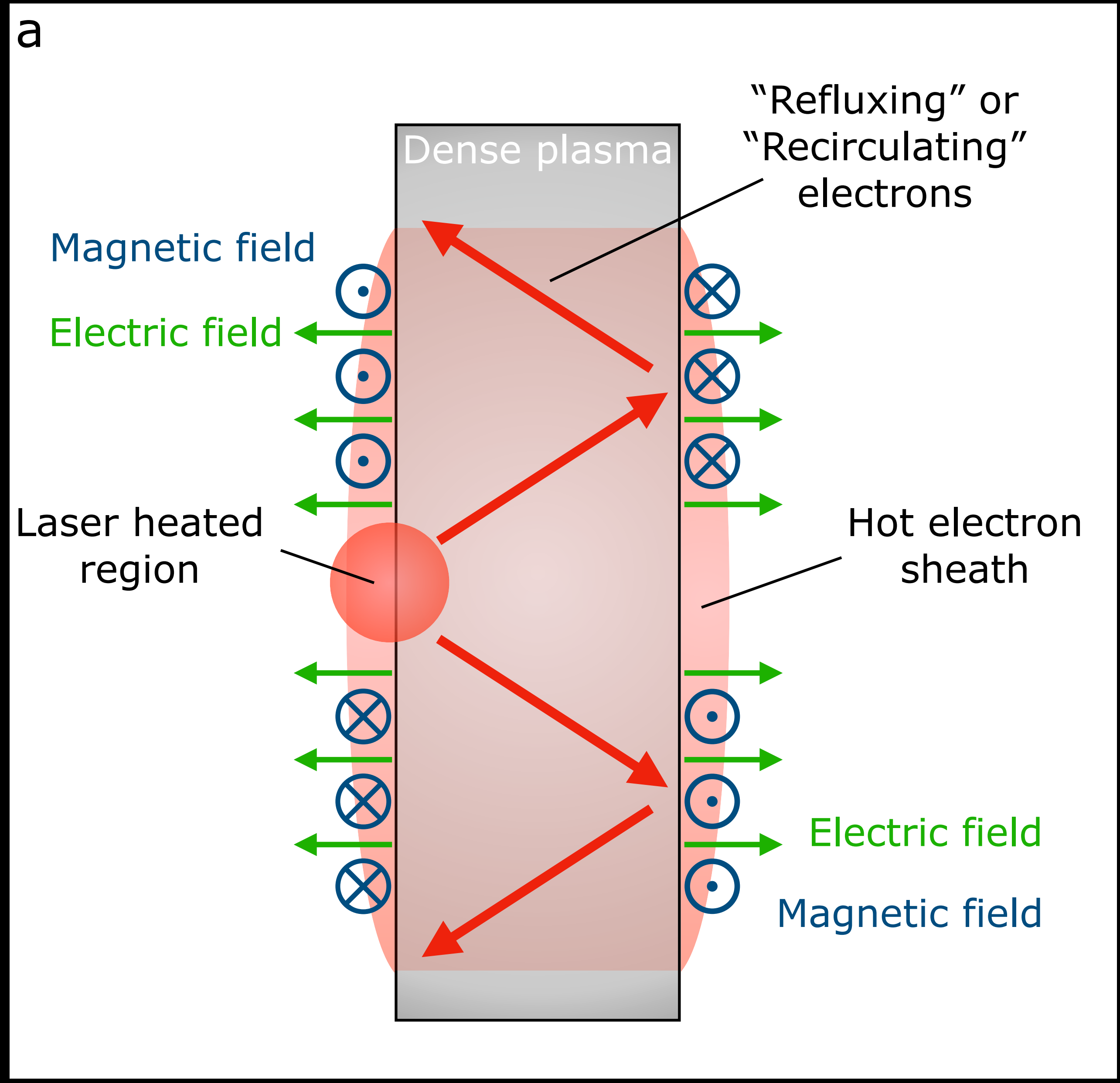
Electric and magnetic fields on the surface of a dense plasma with relativistic electron flows



A. G. R. Thomas[1], D. Uzdensky[2], B. K. Russell[3], S. S. Bulanov[4], S. V. Bulanov[5], P. Campbell[1], G. Grittani[4], K. Krushelnick[1], Q. Qian[1], C. P. Ridgers[6], P. Valenta[5] L. Willingale[1]

[1]Gérard Mourou Center for Ultrafast Optical Science, The University of Michigan, Ann Arbor, Michigan 48109, USA
[2]Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford, OX1 3PU, UK
[3]Department of Astrophysical Sciences, Princeton University, Princeton, New Jersey 08544, USA
[4]Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
[5]ELI Beamlines Facility, The Extreme Light Infrastructure ERIC, Za Radnicí 835, Dolní Břežany, 25241, Czechia
[6]York Plasma Institute, School of Physics, Engineering and Technology, University of York, York YO10 5DD, United Kingdom

Laser heating on surface of a solid target leads to hot electrons that generate sheath fields at the interfaces

- Electric fields relate to TNSA ion acceleration
- Magnetic fields have been used for e.g. magnetic reconnection studies

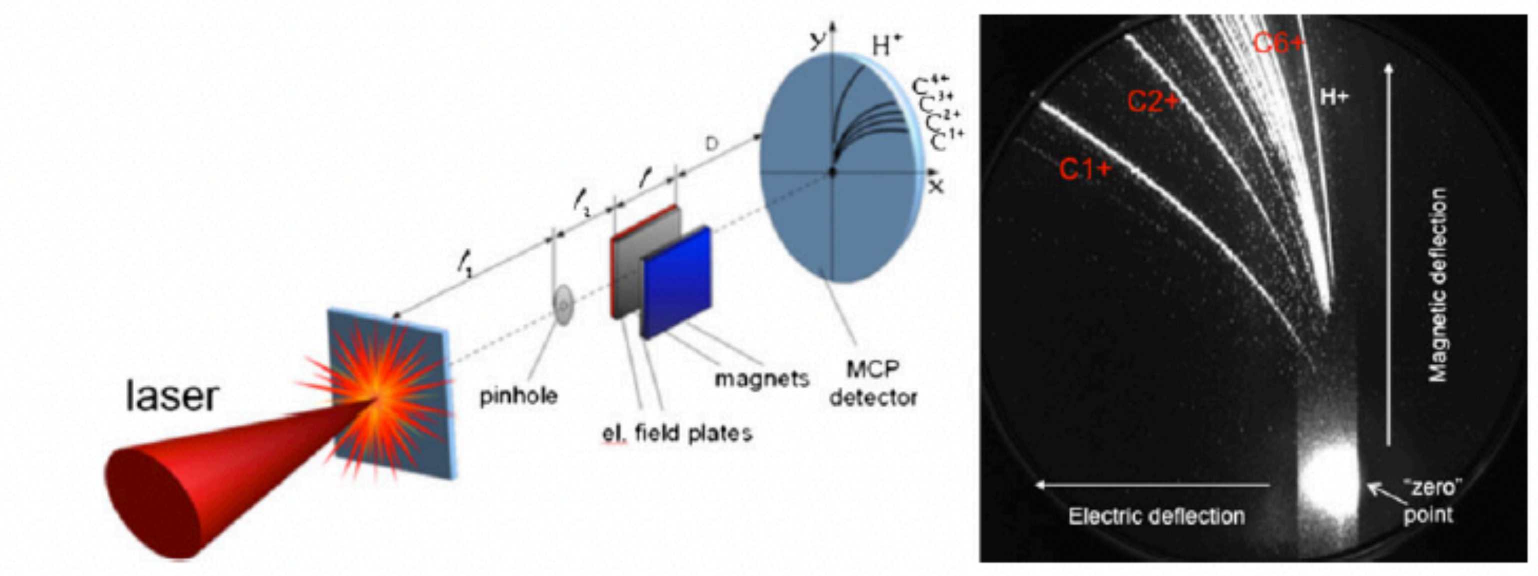
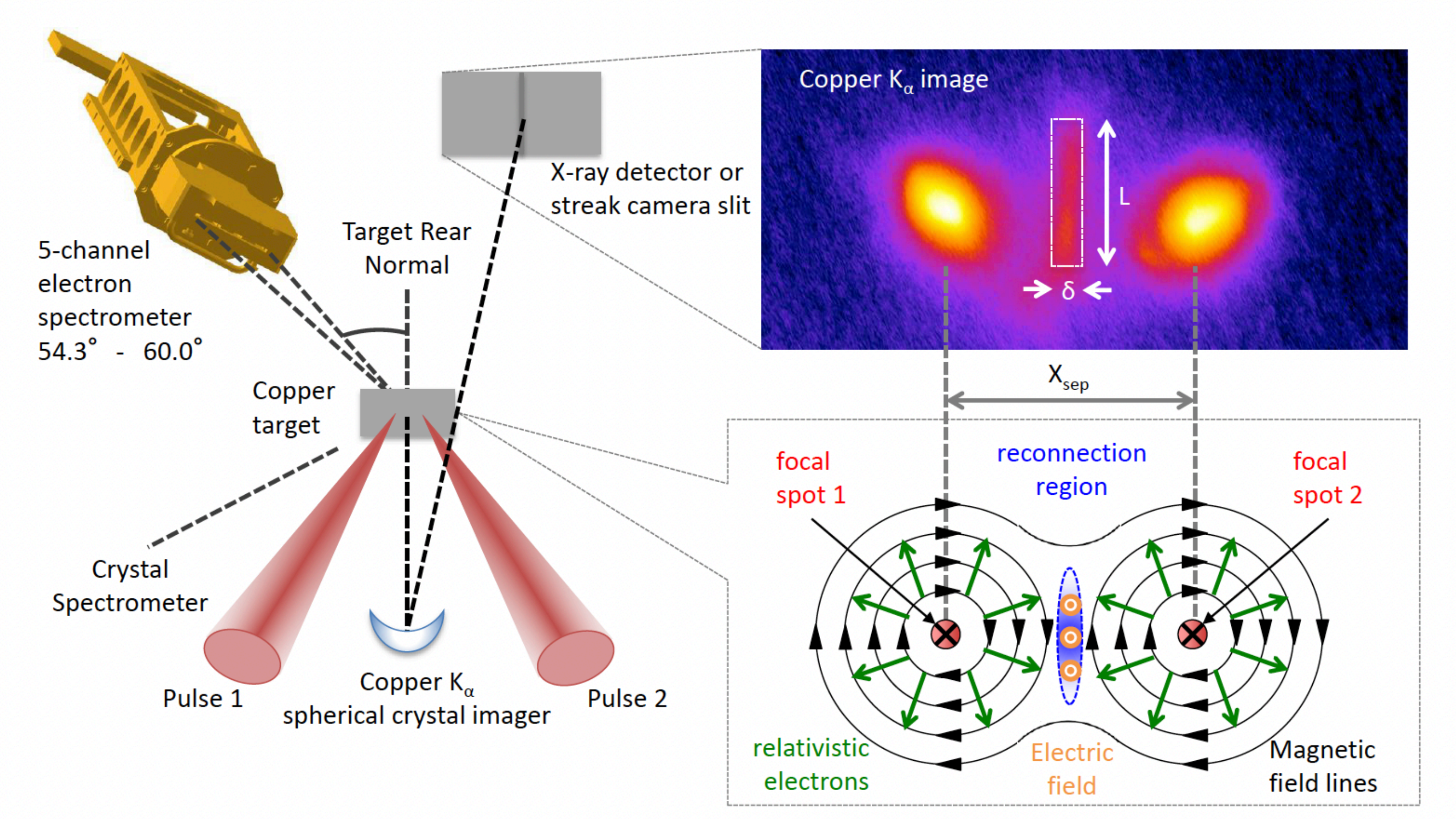


FIG. 10 (color online). Left: Schematic of a Thomson parabola (courtesy of S. ter-Avetisyan). Right: A typical example of ion traces obtained with the Thomson parabola.

A. Macchi Rev. Mod. Phys. (2013)



A. Raymond Phys. Rev. E (2016)

Do electrons "reflux" or skim along surface?

Electron "fountain effect"

Kolodner & Yablonovitch (1979); Forslund & Brackbill (1982); McKenna et al. (2007); Yates et al. (1982); Willingale et al.(2010)

Electron refluxing / recirculation

Ridgers et al. (2011); Green et al. (2018)

Electron surface currents

Quinn et al. (2009); Sarri et al. (2012); Schumaker et al. (2013); Albertazzi et al. (2015); Nakatsutsumi et al. (2018); B.K. Russell et al., Phys. Rev. Research (2025)

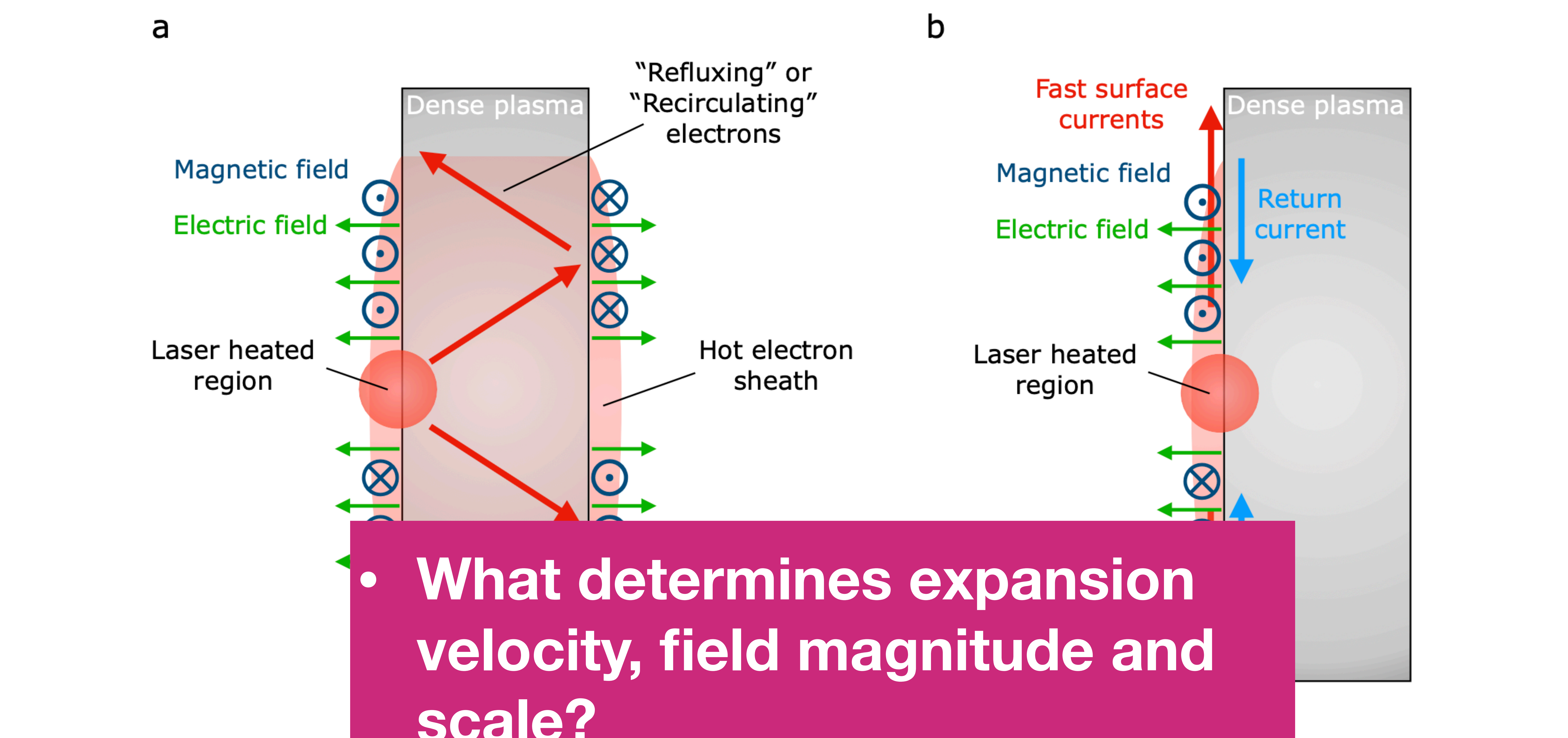


FIGURE 1. Hot elect… egion of electric and magnetic field b… urface currents.

- **What determines expansion velocity, field magnitude and scale?**
- **Why are electrons confined to surface?**

Magnetic field generation

- Consider simple fluid equation for electrons (scalar pressure, no resistivity)

Electron inertial terms - typically ignored

With resistivity, this gives "Ohm's law"

$$m_e\left(\partial_t \mathbf{u}_e + \mathbf{v}_e \cdot \nabla \mathbf{u}_e\right) = -e\left(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}\right) - \frac{\nabla p_e}{n_e}$$

- Take curl:

$$\partial_t \mathbf{B} = m_e \partial_t \left(\nabla \times \mathbf{u}_e\right) + \frac{\nabla p_e \times \nabla n_e}{e n_e^2}$$

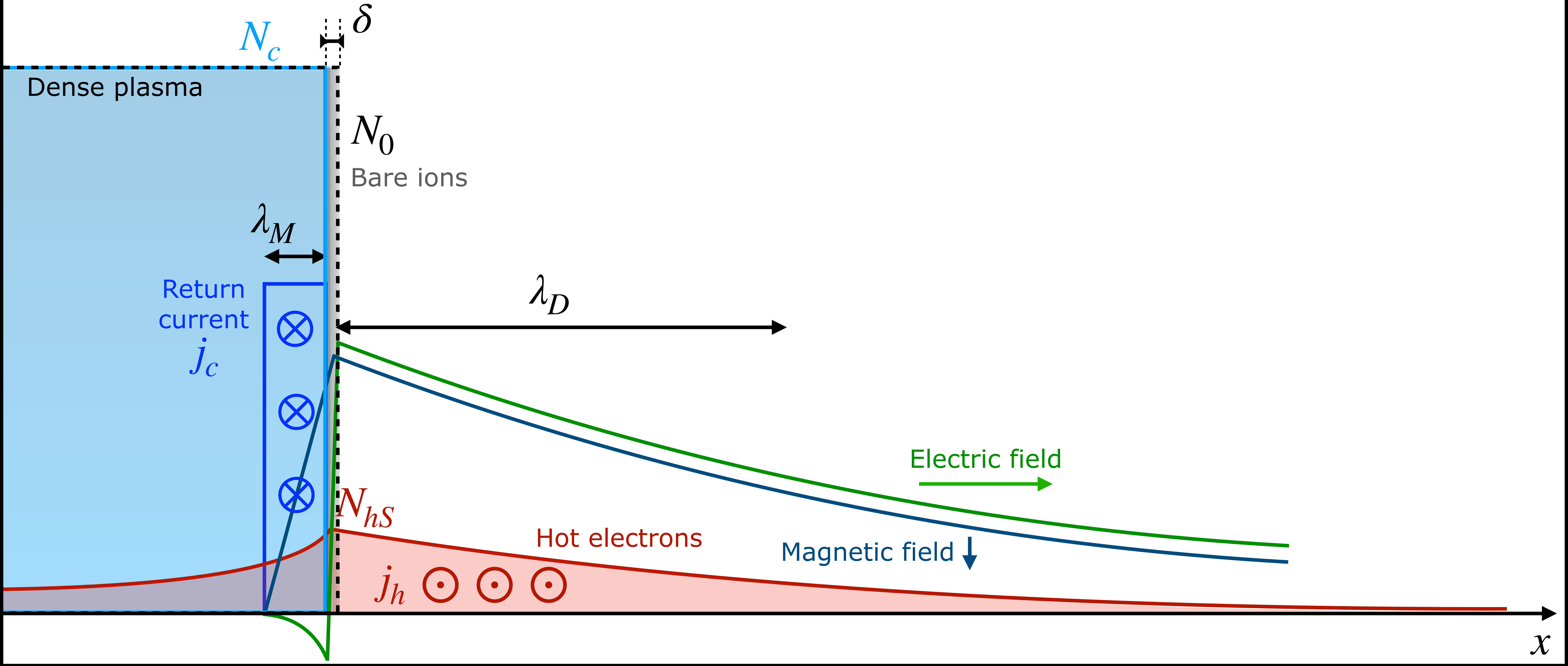
Vorticity of electron fluid

grad n cross grad T mechanism

- Magnetic fields may be generated by non-parallel density and temperature gradients (the "Biermann battery")

- ….Or time varying electron fluid vorticity

Motivation:
Electric / magnetic fields on surface of foil target



Warm relativistic fluid theory for sheath fields

(De Groot, van Leeuwen and van Weert Relativistic Kinetic Theory)

- Assume perfect fluid (no shear stresses, viscosity, or heat conduction)
- Equation of motion for species *s*

This is ∂^μ projected orthogonal to $u_s{}^\mu$ i.e. space like and is gradient in fluid rest frame

$$h_s n_s u_s^\nu \partial_\nu u_s^\mu = \nabla^\mu (n_s \theta_s) + q_s (\partial^\mu A_\nu - \partial_\nu A^\mu) n_s u_s^\nu$$

- Maxwell's equations (Lorenz gauge)

$$-\partial_\nu \partial^\nu A^\mu = \sum_s q_s j_s^\mu$$

$h_s = 4\theta_s + \tau_s / n_s$ Enthalpy per particle

$\theta_s = -\frac{1}{3n_s} \Delta_{s\,\mu\nu} T_s{}^{\mu\nu}$ Relativistic temperature

$n_s = u_{s\mu} j_s^\mu$ Proper density

$u_s^\mu = j_s^\mu / \sqrt{j_{s\nu} j_s{}^\nu}$ (Eckart) fluid velocity

$$j_s^\mu = \int f_s p^\mu d\Omega \,, \quad T_s^{\mu\nu} = \int f_s p^\mu p^\nu d\Omega \,,$$

$\tau_s = \int f_s d\Omega$

Using *natural relativistic plasma* units: $c = e = \varepsilon_0 = m_e = 1$ and time normalized to ω_p

Theory for isothermal hot electron sheath

- In vacuum region only there are only hot electrons:

$$\boxed{\partial_x{}^2 A^\mu = n_h u_h^\mu}$$

- If we assume **constant flow velocity** and **isothermal** steady state; contract with $u_h{}^\mu$

$$\boxed{\partial_x{}^2 \psi_h = n_h}$$

$$\boxed{\psi_h = u_h{}^\mu A_\mu}$$

Electrostatic potential in fluid rest frame

- with the boundary conditions $\psi_h \to -\infty$, $\partial_x \psi_h \to 0$ as $x \to \infty$, this yields the divergent potential

$$\boxed{\psi_h = -2\theta_h \ln\left(1 + \frac{\kappa_h x}{\sqrt{2}}\right)}$$

Nonrelativistic, electrostatic solution Crow et al.(1975)

$$\kappa_h = \sqrt{\frac{n_h}{\theta_h}}$$

A better theory

- After rearrangement of momentum equation:

$$h_s n_s u_s^\nu \partial_\nu u_s^\mu + q_s n_s u_s^\nu \partial_\nu A^\mu - q_s n_s u_s^\mu u_s^\nu u_{s\,\alpha} \partial^\alpha A_\nu = \nabla^\mu (n_s \theta_s) + q_s n_s u_s^\nu \nabla^\mu A_\nu$$

- Note: Space-like gradients on right, time-like on left

- If we assume that the time variation is relatively slow, then to lowest order approximation

$$\nabla (n_s \theta_s) = -q_s n_s {u_s}^\nu \nabla A_\nu$$

→ Fields arise to balance pressure gradients in fluid rest frame

Relativistic fluid theory for sheath fields

- To next level of approximation using $dh_s = d(\theta_s n_s)/n_s$ (Taub 1957)

$$h_s n_s u_s^\nu \partial_\nu u_s^\mu + q_s n_s u_s^\nu \partial_\nu A_\mu + n_s u_s^\mu u_{s\alpha} \partial^\alpha h_s = 0$$

- Or, the conservation law

For non-trivial potential, can't have solution with constant u_s and θ_s

$$\pi_s{}^\mu(x^\mu) = h_s u_s{}^\mu + q_s A_\mu = h_{s0} u_{s0}{}^\mu (x_0^\mu)$$

- (Warm fluid generalization of canonical momentum conservation)

Sketch of solution method

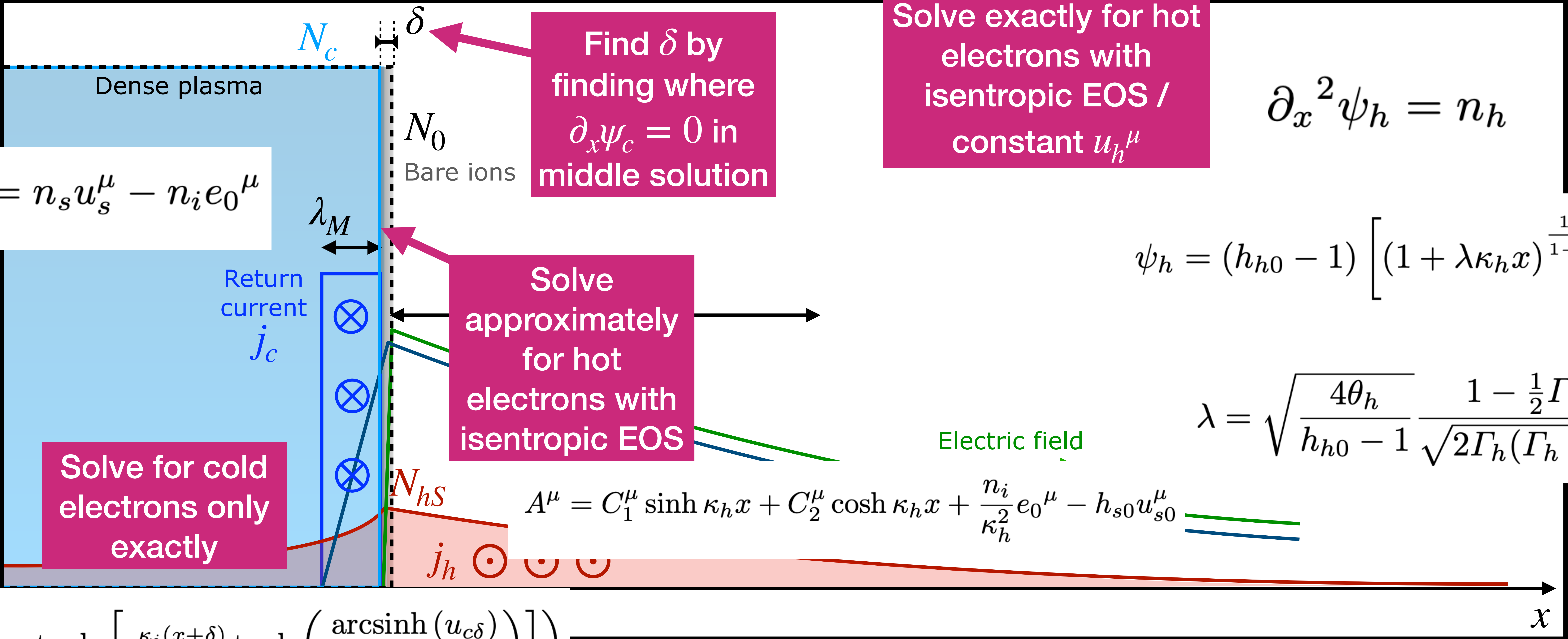
Isentropic hot electrons, zero temperature cold electrons and stationary ions

Solve 1D problem with semi-infinite slab

Potentials are solved for and set to zero at surface

Boundaries at $\pm\infty$

Two distinct interfaces



Find δ by finding where $\partial_x \psi_c = 0$ in middle solution

Solve exactly for hot electrons with isentropic EOS / constant $u_h{}^\mu$

Solve approximately for hot electrons with isentropic EOS

Solve for cold electrons only exactly

$$\partial_x{}^2 A^\mu = n_s u_s^\mu - n_i e_0{}^\mu$$

$$\partial_x{}^2 \psi_h = n_h$$

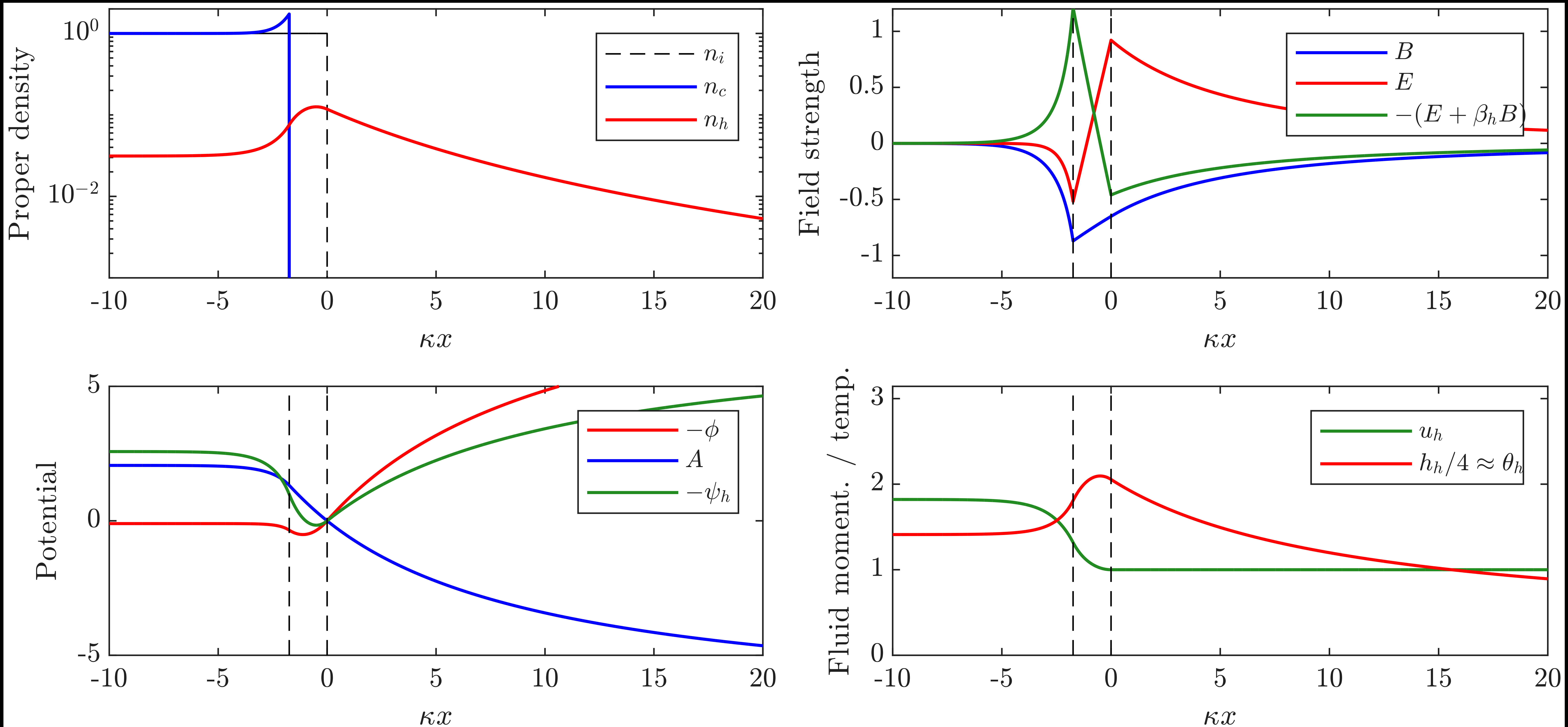
$$\psi_h = (h_{h0} - 1)\left[(1 + \lambda\kappa_h x)^{\frac{1-\Gamma_h}{1-\frac{1}{2}\Gamma_h}} - 1\right]$$

$$\lambda = \sqrt{\frac{4\theta_h}{h_{h0} - 1}}\,\frac{1 - \frac{1}{2}\Gamma_h}{\sqrt{2\Gamma_h(\Gamma_h - 1)}}\,.$$

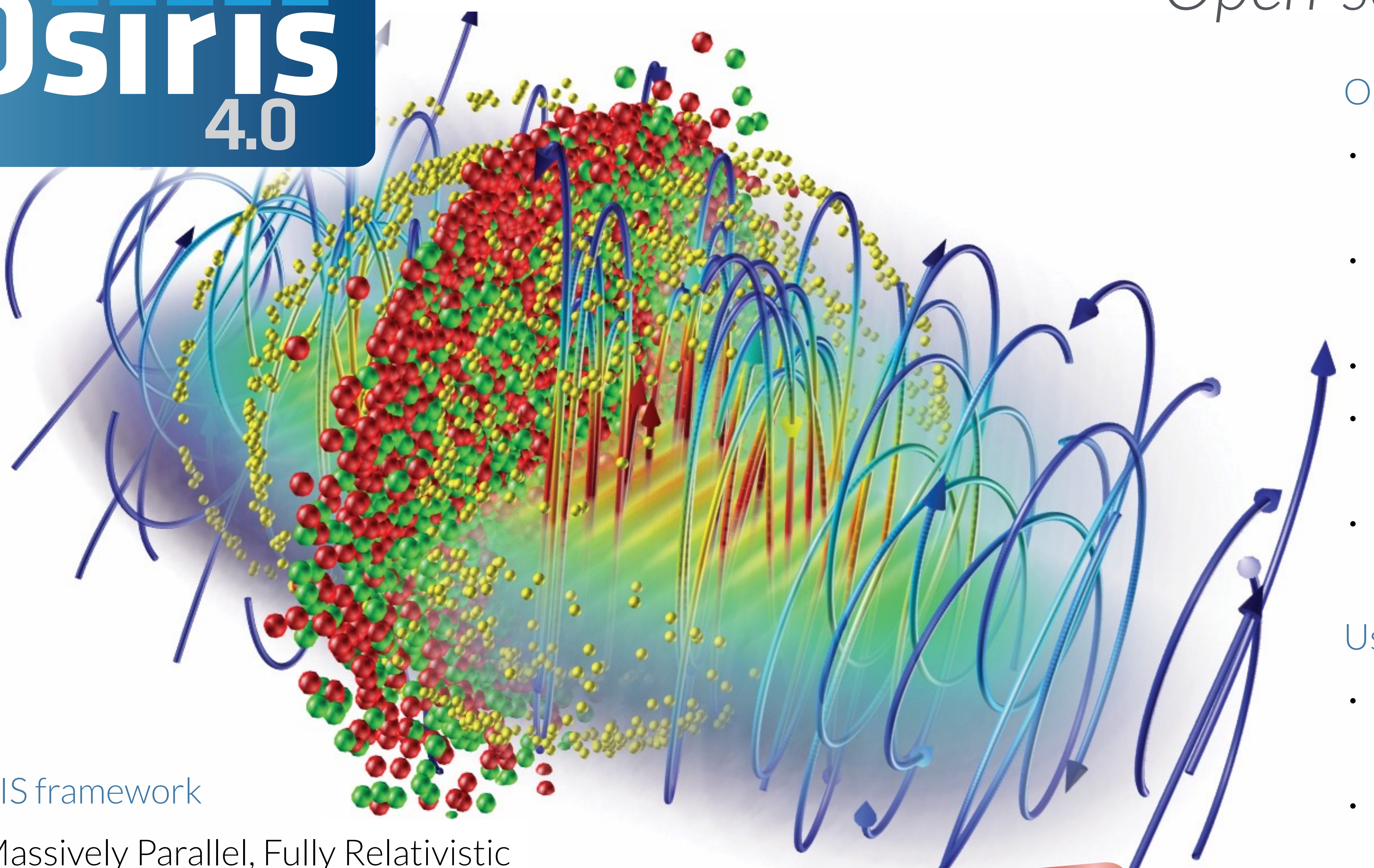
$$A^\mu = C_1^\mu \sinh \kappa_h x + C_2^\mu \cosh \kappa_h x + \frac{n_i}{\kappa_h^2} e_0{}^\mu - h_{s0} u_{s0}^\mu$$

$$u_c = \sinh\left(4\,\mathrm{arctanh}\left[e^{\kappa_i(x+\delta)} \tanh\left(\frac{\mathrm{arcsinh}\,(u_{c\delta})}{4}\right)\right]\right)$$

Theoretical curves for $N_{h0} = 0.15, \theta_h = 2, u_h = 1$



Open-source version available



Open-access model

- 40+ research groups worldwide are using OSIRIS
- 400+ publications in leading scientific journals
- Large developer and user community
- Detailed documentation and sample inputs files available
- Support for education and training

Using OSIRIS 4.0

- The code can be used freely by research institutions after signing an MoU
- Open-source version at:

https://osiris-code.github.io/

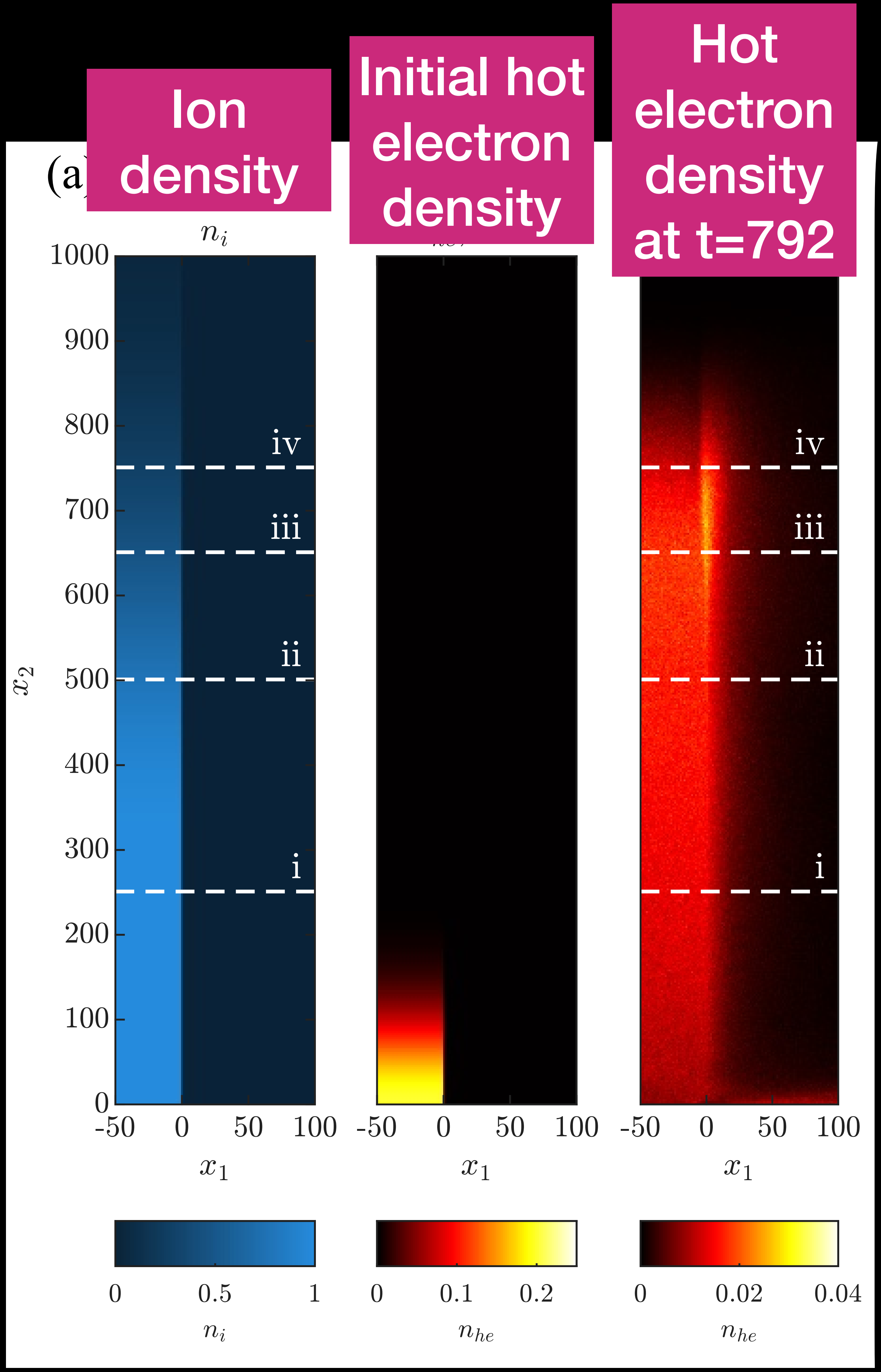
OSIRIS framework

- Massively Parallel, Fully Relativistic Particle-in-Cell Code
- Support for advanced CPU / GPU architectures
- Extended physics/simulation models
- AI/ML surrogate models and data-driven discovery

Ricardo Fonseca: ricardo.fonseca@tecnico.ulisboa.pt

Validation of theory: 2D particle-in-cell simulations with OSIRIS 4.0

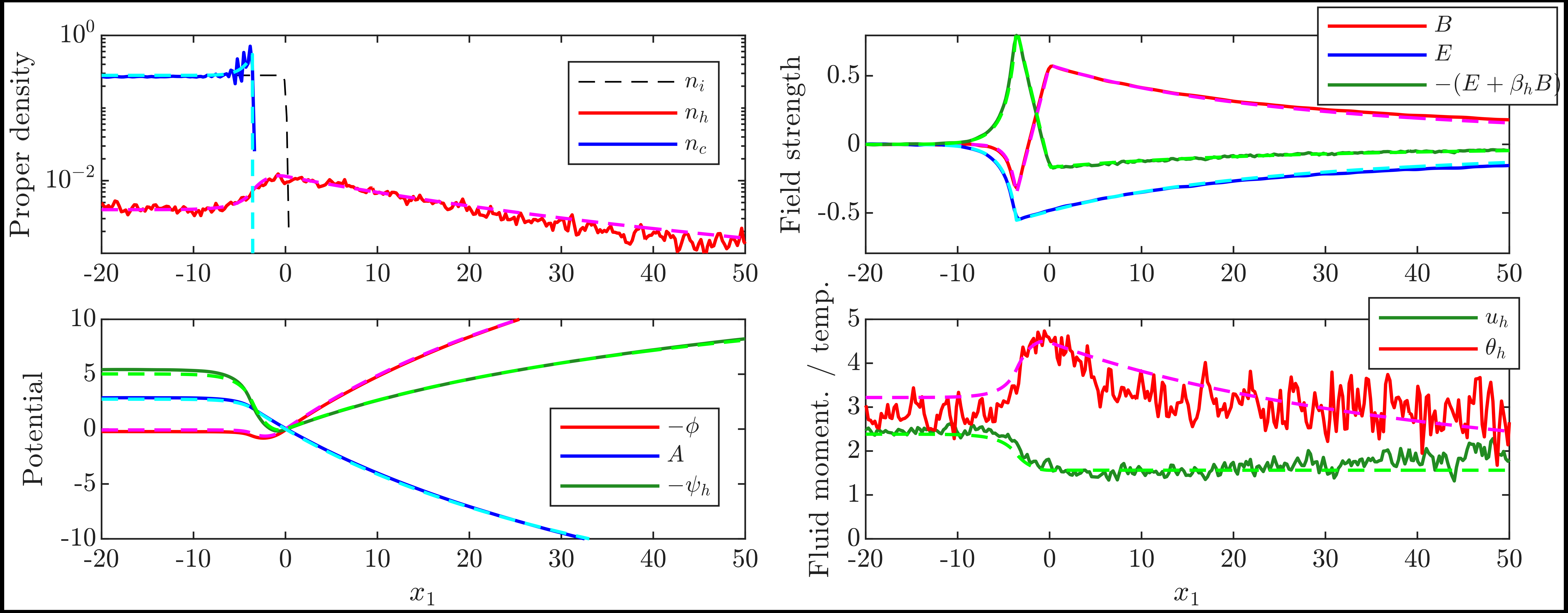
- A thermal hot electron distribution was initialized in a gaussian spatial profile and allowed to expand
- Dense cold plasma had a slow decrease in density to test theory at different ratios of N_h/N_i
- Only parameters in theory are surface density, temperature and fluid momentum (single value, not profile and no fitting!)
    - Extracted at each line position



Theory predicts density, temperature, field, potential and fluid momentum profiles all with good agreement

Profile (iv) at t=792

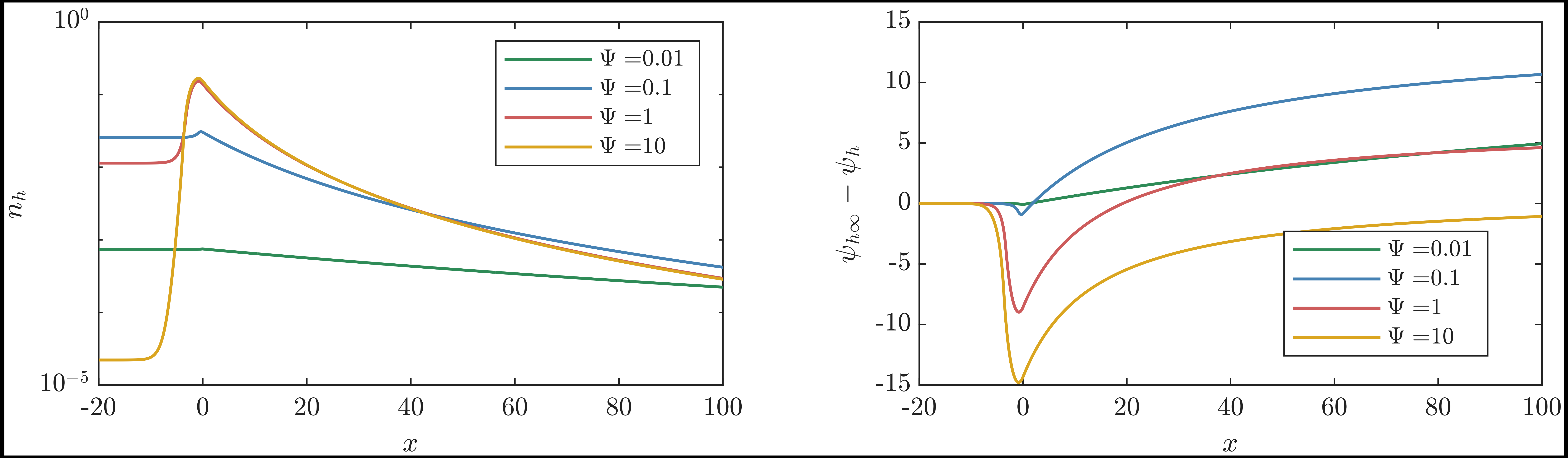
Dotted lines show theory for $n_i = 0.28$, $N_{hS} = 0.020$, $\theta_h = 4.4$, $u_h = 1.6$



Electrons will be confined for large n_h, u_h, small θ_h

$$\frac{\Delta\psi_h}{\theta_h} \sim \Psi = = \gamma_h \beta_h^2 \sqrt{\frac{n_h}{\theta_h}}$$

- For $\Psi \gtrsim 1$ the hot electrons are confined to the surface
- For $\Psi \ll 1$ the hot electrons are not confined (i.e. will reflux for a finite foil)



Summary

- Hot electrons expanding on a dense plasma surface lead to electric and magnetic sheaths
- Analytic theory is developed for two species; an isentropic warm perfect fluid and cold dense electrons in 1D using relativistic fluid theory
- Theory agrees well with 2D simulations of an expanding thermal electron source in dense plasma-vacuum interface; an approximation for a typical laser-solid plasma interaction
- Identification of refluxing vs surface confinement, deceleration of electron fluid in field generation, and scalings for field strength and spatial scales
- Isentropic model leads to finite potentials (c.f. divergent in isothermal model)