Breakdown Analysis and Wakefield Simulation for RF Cavities in the Muon Collider Ionization Cooling Channel

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Acknowledgments

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  Lawrence Berkeley National Laboratory

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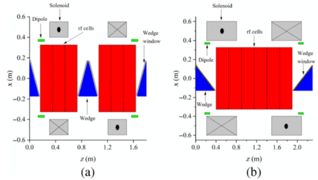
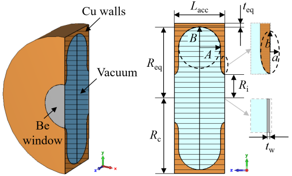
Overview

Overview of the Proposed Rectilinear Cooling Channel



Staged rectilinear cooling channel



Enclosed, windowed RF cavity

Operating context and simulation challenge

- Normal-conducting RF cavities restore p_z lost in absorbers during ionization cooling.
- The pre- and post-merge stages use cavities operating at **352 or 704 MHz**, with nominal gradients of between **21.2–31.7 MV/m**.
- There are strong, spatially varying solenoidal fields, up to 15 T in a cell.
- Beam size and charge evolve throughout the channel.
- Thin windows close the beam apertures while preserving favorable RF performance.

Cooling-channel parameters adapted from R. Zhu et al., *Phys. Rev. Accel. Beams* **28**, 041003 (2025).

Cavity geometry adapted from C. Barbagallo et al., "RF cavity design and beam

Focus of This Talk

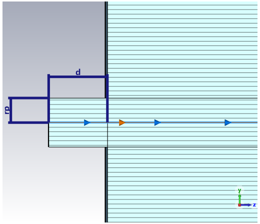
Wakefields

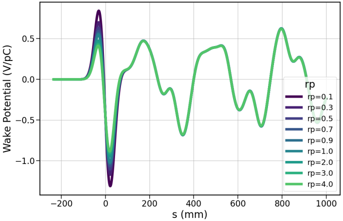
- How can the wake be reconstructed in a windowed, enclosed cavity?
- How does finite transverse bunch size change the balance between cavity modes and direct space charge?

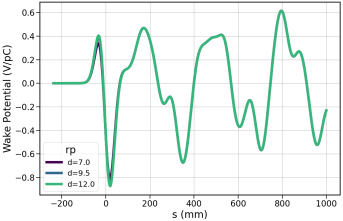
Breakdown

- Do realistic RF and lattice fields preserve localized electron bombardment?
- How do material properties affect the thermomechanical operating limit?

Limitations of the CST Wakefield Solver







Artificial beam-pipe geometry

Pipe-radius sweep

Pipe-extension sweep

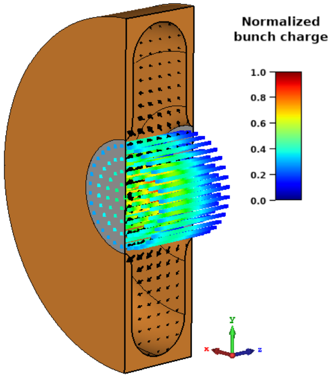
Takeaways

- CST requires beam path $\rightarrow$ closing the pipe with a metallic face works only with zero-thickness
- Adding pipe $\rightarrow$ geometry-independent result
- Source is a line-charge $\rightarrow$ misrepresents large beam waist for μCol

PIC-Based Wake Reconstruction

Transport the source distribution → Export time-dependent fields → Sample along co-moving paths → Integrate and normalize W_z

- The windows remain electromagnetic boundaries but are transparent to the numerical source particles.
- Exported fields are integrated along co-moving test-particle trajectories.



Reconstructed wake

$$W_z(s) = -\frac{1}{Q}\int E_z\left(z, t = \frac{z+s}{\beta c}\right)\,\mathrm{d}z$$

Finite Transverse Extent and Wake Decomposition

Finite transverse form factor

$$F_{n,\perp}(\sigma_\mathrm{t}) = \int_0^{R_\mathrm{i}} 2\pi r_\mathrm{s}\, \rho_r(r_\mathrm{s}) \times \sqrt{(R/Q)_n(r_\mathrm{s})}\, \mathrm{d}r_\mathrm{s}.$$

Eigenmode wake reconstruction

$$W^0_{z,\mathrm{eig}}(t) = H(t) \sum_n \frac{\omega_n}{2} (R/Q)_n e^{-t/\tau_n} \cos(\omega_n t),$$

with

$$\tau_n = \frac{2Q_{\mathrm{L},n}}{\omega_n}.$$

$$W_{z,\mathrm{eig}}(t, \sigma_\mathrm{t}) = \frac{1}{Q_b} \int_{-\infty}^{t} F_{n,\perp}(\sigma_\mathrm{t}) W^0_{z,\mathrm{eig}}(t - t')\lambda(t')\, \mathrm{d}t'.$$

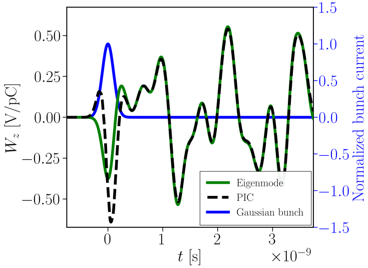
Complete longitudinal wake

$$W_z(t) = \underbrace{W_{z,\mathrm{FM}}(t) + W_{z,\mathrm{HOMs}}(t)}_{W_{z,\mathrm{eig}}(t)} + W_{z,\mathrm{SC}}(t)$$

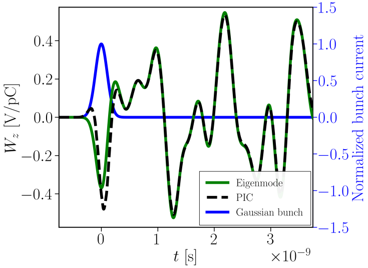
$$W_{z,\mathrm{SC}} = W_{z,\mathrm{ISC}} + W_{z,\mathrm{DSC}}$$

Methods developed at CERN by A. Grudiev and C. Barbagallo

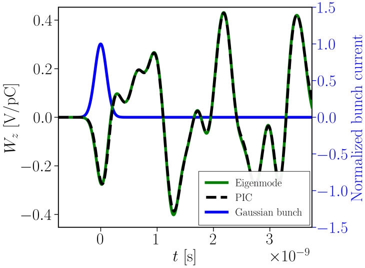
PIC Reconstruction Compared with the Eigenmode Model



Small beam radius
$\sigma_{\mathrm{r}} = 0.1\ mm$



Intermediate beam radius
$\sigma_{\mathrm{r}} = 5.44 mm$



Large beam radius
$\sigma_{\mathrm{r}} = 40\ mm$

Conclusion

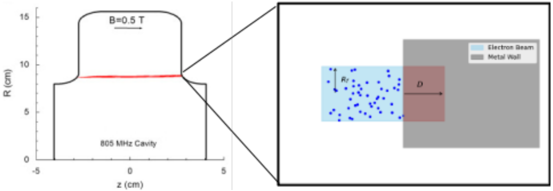
Fixed bunch charge, reducing radius substantially strengthens the space-charge wake $\Rightarrow$ the beam becomes more tightly cooled in the later stages of the channel, space-charge contributions may become non-negligible and should be included, particularly for beams with $\beta \simeq 0.9$.

Temperature-Dependent Breakdown Model in a Magnetic Field

- Local enhancement of the accelerating gradient G produces a surface field $\beta_{FE} G$.
- The magnetic field focuses the emitted current into electron beamlets.
- During the RF pulse, a beamlet of radius R_f deposits energy over a penetration depth D.
- **This work: models real field distributions and performs a material property trade study**

Fowler–Nordheim current density

$$j_{FN} = 6.0 \times 10^{-12} \frac{(\beta_{FE} G)^{2.5}}{\phi^{1.75}} 10^{4.52\phi^{-0.5}} \exp\left[-\frac{6.53\times10^{9}\phi^{1.5}}{\beta_{FE} G}\right]$$



Heat conduction

$$\left(\nabla^2 - \frac{1}{\alpha_d}\frac{\partial}{\partial t}\right) T + \frac{W}{\kappa} = 0$$

Temperature rise

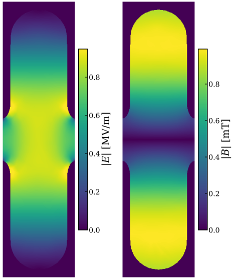
$$\Delta T(r,z,\tau) = \int_0^{\tau}\int_0^{R_f}\int_0^{D} G_{rz}(r,z;r',z';\tau - t')\, W(r',z',t')\, 2\pi r'\, \mathrm{d}z'\, \mathrm{d}r'\, \mathrm{d}t'$$

Damage threshold

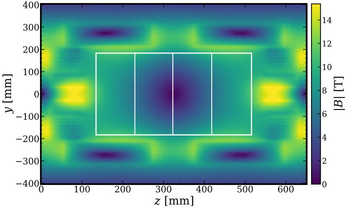
$$\Delta T_s = \frac{(1-\nu)\sigma_t}{E\alpha_{\mathrm{th}}}$$

Model adopted from *Nucl. Instrum. Methods Phys. Res. A* **620**, 147–154 (2010), doi:10.1016/j.nima.2010.03.167.

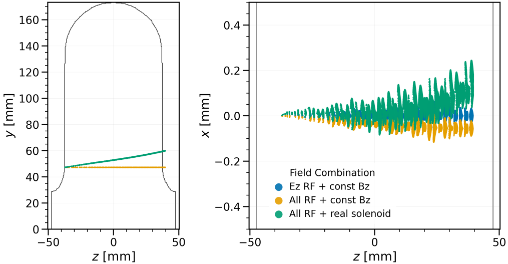
Dark Current Tracking Simulations with RF-Track



Stage-B8 cavity fields
Electric- and magnetic-field magnitudes



Lattice magnetic field
Maximum longitudinal field: ∼15.8 T

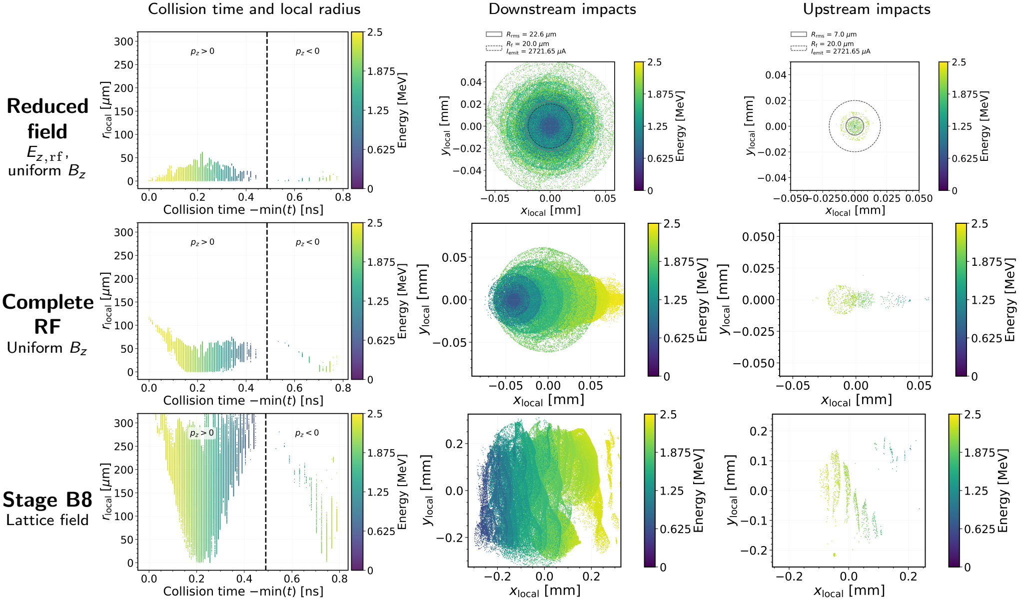


Electron trajectory comparison
Reduced fields, complete RF fields, and realistic lattice fields

Tracking results

- Three field models isolate the effects of transverse RF and lattice fields.
- Transverse RF fields shift the electron trajectories and impact locations.
- The realistic lattice field broadens the distribution, but impacts remain localized.
- Particle source sampled based on β_{FE} from FN

Localized Impacts



Relevance of Thermal Loading

1. Tracking simulations show that the non-uniform fields create complex bombardment profiles in space and time
2. The localization of the bombardments indicate that thermal loading from bombardment is still relevant with transverse field components and fring fields

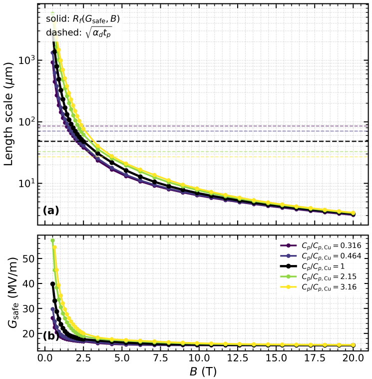
Temperature rise

$$\Delta T(r, z, \tau) = \int_0^{\tau} \int_0^{R_f} \int_0^{D} G_{rz}\left(r, z; r', z'; \tau - t'; \alpha_d = \frac{\kappa}{\rho C_p}\right) W(r', z', t')\, 2\pi r' \,\mathrm{d}z' \,\mathrm{d}r' \,\mathrm{d}t'$$

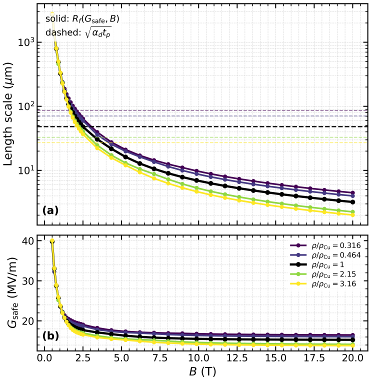
Thermal diffusivity, $\alpha_d = \kappa/(\rho C_p)$, balances spatial spreading of heat and atomic reaction to power density deposit W.

Next we sweep C_p, κ, and ρ to assess what materials could help mitigate bombardment induced damage

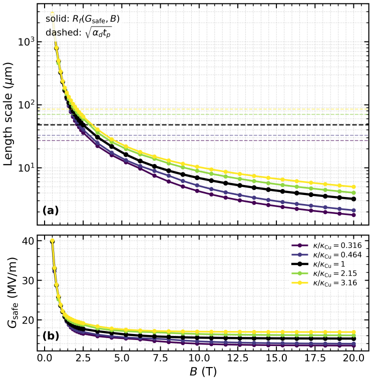
Material-Property Sensitivity



Heat capacity, C_p



Density, ρ

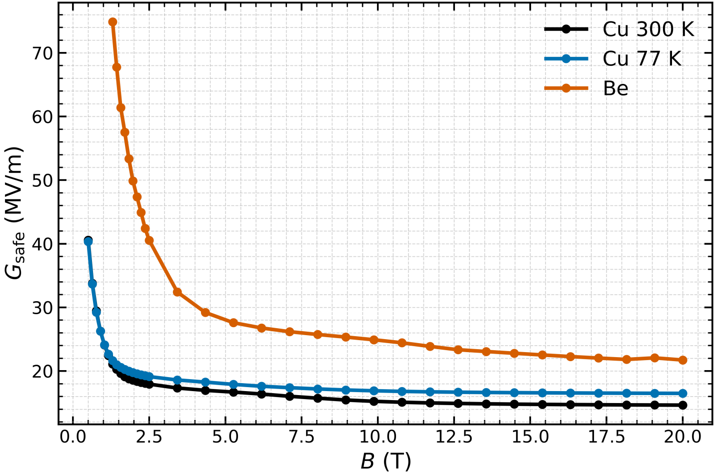


Thermal conductivity, κ

Conclusion

No single property controls the response: ρC_p governs local energy storage, while κ becomes increasingly important as magnetic focusing reduces the impact radius.

Copper and Beryllium Comparison

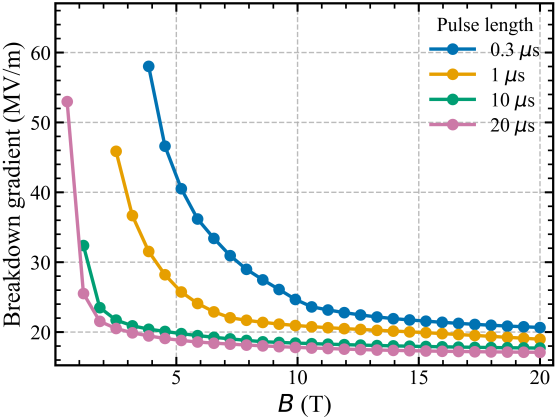


Interpretation

- Cu at 300 K provides the room-temperature reference.
- Be benefits from its high thermomechanical threshold and low energy deposit.
- At high magnetic field, Cu at 77 K benefits from enhanced thermal diffusivity

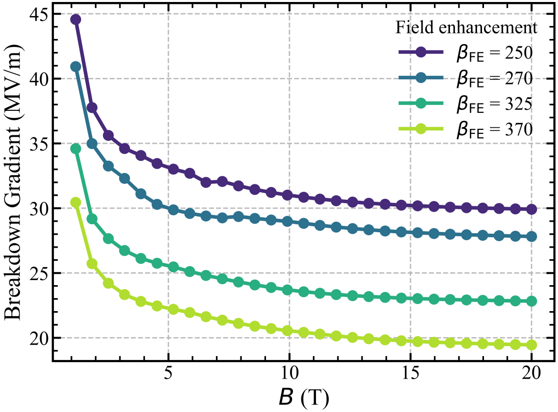
Source: NIST Cryogenic Material Properties Database, Beryllium and OFHC copper data, trc.nist.gov/cryogenics/materials (accessed July 2026).

Sensitivity to RF Pulse and Field Enhancement



RF pulse length

Shorter pulses reduce accumulated heating.



Field-enhancement factor

Lower β_{FE} strongly suppresses field emission. can be associated with cryogenic operation or hardened materials

Conclusions

Wakefield studies

- PIC field exports reconstruct the wake while preserving the closed-window boundaries.
- The projected eigenmode model reproduces the geometric cavity wake.
- At fixed charge, direct space charge grows substantially as the beam radius decreases.
- Later cooling stages therefore require explicit space-charge contributions.

Breakdown studies

- Realistic RF and lattice fields shift and broaden impacts, but bombardment remains localized.
- At high magnetic field, a high diffusivity could be beneficial over strengthened materials.
- Shorter RF pulses reduce heating, although their relative benefit decreases at high field.
- Surface condition through β_{FE} may rival bulk-material effects and requires experimental validation.

Questions?

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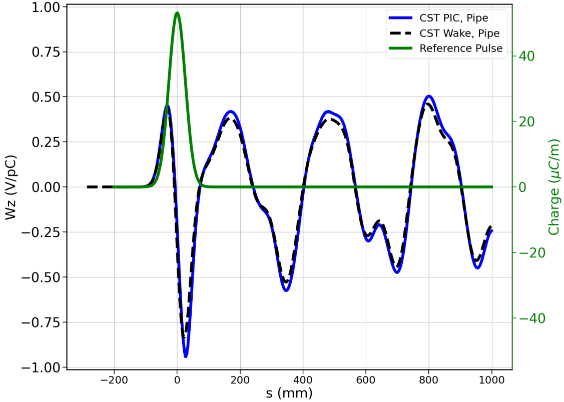
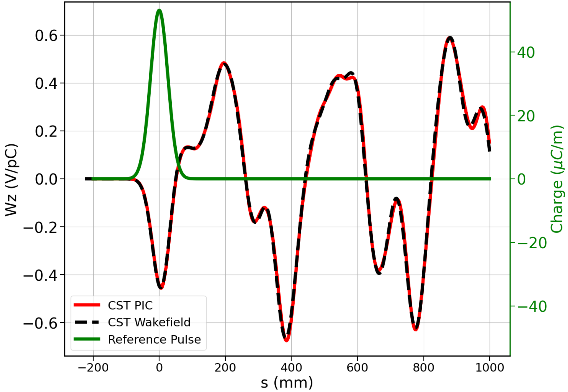
Backup: PIC Wake Reconstruction Validation

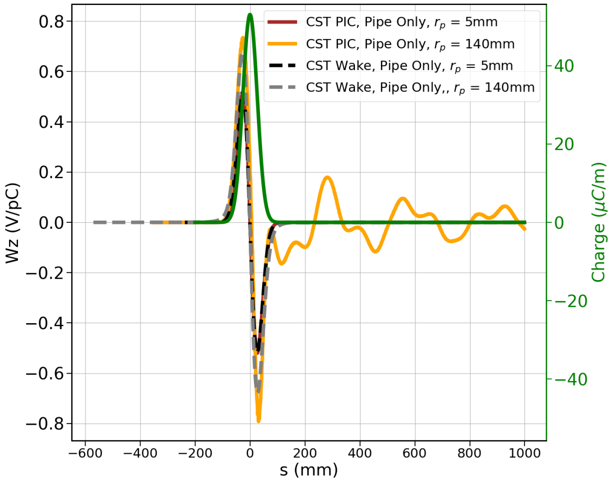


Subrelativistic source: $\beta = 0.9$

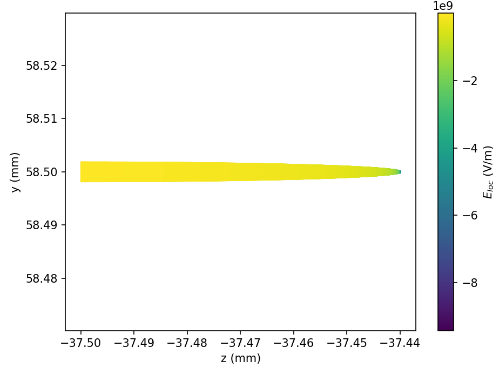


Ultrarelativistic source: $\beta = 1$

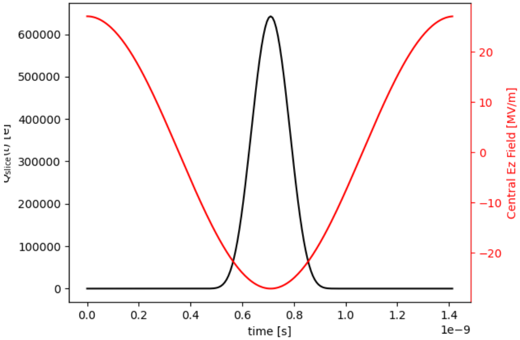
Backup: Independent Direct-Space-Charge Cross-Check



Backup: Field-Emission Source Model

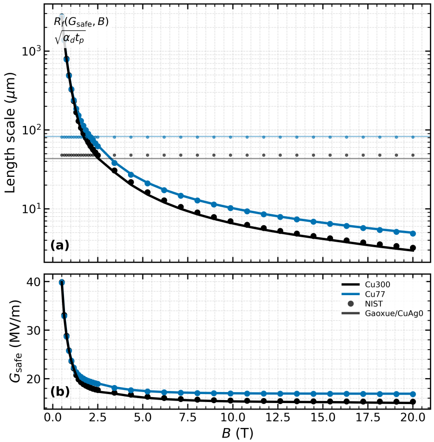


Spatial field enhancement



Time-dependent emission

Backup: Predictions Using DFT Material Models



Source: DFT material-property results provided by Gaoxue Wang and based on G. Wang, E. I. Simakov, and D. Perez, "Ab initio investigation of elastic properties of dilute Cu alloys for high-gradient accelerating structures," *J. Appl. Phys.* **132**, 175112 (2022), doi:10.1063/5.0106880.