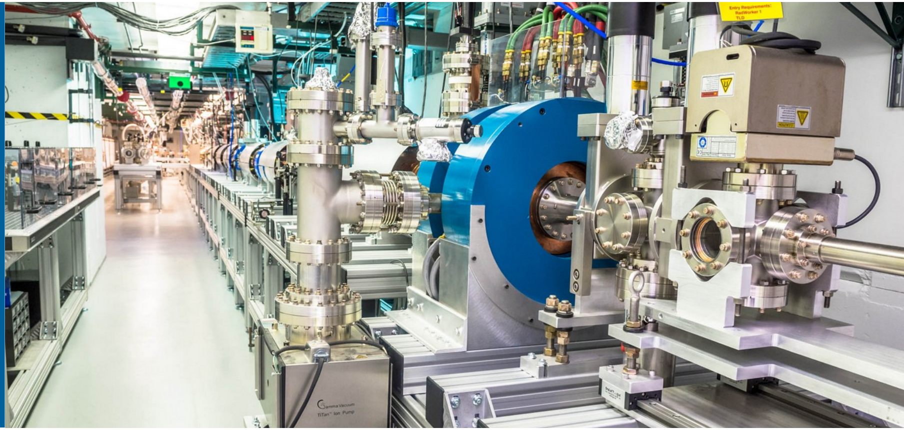
3D Simulation and Analysis of Coherent Synchrotron Radiation with Shielding



O. Ramachandran[1], G. Ha[2], C-K. Huang[3], X. Lu[1,2], J. Power[1], J. Qiang[4]

[1] Argonne National Laboratory, Lemont, IL
[2] Northern Illinois University, DeKalb, IL
[3] Los Alamos National Laboratory, Los Alamos, NM
[4] Lawrence Berkeley National Laboratory, Berkeley, CA

AAC 2026

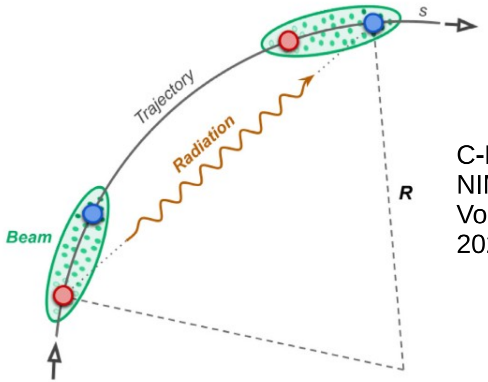
Outline

- Coherent synchrotron radiation, 1D/3D effects, shielding
- Modelling CSR in 3D
- Derbenev criterion and applicability of 1D methods
- Interplay between profile shape and shielding.
- Conclusions

This research was supported by the U.S. Department of Energy, Office of Science, Office of High Energy Physics under Award DE-SC0024445.

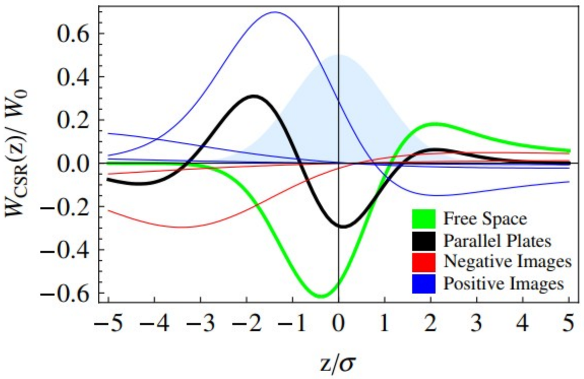
Coherent Synchrotron Radiation

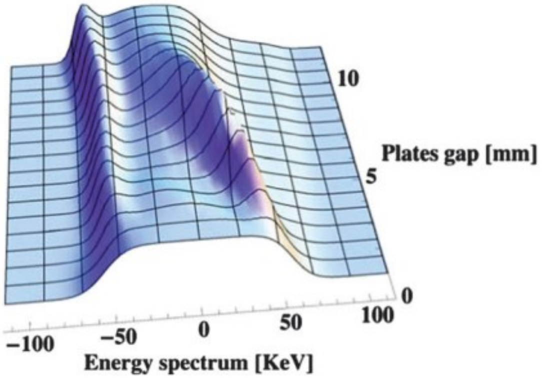
- When a beam bends, it emits synchrotron radiation.
- When the wavelength of this radiation is on the order of the bunch length, it coherently adds as the bunch moves through a bending magnet, causing disruption of the beam quality.
- Mitigating CSR is a bottleneck obstacle in the brightness of beams in storage rings, linear colliders and FELs.



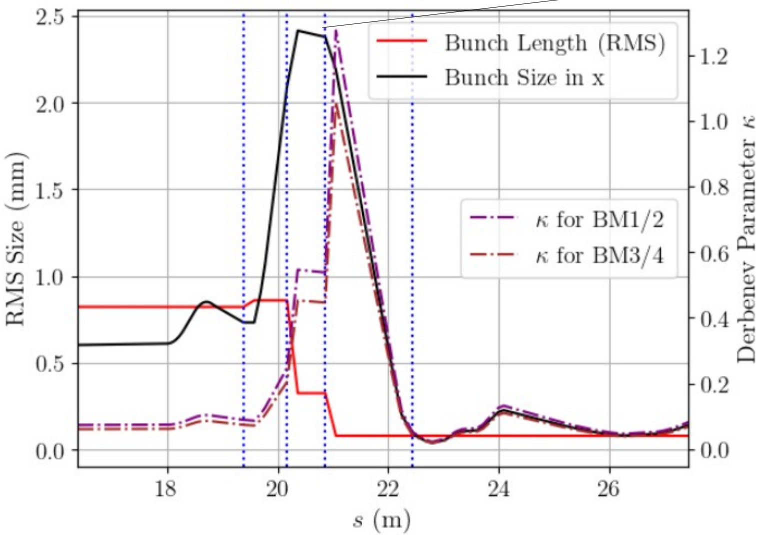
C-K. Huang et al.,
NIMA-A,
Vol. 1034,
2022

- Mitigating CSR through the use of shielding walls in the vacuum chamber has been demonstrated in experiment
(Yakimenko et al, Phys. Rev. Lett. 109, 164802)

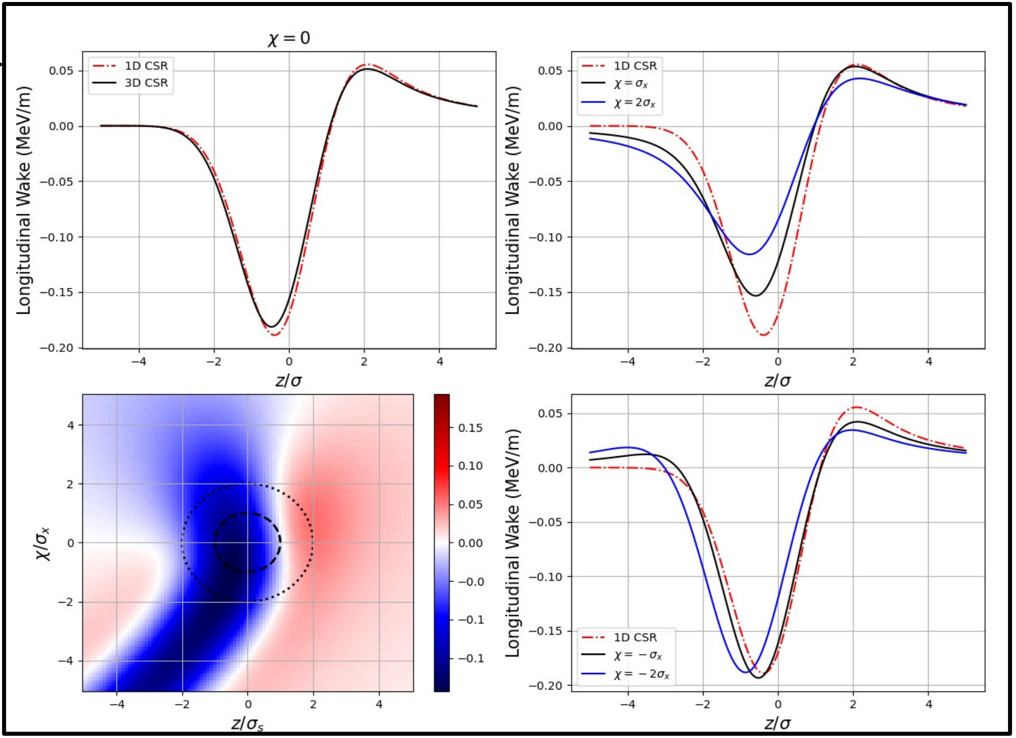




Why do we need 3D?



Dipole 3 in APS Bunch Compressor



Note the spike in transverse beam size, similar to the previous slide, which makes the horizontal variation in the longitudinal wake significant.

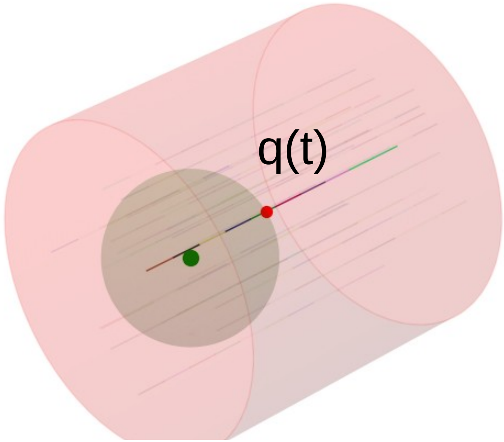
For more details please visit refer to our poster at T26

Modelling CSR in 3D

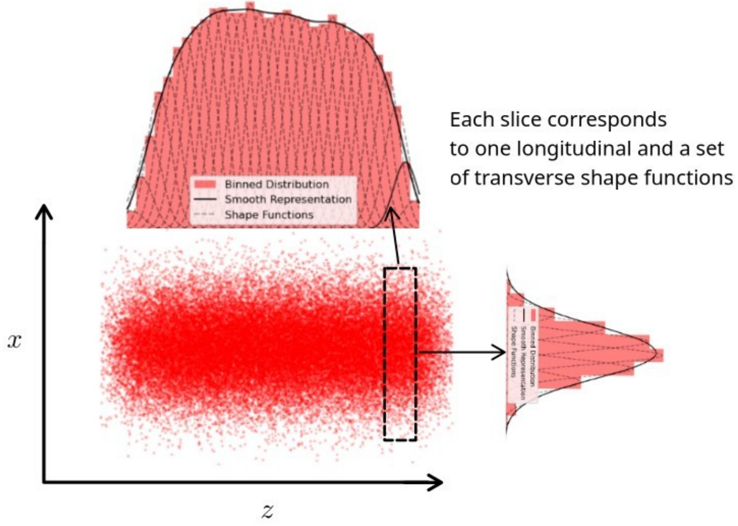
$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int d^3r' \left[\frac{\hat{\mathbf{R}}}{|\mathbf{R}|^2}\,\rho(\mathbf{r}',t_r) + \frac{\hat{\mathbf{R}}}{c|\mathbf{R}|}\,\dot{\rho}(\mathbf{r}',t_r) - \frac{1}{c^2|\mathbf{R}|}\,\dot{\mathbf{J}}(\mathbf{r}',t_r)\right] \tag{1}$$

$$\mathcal{E}(s) = \frac{-1}{4\pi\varepsilon_0}\int_0^{\alpha_{\max}} d\alpha \int dx' \int dy'\; \rho\big(s-\Delta s(\alpha,x',y'),\, x',y'\big)\,\frac{f\big(\mathbf{R}(\alpha,x',y'),\hat{\mathbf{n}}\big)}{|\mathbf{R}(\alpha,x',y')|} \tag{2}$$

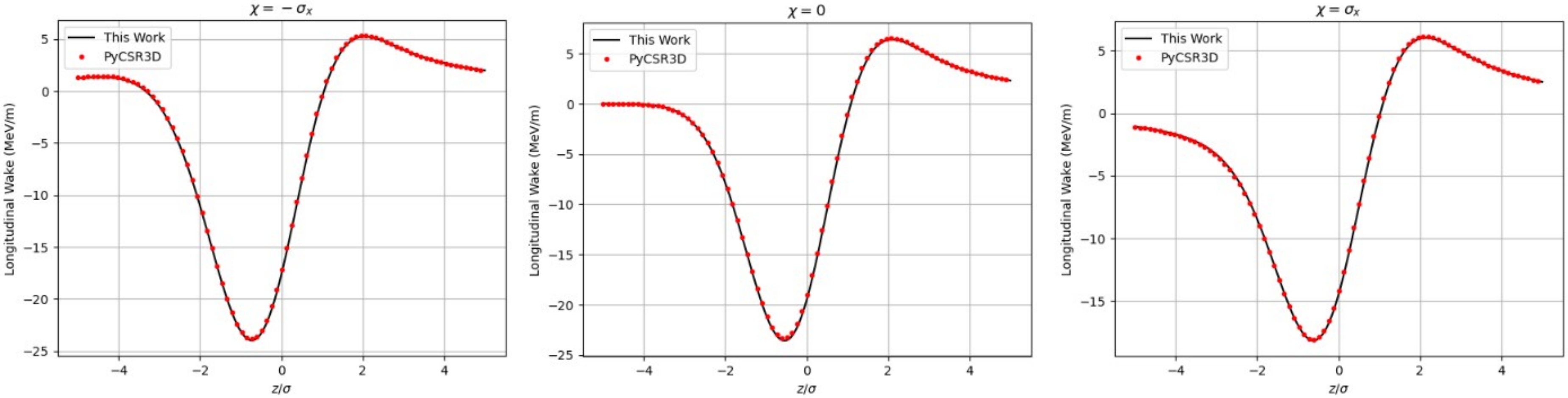
$$|\mathbf{R}|^2 = (x_0-x')^2 + (y_0-y')^2 + 4(R+x_0)(R+x')\sin^2\left(\frac{\alpha}{2}\right)$$



- The bunch is binned into a co-moving 3D voxel grid. (interpolated by Gaussians)
- Any particular grid cell follows a fixed straight or curvilinear path.
- The grid weights are used to calculate the E-field.
- There is a coordinate singularity at $|r-r'|\to 0$, but is avoided by switching to spherical coordinates.
- The space charge terms are left in the integrand.

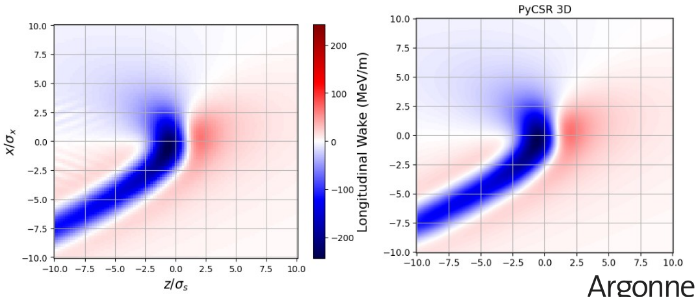


Comparisons against PyCSR3D

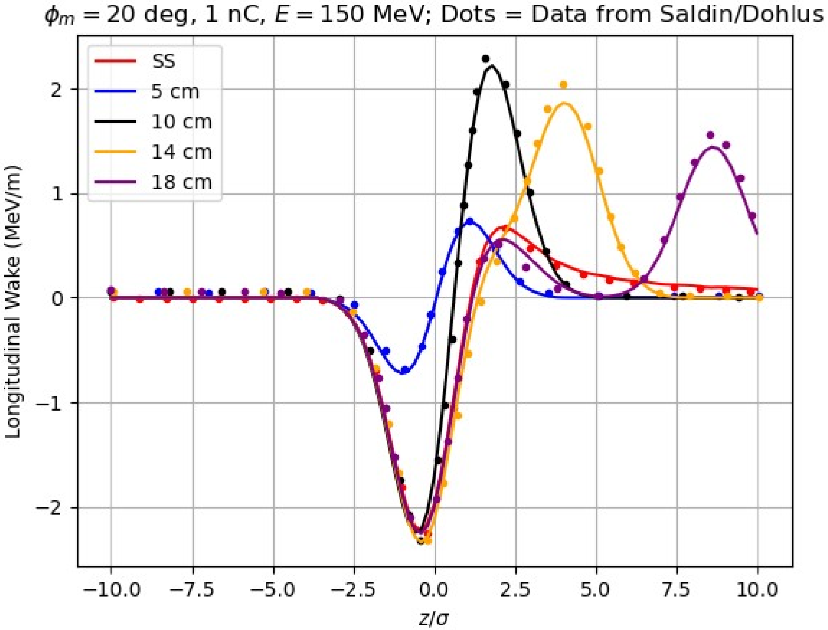


$$\sigma_x = 300\ \mu m \quad \sigma_x = 1\ \mu m \quad \sigma_z = 10\ \mu m$$

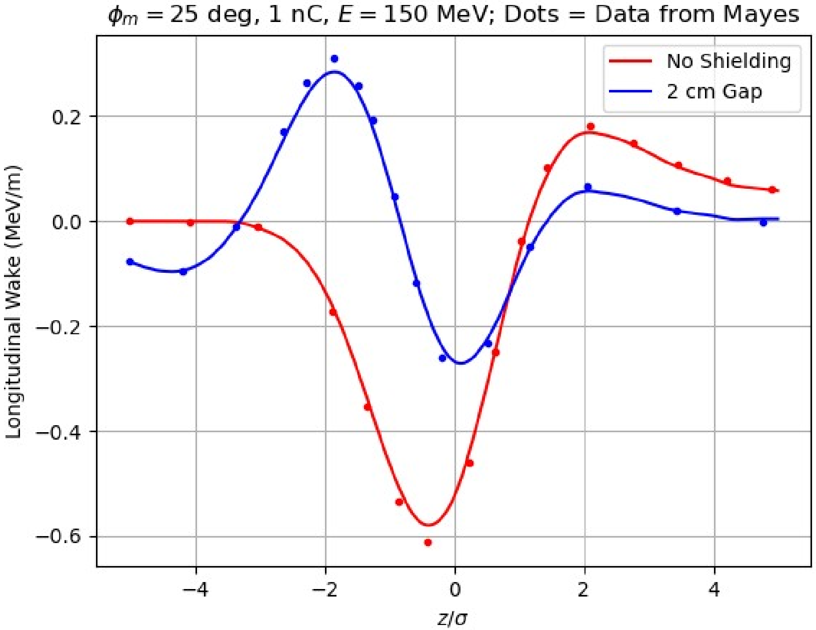
$$\kappa \approx 0.65$$



Analytical Validation at the 1D limit



Entry wakes are calculated using the radiation integrals in the preceeding drift space.

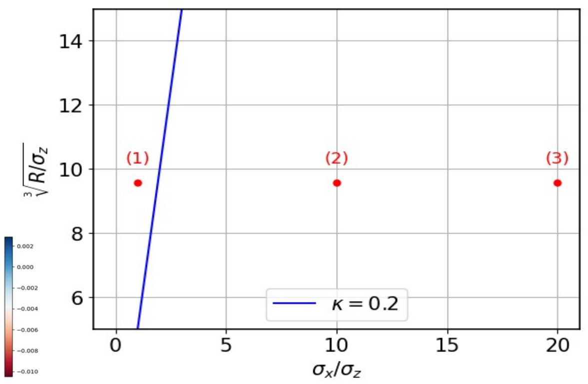


Shielding is accomplished with image charges. No regularization necessary.

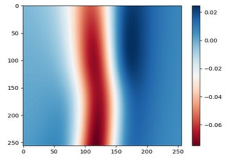
Analysis of the Derbenev Parameter

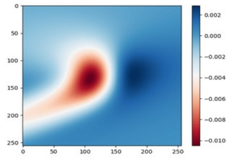
- The Derbenev criterion, $\kappa = \sigma_x/(R\sigma_z^2)^{1/3}$ is used as a benchmark to determine applicability of 1D CSR codes for a given beam/lattice condition.

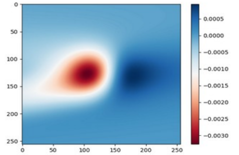
- Generally $\kappa \ll 1$ is considered to imply that a 1D code is sufficient to capture the radiative effects.

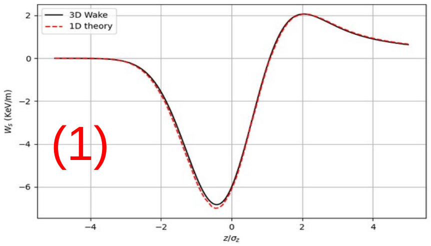


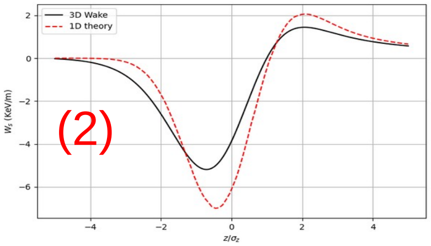
In the next few slides, we try to understand how the Derbenev parameter scales with different beam conditions

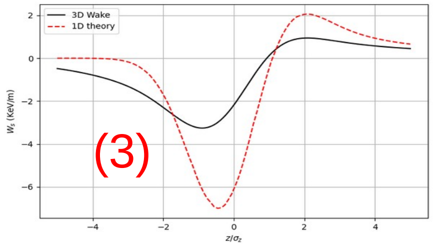




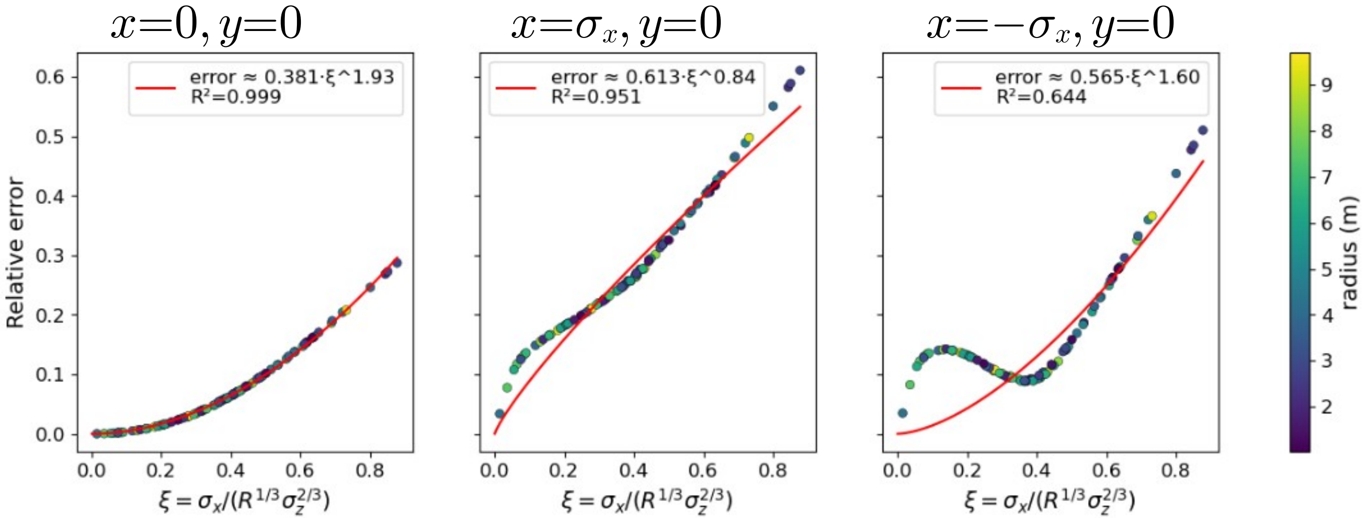






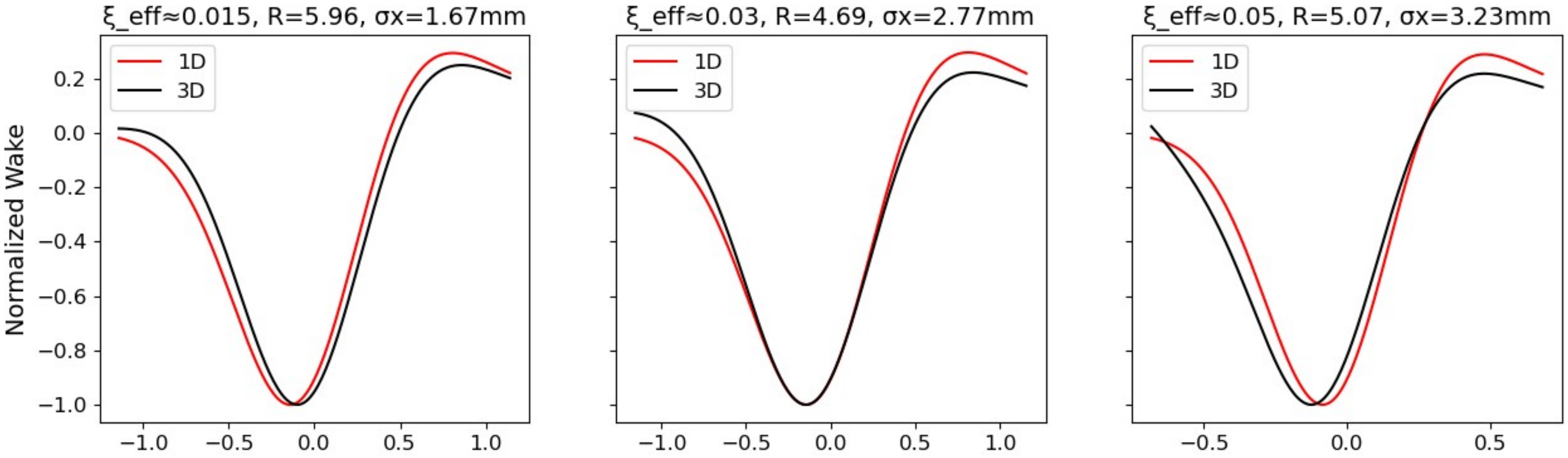


Analysis of the Derbenev Parameter



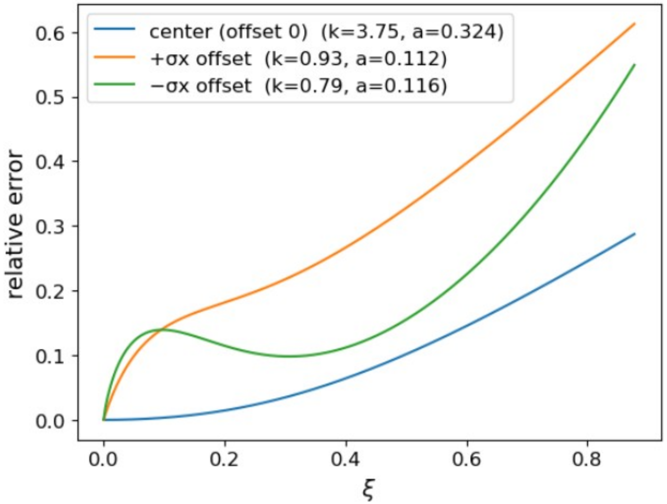
- If the Derbenev parameter is a good estimate of the error between 1D and 3D CSR, it should be possible to fit a power law for a range of R, σ_x, σ_z
- The data points were obtained using an agentic parameter search loop (poster T33)
- Through the centerline, Derbenev is spot-on.

Analysis of the Derbenev Parameter



- Off-axis, the criterion underestimates phase errors.

Analysis of the Derbenev Parameter

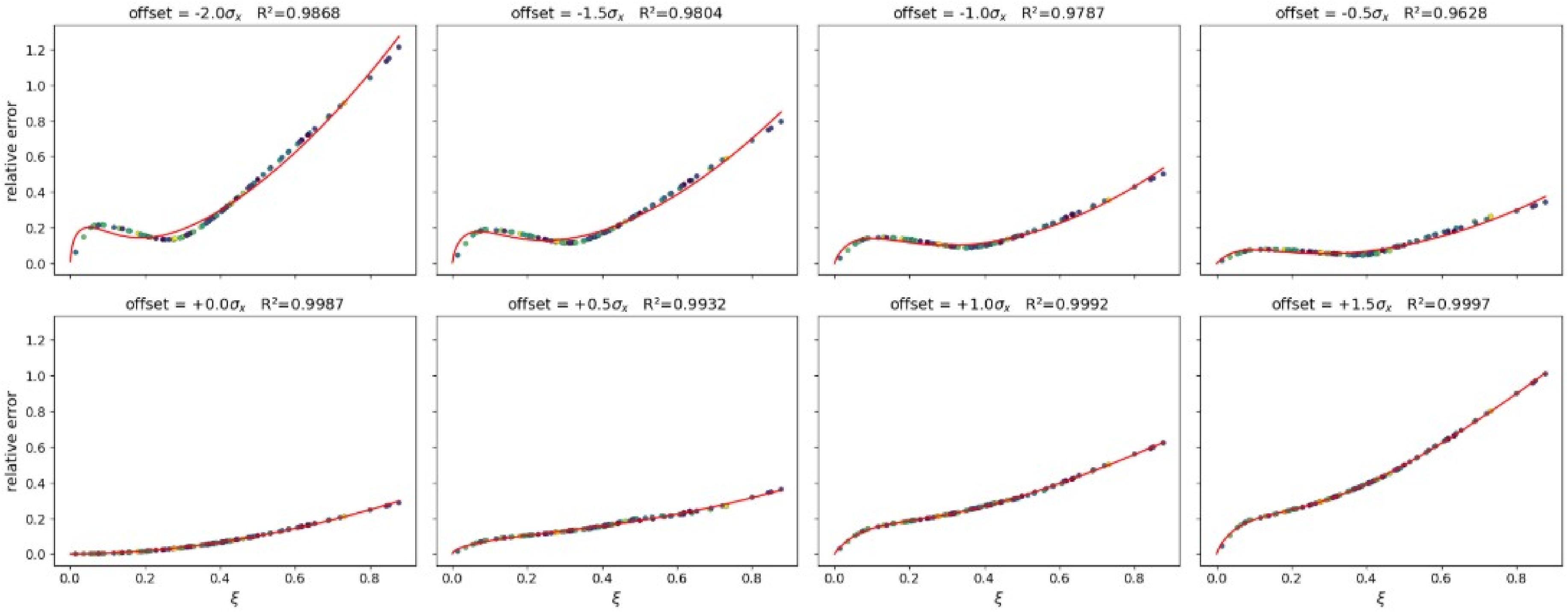


$$\varepsilon(\xi) = C_1\,\xi^k\,e^{-\xi/a} + C_2\,\xi^n$$

There are a few magic numbers here: C_1, C_2, k, n, a, that need to be reconciled with physical parameters, but it explains the trend.

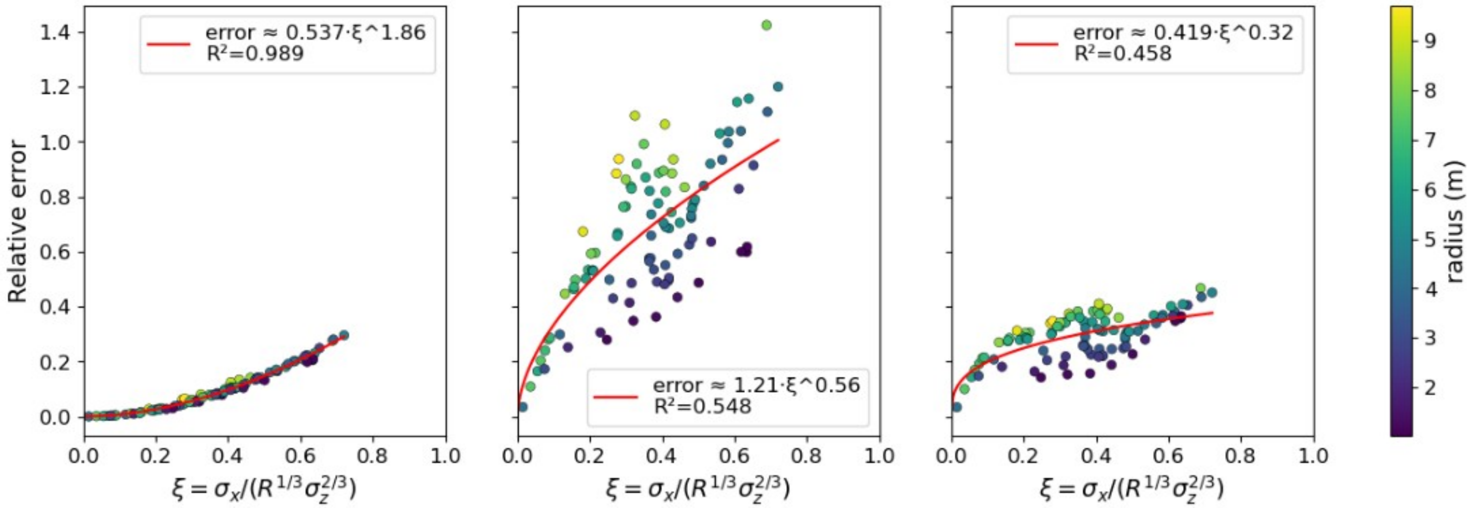
- What about a different scaling law?
- Both positive and negative offsets have that gamma function shaped phase error bump. Perhaps it's possible to add that to the fitting curve...

Analysis of the Derbenev Parameter



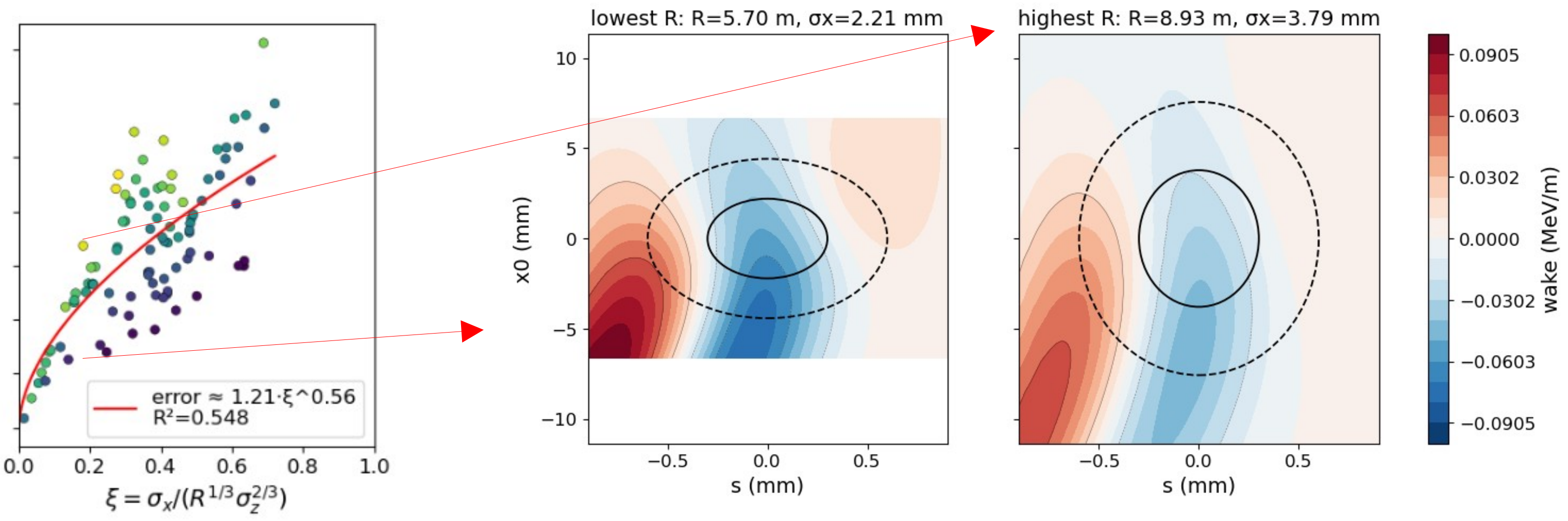
The gamma function guess seems to fit the error data at different offsets, but it still uses curve fits with magic numbers. My instinct is that the retardation condition has to factor in somehow to the value of these constants.

Derbenev Parameter and Shielding



- With shielding on, the story becomes more complicated.
- Through the centerline, Derbenev is still okay, but notice the spread of errors you can get for nominally the same κ

Derbenev Parameter and Shielding



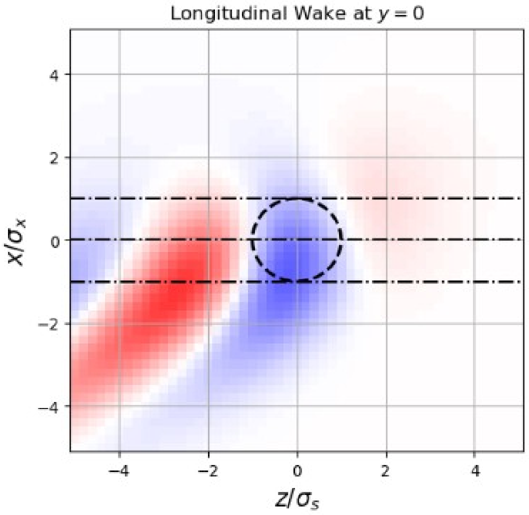
- The wake changes significantly (both in shape and in magnitude) for different bending radii at the same κ
- This is probably an indication that a different (non-Derbenev) power law base is governing things here.

Shielded CSR Wake on the y=0 plane

$$\kappa \approx 1.0$$

$$\Delta = 6\ \sigma_z$$

Total Shielded Wake



Physical Bunch Wake

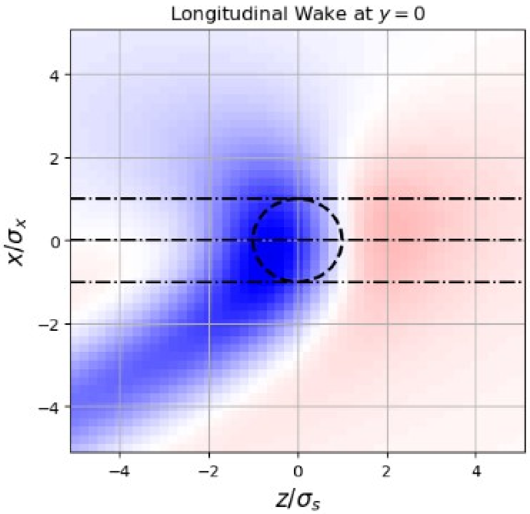
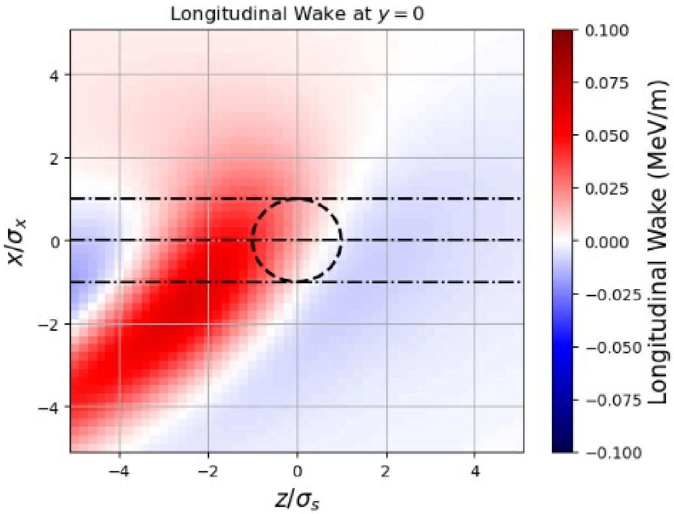


Image Bunch Wake

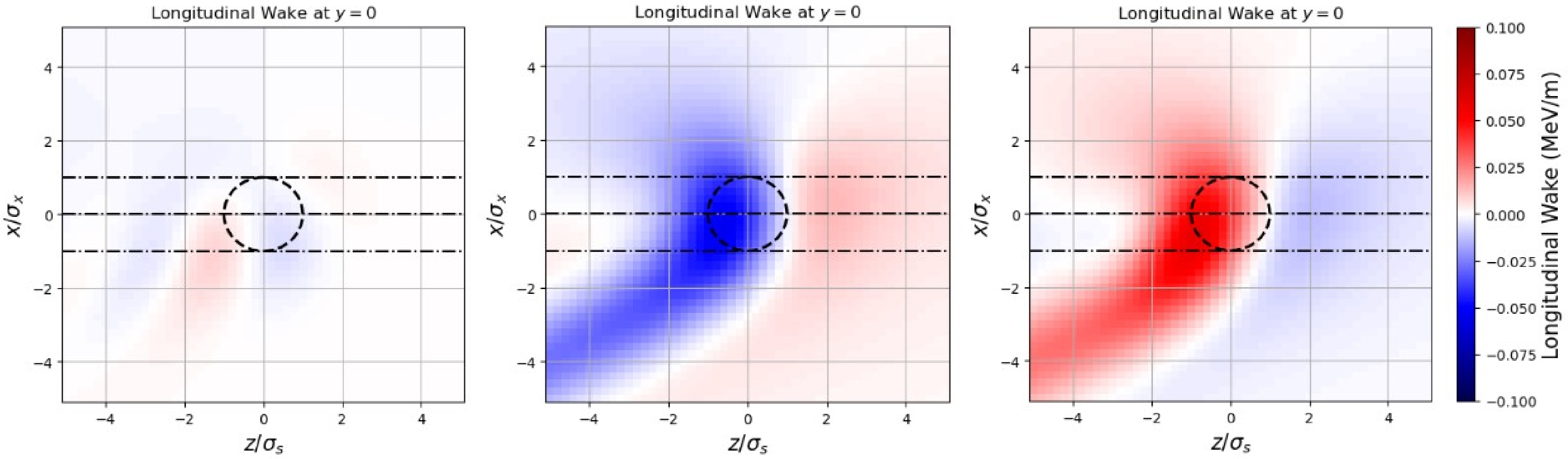


$$\kappa \approx 1.0$$

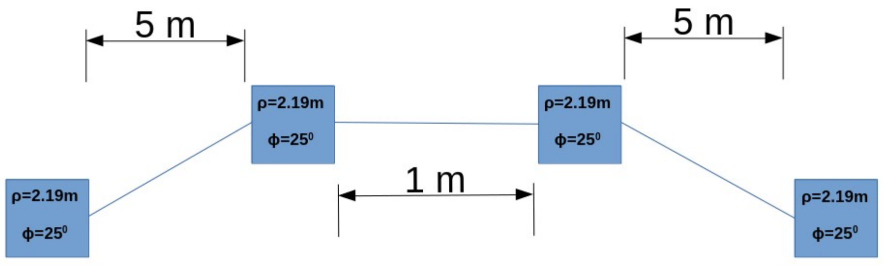
Shielded CSR Wake on the y=0 plane

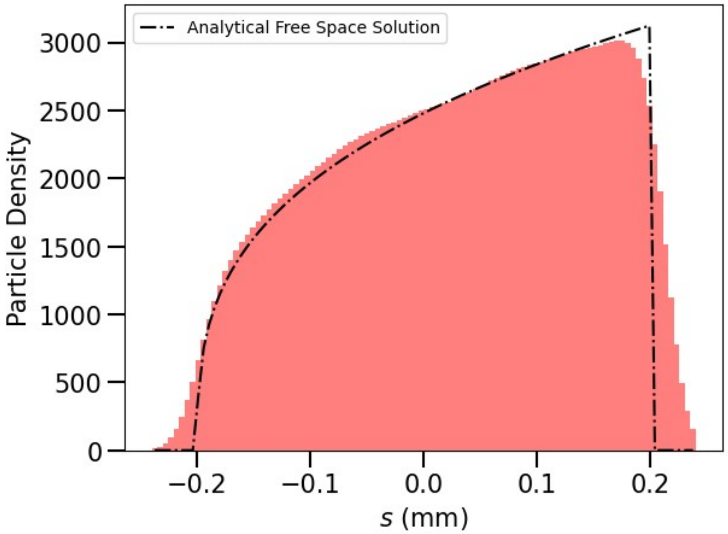
$$\Delta = 3\ \sigma_z$$

Total Shielded Wake | Physical Bunch Wake | Image Bunch Wake



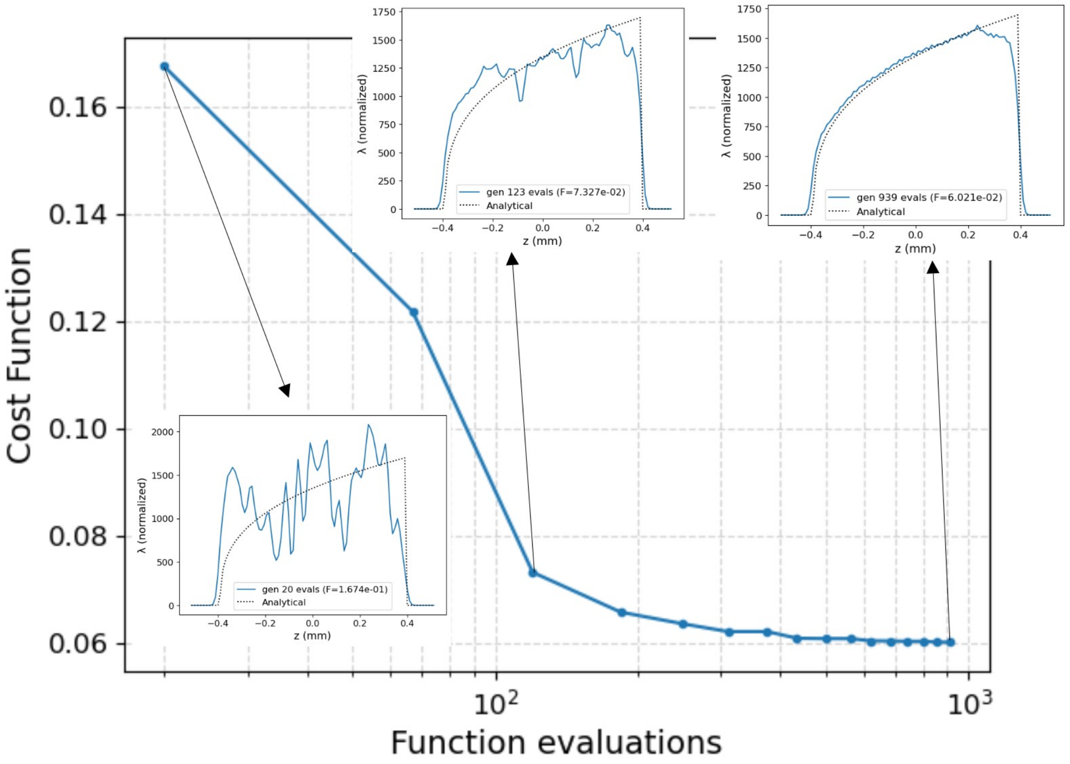
Profile Optimization to Mitigate CSR





- CSR in the last magnet can result in emittance growth in the downstream profile.
- Mitchell et al. 2013, proposed shaping the initial profile to mitigate CSR.
- We expanded his work by also considering shielding

Profile Optimization+Shielding to Mitigate CSR

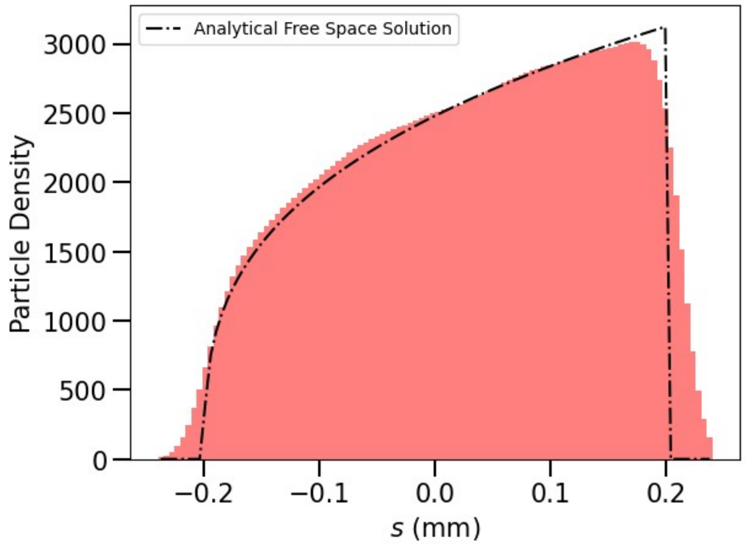


$$F(\mathbf{x}) = F_{\mathrm{wake}}(\mathbf{x}) + \alpha\, F_{\mathrm{rough}}(\mathbf{x}),$$

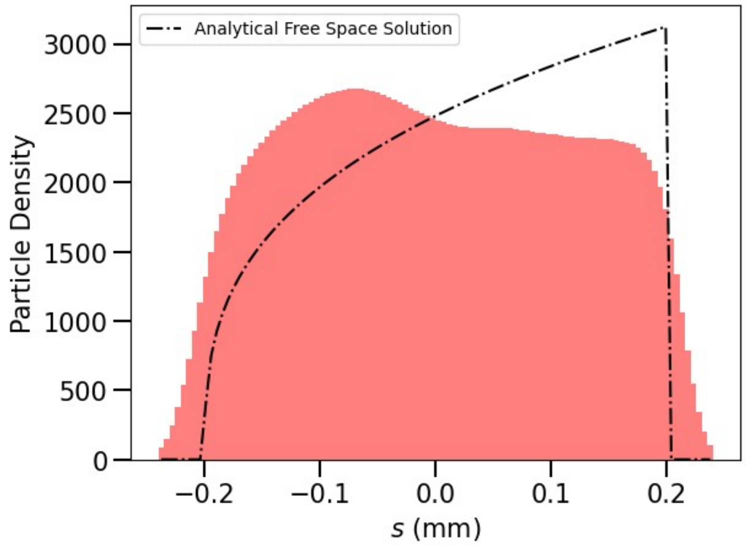
$$F_{\mathrm{wake}}(\mathbf{x}) = \sqrt{\iint \left[\Delta E(\mathbf{r}) - \langle \Delta E\rangle\right]^2 \rho(\mathbf{r})\, d\mathbf{r}},$$

$$F_{\mathrm{rough}}(\mathbf{x}) = \frac{1}{\max_z \lambda}\sqrt{\int \left[\frac{d^2\lambda(z;\mathbf{x})}{dz^2}\right]^2 dz}.$$

Profile Optimization to Mitigate CSR



No Shielding

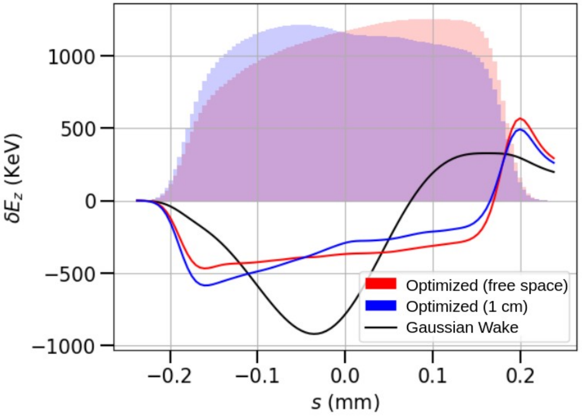


1 cm Gap

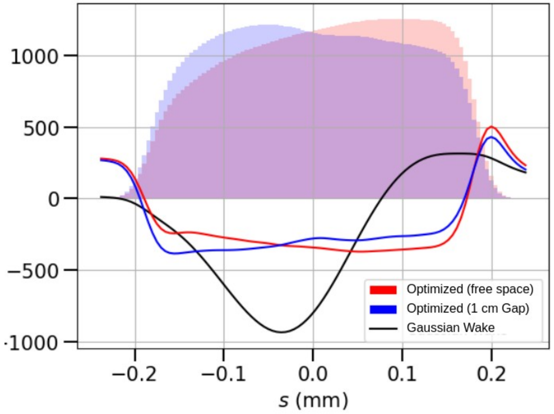
Analytical Soln for β=1: $\lambda(s) = \frac{4}{3}\frac{(s-a)^{1/3}}{(b-a)^{4/3}}$

Profile Optimization to Mitigate CSR

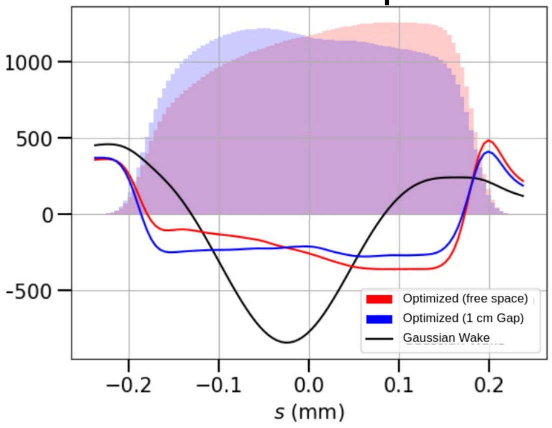
No Shielding



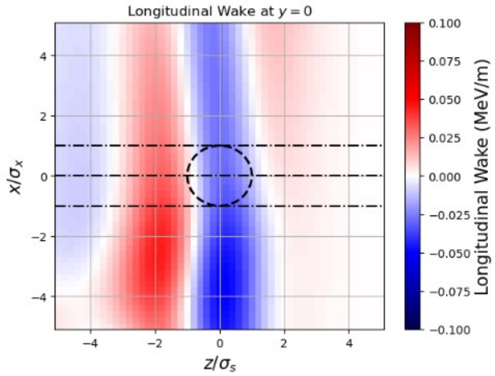
1 cm Gap



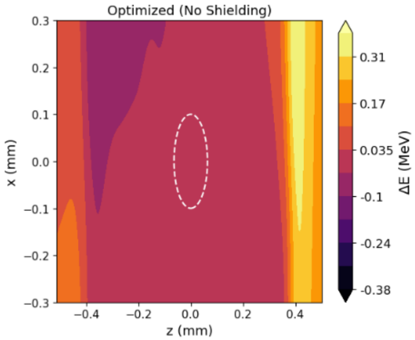
8 mm Gap



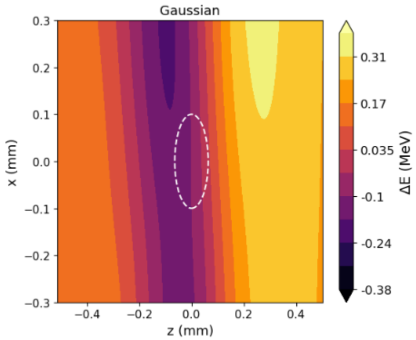
- In free space, the no shielding profile moves particles up front to flatten out the wake in the back.
- However, this compensation stops working with small gap sizes because the image charges are radiating to the back of the bunch.
- By pushing particles back, this overcompensation is corrected for.



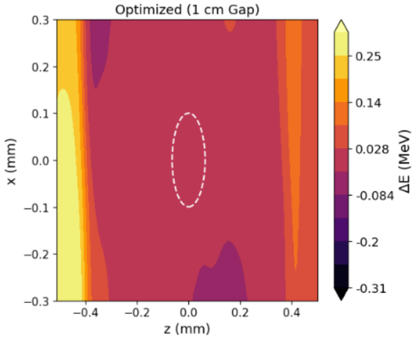
Longitudinal Wakes in 3D



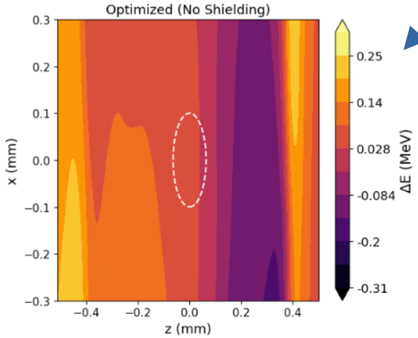
(a) Optimized, no shielding



(b) Gaussian, no shielding



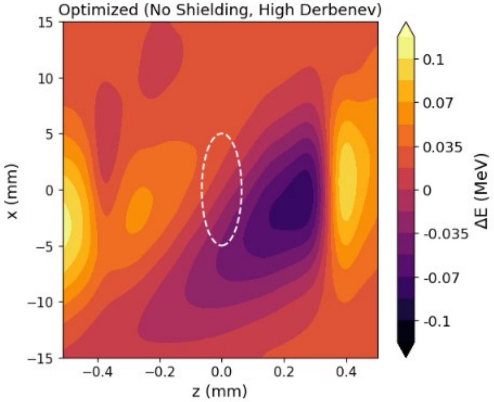
(c) Optimized, 1 cm shielding



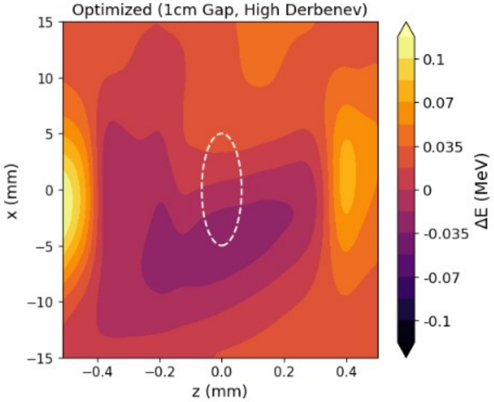
(d) Free-space optimal, 1 cm gap

- The optimized profile flattens the wake over a majority of the bunch.
- Shielding by itself doesn’t work due to the delayed offset in the image charge contributions, requiring a special profile to work well.

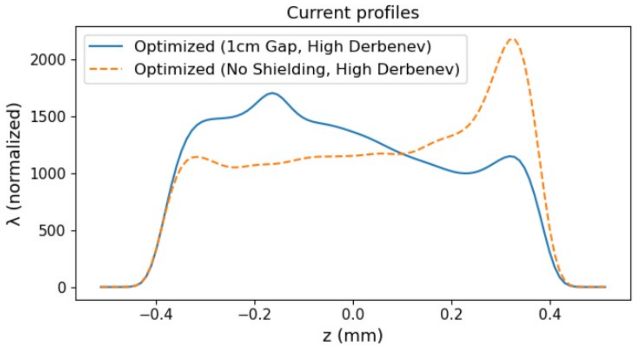
Profile Optimization for High Derbenev



(a) No shielding

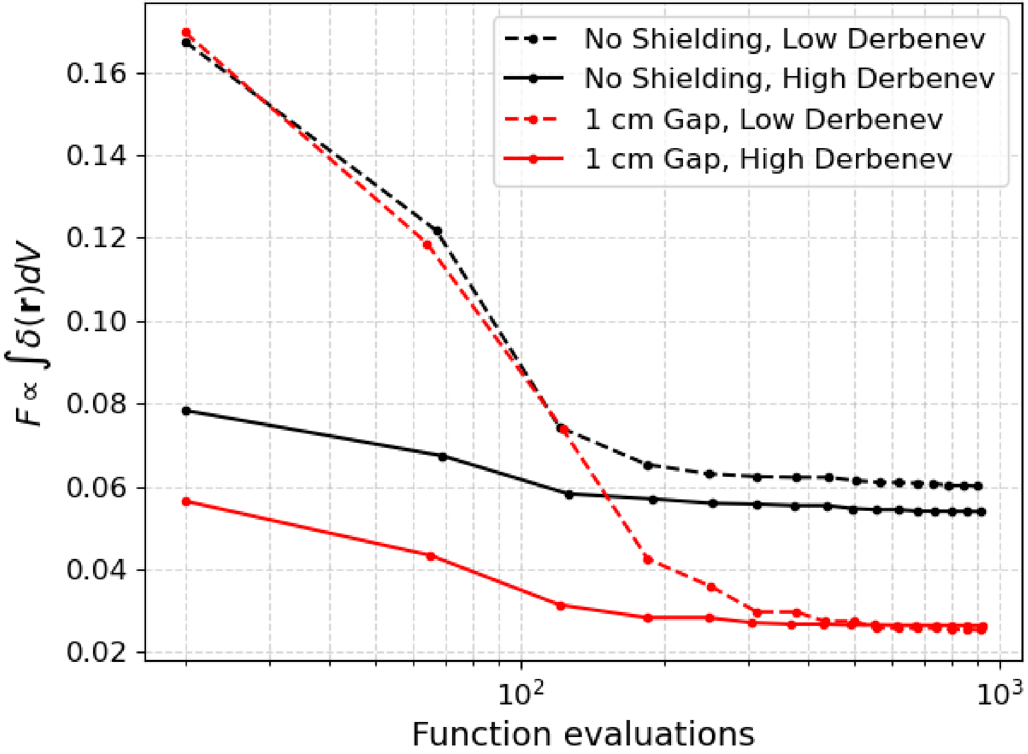


(b) 1 cm shielding gap



- However, for high Derbenev, the transverse variation is hard to compensate with just longitudinal shaping.
- However, since the image charges have high width, shielding+profile shaping does a better job.
- Maybe transverse shaping is another idea to try in the future?

Profile Optimization for High Derbenev



- Profile shaping + shielding does about as well as 1D with a 3D beam.
- Pure profile shaping is not very effective...

Conclusions

- In this work, we have looked into the characteristics of the CSR wake for different beam/bending magnet paremeters and shielding.
- In particular, we looked at the applicability of the Derbenev parameter as a metric to explain the discrepancy between 1D CSR theory and a full 3D calculation.
- Without shielding, the Derbenev parameter is generally accurate (particularly through the centerline of the beam), but does not capture phase errors off-axis.
- This can be fixed by adding a Gamma function to the power law, but several magic numbers need to be reconciled with physical parameters.
- With shielding a different power law base seems to be necessary.
- Combining profile shaping and shielding is effective at flattening the CSR wake, even at relatively large wall separations.
- This work is part of a larger effort (theoretical and experimental) at the AWA to better understand the effects of CSR in complex beams.

Bibliography

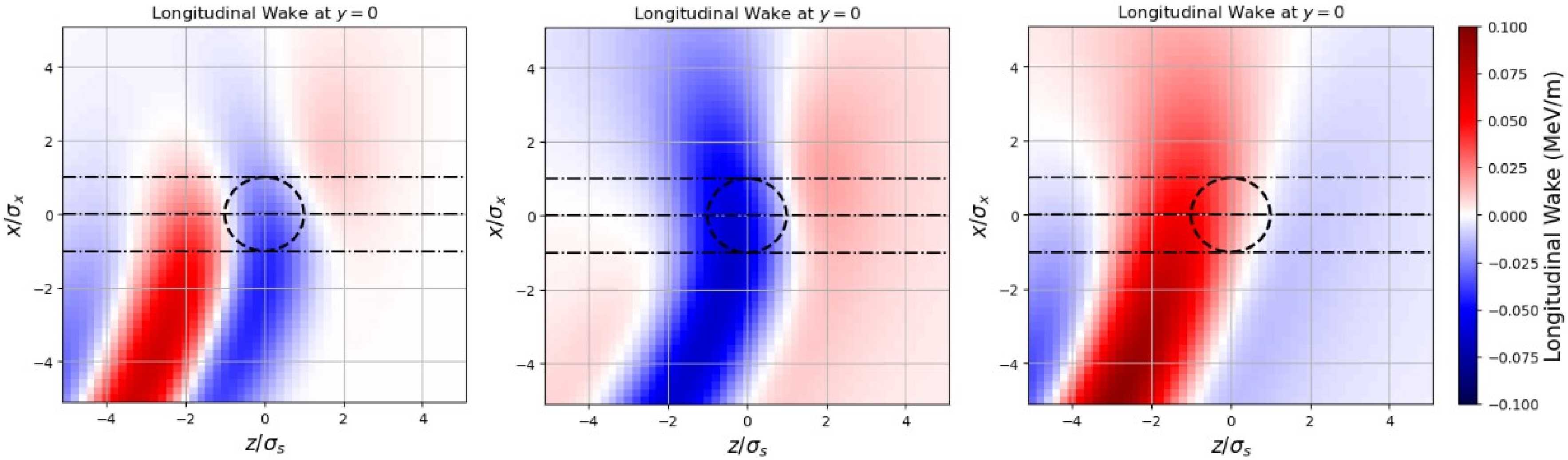
- V. Yakimenko, et al., “Experimental observation of suppression of coherent-synchrotron-radiation–induced beam-energy spread with shielding plates”, Phys. Rev. Lett., vol. 109, no. 16, p. 164 802, 2012
- C. Mayes and G. Hoffstaetter, “Exact 1D model for coherent synchrotron radiation with shielding and bunch compression”, Phys. Rev. ST Accel. Beams, vol. 12, p. 024 401, 2009
- E.L. Saldin, E.A. Schneidmiller, and M.V. Yurkov. “Radiative interaction of electrons in a bunch moving in an undulator.” NIM Phys. Res., Sect. A, 417(1):158–168, 1998
- Ya. S. Derbenev, J. Rossbach, E. L. Saldin, and V. D. Shiltsev. “Microbunch radiative tail-head interaction.” Technical report, 1995.
- Chad Mitchell, Ji Qiang, and Paul Emma. “Longitudinal pulse shaping for the suppression of coherent synchrotron radiation-induced emittance growth.” Phys. Rev. Accel. Beams, 16:060703, Jun 2013.

Backup

Shielded CSR Wake on the y=0 plane

$$\kappa \approx 0.2$$
$$\Delta = 6\ \sigma_z$$

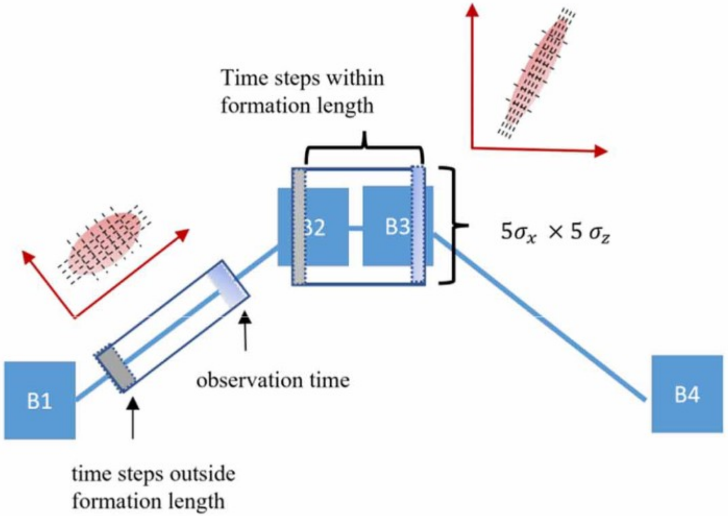
Total Shielded Wake | Physical Bunch Wake | Image Bunch Wake



2/3D Time-Domain CSR Methods

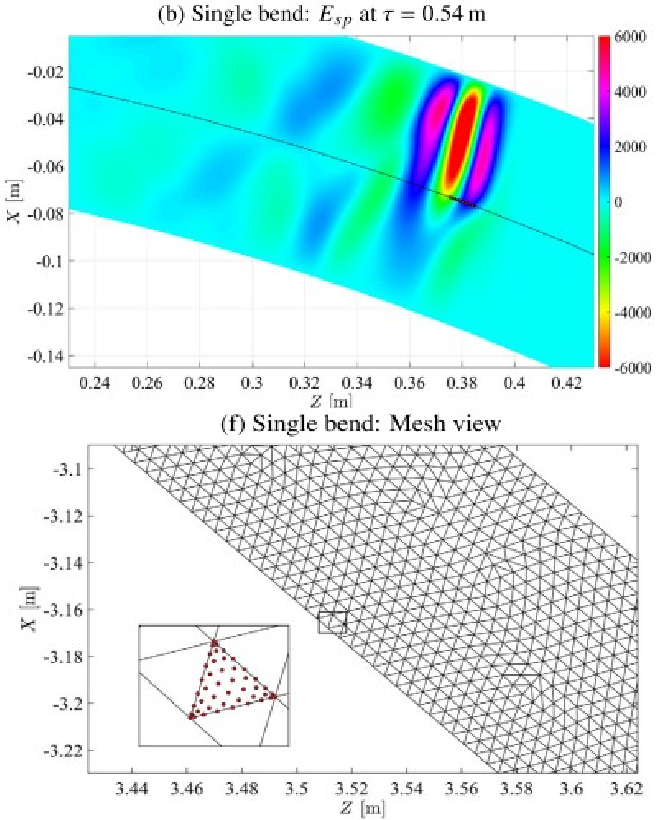
J. Tang, G. Stukapov, Z. Huang, FLS 2023

- Another interesting idea in the literature is to mesh the entire lattice.
- Now, the retardation condition can be done as a forward calculation based on the time-history of the grid.
- Entry/exit wakes come for free.
- Shielding is probably tricky to implement? (Jingyi Tang's thesis has a chapter on shielding, but as far as I know DFCSR does not implement it...)



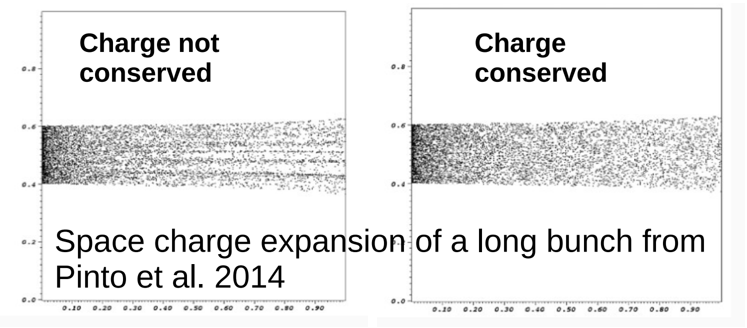
Mesh Based Field Solvers

Bizzozero et al. 2019



- Some groups have tried discontinuous Galerkin methods as well, but usually these methods tend to have many gotchas (charge conservation, grid dispersion etc.).

- But, you get shielding (by any wall shape) for free.



Space charge expansion of a long bunch from Pinto et al. 2014

A Survey of CSR Simulation Techniques

Code	Type	Shielding
LW3D	3D (Lienard-Wiechert)	No
Elegant/IMPACT/ OCELOT	1D (Analytical Wake)	No
GPT	1D (Analytical Wake)	Yes
CSRTrack	1D/2D (IGF)	Yes
CoSyR	2D (Lienard-Wiechert)	No
PyCSR2D/3D	2D/3D (IGF, Wake only)	No
PyDFCSR	2D (Uses a global mesh)	No

"Analytical Wake": The code reduces the bunch to a 1D line charge. You can then analytically write down the longitudinal/transverse forces.

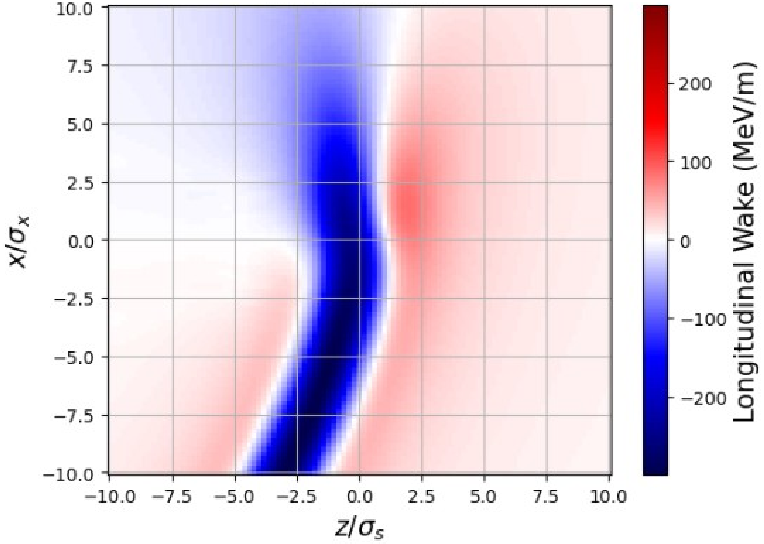
"Lienard-Wiechert": Self-consistently calculates the E/B-fields for a collection of point particles and uses a PIC style method to evolve the bunch.

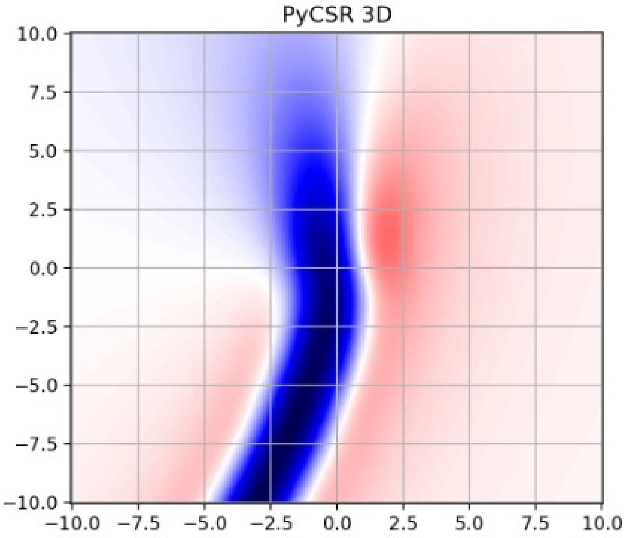
"IGF": Integrated Green's function, similar to 1D wake, but the convolution is done numerically.

There's no code that we're aware of that does full 3D CSR and accounts for shielding

3D Comparisons against PyCSR3D

This Work

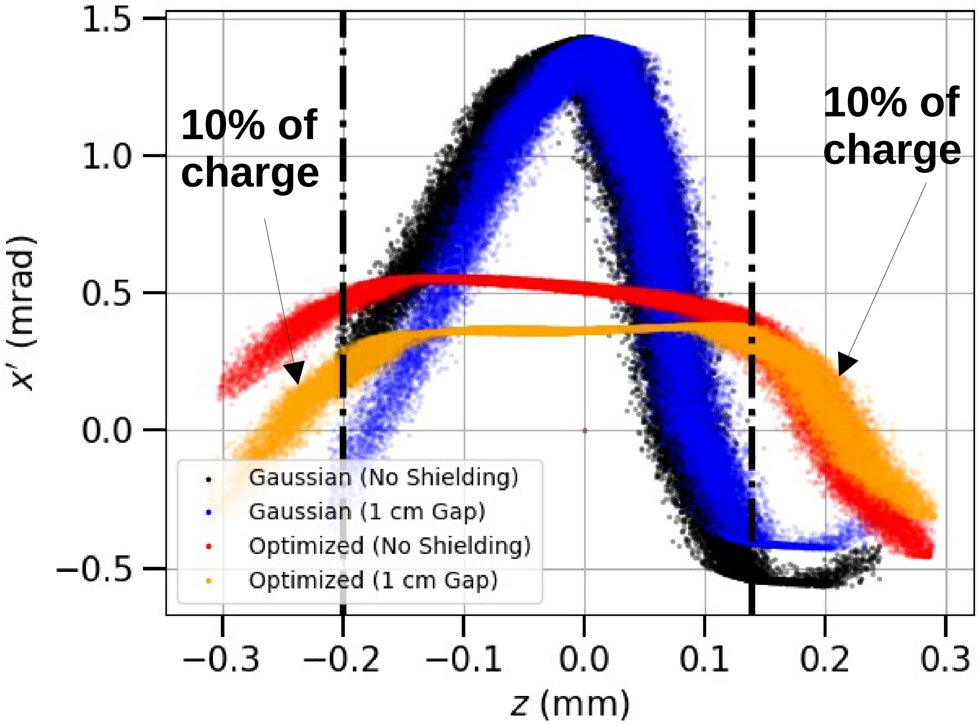




$$\sigma_x = 100\ \mu m \quad \sigma_x = 1\ \mu m \quad \sigma_z = 10\ \mu m$$

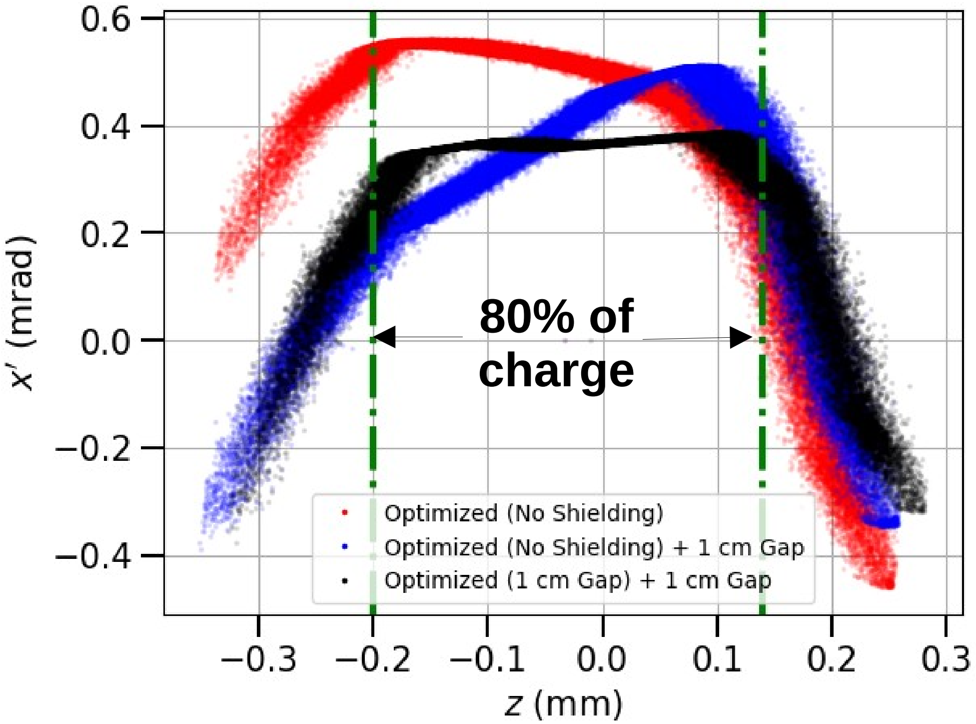
$$\kappa \approx 0.2$$

Phase Space Comparisons



Profile (Shld. config)	Projected Emittance (μm)
Gaussian (No Shielding)	5.65
Gaussian (1cm Gap)	5.50
Optimized for Free Space (No Shielding)	2.24
Optimized for 1 cm Gap (1cm Gap)	1.51

Phase Space Comparisons



Profile (Shld. config)	Projected Emittance (μm)
Optimized for Free Space (No Shielding)	2.24
Optimized for Free Space (1cm Gap)	1.93
Optimized for 1 cm Gap (1cm Gap)	1.51

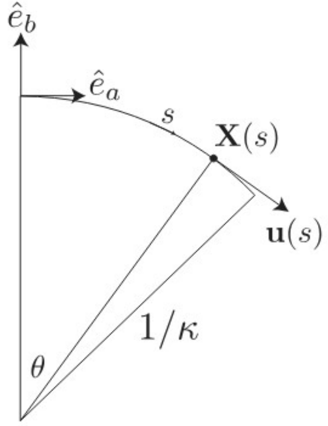
1D Time-Domain CSR Methods

(From Mayes and Hoffstaeter, 2009)

$$\mathbf{E}(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\int d^3x' \Big[\frac{\mathbf{r}}{r^3}\rho(\mathbf{x}',t') + \frac{\mathbf{r}}{cr^2}\partial_{t'}\rho(\mathbf{x}',t') - \frac{1}{c^2 r}\partial_{t'}\mathbf{J}(\mathbf{x}',t')\Big]_{t'=t-r/c},$$

$$\rho(s,t) = Q\lambda(s - s_b - \beta ct),$$

$$\mathbf{J}(s,t) = Q\beta c\mathbf{u}(s)\lambda(s - s_b - \beta ct),$$



$$\frac{d\mathcal{E}}{ds}(s)\Big|_B = Nr_c mc^2\Big\{\frac{-\kappa\lambda(s_\alpha)}{\sqrt{2-2\cos\alpha}}\Big|_{\alpha=-(\kappa B-\theta)}^{\alpha=\theta} + \int_{-(\kappa B-\theta)}^{\theta} d\alpha\, \frac{\beta^2\cos(\alpha)-1}{\sqrt{2-2\cos\alpha}}\lambda'(s_\alpha)\Big\}.$$

These three equations encode the so-called ‘retardation’ condition

$$\alpha \equiv \theta - \theta',$$

$$s_\alpha \equiv \frac{1}{\kappa}(\theta - \theta_0 - \alpha) + \beta r_\alpha,$$

$$r_\alpha \equiv \frac{1}{\kappa}\sqrt{2-2\cos\alpha}.$$

(This needs to be regularized to remove space charge coordinate singularity)