

Magnetic sextupole spectrometers

B. Naranjo, G. Andonian, A. Fukasawa, Y. Kang, P. Manwani, J. Phillips, J. Rozells,
O. Williams, J. Rosenzweig

UCLA Dept. of Physics & Astronomy

M. Yadav

Old Dominion University School of Data Science

P. Alva Rosa

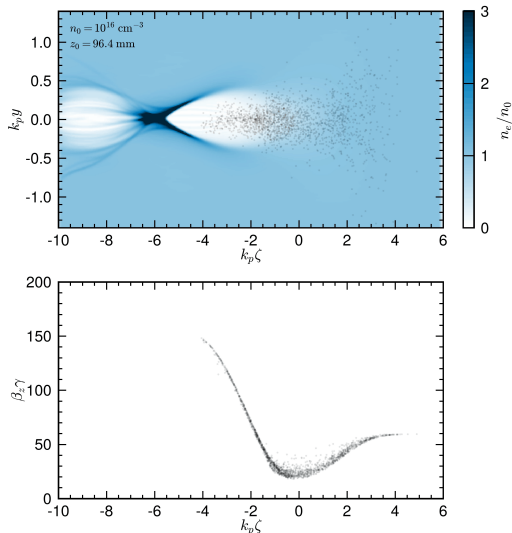
Augsburg University

C. Hast, I. Rajkovic, D. Storey

SLAC National Accelerator Laboratory

AAC 2026
Los Angeles, 2026 July 27

Motivation for broadband electron spectrometer



- ▶ Long 30 MeV driver beam run to nearly full depletion in PWFA at MITHRA.
- ▶ Want to record 1-100 MeV in a single shot to fully capture depleted head and accelerated tail.
- ▶ Repurpose FACET-II CPT sextupole design.

Multipole field

For magnetostatic current-free quasi-2D regions,

$$\left. \begin{array}{l} \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = 0 \end{array} \right\} \implies \left. \begin{array}{l} \mathbf{B} = \nabla \times \mathbf{A} \\ \mathbf{B} = -\nabla \phi \end{array} \right\} \implies \begin{array}{l} B_x = -\frac{\partial \phi}{\partial x} = \frac{\partial A}{\partial y} \\ B_y = -\frac{\partial \phi}{\partial y} = -\frac{\partial A}{\partial x} \end{array}$$

With $z = x + iy$, we may write the complex potential as

$$w(z) = -A(z) - i\phi(z).$$

Then, the Cauchy-Riemann equations give the fields

$$\frac{dw}{dz} = B_y + iB_x.$$

For a multipole of order k , the complex potential is a **homogeneous function** of degree k ,

$$w(z) = \frac{b_k z^k}{k} \implies \phi = -\frac{b_k}{k} r^k \cos(k\theta), \text{ etc.}$$

A relativistic particle of charge q and mass m obeys Lorentz force law

$$\frac{d^2 x^\mu}{d\tau^2} = \frac{q}{m} F^{\mu\nu} \frac{dx_\nu}{d\tau}$$

which follows from a Lagrangian of the form

$$L = -\frac{1}{2} m u^\mu u_\mu - q u^\mu A_\mu.$$

Consider a 4-potential that, for some choice of gauge, is a homogeneous function of degree k ,

$$A^\nu(\alpha x) = \alpha^k A^\nu(x).$$

Given a worldline solution $x^\mu(\tau)$, propose a new solution $\tilde{x}^\mu(\tilde{\tau})$ obtained by scaling

$$\tilde{x}^\mu = \alpha x^\mu \quad \text{and} \quad \tilde{\tau} = \alpha^{1-k} \tau.$$

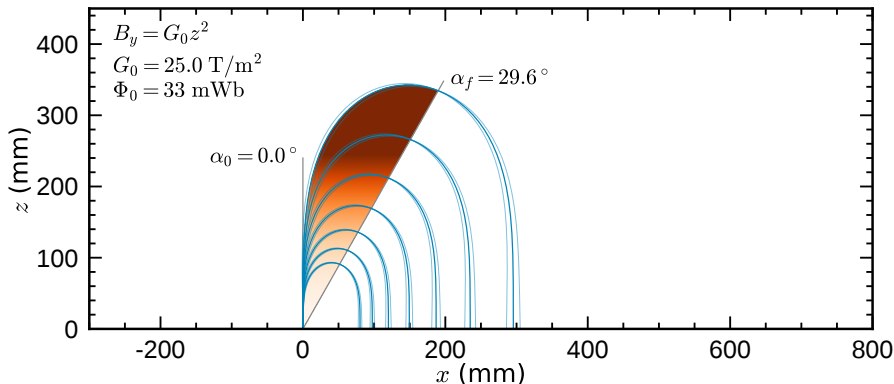
Under this coordinate transformation, the overall Lagrangian scales as α^{2k} . Because multiplying the Lagrangian by an overall constant does not affect the equations of motion, we conclude that $\tilde{x}^\mu(\tilde{\tau})$ is a solution to the original equations of motion.

We thus find **mechanically similar** solutions with

$\text{length} \propto \text{momentum}^{1/k}$

Sextupole field

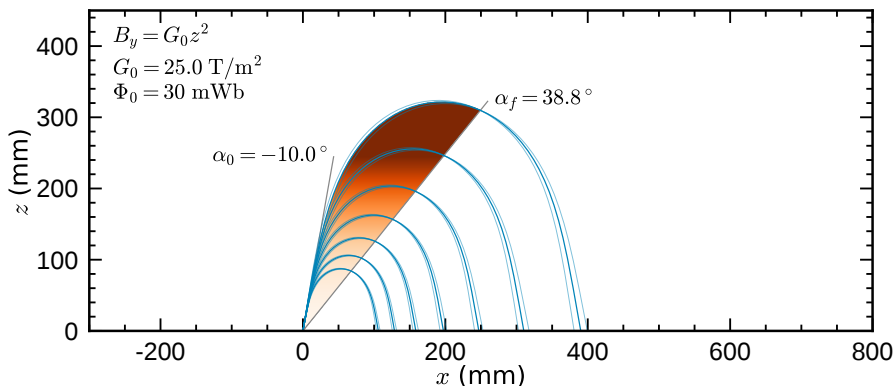
Taking a 2D sextupole ($k = 3$) potential gives similar trajectories whose lengths scale with their cube roots of momenta. In other words, scaling a trajectory's momentum by a factor of eight only scales its path length by a factor of 2.



- ▶ Trajectory energies: [100, 50, 25, 12.5, 6.25, 3.125, 1.5625] MeV
- ▶ Gap field strength is 1 Tesla at $z = 200 \text{ mm}$
- ▶ Magnetic design constraints: 1) Flux budget and 2) Peak field strength
- ▶ By mechanical similarity, the focal point of each trajectory family lies in the same plane, where we then locate the scintillator.

Sextupole field

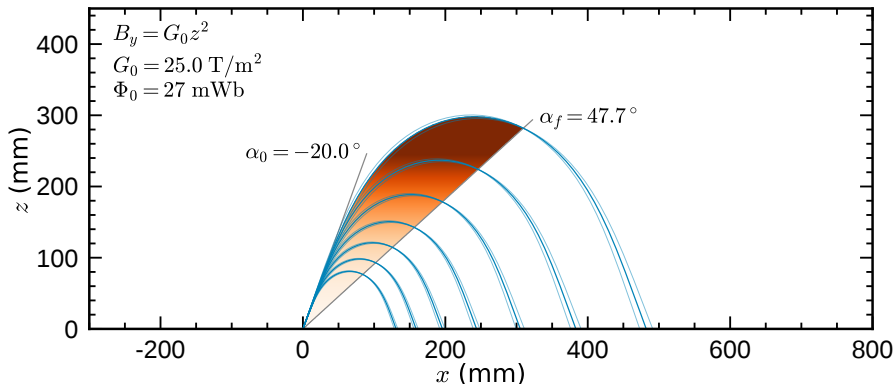
Taking a 2D sextupole ($k = 3$) potential gives similar trajectories whose lengths scale with their cube roots of momenta. In other words, scaling a trajectory's momentum by a factor of eight only scales its path length by a factor of 2.



- ▶ Trajectory energies: [100, 50, 25, 12.5, 6.25, 3.125, 1.5625] MeV
- ▶ Gap field strength is 1 Tesla at $z = 200 \text{ mm}$
- ▶ Magnetic design constraints: 1) Flux budget and 2) Peak field strength
- ▶ By mechanical similarity, the focal point of each trajectory family lies in the same plane, where we then locate the scintillator.

Sextupole field

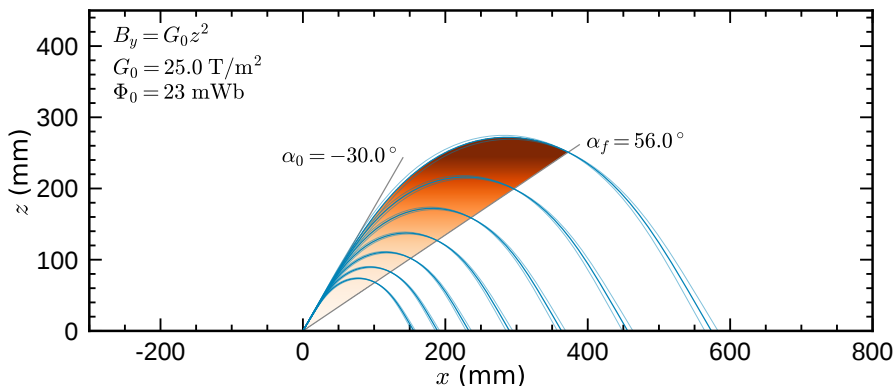
Taking a 2D sextupole ($k = 3$) potential gives similar trajectories whose lengths scale with their cube roots of momenta. In other words, scaling a trajectory's momentum by a factor of eight only scales its path length by a factor of 2.



- ▶ Trajectory energies: [100, 50, 25, 12.5, 6.25, 3.125, 1.5625] MeV
- ▶ Gap field strength is 1 Tesla at $z = 200 \text{ mm}$
- ▶ Magnetic design constraints: 1) Flux budget and 2) Peak field strength
- ▶ By mechanical similarity, the focal point of each trajectory family lies in the same plane, where we then locate the scintillator.

Sextupole field

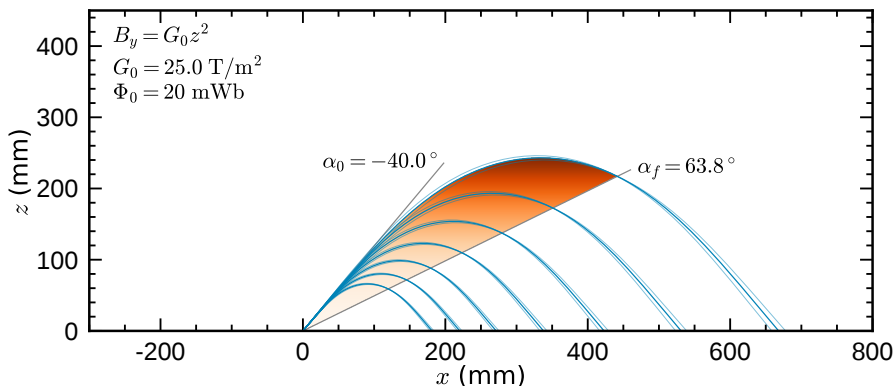
Taking a 2D sextupole ($k = 3$) potential gives similar trajectories whose lengths scale with their cube roots of momenta. In other words, scaling a trajectory's momentum by a factor of eight only scales its path length by a factor of 2.



- ▶ Trajectory energies: [100, 50, 25, 12.5, 6.25, 3.125, 1.5625] MeV
- ▶ Gap field strength is 1 Tesla at $z = 200 \text{ mm}$
- ▶ Magnetic design constraints: 1) Flux budget and 2) Peak field strength
- ▶ By mechanical similarity, the focal point of each trajectory family lies in the same plane, where we then locate the scintillator.

Sextupole field

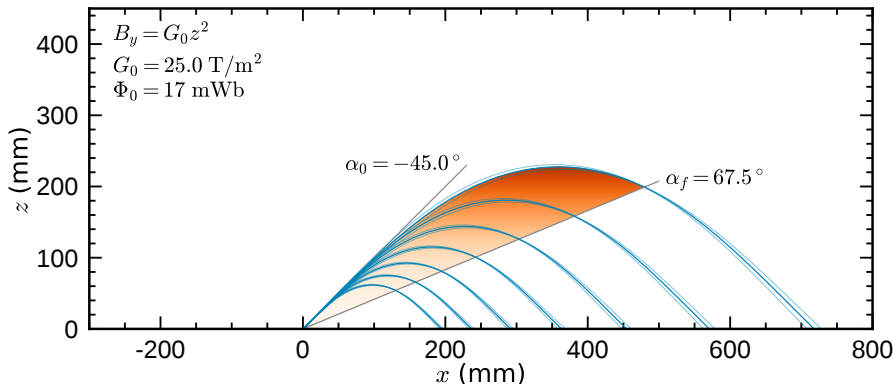
Taking a 2D sextupole ($k = 3$) potential gives similar trajectories whose lengths scale with their cube roots of momenta. In other words, scaling a trajectory's momentum by a factor of eight only scales its path length by a factor of 2.



- ▶ Trajectory energies: [100, 50, 25, 12.5, 6.25, 3.125, 1.5625] MeV
- ▶ Gap field strength is 1 Tesla at $z = 200 \text{ mm}$
- ▶ Magnetic design constraints: 1) Flux budget and 2) Peak field strength
- ▶ By mechanical similarity, the focal point of each trajectory family lies in the same plane, where we then locate the scintillator.

Sextupole field

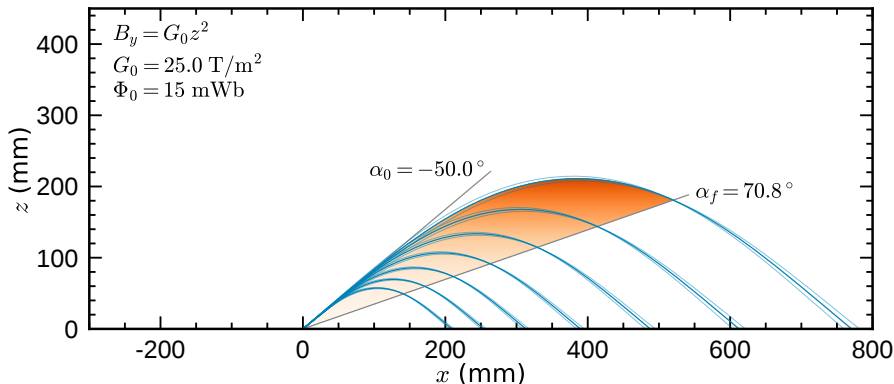
Taking a 2D sextupole ($k = 3$) potential gives similar trajectories whose lengths scale with their cube roots of momenta. In other words, scaling a trajectory's momentum by a factor of eight only scales its path length by a factor of 2.



- ▶ Trajectory energies: [100, 50, 25, 12.5, 6.25, 3.125, 1.5625] MeV
- ▶ Gap field strength is 1 Tesla at $z = 200 \text{ mm}$
- ▶ Magnetic design constraints: 1) Flux budget and 2) Peak field strength
- ▶ By mechanical similarity, the focal point of each trajectory family lies in the same plane, where we then locate the scintillator.

Sextupole field

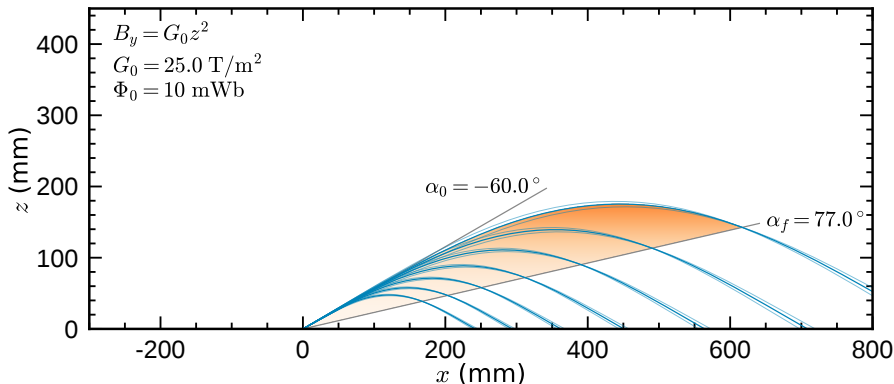
Taking a 2D sextupole ($k = 3$) potential gives similar trajectories whose lengths scale with their cube roots of momenta. In other words, scaling a trajectory's momentum by a factor of eight only scales its path length by a factor of 2.



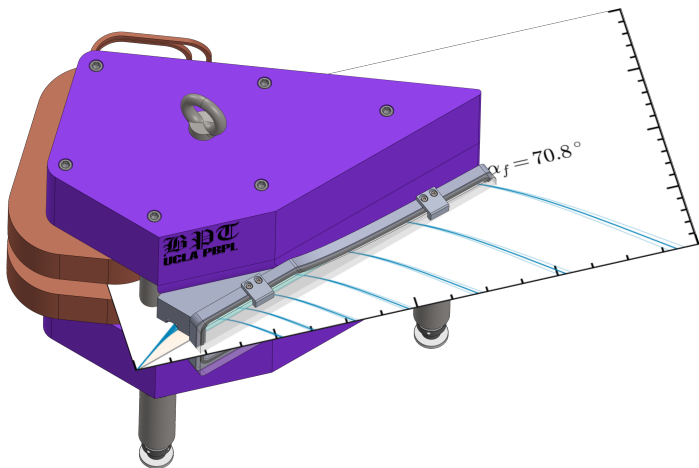
- ▶ Trajectory energies: [100, 50, 25, 12.5, 6.25, 3.125, 1.5625] MeV
- ▶ Gap field strength is 1 Tesla at $z = 200 \text{ mm}$
- ▶ Magnetic design constraints: 1) Flux budget and 2) Peak field strength
- ▶ By mechanical similarity, the focal point of each trajectory family lies in the same plane, where we then locate the scintillator.

Sextupole field

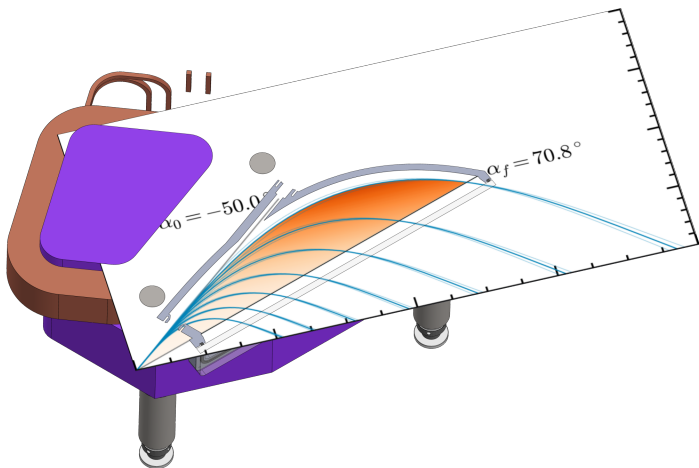
Taking a 2D sextupole ($k = 3$) potential gives similar trajectories whose lengths scale with their cube roots of momenta. In other words, scaling a trajectory's momentum by a factor of eight only scales its path length by a factor of 2.



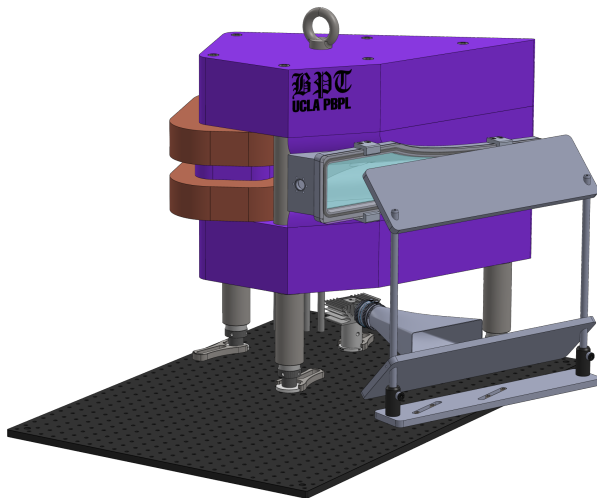
- ▶ Trajectory energies: [100, 50, 25, 12.5, 6.25, 3.125, 1.5625] MeV
- ▶ Gap field strength is 1 Tesla at $z = 200 \text{ mm}$
- ▶ Magnetic design constraints: 1) Flux budget and 2) Peak field strength
- ▶ By mechanical similarity, the focal point of each trajectory family lies in the same plane, where we then locate the scintillator.



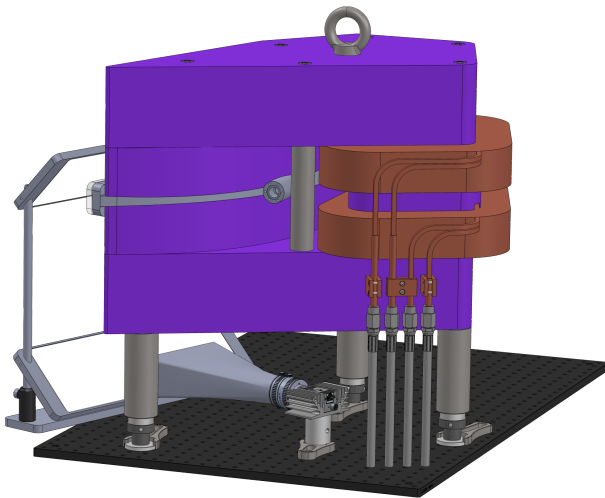
- ▶ Choose $\alpha_0 = 50^\circ$ as a good compromise among several factors, including peak magnetic field, magnetic flux consumed, size and mass of spectrometer, and spectrometer energy resolution.
- ▶ Coil shape and size constrained to match that of CPT to keep down cost.



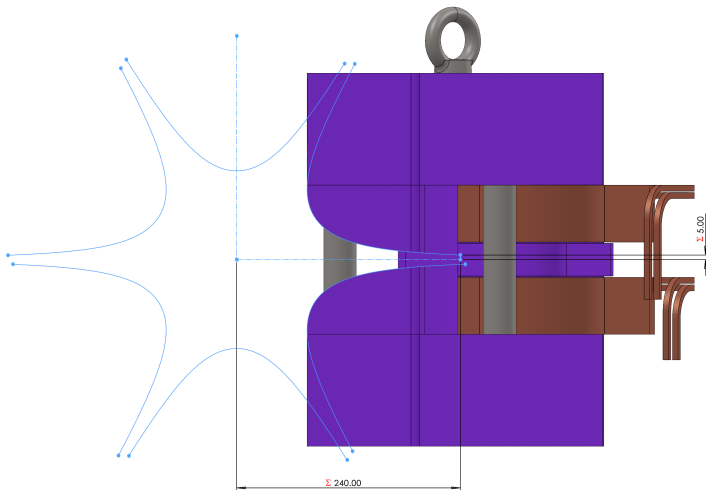
- ▶ Choose $\alpha_0 = 50^\circ$ as a good compromise among several factors, including peak magnetic field, magnetic flux consumed, size and mass of spectrometer, and spectrometer energy resolution.
- ▶ Coil shape and size constrained to match that of CPT to keep down cost.



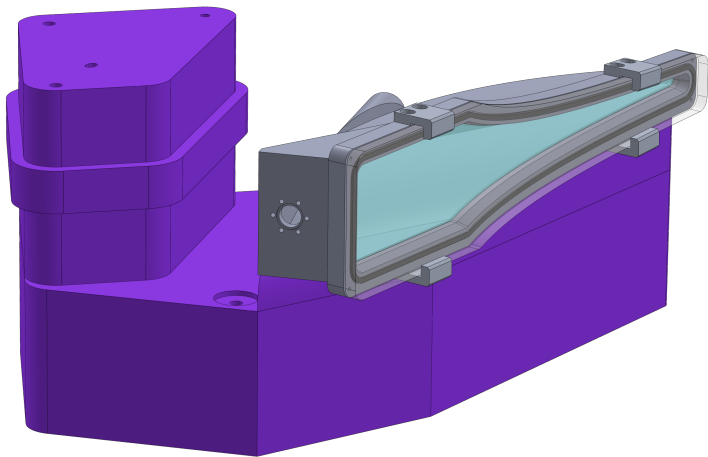
- ▶ Scintillator readout via camera and folded mirror arrangement.
- ▶ Total weight of spectrometer is 800 pounds.
- ▶ At maximum field, each pole surface pulls into the gap with over 2000 pounds net force, braced by two stainless support columns.



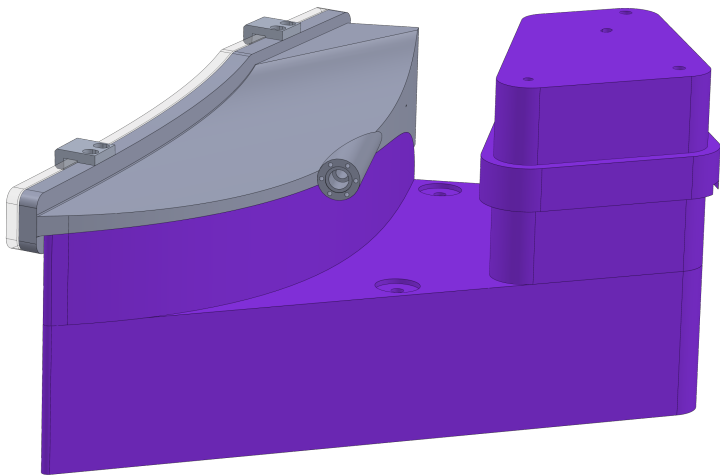
- ▶ Scintillator readout via camera and folded mirror arrangement.
- ▶ Total weight of spectrometer is 800 pounds.
- ▶ At maximum field, each pole surface pulls into the gap with over 2000 pounds net force, braced by two stainless support columns.



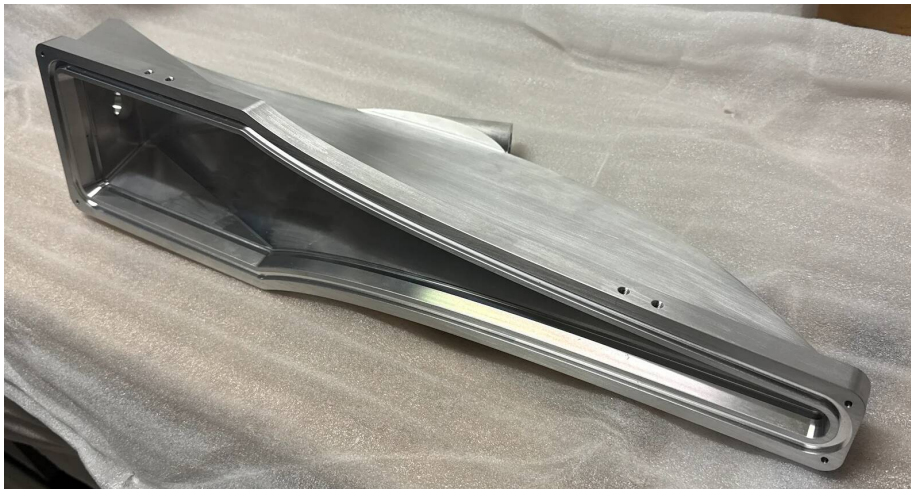
- ▶ Just two of the six poles are included. To compensate for excess fringe field, the inlet beam tube is magnetically shielded.



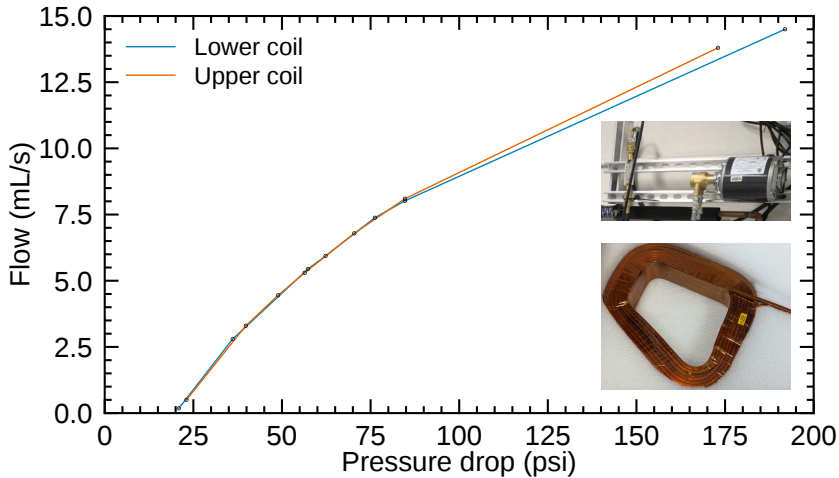
- ▶ Beamline vacuum is maintained throughout the spectrometer. Integrated vacuum chamber with viewport and upstream/downstream beamline ports.
- ▶ Chamber is milled from a single block of aluminum.



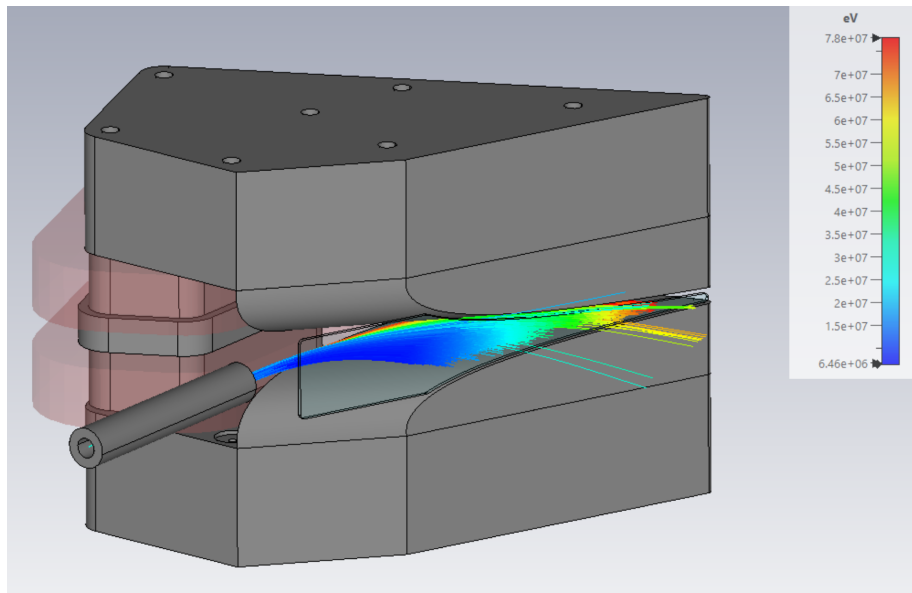
- ▶ Beamline vacuum is maintained throughout the spectrometer. Integrated vacuum chamber with viewport and upstream/downstream beamline ports.
- ▶ Chamber is milled from a single block of aluminum.



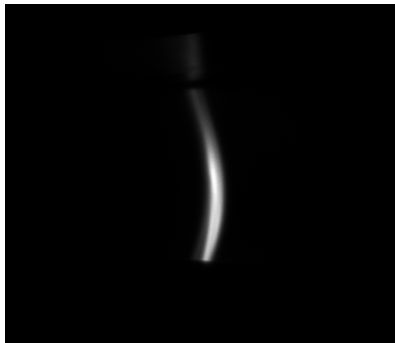
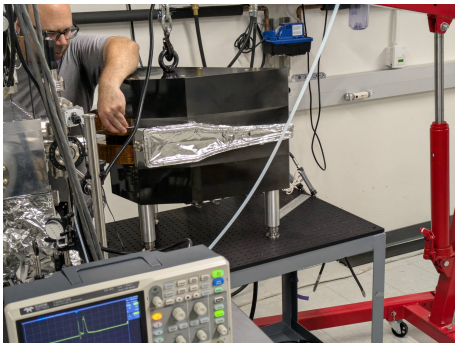
- ▶ Beamline vacuum is maintained throughout the spectrometer. Integrated vacuum chamber with viewport and upstream/downstream beamline ports.
- ▶ Chamber is milled from a single block of aluminum.



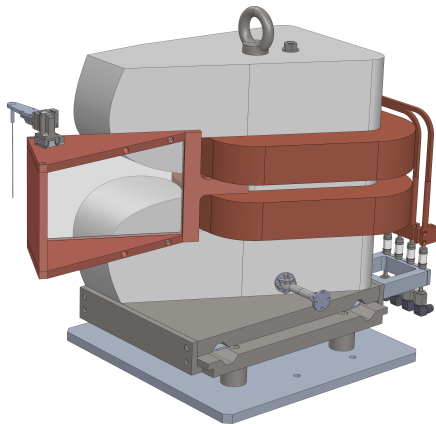
- ▶ At an operating current of 120 amps, each coil has 500 watts of heat production. Need water cooling.
- ▶ Each coil has an inner diameter of 1/8 inch and a length of approximately 70 meters.
- ▶ Booster pump provides 170 psi pressure drop and sufficient flow.



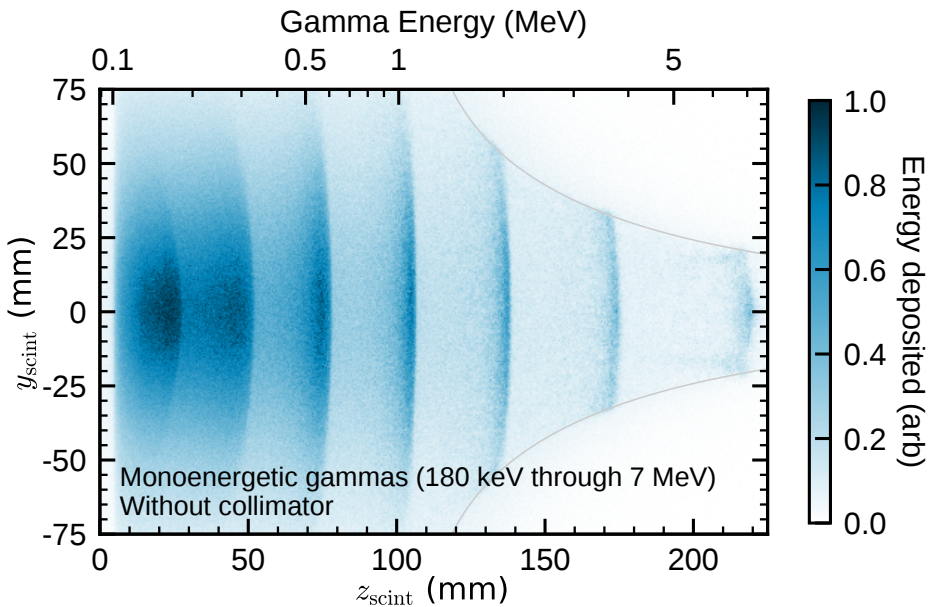
- MITHRA PWFA electron spectrum simulated in CST.

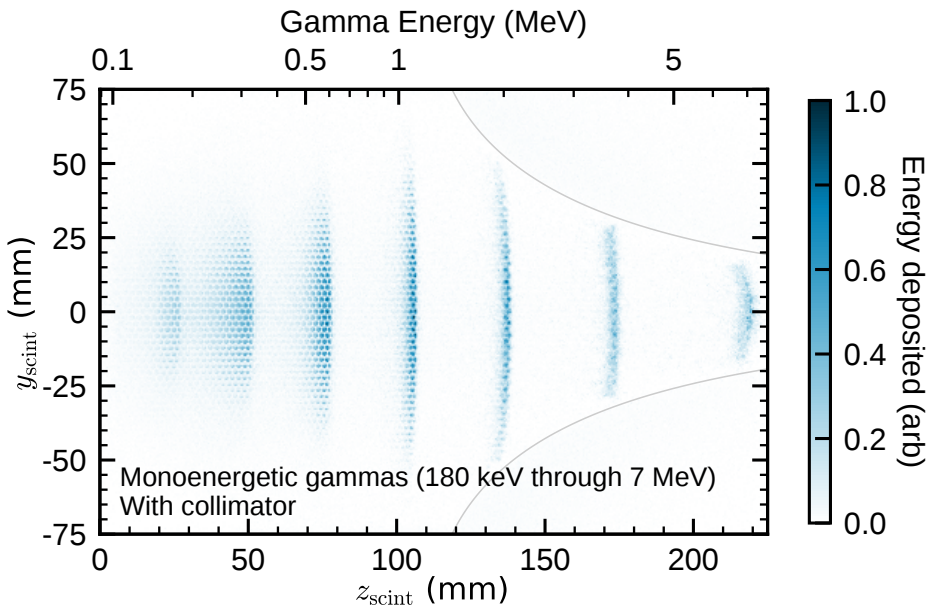


- ▶ Transported spectrometer from main campus to STRB last week.
- ▶ Reassembled and installed on beamline.
- ▶ First light with 30 MeV electron beam!

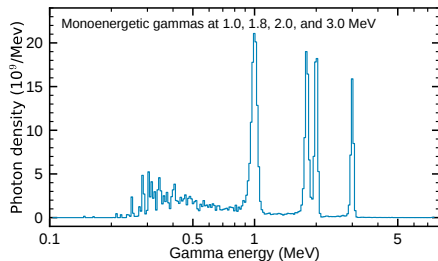
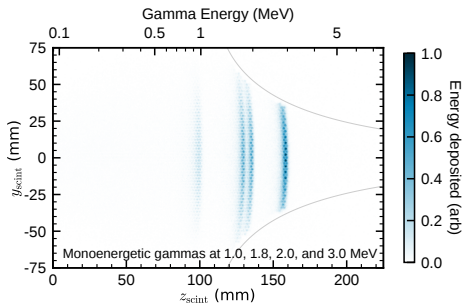


- ▶ 0.2-10 MeV single-shot Compton gamma spectrometer for FACET-II
- ▶ Gammas passing through beryllium converter rod Compton-scatter electrons forward for subsequent magnetic analysis.
- ▶ Spectrometer rolls on 1 inch diameter ceramic bearings so that it can move off beam when not in use. Total amount of in-vacuum translation is 75mm.
- ▶ Coils are water cooled in vacuum.
- ▶ Total weight of magnet plus stage = 650 pounds.





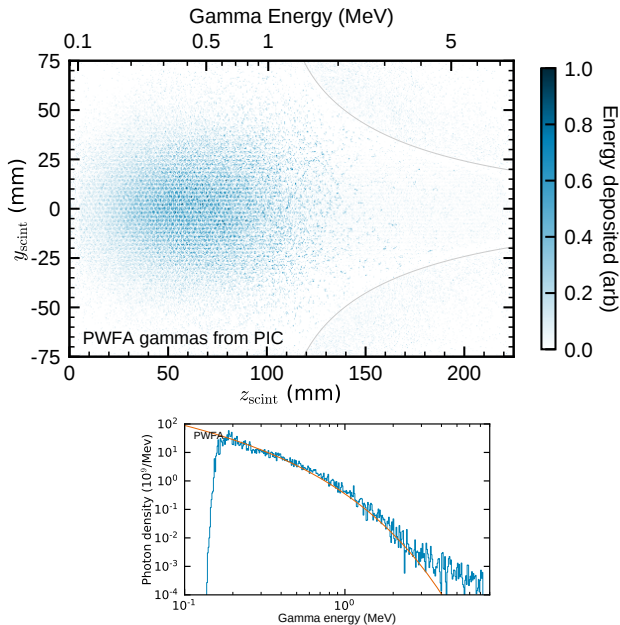
CPT: Maximum Likelihood-Expectation Maximization (ML-EM)

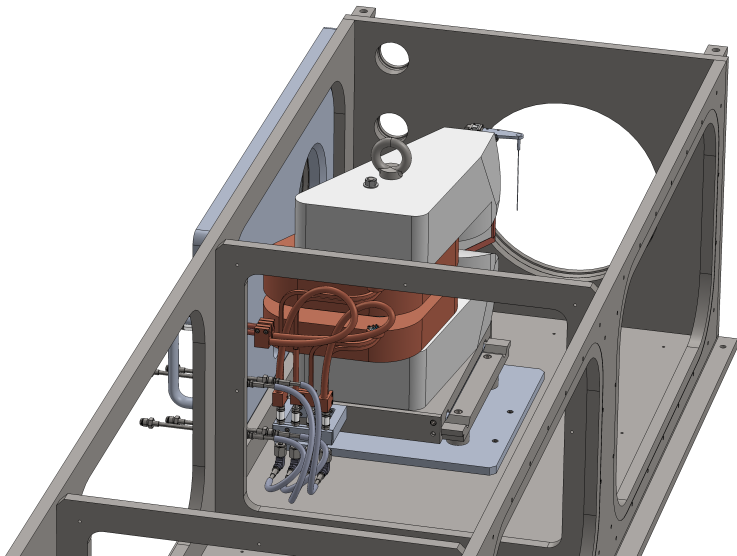


- ▶ ML-EM¹ is used as baseline for spectral reconstruction.
- ▶ Recovers gamma lines with energy resolution on the order of 1%.

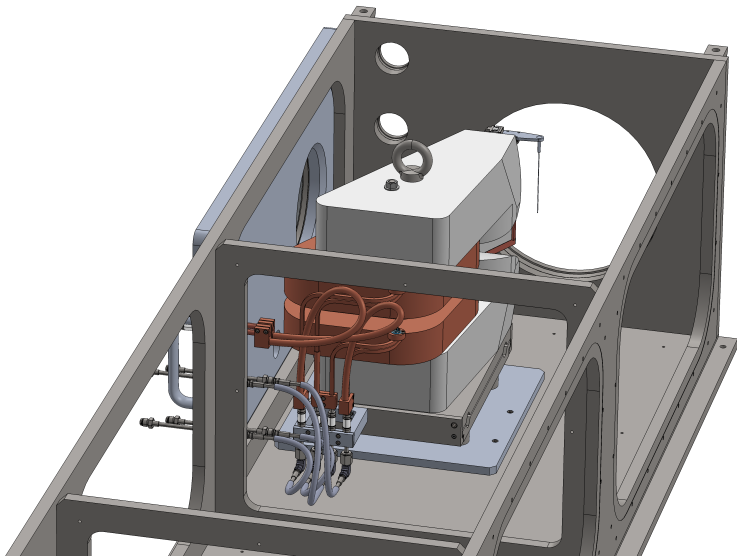
¹Shepp and Vardi, *IEEE Trans. Med. Imaging* **1**, 113 (1982)

CPT: PWFA - Maximum likelihood reconstruction

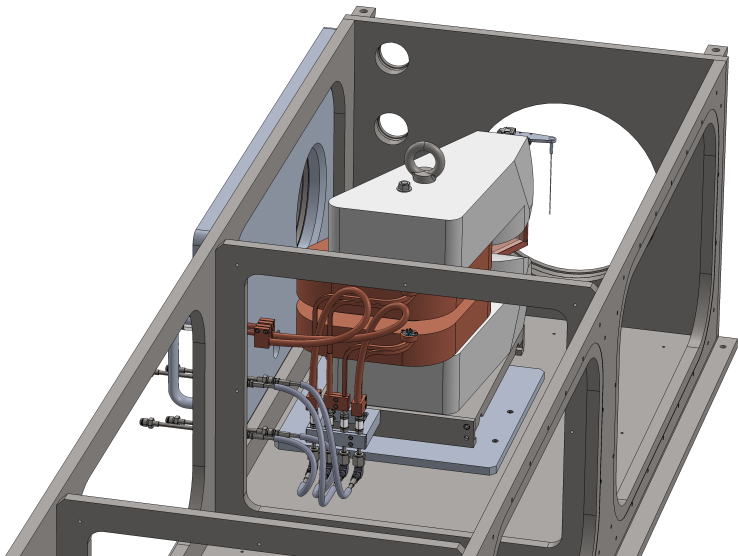




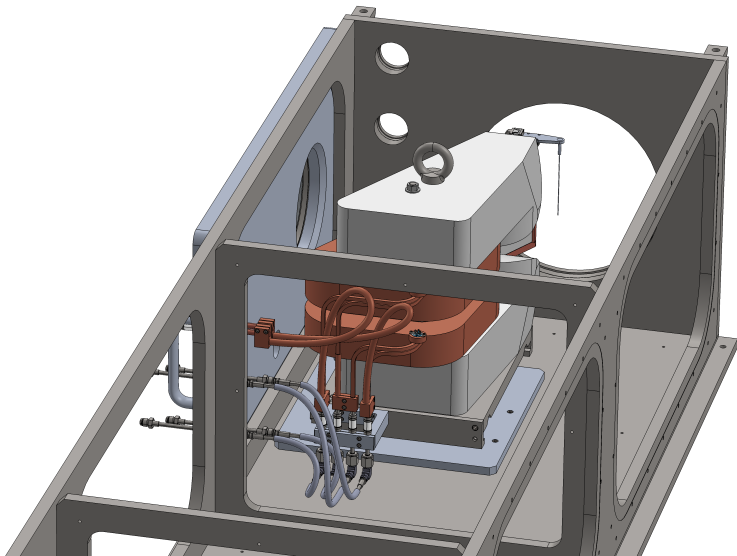
Fully retracted



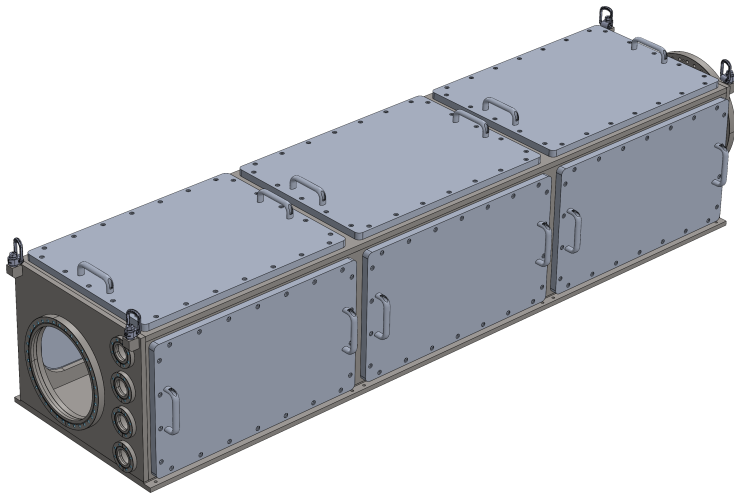
Neutral position



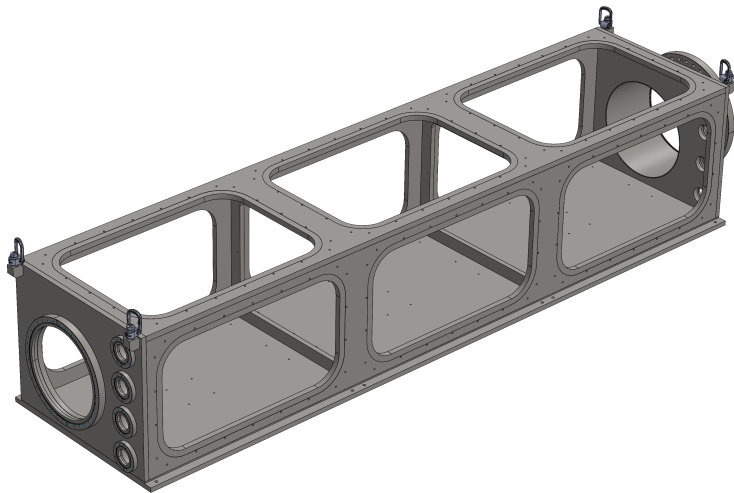
On beam



Fully inserted



- ▶ GDC = Gamma Detection Chamber
- ▶ Stainless frame with nine O-ring sealed aluminum panels (similar to EDC/PDC).



- ▶ Interior stiffening ribs
- ▶ Upstream 16" Conflat flange and downstream 19" Wireseal flange
- ▶ Eight auxiliary CF450s



- Both CPT and the GDC shown here at End Station B.



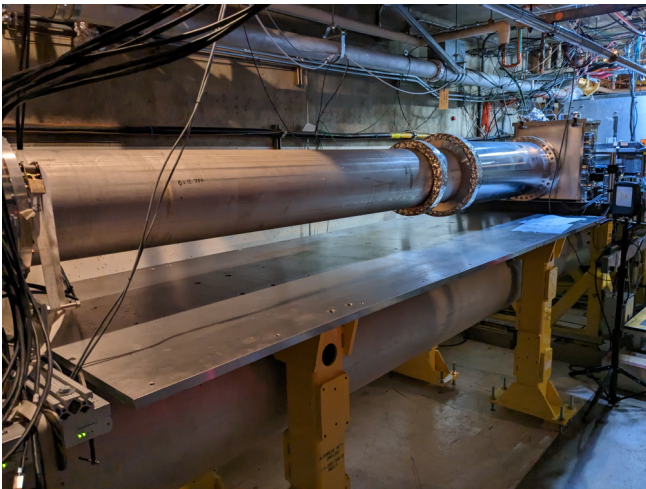
- Both CPT and the GDC shown here at End Station B.



- Both CPT and the GDC shown here at End Station B.



- Both CPT and the GDC shown here at End Station B.



- ▶ To be installed here, between the EDC and the Butterfly Chamber.
- ▶ Support table is new. Same design as the PDC/EDC support table.
- ▶ Chamber and spectrometer installation in July.